

# Optimization of Sampling of Small Pelagic Fishes in the Exclusive Economic Zone of Senegal under the Climate Impact

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## Abstract

Senegal is in a very favourable geographical position for sea fishing. Its coast has an upwelling favouring a good development of phytoplankton very appreciated by the various fish families that populate its Exclusive Economic Zone (EEZ). The little pelagic fish make up the majority of landings. The dynamics of this family of resources is very complex while its perfect mastery is essential for a fishing well controlled maritime. The mathematical models that exist in the literature have not address the different issues related to maritime fisheries and climate change in the Senegalese fishing areas. The linear programming model in integer numbers has been developed after calculation of equilibrium biomass, catches at equilibrium catchability by the application of Schaefer and Freon models in the Senegalese Economic Exclusive Zone. Two proposals have been developed to better explain the tools used in the writing of the mathematical model. The objective is to maximize the biomass of this family of fish resources on the Petite Côte, Grande Côte and Cape Verde depending on samples and climate change. In the application of the model, real data made it possible to test the Linear program in integer numbers obtained. This optimization study allowed us to find an effective way to maximize recruitment within this resource family. This consists in setting up several less expensive marine refuges to build in the fishing zones targeted by the study. The simulation computer program of the model is presented in the appendix.

**Keywords:** Linear Program in integer numbers, Senegal, Sea fishing, Upwelling, phytoplankton, Small pelagic fishes, Dynamics, Biomass, Climate change

## 1. Introduction

A marine ecosystem is a natural unit made up of all its components (animals, plants, microorganisms) and physico-chemical (abiotic) factors with which they interact. It includes three elements which are the biotope<sup>1</sup>, biocenosis<sup>2</sup> and the relationships that can exist and develop within this system. Many scientific teams are studying the impact of anthropogenic pressures and global warming on marine biodiversity, laying and recruitment. Small coastal pelagic fish are all small fish (anchovy, ethmos/bonga, horse mackerel, mackerel, sardine and sardinella) which pass most or almost all of their adult phase on the surface or in full water. These resources constitute the major part of landings in the North-West African sub-region, with annual catches of about 2 million tonnes out of a total of 2.8 million tonnes (Eichelsheim, 2014). These types of fish have a very complex organization and dynamics (Lepage et al. 2012). In 1975, Nihoul used a coupled physical model (salinity, temperature,...)/biology (biomass, energy, ...) three-dimensional to express the change in time and space of a fish resource biomass B. Models such as the so-called global models (Schaefer, 1954), (Fox, 1970) and (Pella and Tomlinson, 1969) have been developed in the same period to meet a pressing need for resource management fisheries. These models differ only in their production function but generally they make a stock assessment over time. The climate aspect is not at the base considers in the parameters of these models but scientists in the field of marine fisheries as (Griffin et al. 1976) and (Freon et al. 1988) introduced a climatic variable (the Eupwelling index) based on these global models. Matrix models making a growth assessment Structured populations in length were developed by Drouineau et al. in 2008. The latter thus presented a model of population dynamics of Merlu under constraints. They constructed a matrix

<sup>1</sup>Region with coherent environmental conditions,forming an abiotic support of an ecosystem.

<sup>2</sup>Set of living things that interact with each other in an ecosystem.

containing abundances in each area and each length class in time steps, a matrix containing survival rates in a time step, a matrix containing migration rates for each length class between different areas, a growth matrix and a recruitment matrix in time steps. The fishing mortality caused by a fishing subunit on the size of a population length class at a time step is the product of the catchability of the fishing subunit. In order to limit the parameters (Froysa et al., 2002) and (Deriso et al., 1985), have modelled catch-ability by a sigmoidal mathematical function representing an increasing rate of retention for trawlers and a gamma law for long-liners and gillnetters. The study of renewable resource management can lead to a series of optimal control problems (Lavigne, 2008). These mathematical questions have been studied by (Clark, Clarke, Munro, 1979) and have given rise to several scientific publications on the same topic. They have based this approach of optimal control in order to find a fishing effort rate so that the current value of the fishery is maximized. Brochier and al. (2015) modelled the importance of establishing an artificial housing area in a marine protected area by the system (predator/prey)<sup>3</sup> The importance of establishing an artificial housing area in a protected marine area.

### 1.1 Presentation of the Senegalese Pelagic Zone

The Senegalese maritime zone is rich in two kinds of pelagic resources. On the high seas, of shore pelagic resources (tropical and small coastal tunas), coastal pelagic resources (sardinella, horse mackerel, mackerel, etc.), which constitute more than 70% of catches made in the exclusive economic zone of Senegal, mainly via artisanal fishing, for the most important of the local consumption (Department of Marine Fisheries, 2016). This distribution of pelagic resources is shown in Fig.1.

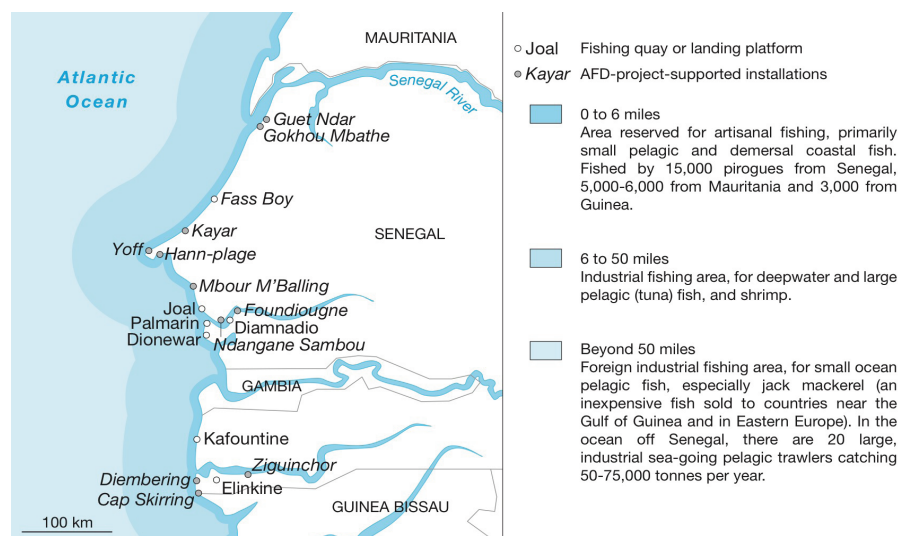


Figure 1. Zones Plagiques du Sngal  
Source: Henry et simon, 2011.

### 1.2 Introduce the Problem

The different models converge towards the same objective which is to make an estimation and an optimal distribution of stock. Climate change is not insignificant nowadays, its consideration in mathematical or environmental models is more than a challenge in the research world. Senegal is involved in several programmes to protect small pelagic fish. Thus, the DPM<sup>4</sup> and the CRODT<sup>5</sup> work together for a participative management because this resource constitutes the main catch of our artisanal fishermen. These small pelagic fish are migrating towards the Mauritanian coast. In 2011, 29% of marine fish stocks were overfished (FAO, 2014). The purpose of our study is to model the effect of overfishing and the impact of climate change on the dynamics of these small pelagic fish in Senegal, then to propose optimal management solutions.

## 2. Method

There are many mathematical models developed according to different parameters. These models make it possible to make projections on the evolution of the fishery and stocks of natural populations. The dynamics of these populations depend strongly on biological factors, such as the rate of population growth, which includes birth and death as well as the movements of individuals (imigration and emigration). Human activity is also an important factor influencing population

<sup>3</sup> Lotka-Volterra model (1960-1940).

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dynamics. Variables are the rate of growth, the effort that corresponds to the number and duration of exploitation, the costs related to effort and catchability.

The parameters and indices in Table 1 will be used throughout.

### Index and parameters

Table 1. Parameter of model

index and parameters	Definitions
$i$	area index
$t$	time index
$Z$	set of zones
$T$	set of times
$B_{t,i}$	biomass of the stock at time $t$ in zone $i$
$B_i$	Biomass of stock in zone $i$
$B_i^e$	Biomass of the zone stock $i$ to equilibrium
$k_i$	constant biomass rate of increase
$B_i^\infty$	biomass tolerated in zone $i$
$B_t$	biomass of the stock at time $t$
$q_i$	catchability $q$ in a zone
$f_{t,i}$	fishing effort at time $t$ in zone $i$
$f_i^{opt}$	optimal fishing effort in each zone $i$
$v_i$	climatic parameter in zones $i$
$U_i^e$	Taken equilibrium in each zone $i$
$U_i^{opt}$	optimale catch in a zone $i$
$Y_i^e$	average catches of area $i$
$Y_{i,j}$	unitary capture in zones $i$ and sub zone $j$
$Y_i^{max}$	maximum capture in zone $i$
$h_i = \frac{k_i}{B_i^\infty}$	Parameter of habitat change
$g(v_i)$	mathematic function connecting a climate variable $v_i$ at $B_i^\infty$
$A_i$	matrix of the parameters of feeds of the species
$d_{i,j}$	migration coefficient between zone $i$ and zone $j$
$b_i$	constant of the rate of climatic variation in a zone $i$
$Alim$	Diet of pelagic resource
$R$	recruitment

### 2.1 Proposition 1

The biomass variation  $B_i$  with  $B_i \in \mathbb{R}_+^*$ , a pelagic resource exploited and impacted by natural variations in a zone  $i \in \mathbb{N}$  follows the following ordinary dynamic equation

$$\frac{\partial B}{\partial t} = k_i \cdot B_i - h_i \cdot B_i^2 - q_i \cdot f_i \cdot B_i = h_i \cdot B_i \cdot (B_i^\infty - B_i) - q_i \cdot f_i \cdot B_i \quad (1)$$

and  $\forall h_i \in \mathbb{R}_+^*$

$$B_i^e = B_i^\infty - \frac{q_i \cdot f_i}{h_i} = g(v_i) - b_i \cdot q_i \cdot \frac{f_i}{h_i} \quad (2)$$

$$U_i^e = q_i \cdot B_i^e = q_i \cdot B_i^\infty - q_i^2 \cdot \frac{f_i}{h_i} = b_i \cdot q_i \cdot g(v_i) - b_i \cdot q_i^2 \cdot \frac{f_i}{h_i} \quad (3)$$

$$Y_i^e = f_i \cdot U_i^e = q_i \cdot B_i^\infty \cdot f_i - q_i^2 \cdot \frac{f_i^2}{h_i} = b_i \cdot q_i \cdot g(v_i) \cdot f_i - b_i \cdot q_i^2 \cdot \frac{f_i^2}{h_i} \quad (4)$$

$$f_i^{opt} = B_i^\infty \cdot \frac{h_i}{2 \cdot q_i} = g(b_i) \cdot \frac{h_i}{2} \cdot b_i \cdot q_i \quad (5)$$

$$U_i^{opt} = q_i \cdot \frac{B_i^\infty}{2} = b_i \cdot q_i \cdot \frac{g(v_i)}{2} \quad (6)$$

$$Y_i^{max} = B_i^{\infty,2} \cdot \frac{h_i}{4} = g^2(v_i) \cdot \frac{h_i}{4} \quad (7)$$

### Proof

The population density of a stock follows the following equation:

$$B_{t+1} = B_t + \frac{dB}{dt}$$

The models of linear synthetic production are based on the description of the evolution of the relative instantaneous growth rate of the biomass by the logistic curve (Freon et al., 1988). The graphic description of the state the stock in each fishing zone corresponds to the following discrete state representation in discrete time:  $B_{t+1,i} = B_{t,i} + \frac{\partial B}{\partial t}$ .

The knowledge of  $\frac{\partial B}{\partial t}$  is crucial in the evaluation and optimization of the abstractions performed on a fish stock. In the absence of exploitation in each fishing zone and in modifying the model of Schaefer and Freon by applying them a variable zone  $i$ , is obtained the following relation in a known time:  $\frac{\partial B}{\partial t} \frac{1}{B_i} = k_i \cdot \frac{B_i^\infty - B_i}{B_i^\infty} = k_i \cdot (1 - \frac{B_i}{B_i^\infty})$ . Result from the combination of natural variations and fishing levies, is obtained the basic equation of the so-called Schaefer model which, applied to a zone  $i$  of Senegal, give the relation (1):

$$\frac{\partial B}{\partial t} = k_i \cdot B_i - h_i \cdot B_i^2 - q_i \cdot f_i \cdot B_i = h_i \cdot B_i \cdot (B_i^\infty - B_i) - q_i \cdot f_i \cdot B_i$$

With  $h_i = \frac{k_i}{B_i^\infty} = cte$ , modelise the fact that the habitat change affects both  $B_i^\infty$  and  $k_i$ .

This relationship applied to fishing areas, allows to identify the highly threatened areas and potentially rich areas. The hydroclimatic phenomena, according to this formulation, can intervene only at two levels, on  $q_i$  if the catchability varies, or on the pair  $k_i - B_i^\infty$  (the ratio of these two quantities being constant), if These are natural variations of abundance. The relation (1) to equilibrium makes it possible to estimate the current recruitment<sup>6</sup> in each fishing zone of Senegal from the data of the Oceanographic Research Center of Dakar Thiaroy (CRODT). If the stock has the equilibrium state and the increase of the biomass is zero, this leads from equation (1):

$$B_i^e = B_i^\infty - \frac{q_i \cdot f_i}{h_i} = b_i g(v_i) - b_i \cdot q_i \cdot \frac{f_i}{h_i}$$

Replacing  $U_i^e$  and  $Y_i^e$  respectively by  $q_i \cdot B_i^e$  and  $f_i \cdot U_i^e$ , we obtain:

$$U_i^e = q_i \cdot B_i^e = q_i \cdot B_i^\infty - q_i^2 \frac{f_i}{h_i} = b_i \cdot q_i \cdot g(v_i) - b_i \cdot q_i^2 \frac{f_i}{h_i}$$

$$Y_i^e = f_i \cdot U_i^e = q_i \cdot B_i^\infty \cdot f_i - q_i^2 \cdot \frac{f_i^2}{h_i} = b_i \cdot q_i \cdot g(v_i) \cdot f_i - b_i \cdot q_i^2 \frac{f_i^2}{h_i}$$

$f_i^{opt}$  is obtained by looking for the value of  $f_i$  which cancels the derivative of equation (3), ie:

$$f_i^{opt} = B_i^\infty \cdot \frac{h_i}{2q_i} = \frac{g(v_i)}{q_i} \cdot \frac{h_i}{2}$$

For obtain the maximum values of  $U_i^{opt}$  and  $Y_i^{max}$ , we replace them in  $q_i \cdot B_i^e$  and  $f_i \cdot U_i^e$ ,  $f_i$  is replaced by the formula of  $f_i^{opt}$  in these equations. We obtain the following result:

$$U_i^{opt} = q_i \cdot \frac{B_i^\infty}{2} = b_i \cdot q_i \cdot \frac{g(v_i)}{2}$$

$$Y_i^{max} = B_i^{\infty,2} \cdot \frac{h_i}{4} = g^2(v_i) \cdot \frac{h_i}{4} \quad \blacksquare$$

<sup>6</sup>All births and new birds arriving in a fish resource family

## 2.2 Proposition 2

Suppose catchability  $q_i$ , with  $q_i \neq 0$  varies from area to the other and that  $B_i \in \mathbb{R}_+^*$  and  $h_i \in \mathbb{R}_+^*$ , the average catches per zone  $U_i^e$  is obtained for the following relation:

$$U_i^e = b_i \cdot q_i \cdot v_i \cdot B_i^\infty - q_i^2 \cdot b_i \frac{f_i}{h_i} \quad (8)$$

We obtain  $f_i^{opt}$ ,  $U_i^{opt}$  and  $Y_i^{max}$  from the relation (8).

$$f_i^{opt} = \frac{v_i \cdot k_i}{2q_i} \quad (9)$$

$$U_i^{opt} = b_i \cdot q_i \cdot v_i \cdot (B_i^\infty - \frac{k_i}{2h_i}) \quad (10)$$

$$Y_i^{max} = \frac{B_i^\infty \cdot b_i \cdot v_i^2 \cdot k_i}{4} \quad (11)$$

*Proof* Catchability can affect hydroclimatic phenomena at either of its two components, which are accessibility and vulnerability (Freon et al. 1988). To give just a few examples, in a given fishery, movements of water masses may induce coastal parallel or perpendicular migrations, which will affect the accessibility of the stock. Consider a stock in equilibrium conditions not only with the fishery, but also with the environment. Assuming that catchability varies from one zone  $i$  to the other and that  $g(vi) = B_i^\infty \cdot v_i$ , the mean catches per zone  $U_i^e$  are obtained by the following relation:

$$U_i^e = b_i \cdot q_i \cdot v_i \cdot B_i^\infty - q_i^2 \cdot b_i \frac{f_i}{h_i}$$

We obtain  $f_i^{opt}$ ,  $Y_i^{max}$  and  $U_i^{opt}$  from this last relation.

$f_i^{opt}$  is obtained by canceling the derivative with respect to  $q$  of  $U_i^e$ :

$$f_i^{opt} = \frac{v_i \cdot k_i}{2q_i}$$

By replacing in  $U_i^e = b_i \cdot q_i \cdot v_i \cdot B_i^\infty - q_i^2 \cdot b_i \frac{f_i}{h_i}$

$$U_i^{opt} = b_i \cdot q_i \cdot v_i \cdot (B_i^\infty - \frac{k_i}{2h_i})$$

We obtain  $Y_i^{max}$  by  $f_i^{opt}$  and  $U_i^{opt}$

$$Y_i^{max} = f_i^{opt} \cdot U_i^{opt} = \frac{B_i^\infty \cdot b_i \cdot v_i^2 \cdot k_i}{4}. \quad \blacksquare$$

## 2.3 Data Analysis

Table (2) is a result of analysis of real data over a period of thirty years relating to the fishing effort of Petite Cote, Grande Cote and Cape Verde. Table (3) is a result of analysis of actual catch data for the Petite Cote, Grande Cote and Cape

Table 2. Materials and Fishing Gear

Fishing gear		Gear group	
Name	Number	Name	Number
Surface fixed net	3520	Fixed net	5582
Purse seiner	2693	Purse seiner	2693
Surface gill netting	828	Line	676
Protective gill net	779	Gillnet gillant	501
Other	1418	Other	287

Verde.

Table (4) shows the results of the analyzes and calculations above. The CRODT data allowed us to obtain answers reflecting the reality of the catches. The data were classified according to the three target areas of the study. There are slight differences in some physical parameters.

Table 3. Fishing Efforts and Catches

Capture (Ton) and Effort (number of output)		
Indices	Effort	Captures (Tonne)
Min	1.0	0.0
Max	26292.6	21577.6
25% of observations	278.6	0.3
Median	591.0	7.7
Average	873.1	427.9
75% of observations	1045.0	172.0

Table 4. Statistical results

Results of statistical calculations		
Regions	Parameters	Value
Cape Verde	$q_1$	0.378
Cape Verde	$f_1$	710.0
Cape Verde	$B_1^\infty$ (Ton)	6665.509
Cape verde	$B_1^e$ (Ton)	5437.0106
Cape Verde	$Y_1^{max}$ (Ton)	136.6911
Grande Côte	$q_2$	0.093
Grande Côte	$k_2$	3.0316
Grande Côte	$f_2$	329.6
Grande Côte	$B_2^\infty$ (Ton)	10257.333
Grande Côte	$B_2^e$ (Ton)	9969,424
Grande Côte	$Y_2^{max}$ (Ton)	120.1978
Petite Côte	$q_3$	0.317
Petite Côte	$k_3$	3.99465
Petite Côte	$f_3$	748.00
Petite Côte	$B_3^\infty$ (Ton)	12623.882
Petite Côte	$B_3^e$ (Ton)	10541.66394
Petite Côte	$Y_3^{max}$ (Ton)	60.2942

The evolution of catches from 1981 to 2014 can be materialized by the number of trips based on the linear regression function (12) obtained from Table (4). The outputs moved from  $1.10^4$  to  $3.10^4$  during at period. A simulation of (12) on matlab gives figure 2.

$$Y = 0.06119X + 375 \quad (12)$$

A simulation of the linear regression function (12) on matlab give the figure 2

Despite a large evolution in the number of trips at sea over the years, catches have increased slightly for this species and the higher the Eupwelling index, the more fish there is. These results led to the design of the optimization model below, making it possible to locate the areas in which spatial management measures can be applied.

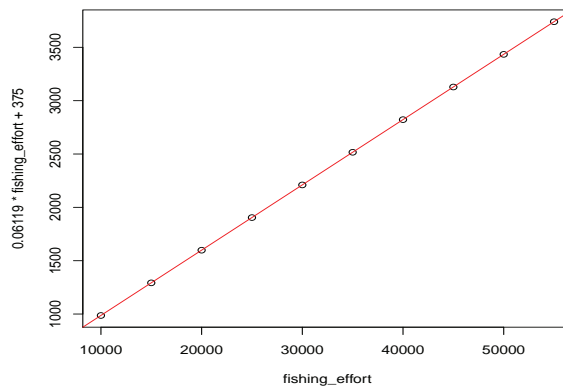


Figure 2. Catch's evolution by output area

### 3. Location of Marine Refuges

This part is a consequence of part (2) The aim is to locate highly vulnerable areas and to introduce management policies such as the creation of marine refuges, the establishment of artificial reefs.

#### 3.1 Hypotheses

- The number of sub-zones for developing marine refuges is fixed at 6 in each region.

#### 3.2 Model

In this model we will maximize the function

$$R = \sum_{i \in Z} k_i \cdot B_i^e \cdot \left(1 - \frac{B_i^e}{B_i^\infty}\right) \quad (13)$$

This represents the recruitment of the species in the three fishing zones of the study to maximize its value in these areas.

$$X_{i,j} = \begin{cases} 1 & \text{if the sub-zone } j \text{ of zone } i \text{ meets the criteria for the establishment of a marine refuge} \\ 0 & \text{otherwise} \end{cases}$$

##### 3.2.1 Mathematical Formulation

The objective is to maximize the biomass of pelagic resources in the Senegalese coast by optimizing the recruitment function (13).

$$\text{maximize } \sum_{i \in \text{zone}} \sum_{j \in S} \left( B_i^e + k_i \cdot B_i^e \cdot \left(1 - \frac{B_i^e}{B_i^\infty}\right) \right) X_{i,j} \quad (a)$$

Under the constraints:

$$\begin{cases} \sum_{i \in Z} X_{i,j} = 1 & \forall j \in S & (b) \\ B_i^e X_{i,j} \geq Cap_i & \forall i \in Z, \forall j \in S & (c) \\ Y_i^{max} \times X_{i,j} \geq Y_{i,j} & \forall i \in Z, \forall j \in S & (d) \\ Alim_i \times X_{i,j} \leq A_i & \forall i \in Z, \forall j \in S & (e) \\ X_{i,j} \in \{0, 1\} & \forall i \in Z, \forall j \in S & (f) \end{cases}$$

The objective (a) is to maximize the recruitment of this resource in the different fishing zones.

The constraint (b) shows that a marine refuge has a unique location.

The constraint (c) informs us that marine refuge development areas should be areas where catches exceed a certain threshold. The constraint (d) shows that areas favorable to marine refuges must exceed a certain amount per unit catch.

Stress (e) tells us that a selected area must have a bathymetry not very favorable to the development of these small pelagic fish with the aim of introducing artificial reefs <sup>7</sup>.

<sup>7</sup>Mechanisms and tools for restoring favorable conditions for feeding and reproduction of fish resources

The constraint (f) is that of integrity.

#### 4. Numerical Results

The localization part of the model was simulated in OPl<sup>8</sup> presented in the appendix. a programming language of mathematical models. The results are shown in Table 7. The optimal recruitment for sustainable management of this fishery resource from the data is  $R = 2088.8476$  Tonnes. *4.1 Graphic Interpretation*

Table 5. Marine Refuges

Number of marine refuge per zone			
Sub-Zone	Cape Verde	Petite Côte	Grande Côte
1	4	4	4
2	2	4	4
3	4	4	4
4	2	4	2
5	4	2	4
6	0	2	2

Graph 3 shows the distribution of marine refuge (RM) creation zones in the different fishing zones. We find a narrow response in terms of the number of MR. The different areas have similar problems related to the fishing effort. Therefore, similar management measures need to be applied in these areas in order to balance recruitment.

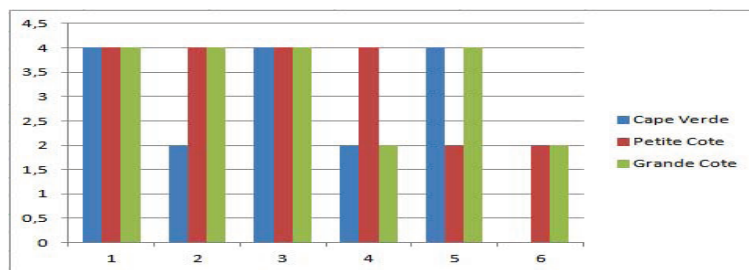


Figure 3. Distribution of marine refuges in Cape Verde, Grande Cote and Petite Cote

#### 5. Conclusion and Discussions

The proposed model allows to integrate the action of Eupwelling on the Biomass of small pelagic coastal. It presents constraints related to the richness of the interior environment and the destruction of residential areas. Despite these constraints, these models often remain the least bad solution, particularly in tropical regions where hydroclimatic conditions are predominant over the variability of production (Freon, 1985). This model, built from a differential equilibrium equation, allowed us to achieve an improvement in biomass in six fishing zones on the Grande Cote, Petite Cote and Cape Verde. The linear integer program allowed for an optimal distribution of marine refuges at the three fishing sites. This activity must be effectively controlled so that consumers and fishermen are protected. For this study only one climatic factor was when climatic changes are observed through fluctuations of chlorophyll A, salinity, surface temperature, wind speed, melting glaciers ... All these factors could object of a complex modeling. This work is a rough draft for this type of approach combining Differential equations and Linear Programming in integers

#### References

- Arnason. (1993). The Icelandic Individual Transferable Quota System. *A Descriptive account, Marine Ressource Economics*, 8, 201-218.
- Brochier. (2015). Implementation of artificial habitats: Inside or outside the marine protected areas? Insights from a mathematical approach. *Ecological Modelling*, 297, 98-106.
- Camara. (2009). What is the management of artisanal fisheries in West Africa? *Study of the complexity of the fishing area in the Senegalese littoral zone*, Doctoral Thesis: UCAD, 339.
- Clarke, C. W., Clarke, F. H., & Munro, G. R. (1979). The optimal exploitation of the renewable resources stocks: Problems of irreversible. *Econometrica*, 47, 25-47.

<sup>8</sup>Optimization Linear Programing version 12.0. Developed by IBM that incorporates simplex



- DPM. (2015). Fishing and fishery resources in Senegal. *from the guide of fisheries resources of Senegal, FAO, www.senegal-export.com*
- Drouineau. (2008). Development and adjustment of a population dynamics model length structure and specialty applies to northern stock of hake (*Merluccius merluccius*). *AGROCAMPUS, ifremer, Doctoral School Life-Agro-Health*, 62.
- Eichelsheim. (2014). Sustainable Development of Coastal Areas (Senegal, Guinea-Bissau, Guinea). *Towards citizen governance of the territories for IDEE Casamance September*, 1.
- Fron. (1988). The introduction of a climate variable in global production models *ORSTOM, Caribbean Pole*, 3-4, 396.
- VIAL, G. The prey-predator system of Volterra-Lotka. *University of Reindeer I, Preparation for Mathematical Aggregation*, 1-6.
- Murray, J. D. (2013). Biology of exponentials and sigmoids. Introduction to the models Malthusian, logistics and Gompertz, from the point of view of the biologist. *HEC 2001 :Volterra, discrete and continuous different equations., Ens 2012 et ENS 2009: Volterra-Lotka*, 1-123.
- Lavigne. (2008). Concept of optimal control applied to the management of a renewable resource. *Royal Military College of Saint Jean. Quebec Mathematical Association*, 41-49.
- Lepage, A. J. (2012). Conic hyperspectral dispersion mapping. *Science et application*, 1-15.
- Munro, (1979). The optimal management of transboundary renewable resources, *Canadian journal of Economics*, XII(3), (Aug), 355-376.
- Pella, T. (1969). Pôle Halieutique Agrocampus Ouest *halieutique.agrocampus-ouest.fr/publi/644/Chapitre5.pdf:97*
- Robert, M. G. (1976). Oster Bifurcations and dynamic complexity in simple ecological models, *The American Naturalist*, 110(974), Juillet-août, 573-599.
- Shaefer, & fox (1972). Laws of Linear and Weight Growth. Mortality-structure demographic. *Linear model and exponential model, ORSTOM: 7*.
- Shields. (2017). Integrating the notion of "shelters" helps to achieve its goal (Ottawa). *Environmental News, www.ledevoir.com*.
- THIAW. (2010). Dynamic fisheries resources with short life span: casdesstocks of octopus and shrimps exploited in Senegal These. *Thse/AGROCAMPUS, Graduate School: Life-Agronomy-Health, under the seal of the European University of Brittany*, 8-10.

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