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Mathematical Modelling of COVID-19 and Solving Riemann Hypothesis, Polignac's and Twin Prime Conjectures Using Novel Fic-Fac Ratio With Manifestations of Chaos-Fractal Phenomena

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Abstract

COVID-19 originated from Wuhan, China in December 2019. Declared by the World Health Organization on March 11, 2020; COVID-19 pandemic has resulted in unprecedented negative global impacts on health and economy. International cooperation is required to combat this "Incompletely Predictable" pandemic. With manifestations of Chaos-Fractal phenomena, we mathematically model COVID-19 and solve [unconnected] open problems in Number theory using our versatile Fic-Fac Ratio. Computed as Information-based complexity, our innovative Information-complexity conservation constitutes a unique all-purpose analytic tool associated with Mathematics for Incompletely Predictable problems. These problems are literally "complex systems" containing well-defined Incompletely Predictable entities such as nontrivial zeros and two types of Gram points in Riemann zeta function (or its proxy Dirichlet eta function) together with prime and composite numbers from Sieve of Eratosthenes. Correct and complete mathematical arguments for first key step of converting this function into its continuous format version, and second key step of using our unique Dimension (2x - N) system instead of this Sieve result in primary spin-offs from first key step consisting of providing proof for Riemann hypothesis (and explaining closely related two types of Gram points), and second key step consisting of providing proofs for Polignac's and Twin prime conjectures.

Keywords: COVID-19, Dirichlet Sigma-Power Laws, Plus Gap 2 Composite Number Continuous and Plus-Minus Gap 2 Composite Number Alternating Laws, Polignac's and Twin prime conjectures, Riemann hypothesis

Mathematics Subject Classification (2010): Primary 11M26, Secondary 11A41

1. Introduction

Gram[x=0,y=0] points, Gram[y=0] points and Gram[x=0] points are three types of Gram points (GP) dependently computed directly from Riemann zeta function (RζF or ζ(s)) [or its *proxy* Dirichlet eta function (DηF or η(s))]. Respectively, these three entities are based on Origin intercept points, x-axis intercept points and y-axis intercept points. Nontrivial zeros (NTZ) are synonymous with Gram[x=0,y=0] points. Prime and composite numbers are two other entities dependently computed, respectively, directly and indirectly from Sieve of Eratosthenes. We justify below how all these five well-defined entities will manifest Incompletely Predictable properties. For RζF, its equivalent Euler product formula using product over prime numbers [instead of summation over natural numbers] will faithfully represent this function. Consequently, all prime [and, by default, composite numbers] are intrinsically "encoded" in this function.

Regarded as **primary spin-offs**, correct and complete mathematical arguments for solving Riemann hypothesis (viz, conjecture that all NTZ are located on $\sigma = \frac{1}{2}$ critical line of RζF) and explaining closely related two types of GP (viz, Gram[y=0] points and Gram[x=0] points) can inherently be explained to belong to Mathematics for Incompletely Predictable problems. Correct and complete mathematical arguments on solving Polignac's and Twin prime conjectures also belong to this brand of Mathematics. Containing pure and applied mathematics, treatise on relevant Mathematics required to solve and explain the former is outlined first; and to solve the later is outlined subsequently. Regarded as novel method, exact and inexact Dimensional analysis homogeneity applied to derived Dirichlet Sigma-Power Law (DSPL) symbolize rigorous proof for Riemann hypothesis and precise explanation for two types of GP. Plus Gap 2 Composite Number Continuous Law and Plus-Minus Gap 2 Composite Number Alternating Law are descriptive Laws derived using Dimension (2x - N) system. They symbolize rigorous proofs for Polignac's and Twin prime conjectures.

We will elaborate upon the profound statement "*With this one solution, we have proven five hundred theorems or more at once*". Regarded as **secondary spin-offs** arising out of solving Riemann hypothesis, this statement apply to many important theorems in Number theory (mostly on prime numbers) that rely on properties of RζF such as where trivial

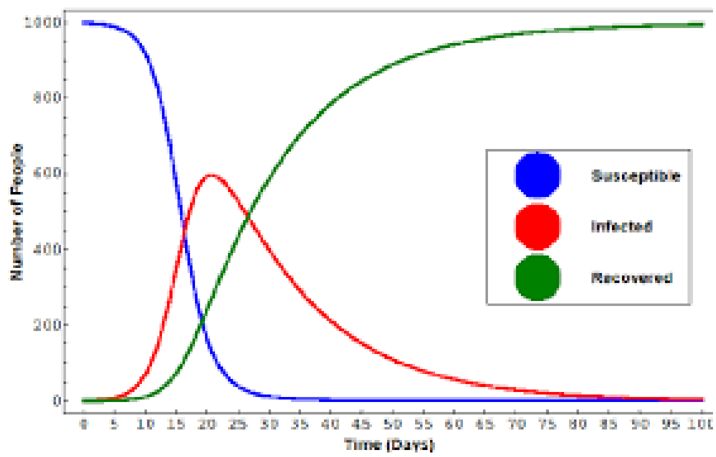

Figure 1. Schematically depicted SIR model with three compartments

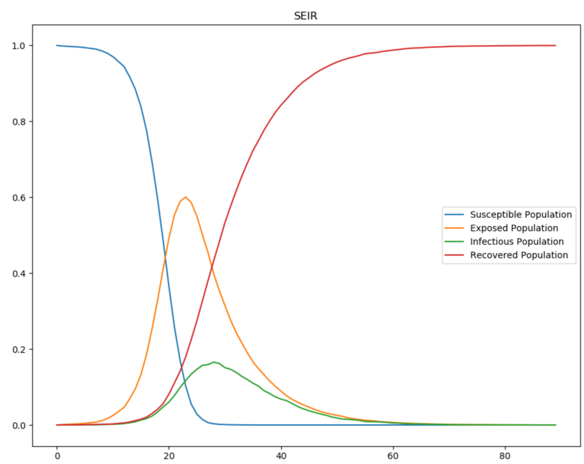

Figure 2. Schematically depicted $SEIR$ model with four compartments

zeros and NTZ are / are not located. Derived innovative *Fic-Fac Ratio* is regarded as **tertiary spin-offs** potentially serving as medical or epidemiological tool to help understand highly contagious SARS-CoV-2 causing COVID-19 and 2020 Coronavirus Pandemic. This event has resulted in unprecedented negative global health and economic impacts. Fic-Fac Ratio connects seemingly unrelated subject of Medicine with frontier Mathematics from Number theory.

COVID-19 is an acronym that stands for Coronavirus Disease 2019 with severity ranging from asymptomatic, mild, moderate, severe to life-threatening with potential to result in chronic residual debilitating symptoms after recovery. It is a proven multi-organ disease generally affecting human lungs to the worst degree. To help ease time constraint of front-line health workers interested in reading this paper, its content is mindfully composed to succinctly include selected materials relevant to COVID-19 pandemic which was officially declared by World Health Organization (WHO) on March 11, 2020. Caused by highly contagious and moderately virulent SARS-CoV-2 [originating from Wuhan, China in December 2019]; this deadly pandemic has resulted in unprecedented negative global impacts from health and economic crisis with numerous deaths and widespread job losses. Similar to most respiratory virus, spread of infection could occur through contact (direct or indirect), > 5 μm size droplet spray in short-range transmission, < 5 μm size aerosol in long-range transmission (airborne transmission). International cooperation to effectively combat the pandemic is required with China and US playing crucial roles aided by other big and small countries alike such as Russia, Canada, Great Britain, Germany, Australia, New Zealand, Vietnam and Thailand.

We devote the initial few pages of this paper to highlight importance of mathematics in understanding infectious disease outbreak. SIR model in Figure 1 and $SEIR$ model in Figure 2 are epidemiological (compartmental) models commonly

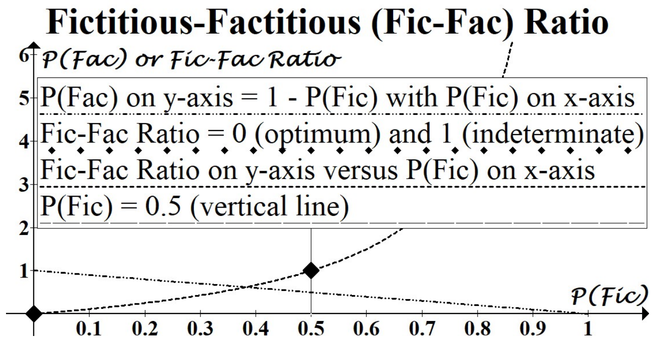

Figure 3. Schematic representation of Fic-Fac Ratio

Table 1. The 2x2 contingency table for mathematical arguments (MA) and diagnostic tests (DT)

Gold standard for MA or DT:			
MA results		Positive	Negative
or	Positive	True +ve [a]	False +ve [b]
DT results:	Negative	False -ve [c]	True -ve [d]

used by mathematicians to compute theoretical number of people inflicted with an infectious illness such as COVID-19 in a closed population over time. Respectively, they consist of three and four compartments derived from: S for Susceptible Population, E for Exposed Population, I for Infectious Population, and R for Recovered Population [including deceased &/or immune individuals]. Both models utilize (deterministic) ordinary differential equations. Aspects of modelling this pandemic in terms of our derived Fic-Fac Ratio are loosely and intuitively perceived as "Incompletely Predictable" – a term also used when solving the [unconnected] above-mentioned open problems in Number theory.

Factitious versus Fictitious and the novel & versatile Fic-Fac Ratio:

The adjective **factitious** derives from factus and therefore facere means *to correctly make or utilize something based on (true) fact* whereas **fictitious** derives from fictus and therefore fingere means *to incorrectly make or utilize something based on (false) fiction*. We predominantly refer the "something" here to mathematical arguments (MA) and diagnostic tests (DT). These two adjectives with their given meanings are used to help create Fic-Fac Ratio, which is an acronym that stands for Fictitious-Factitious Ratio. DT 'Accuracy' refers to ability of that test to distinguish between patients with disease, and those without. Roughly considered as ***'Inverse Accuracy'*** [with higher Accuracy corresponding to lower Fic-Fac Ratio and *vice versa*], we advocate this Ratio be universally applicable to all well-defined mathematical models.

Equating to '<100% accuracy' (with a "pseudo-component") or '100% accuracy' (without a "pseudo-component"), we categorize all synthesized mathematical models to be broadly associated with either "proposed states" such as Riemann hypothesis or "natural states" such as COVID-19 pandemic. During epidemiological modelling of COVID-19 pandemic, less accurate "Pseudo-SIR" model [as opposed to more accurate $SEIR$ model] is the relevant pseudo-component as it does not contain compartment E for Exposed Population. Modelling concepts from open problems, COVID-19 and its resulting pandemic using derived Fic-Fac Ratio are outlined next whereby we provide concrete examples of ideal gold standard MA and ideal gold standard DT with their associated MA and DT results corresponding to Fic-Fac Ratio = 0.

1.1 Fic-Fac Ratio for Open Problems, COVID-19 and Its Resulting Pandemic

Abbreviations: MA = mathematical arguments, DT = diagnostic tests, P = Probability (or Proportion), R = Fic-Fac Ratio. We supply definitions, equations and schematic diagram of Fic-Fac Ratio (Figure 3) on its important inter-relationships applicable to MA and DT. Required MA giving [abstract] positive and [abstract] negative MA results in a specified conjecture or hypothesis must be implemented to, respectively, fully **confirm** a "proposed state" to be *correctly valid* and *correctly not invalid*. Required DT giving positive and negative DT results in a specified subject group or population must be implemented that, respectively, aim to fully **support** a "natural state" to *correctly occur* and *correctly not occur*.

Based on 2x2 contingency table in Table 1, both MA and DT have parameters forming "stable properties" and "frequency-dependent properties" as depicted below. Fic-Fac Ratio (range: 0 - ∞) is roughly ***'Inverse Accuracy'*** since it varies in opposite direction to that for Accuracy (range: 0 - 1).

Two stable properties: Sensitivity (Sen) = a/(a+c); Specificity (Spec) = d/(b+d)

Four frequency-dependent properties: Positive predictive value (+ve Pred value) = a/(a+b); Negative predictive value (-ve Pred value) = d/(c+d); Accuracy (Accu) = (a+d)/(a+b+c+d); Prevalence (Prev) = (a+c)/(a+b+c+d)

Using Bayes' theorem, +ve Predictive values can also be calculated as

$$\frac{[(Prev)(Sen)]}{[(Prev)(Sen) + (1 - Prev)(1 - Spec)]}$$

Factitious (Fac) = Number of (true) fact = P(Fac) = (a+d).

Fictitious (Fic) = Number of (false) fiction = P(Fic) = (b+c).

P(Fic) + P(Fac) = 1 $\Longrightarrow$ P(Fac) = 1 - P(Fic) ... Equation 1

R = $\frac{P(Fic)}{P(Fac)}$ with range of values from 0 to ["undefined"] ∞ ... Equation 2

Target: Accept R < 1 and Reject R > 1 with R = 1 being indeterminate.

Note: For a well-defined "proposed state" or "natural state", P(Fic) and P(Fac) may each be constituted by ≥1 MA or ≥1 DT that are mutually independent and/or dependent. Using parameter R (Equation 2), Equation 1 is equivalent to two parametric equations P(Fic) = $\frac{R}{R+1}$ & P(Fac) = $\frac{1}{R+1}$ with R+1 ≠ 0 & R ≠ 0.

In $SEIR$ model, extra compartment E for Exposed Population allows modelling to incorporate **incubation period**. This is time from exposure to causative agent until first symptoms develop and is characteristic for each disease agent. WHO estimated in early 2020 the incubation period for COVID-19 ranges from 1 to 14 days with a median incubation period of 5 to 6 days. One useful way to determine the infectiousness of COVID-19 is the reproductive rate of its causative agent SARS-CoV-2, or R_0. R_0 measures the average number of secondary infections caused by a single case and is initially estimated by WHO to be 1.4 - 2.5 (average 1.95). Higher in countries that do not implement strong public measures such as extensive [and repeated] testing, contact tracing, case isolation and contact quarantine; R_0 is a context specific measurement which will fall to < 1 with successful control of outbreaks. Another [less useful] measure of infectiousness is household secondary attack rate, or the proportion of household members who are likely to get infected from a case. Estimates of this rate have not unexpectedly varied significantly between studies in 2020 [not quoted here], ranging from as low as 3 - 10% to as high as 100% for COVID-19. This suggests that there may be factors that vary considerably between different groups, such as types of activities, duration of event, ventilation of the household and viral shedding of the case. All the above estimates can be subsequently refined as more data becomes available.

Applying **Artificial Intelligence** technology to contact tracing has been demonstrated to provide markedly improved efficiency for this important process. We now give four concrete examples utilizing Fic-Fac Ratio (R) whereby their corresponding false positive and false negative MA and DT results do not exist and consequently from Table 1 with (a+d) = 1, (b+c) = 0, and R = 0. For optimal understanding, we discuss [hypothetical] test subject group on MA and patient group on DT with total number of each group and its two subgroups denoted (respectively) by N_T=100 and N_1=N_2=50.

Obtaining MA results for a hypothesis or conjecture using ideal gold standard MA to rigorously prove:

(I) Riemann hypothesis to be true via (i) all nontrivial zeros are located on critical line [true positive MA result] and (ii) all nontrivial zeros are not located away from critical line [true negative MA result];
(II) Twin prime conjecture to be true via (i) twin primes are infinite in magnitude [true positive MA result] and (ii) twin primes are not finite in magnitude [true negative MA result]; and
(III) Conjecture "Ubiquitous human angiotensin-converting enzyme 2 (ACE2) receptor is sole entry receptor for SARS-CoV-2 causing COVID-19 when susceptible test subjects N_T = 100 are [unethically] experimentally exposed to this virus with assumed 100% infectivity rate in ideal world (but likely, say, up to around 59% infectivity rate (Ing, Cocks & Green, 2020) in real world)" to be true via (i) COVID-19 infection will occur in test subjects N_1 = 50 exposed to SARS-CoV-2 while not taking novel drug *'irreversible ACE2 blocker'* with 100% efficacy and acceptable "safety profile" [true positive MA result] and (ii) COVID-19 infection will not occur in test subjects N_2 = 50 exposed to SARS-CoV-2 while taking this same novel drug [true negative MA result].

Obtaining DT results for a group of individuals (patient group with N_T = 100) employing [hypothetical] ideal gold standard DT [with Sensitivity = 100% and Specificity = 100%] to definitively determine:
(IV) proportion of patient (i) having [with true positive DT result] COVID-19 infection with N_1 = 50 and (ii) not having [with true negative DT result] COVID-19 infection with N_2 = 50.

The gene that encodes Transmembrane Serine Protease 2 (TMPRSS2) is activated when male hormones bind to androgen receptor. It can be experimentally shown that TMPRSS2 enzyme (Hoffmann et al, 2020) is required to cleave SARS-CoV-2's spike protein – a process known as proteolytic priming – before the virus could enter cells via its spike protein binding to ACE2 receptor. Pharmacologically targeting (e.g.) ACE2 could theoretically be key to unlocking effective vaccines based on (e.g.) mRNA & DNA nucleic acid, weakened or inactivated viral forms, protein subunits and viral vectors; and effective drugs (e.g.) antiviral medication Remdesivir [by inhibiting viral replication thus shortening time to clinical recovery], 'androgen deprivation therapy', 'irreversible ACE2 blocker' and 'TMPRSS2 inhibitor'. Another hypothetical novel drug *'floating version of ACE2'* could trick the virus to preferably bind with this drug rather than ACE2 on human cells thus potentially treating COVID-19 infection, preventing viral replication and spread. As an unwanted profile of COVID-19 viral vector vaccine, we anticipate auto-immune mediated side effect 'vaccine induced prothrombotic immune thrombocytopenia' (thrombosis with thrombocytopenia syndrome) with unusual blood clots in brain (cerebral venous sinus thrombosis) or in other parts of body associated with low platelet levels to rarely occur in vaccinated subjects.

With main effect of increasing vasoconstricting angiotensin II hormone, ACE acts as a key regulatory peptide in renin-angiotensin-aldosterone system (RAAS); and with main effect of decreasing vasoconstricting angiotensin II hormone, its counterpart ACE2 acts as key counterregulatory peptide via its dual actions of firstly, acting as an ubiquitous functional receptor present in many parts of our body and secondly, simultaneously acting as an enzyme that predominantly degrade angiotensin II (and to a lesser extent cleaves angiotensin I and participates in hydrolysis of other peptides). In patients with RAAS blockade such as on ACE inhibitor (ACEI) or angiotensin II receptor blocker (ARB) therapy for hypertension or diabetes, health workers are dealing here with a double-edged sword depending on the phase of disease. Increased baseline ACE2 expression in these patients could potentially increase SARS-CoV-2 infectivity and ACEI/ARB use would be an addressable risk factor. Conversely, once infected, downregulation of ACE2 may be the hallmark of COVID-19 progression. Consequently, upregulation by preferentially employing RAAS blockade and ACE2 replacement in acute respiratory distress syndrome phase may turn out to be beneficial.

"Proposed states" such as modelling Riemann hypothesis when formulated as equation can and must be error-free. All "proposed states" can and must have their Fic-Fac Ratio = 0 with P(Fac) = 1 and P(Fic) = 0. This is equivalent to stating mathematical-based proofs for "proposed states" must always be mathematically rigorous and error-free.

Loosely speaking, "natural states" such as Pseudo-SIR model or $SEIR$ model for COVID-19 pandemic are *"Incompletely Predictable"* in the sense that their statistical-based proofs should be statistically significant but they can never be error-free. [Here, we will omit outlining common ordinary differential equations associated with the two models.] This is because both models as schematically displayed will (1) intrinsically be affected by obtained DT results using relevant DT e.g. never having, in practice, 100% accuracy and (2) extrinsically be affected by incorrect DT results obtained due to [unintentional] e.g. sampling errors (likely causing false negative DT results in COVID-19 patients potentially due to obtained saliva samples being insufficient, collected too early during infection or too late during recovery), observational errors, blunders, under- and over-reporting or [intentional] e.g. data fabrication and manipulation. We give an extreme "counter-example" of data fabrication and manipulation: Having ulterior motive, local investigator Mr. CB decided to intentionally send an e-mail containing (say) important test results at (say) 3:45 PM Friday February 8, 2019 to a fabricated email address XYZ. Consequently, these results will never reach the intended recipient (statistician / epidemiologist) for analysis. **Note:** Medico-legally in terms of Fic-Fac Ratio, (i) XYZ is ['positively'] a fabricated email address for recipient when used by Mr. CB since it never belong to recipient = (abstract) True Positive MA and (ii) XYZ used by Mr. CB is ['negatively'] a non-existing email address for recipient since it was never created by recipient = (abstract) True Negative MA. This unjustifiable action will lead to failure of these results to be properly incorporated into modelling an "old" epidemic occurring from (say) October 29, 2018 to February 8, 2019. Both (1) and (2) will lead to some quantifiable increase of P(Fic) values [with reciprocal decrease of P(Fac) values] affecting, for instance, I for Infectious Population. Since we reject [or accept] probability based Fic-Fac Ratio > 1 [or < 1], the overall goal is to always minimize P(Fic) &/or maximize P(Fac).

Gold standard MA must always be an (error-free) ideal gold standard MA. Gold standard DT refers to its use in achieving a definitive diagnosis obtained by biopsy, surgery, autopsy, long-term follow-up or another acknowledged standard. In theory, an ideal gold standard DT designed to detect SARS-CoV-2 is error-free having Sensitivity = 100% (it identifies all individuals with the disease) and Specificity = 100% (it does not falsely identify individuals without the disease); and consequently will also have +ve Predictive values, -ve Predictive values, and Accuracy all = 100%. In practice, there are no ideal gold standard DT, and one tries to use a DT that is as close as possible to the ideal test. The commonly available reverse transcription-polymerase chain reaction (PCR) test on a nasal (oro/nasopharyngeal) swab detects presence of genetic material of SARS-CoV-2 causing COVID-19. Results on Sensitivity and Specificity of this newly developed test depend critically on how closely it approaches the ideal test. It likely has intrinsic Sensitivity & Specificity in the range of (say) 90 - 95%. Then assuming a high Sensitivity & Specificity of 95% meant that the test could still miss about 5% of infected people and falsely diagnose about 5% of non-infected people. If required, whole genome sequencing can additionally be performed on selected positive reverse transcription-PCR samples to detect phylogenetic clusters of SARS-CoV-2 and rapidly identify SARS-CoV-2 transmission chains. Notwithstanding potential for some false-positive test results perhaps due to people previously exposed to other less dangerous coronaviruses, IgG anti-coronavirus antibodies

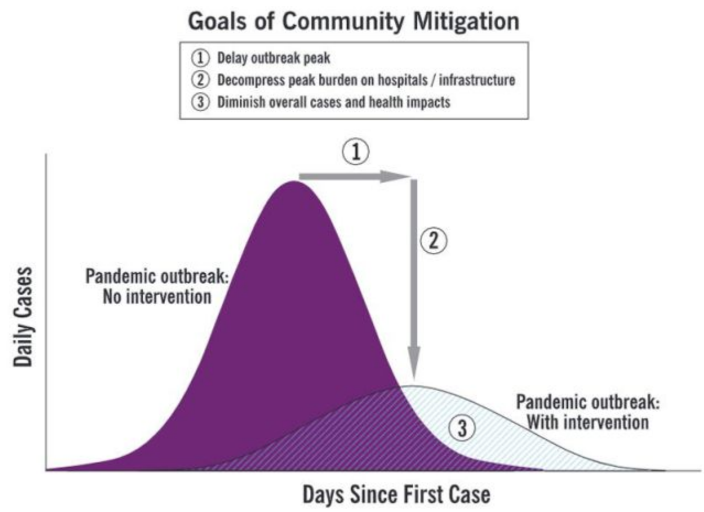

Figure 4. Epidemiological model of pandemic (US Centers for Disease Control and Prevention)

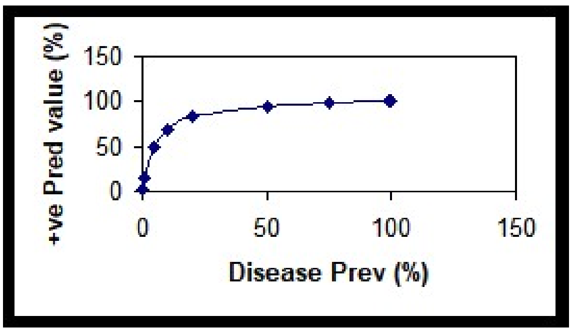

Figure 5. Graphical relationship: Prevalence vs Positive Predictive value with Sensitivity of 95%

could be used to detect past COVID-19 infection and measure community immunity. Future development of potential tests using different methodology may be based on detecting viral components such as proteins, nucleic acids or combinations of these in patient samples. Significant mutations in SARS-CoV-2 genome over time leading to viral strains of increased contagiousness and/or virulence will inevitable occur.

In a study of all 217 passengers and crew on a cruise ship (Ing et al, 2020), 128 tested positive for COVID-19 on reverse transcription-PCR (59%). Of these infected patients, 19% (24) were symptomatic; 6.2% (8) required medical evacuation; 3.1% (4) were intubated and ventilated; and the mortality was 0.8% (1). The majority of infected patients were asymptomatic (81%, 104 patients). Thus prevalence of COVID-19 on affected [isolated] cruise ships [and tentatively projected by us to happen in some "hotspot" outbreak places on planet Earth] is likely to be significantly underestimated.

Remark 1 Difference mitigation measures *with full compliance by everyone* could make to severity of COVID-19 pandemic can be clearly illustrated by epidemiological modelling in Figure 4 courtesy of Centers for Disease Control and Prevention (CDC) *[with arising mental health problems being an addressable issue]*.

Dynamic staged implementation and subsequent staged easing of [beneficial] mitigation measures such as *lockdowns*, *border closures*, *social distancing* (with practising good hand and sneeze / cough hygiene; obeying more than 1.5 - 2 metres distance between people; using Personal Protective Equipment (PPE), importantly, in the correct manner when deemed appropriate to do so by authorized health officials for public and health-care settings e.g. eye protection which included visors or face shields or goggles, among others, and three layered homemade cloth face masks or surgical masks or P2 / N95 respirators (Chu et al, 2020); and limiting indoor / outdoor *mass gatherings* is based on experiences, expert opinions, statistical analysis of collected data or previous and recent research studies thus complying with Evidence-based Medicine (EBM) and Practice (EBP). Generally speaking, EBP = EBM + Clinician Expertise + Patient Values with EBM often depicted as Pyramidal Hierarchy (from top with highest Quality of Evidence to bottom with lowest Quality of Evidence): Meta-Analysis Systematic Review → Randomized Controlled Trial → Cohort Studies → Case Control Studies → Case Report or Case Series → Expert Opinions.

Table 2. Tabulated relationship: Prevalence vs Positive Predictive value with Sensitivity of 95%

Disease Prev (%)	**+ve Pred value (%)**
0	0
0.1	2
1	16
5	50
10	68
20	83
50	95
75	98
99	99.9
100	100

Ability of a test to discriminate between normal (without disease) and abnormal (with disease) individuals is described by its Specificity and Sensitivity. Generally, they are inversely related to each other and may be altered by changing reference interval or normal range. In other words, one can only be improved at the expense of the other. Example, prostate specific antigen (PSA) cut-off of 4.0 ng/mL [and 3.0 ng/mL] is often given as having a sensitivity of 21% [and 32%] with specificity of 91% [and 85%] for detection of any prostate cancer. A study (Karimi-Zarchi et al, 2013) on conventional Pap smear method to detect cervical cancer in women had Sensitivity 51%, Specificity 66.6%, +ve Predictive value 96%, -ve Predictive value 8% and Accuracy 92%. When a DT has Sensitivity of 95% (5% false -ve) and Specificity of 95% (5% false +ve), for a disease with 1% Prevalence, its +ve Predictive value is only 16% but its -ve Predictive value is 99%. Relationship between Prevalence and +ve Predictive value with Sensitivity of 95% is numerically and graphically depicted in Table 2 and Figure 5.

Lymphocytes include natural killer cells which function in cell-mediated, cytotoxic innate immunity; T cells for cell-mediated, cytotoxic adaptive immunity; and B cells for humoral, antibody-driven adaptive immunity (which is mostly mediated by differentiated B cells called plasma cells secreting Immunoglobulins G, A, M, D and E). Memory B cells are a B cell sub-type that are formed within germinal centers following primary infection. Memory B cells can survive for decades and repeatedly generate an accelerated and robust antibody-mediated immune response in the case of re-infection (also known as a secondary immune response). Memory T cells are a subset of T lymphocytes that might have some of the same functions as memory B cells e.g. Antigen-specific memory T cells against viruses or other microbial molecules can be found in both T_{CM} and T_{EM} subsets. T_{VM} subset also function in production of various cytokines. Then for a COVID-19 vaccine candidate targeting sufficient level of immunity against SARS-CoV-2 spike protein, it must generate the appropriate type of antibody and T cell response.

CDC indicated six stages of vaccine development against a new infection or disease: (i) exploratory, (ii) pre-clinical, (iii) clinical development, (iv) regulatory review and approval, (v) manufacturing and (vi) quality control. Clinical development is a three-phase process.

Phase I: Small groups of people receive the trial vaccine. Is the vaccine safe? What dose should it be used?
Phase II: Clinical study is expanded. Vaccine is given to people who have characteristics (such as age & physical health) similar to those for whom vaccine is intended. Can the vaccine generate an immune response?
Phase III: Vaccine is given to thousands of people and tested for efficacy and safety. Can the new vaccine protect from infection or disease?

Many vaccines will undergo Phase IV formal, ongoing studies after the vaccine is approved and licensed. Through international collaboration, researchers are urgently speeding up the process to obtain effective and safe COVID-19 vaccines – mainly mRNA vaccines and viral vector vaccines – by running one trial while simultaneously recruiting for next phase.

For optimal management of critically ill COVID-19 patients, it is vitally important for world-wide medical communities to share extra knowledge [for instance] that giving nebulized Heparin (van Haren et al, 2020) should reduce severity of COVID-19 induced acute respiratory distress syndrome (ARDS) through its anti-viral effects, anti-inflammatory effects, anti-coagulant effects, and mucolytic effects; and that selectively giving corticosteroid Dexamethasone (Mahase, 2020) through its anti-inflammatory [likely via inhibiting CD4+ Helper T cell subsets in production of various cytokines] and immunosuppressant effects [likely via inhibiting CD8+ Cytotoxic T cell subsets in destroying virus-infected cells, tumor cells and transplant rejection] to appropriate COVID-19 patients with excessive immune response called 'cytokine storm' causing harmful hyper-inflammation will reduce the risk of death.

We introduce the educational concept of 'Top-Down Approach' versus 'Bottom-Up Approach' to therapy on COVID-19 induced 'cytokine storm' causing hyper-inflammation. Assuming the simplistic but not totally accurate caveat expressed through the following statement to be true: *'Cytokine storm' is largely caused by imbalance of two broad classes of identifiable cytokines known as pro-inflammatory cytokines and anti-inflammatory cytokines whereby there is supramaximal elevation of the former class with or without supramaximal fall of the later class.* Then giving Dexamethasone acting through the non-specific (increased) anti-inflammatory effect [likely via acting non-specifically on various cytokines] constitutes 'Top-Down Approach' to therapy whereby giving novel synthetic drugs 'pro-inflammatory cytokine X blocker' and/or 'anti-inflammatory cytokine Y' acting through, respectively, their specific (reduced) pro-inflammatory effect and (increased) anti-inflammatory effect constitutes 'Bottom-Up Approach' to therapy. Finally, only globally available safe and effective COVID-19 vaccines that are successfully developed can ultimately control COVID-19 pandemic. This is achieved by providing mass immunization for every eligible person, initially targeting sufficient herd immunity threshold = 1 - $\frac{1}{R_0}$ [estimated to be around 60 - 70%] at community level to prevent on-going transmission of infection.

1.2 Full Compliance with Information-Complexity Conservation by Completely and Incompletely Predictable Entities

Useful preliminary information: A variable e.g. n in f(n) = RζF, n_1 in f(n_1), and n_2 in f(n_2) represents a model state and may change during simulation. A parameter, e.g. σ and t in RζF, is normally a constant in a single simulation used to describe objects statically and is changed only when we need to adjust our model behavior. A single-variable function e.g. f(n_1) or multiple-variable function e.g. f(n_1, n_2) is a set of input-output pairs that follows a few particular rules. An expression usually contains number(s), parameter(s), variable(s) and operator(s). A particular function e.g. f(n_1) is an expression involving variable n_1 that is defined for interval [a,b]. An equation is an assertion that two expressions are equal from which one can determine a particular quantity. An algorithm is a precise step-by-step plan for a computational procedure that possibly begins with an input value and yields an output value in a finite number of steps. A complex algorithm e.g. for generating prime numbers is only defined at two end-points a,b (but not for interval [a,b] as it is not a function). A formula is a fact or a rule written with mathematical symbols, and usually connects two or more quantities with an equal to sign. The terms 'variable', 'parameter', 'function', 'algorithm', 'equation' and 'formula' could be loosely used in some situations of this paper. Colloquially, we insightfully employ "$\Delta x \longrightarrow 0$" to usefully indicate continuous-type equations or functions and "$\Delta x \longrightarrow 1$" to usefully indicate discrete-type equations or functions. Antiderivative $F(n)$ denotes the result obtained when performing integration on function $f(n)$; viz, $\int f(n)dn = F(n) + C$ with $F'(n) = f(n)$.

The word "number" [singular noun] or "numbers" [plural noun] in reference to prime and composite numbers, NTZ and two types of GP can interchangeably be replaced with the word "entity" [singular noun] or "entities" [plural noun]. We propose an innovative method to validly classify certain appropriately chosen equations or algorithms in two ways by using relevant locational properties of its output. This output consist of generated entities either from function-based equations or from algorithms. Our classification system is formalized by providing definitive definitions for Completely Predictable (CP) entities obtained from CP equations or algorithms, and Incompletely Predictable (IP) entities obtained from IP equations or algorithms. 'Container' is a useful analogy that metaphorically group CP entities and IP entities to be exclusively located in, respectively, 'Simple Container' and 'Complex Container'.

Definitions for CP numbers and IP numbers: Respectively, using CP simple equation or algorithm, and IP complex equation or algorithm; a generated CP number, and a generated IP number, is locationally defined as a number whose position is *independently* determined by simple calculations without, and *dependently* determined by complex calculations with, needing to know related positions of all preceding numbers in neighborhood. Simple properties are inferred from a sentence such as "This simple equation or algorithm by itself will intrinsically incorporate actual location [and actual positions] of all CP numbers". Solving CP problems with simple properties amendable to *simple* treatments using *usual* mathematical tools such as Calculus gives 'Simple Elementary Fundamental Laws'-based solutions. Complex properties, or "meta-properties", are inferred from a sentence such as "This complex equation or algorithm by itself will intrinsically incorporate actual location [but not actual positions] of all IP numbers". Solving IP problems with complex properties amendable to *complex* treatments using *unusual* mathematical tools such as exact and inexact Dimensional analysis homogeneity, and Dimension (2x - N) system as well as using *usual* mathematical tools such as Calculus gives 'Complex Elementary Fundamental Laws'-based solutions.

Classified as IP problems, solving and explaining our nominated open problems is intuitively perceived as burdened with "Supramaximal Complexity". Prime numbers are defined as "All Natural numbers apart from 1 that are evenly divisible by itself and by 1" and composite numbers are defined as "All Natural numbers apart from 1 that are evenly divisible by numbers other than itself and 1". We conduct [complex] exercise of solving IP problem involving prime and composite numbers by proving their gaps are always varying (see P-C Pairing). This is a mathematician's paradigm of an ideal example for this type of problem and is in sharp contrast to solving CP problem endowed with "Supraminimal

Complexity" as demonstrated by [simple] exercise of proving even and odd number gaps always equal to 2 (see E-O Pairing) whereby even number (**n**) is defined as "Any integer that can be divided exactly by 2 with last digit always being 0, 2, 4, 6 or 8" and odd number (**n**) is defined as "Any integer that cannot be divided exactly by 2 with last digit always being 1, 3, 5, 7 or 9". Congruence **n** $\equiv$ 0 (mod 2) holds for even **n** and congruence **n** $\equiv$ 1 (mod 2) holds for odd **n**. Thus, '0' is an even **n** when we consider all (non-negative) positive even and odd **n** obtained from whole numbers = 0, 1, 2, 3, 4, 5,.... For convenience, we shall only consider all positive even and odd **n** obtained from natural numbers = 1, 2, 3, 4, 5, 6,... in this paper with following implication: The phrase "all even numbers" is generally taken to denote 2, 4, 6, 8, 10, 12,...; viz, this phrase is equivalent to the expression "all even numbers 2, 4, 6, 8, 10, 12,... equate to all positive even numbers 0, 2, 4, 6, 8, 10, 12.... but with even number '0' intentionally and conveniently ignored".

E-O Pairing: For i = 1, 2, 3,..., ∞; let ith Even and ith Odd numbers = E_i and O_i, and ith even and ith odd number gaps = eGap$_i$ and oGap$_i$. The positions of E_i and O_i are CP and their independence from each other is shown below.

E_i	2		4		6		8		10		12	.....
eGap$_i$		2		2		2		2		2		2

We can precisely, easily and independently calculate E_5 = (2 X 5) = 10 and O_5 = (2 X 5) - 1 = 9.

O_i	1		3		5		7		9		11	.....
oGap$_i$		2		2		2		2		2		2

Finite calculations shown here depict even and odd number gaps [= 2] are constant but even and odd numbers are infinite in magnitude requiring an infinite number of calculations ("mathematical impasse") in order to show these gaps will always be constant and non-varying. Obtaining rigorous proof for this property then consist of recognizing it as CP problem which requires deriving a CP 'non-varying' equation for confirming all even and odd numbers will [intrinsically] contain simple property "All even and odd number gaps = [constant] 2". The [colloquially expressed-"$\Delta x \longrightarrow 1$"] discrete-type equation E_i = 2 X i and O_i = (2 X i) - 1 are, respectively, for computing even and odd numbers [as zero-dimensional points]. When converted into [colloquially expressed-"$\Delta x \longrightarrow 0$"] continuous-type equation E = 2 X i and O = (2 X i) - 1 for i = all real numbers $\geq$ 0 [as one-dimensional lines which include even number '0' when i = 0 in E = 2 X i], calculating gradient ΔE/Δi or ΔO/Δi (= 2) and differentiating dE/di or dO/di (= 2) is precisely equivalent to all even and odd number gaps = 2. Using here notation x [instead of i] to illustrate computation of Area under the Curve (AUC) over interval [a,b] for f(x) [instead of f(i)]: AUC ("precisely") = $\int_a^b f(x)dx$ [viz, definite integral]. AUC ("approximately") = $\lim_{n\to\infty} \sum_{i=1}^{n} \Delta x \cdot f(x_i)$ [viz, $\lim_{n\to\infty}$ Riemann sum] whereby i = 1 to n [and not i = 1 to ∞], $\Delta x = \frac{b-a}{n}$ and $x_i = a + \Delta x \cdot i$. Our precise AUC [straight line] between a and b [as two-dimensional area] is given by $\int_a^b (2i)di = [i^2 + C]_a^b = (b^2 - a^2)$ for even numbers and $\int_a^b (2i-1)di = [i^2 - i + C]_a^b = (b^2 - b - a^2 + a)$ for odd numbers. These discrete-type equations [given by 'Even number' variable vs 'Integer' variable and 'Odd number' variable vs 'Integer' variable] represent two mutually exclusive 'Simple Containers' that contain or generate all even and odd numbers with knowing their **overall actual location [and their actual positions].**

P-C Pairing: For i = 1, 2, 3,..., ∞; let ith Prime and ith Composite numbers = P_i and C_i, and ith prime and ith composite number gaps = pGap$_i$ and cGap$_i$. The positions of P_i and C_i are IP and their dependence on each other is shown below.

P_i	2		3		5		7		11		13	.....
pGap$_i$		1		2		2		4		2		4

We can precisely, tediously and dependently compute P_6 = 13: 2 is 1st prime number, 3 is 2nd prime number, 4 is 1st composite number, 5 is 3rd prime number, 6 is 2nd composite number, 7 is 4th prime number, 8 is 3rd composite number, 9 is 4th composite number, 10 is 5th composite number, 11 is 5th prime number, 12 is 6th composite number, and our desired 13 is 6th prime number.

C_i	4		6		8		9		10		12	.....
cGap$_i$		2		2		1		1		2		2

Finite calculations shown here depict prime number gaps = 1, 2, 2, 4, 2, 4,... and composite number gaps = 2, 2, 1, 1, 2, 2,... are varying but prime and composite numbers are infinite in magnitude requiring an infinite number of calculations ("mathematical impasse") in order to show these gaps will always be varying. Obtaining rigorous proof for this property then consist of recognizing it as IP [and not CP] problem which requires deriving two IP ["$\Delta x \longrightarrow 1$"] 'varying' [and not 'non-varying'] discrete-type algorithms $P_{i+1} = P_i$ + pGap$_i$ and $C_{i+1} = C_i$ + cGap$_i$ for calculating all prime and composite numbers [as zero-dimensional points]. With P_1 = 2 and C_1 = 4, '1' is neither prime nor composite. For (arbitrarily) i = all real numbers $\geq$ 1, 'varying' continuous-type algorithm $P_{i+1} = P_i$ + pGap$_i$ and $C_{i+1} = C_i$ + cGap$_i$ [as one-dimensional lines] incorporate all prime and composite numbers. Corresponding AUC ['varying' line] between a and b [as two-dimensional area] can be metaphorically visualized as [mathematically invalid] $\int_a^b (P_i + pGap_i)di$ for prime numbers and $\int_a^b (C_i + cGap_i)di$ for composite numbers. Respectively, the two algorithms will [intrinsically] contain complex property "Apart from first prime gap = 1, all prime gaps consist of perpetually varying even numbers" and "All composite gaps consist of perpetually varying numbers constituted by 1 and 2". Note: The integration is strictly invalid as these algorithms (and not functions) are only defined at end-points a, b (and not defined in interval [a,b]). Given by 'Prime number' variable vs 'Prime gap' variable and 'Composite number' variable vs 'Composite gap' variable, they represent two mutually exclusive 'Complex Containers' that contain or generate all prime and composite numbers with knowing their **overall actual location [but not actual positions].**

Remark 2 We can validly embrace 'Complex Container' concept for discrete format versions ["$\Delta x \longrightarrow 1$"] D$\eta$F via zeroes [*proxy* for ["$\Delta x \longrightarrow 1$"] R$\zeta$F via zeroes] and ["$\Delta x \longrightarrow 1$"] simplified D$\eta$F (sim-D$\eta$F or sim-$\eta$(s)) via zeroes whereby these functions (and the derived continuous format version ["$\Delta x \longrightarrow 0$"] DSPL via pseudo-zeroes to zeroes conversion) are literally '$\sigma = \frac{1}{2}$ Complex Containers' containing three types of mutually exclusive entities constituted by (i) NTZ viz, Origin intercept points or Gram[x=0,y=0] points (G[x=0,y=0]P), (ii) x-axis intercept points viz, 'usual' Gram points ('usual' GP) or Gram[y=0] points (G[y=0]P) and (iii) y-axis intercept points viz, Gram[x=0] points (G[x=0]P). Here, the term Gram points [as zeroes] conveniently encompass (i), (ii) and (iii). The corollary is that '$\sigma \neq \frac{1}{2}$ Complex Containers' [e.g. for $\sigma = \frac{2}{5}$ or $\frac{3}{5}$] will never contain any of these listed entities but instead contain two types of mutually exclusive entities constituted by (i) virtual G[y=0]P and (ii) virtual G[x=0]P. Here, the term virtual Gram points [as virtual zeroes] conveniently encompass (i) and (ii). These mutually exclusive 'Complex Containers' contain or generate all relevant IP entities with knowing their **overall actual location [but not actual positions]**.

Corresponding [perpetually varying] GP gaps or virtual GP gaps [both being transcendental numbers] are defined as the difference between any two nominated [consecutive] $\sigma = \frac{1}{2}$ GP or $\sigma \neq \frac{1}{2}$ virtual GP. All NTZ in RζF are proposed by Riemann hypothesis to only be located at $\sigma = \frac{1}{2}$ (critical line) of this function. Remark 1 above connotes $\sigma = \frac{1}{2}$ and $\sigma \neq \frac{1}{2}$ situations represent two mutually exclusive 'Complex Containers'. In principle, the preceding two sentences when combined together should allow rigorous proof for Riemann hypothesis to materialize. This rigorous proof is successfully obtained by rigidly applying exact [denoting $\sigma = \frac{1}{2}$] and inexact [denoting $\sigma \neq \frac{1}{2}$] Dimensional analysis homogeneity to DSPL (as pseudo-zeroes to zeroes conversion) which is rigorously derived from DηF (as zeroes) [*proxy* for RζF (as zeroes)] via intermediate sim-DηF (as zeroes). An identical procedure is used to precisely explain two types of GP.

Net Area Values and Total Area Values: $\int f(n)dn = F(n) + C$ with $F'(n) = f(n)$. Consider a nominated function $f(n)$ for interval [a,b]. We define Net Area Value (NAV) calculated using its antiderivative $F(n)$ as the net difference between positive area value(s) [above horizontal x-axis] and negative area value(s) [below horizontal x-axis] in interval [a,b]; viz, NAV = all +ve value(s) + all -ve value(s). Again calculated using $F(n)$, we define Total Area Value (TAV) as the total sum of (absolute value) positive area value(s) [above horizontal x-axis] and (absolute value) negative area value(s) [below horizontal x-axis] in interval [a,b]; viz, TAV = all |+ve value(s)| + all |-ve value(s)|. Calculated NAV and TAV are precise using antiderivative $F(n)$ obtained from integration of $f(n)$ but are only approximate when using Riemann sum on $f(n)$.

Zeroes and Pseudo-zeroes: [We will progressively explain these two entities below.] There are three types of stationary points in a given periodic f(n) involving sine and/or cosine functions that could act as x-axis intercept points via three types of f(n)'s zeroes with corresponding three types of F(n)'s pseudo-zeroes: maximum points e.g. with f(n) or F(n) = sin n - 1; minimum points e.g. with f(n) or F(n) = sin n + 1; and points of inflection e.g. with f(n) or F(n) = sin n [**which also has Origin intercept point as a zero or pseudo-zero**]. A fourth type of f(n)'s zeroes and F(n)'s pseudo-zeroes consist of non-stationary points occurring e.g. with f(n) or F(n) = sin n + 0.5.

With (j - i) = (l - k) = 2π [viz, one Full cycle], let a given zero be located in f(n)'s interval [i,j] viz, i < zero < j; and its corresponding pseudo-zero be located in F(n)'s pseudo-interval [k,l] viz, k < pseudo-zero < l. For this zero and pseudo-zero characterized by either point of inflection or non-stationary point; both will comply with preserving positivity [going from (-ve) below x-axis to (+ve) above x-axis] as explained using the zero case [with the pseudo-zero case following similar lines of explanations]. This can be stated as follow for interval [i,j]: If j > i, then computed f(j) > computed f(i). In particular, the condition "If i ≥ 0, then computed f(i) ≥ 0" must not be present for these two particular types of zero to validly exist in interval [i,j]. With reversal of inequality signs, the converse situation for j < zero < i and corresponding l < pseudo-zero < k will be equally true in preserving negativity [going from (+ve) above x-axis to (-ve) below x-axis]. These are useful observed properties for zeroes and pseudo-zeroes.

Preservation or conservation of Net Area Value and Total Area Value: For f(n)'s interval [a,b] whereby a = initial zero and b = next zero, and F(n)'s pseudo interval [c,d] whereby c = initial pseudo-zero and d = next pseudo-zero; we

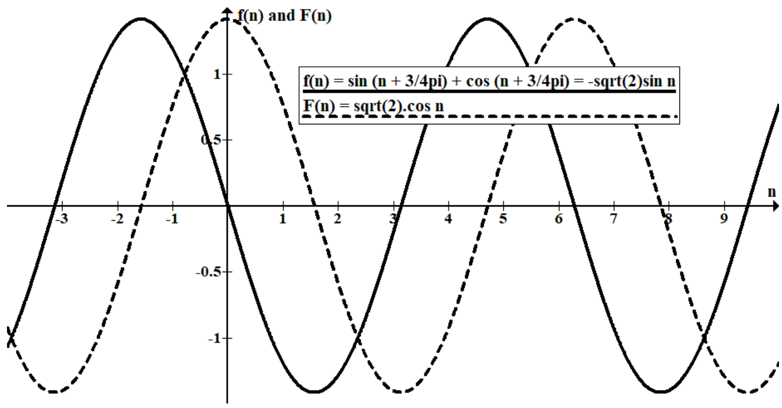

Figure 6. Plot of $f(n) = sin(n + \frac{3}{4}\pi) + cos(n + \frac{3}{4}\pi) = -\sqrt{2}\sin n$ and $F(n) = \sqrt{2}\cos n$ [+ C].

show below how compliance with preservation or conservation of NAV and TAV will simultaneously occur in both f(n)'s zeroes and F(n)'s pseudo-zeroes given by their sine and/or cosine functions only when zero gap = (b - a) = pseudo-zero gap = (d - c) = 2π [viz, involving one Full cycle].

Consider a (first) randomly chosen example of single-variable [simple] periodic sin and/or cosine function $f(n) = (\cos n)^{\frac{1}{3}} - (\sin n)^{\frac{1}{3}}$ which has zeroes and two individual exponents $\frac{1}{3}$ [with their sum $\frac{2}{3}$] being persistently fractional numbers. Observed characteristics of exponents from this first example [and second example below] as CP problems with perpetually present [intrinsic] simple properties could hint at possible invalidity of exact and inexact DA homogeneity as useful analytic tool used for IP problems. However, these are irrelevant (counter)examples since exact and inexact DA homogeneity in this research paper for our IP problems are perpetually present [intrinsic] complex properties that clearly do not refer to these CP recurring zeroes but instead validly refer to totally different IP recurring zeroes calculated as axes-intercept points in DηF and approximate NAV = 0 from Riemann sum interpretation of sim-DηF.

Derived complex properties of exact and inexact DA homogeneity in our IP problems may be assumed by some to be just associations and not proper explanations. We advocate this assumption to be incorrect since these perpetually present [intrinsic] complex properties are, nevertheless, valid properties fully supported by irrefutable [albeit convoluted] correct and complete set of mathematical arguments with proper explanations behind them based on, for instance, valid analysis on modulus of DηF, valid mathematical definition for NTZ, and valid compliance with preserving or conserving NAV = 0 condition.

Consider a (second) randomly chosen example of single-variable [simple] periodic sin and/or cosine function f(n) = (sin (n+$\frac{3}{4}\pi$))^1 + (cos (n+$\frac{3}{4}\pi$))^1 which has zeroes [**that also include the Origin intercept point as a zero**]. Depicted in Figure 6, this function is also equivalently simplified to f(n) = $-\sqrt{2}(\sin n)^1$ with non-negotiable trigonometric identity $\cos n - \sin n = \sqrt{2}\sin\left(n + \frac{3}{4}\pi\right)$ application. We note the individual or sum exponents of all involved sine and/or cosine terms is 1 or 2, being persistently whole numbers.

We compare and contrast the f(n)'s sine and cosine terms in the above two mentioned (unrelated) examples resulting in following two non-specific observations [without detailed discussion on their limited generalization to two other unrelated examples below]: [I] The dual sine and cosine f(n) with individual exponent = whole number 1 and sum exponent = whole number 2 in second example, which has zeroes [**that also include the Origin intercept point as a zero**], can only be **non-negotiably simplified (using trigonometric identity)** to be expressed in [solitary] sine term with exponent = whole number 1. [II] The dual sine and cosine f(n) with individual exponent = fractional number $\frac{1}{3}$ and sum exponent = fractional number $\frac{2}{3}$ in first example, which has zeroes, cannot be simplified further and remains expressed in [combined] sine and cosine terms. Two other unrelated examples: The f(n) = (sin n)^1 + (cos n)^1 which has zeroes and endowed with whole number 1 for its individual exponent with their sum = whole number 2 contradicts observation [I] as this function cannot be simplified further and remains expressed in [combined] sine and cosine terms. The f(n) = (sin n)$^{\frac{1}{2}}$ + (cos n)$^{\frac{1}{2}}$ [conveniently considered here as a different f(n) variety that *do not have zeroes*] endowed with two fractional exponent $\frac{1}{2}$ but with their sum = whole number 1, cannot be simplified further and remains expressed in [combined] sine and cosine terms – this case validly comply with observation [II] but sum of exponents being a whole number, and not a fractional number, could also be non-specifically interpreted to (partially) contradict observation [I].

Two useful points about f(n)'s Zeroes and F(n)'s Pseudo-zeroes involving sine and/or cosine terms; and exact and

inexact Dimensional analysis homogeneity:
(I) With (Zeroes) = (Pseudo-zeroes) – ($\frac{\pi}{2}$) [given in terms of Full cycles] being valid in both CP problems and IP problems, the zeroes obtained from IP DηF via axes-intercept points and IP sim-DηF via approximate NAV = 0 are exactly related to the pseudo-zeroes obtained via precise NAV = 0 calculated using IP DSPL [the antiderivative for integration of sim-DηF]. **Summary of useful shorthand notations: DηF [as Zeroes], sim-DηF [as Zeroes], and DSPL [as Pseudo-zeroes to Zeroes conversion].**
(II) Parameter σ = (i) $\frac{1}{2}$ [viz, exact Dimensional analysis homogeneity for IP problem with full presence of Origin intercept points that **compulsorily involve trigonometric identity** $\cos n - \sin n$] and (ii) $\neq \frac{1}{2}$ [viz, inexact Dimensional analysis homogeneity for IP problem with full absence of Origin intercept points] situations are, respectively, the non-specific analogy of (i) CP f(n) from second example with individual / sum exponent = whole number [*ala* exact Dimensional analysis homogeneity for CP problem with full presence of Origin intercept point that **compulsorily involve trigonometric identity** $\cos n - \sin n$] and (ii) CP f(n) from first example with individual / sum exponent ≠ whole number [*ala* inexact Dimensional analysis homogeneity for CP problem with full absence of Origin intercept point]. **The *Supplementary materials* in Conclusion include mathematical explanation why nontrivial zeros must inevitably exist in sim-DηF [as zeroes] uniquely and only at [fractional number] individual exponent $\sigma = \frac{1}{2}$ (viz, complying with exact Dimensional analysis homogeneity for IP problem based on [whole number] sum of two exponents = $2(1-\sigma) = \frac{1}{2} + \frac{1}{2} = 1$) in Dirichlet Sigma-Power Law [as pseudo-zeroes to zeroes conversion].**

Full cycle-zeroes and Half cycle-zeroes of f(n) = $-\sqrt{2}\sin n$ from above second example are its recurring x-axis intercept points as seen in Figure 6. The term Full cycle symbolizes "non-varying CP full loop from 0π to 2π"; and "Half cycle" symbolizes "non-varying CP half loop from 0π to 1π or from 1π to 2π". From +ve to -ve; x-axis line is denoted by 0π to 1π, and y-axis line is denoted by $\frac{\pi}{2}$ to $\frac{3\pi}{2}$. In interval [0,2π], $f(n) = 0$ when n = 0π [Full cycle-zero], 1π [Half cycle-zero] and 2π [Full cycle-zero]. Ignore the 1π [Half cycle-zero] and conveniently name 0π [Full cycle-zero] and 2π [Full cycle-zero] as the initial zero and next zero. $F(n)$ is the antiderivative of integral $\int f(n)dn$ since $F'(n) = f(n)$. Precise NAV is given by $\int_0^{2\pi} f(n)dn = F(n)$ for the same interval [0,2π]. This NAV = 0 as $F(n) = [\sqrt{2}\cos n + C]_0^{2\pi} = (\sqrt{2}\cdot 1) - (\sqrt{2}\cdot 1) = 0$. In pseudo interval [$\frac{\pi}{2}$,$\frac{5\pi}{2}$] of $F(n)$, $\frac{\pi}{2}$ and $\frac{5\pi}{2}$ are its pseudo x-axis intercept points which we conveniently name here as initial pseudo-zero and next pseudo-zero. Both f(n)'s zero gap (initial zero minus next zero) and F(n)'s pseudo-zero gap (initial pseudo-zero minus next pseudo-zero) = 2π which resulted in f(n)'s interval gap and F(n)'s pseudo interval gap being also of identical magnitude 2π. As a direct consequence, calculated precise NAV = 0 will also apply to $F(n)$ on this pseudo interval with $(\sqrt{2}\cdot 0) - (\sqrt{2}\cdot 0) = 0$. Since NAV = $-\sqrt{2} + +\sqrt{2} = 0$ for (different) interval [0,π] and NAV = $-\sqrt{2} + -\sqrt{2} = -2\sqrt{2}$ for its corresponding pseudo-interval [$\frac{\pi}{2}$,$\frac{3\pi}{2}$], we observe in Figure 6 that NAV is not preserved or conserved when the interval and pseudo-interval do not involve a Full cycle. **Hence, NAV is preserved or conserved for corresponding CP f(n)'s zeroes and CP F(n)'s pseudo-zeroes only if the interval and pseudo-interval involve a Full cycle.**

With $f(n)$ and $F(n)$ involving [cofunctions] sine and cosine, we see in Figure 6 that calculated precise TAV ≠ 0 for given interval e.g. [0,$\frac{\pi}{2}$] with interval gap = $\frac{\pi}{2}$ when derived using $F(n) = \sqrt{2}\cos n + C$ [= $|\sqrt{2}\cdot 0| + |\sqrt{2}\cdot 1| = \sqrt{2}$] will be identical in magnitude to corresponding pseudo-interval [$\frac{\pi}{2}$,π] with interval gap = $\frac{\pi}{2}$ when derived using $F(n) = \sqrt{2}\cos n + C$ [= $|\sqrt{2}\cdot -1| + |\sqrt{2}\cdot 0| = \sqrt{2}$]. Similarly for one Full cycle with interval [0,2π] and corresponding pseudo-interval [$\frac{\pi}{2}$,$\frac{5\pi}{2}$], the calculated precise TAV = $4\sqrt{2}$ for both. **Hence, TAV is preserved or conserved for corresponding CP $f(n)$'s zeroes and CP $F(n)$'s pseudo-zeroes that involve any given interval and pseudo-interval.**

Always being $\frac{\pi}{2}$ out-of-phase with each other, sine and cosine are cofunctions with sin n = cos ($\frac{\pi}{2}$ - n), cos n = sin ($\frac{\pi}{2}$ - n), $\frac{d(\sin n)}{dn} = \cos n$, $\frac{d(\cos n)}{dn} = -\sin n$, $\int \sin n \cdot dn = -\cos n + C$ [= sin (n - $\frac{\pi}{2}$) + C] and $\int \cos n \cdot dn = \sin n + C$ [= cos (n - $\frac{\pi}{2}$) + C]. Last two integrals explain valid relation between f(n)'s zeroes and F(n)'s pseudo-zeroes when they involve sine and/or cosine terms viz, **f(n)'s CP Zeroes = F(n)'s CP Pseudo-zeroes – $\frac{\pi}{2}$.**

1.3 Valid application of both f(n)'s Zeroes and F(n)'s Pseudo-zeroes [when converted to Zeroes] to represent Gram points and virtual Gram points

Whereby f(n) and F(n) have parameters σ and t; the above crucial findings are validly extrapolated to single-variable [complex] periodic (sine and cosine) functions: (i) f(n) DηF (*proxy* for RζF) = 0 to obtain zeroes, (ii) f(n) sim-DηF = 0 to obtain zeroes, and (iii) F(n) DSPL = 0 to obtain pseudo-zeroes [which can be converted to zeroes]. At $\sigma = \frac{1}{2}$ [critical line], the GP consisting of NTZ, G[y=0]P and G[x=0]P precisely correspond to t values for these f(n)'s zeroes. The t values for F(n)'s pseudo-zeroes can be used to calculate t values for f(n)'s zeroes since **f(n)'s IP Zeroes (t values) = F(n)'s IP Pseudo-zeroes (t values) – $\frac{\pi}{2}$.** In addition, NAV and TAV are preserved or conserved for corresponding IP f(n)'s zeroes and IP F(n)'s pseudo-zeroes since both f(n)'s [varying] zero gap (initial zero minus next zero) and F(n)'s [varying] pseudo-zero gap (initial pseudo-zero minus next pseudo-zero) is given by 2π [denotes one Full cycle traversed by parameter t].

At $\sigma \neq \frac{1}{2}$ [or non-critical lines], the virtual GP consisting of virtual G[y=0]P and virtual G[x=0]P precisely correspond to t values for these f(n)'s virtual zeroes. [Virtual NTZ do not exist.] The t values for F(n)'s virtual pseudo-zeroes can be used to calculate t values for f(n)'s virtual zeroes since **f(n)'s IP Virtual Zeroes (t values) = F(n)'s IP Virtual Pseudo-zeroes (t values) –** $\frac{\pi}{2}$. NAV and TAV will also be preserved or conserved for these corresponding IP f(n)'s virtual zeroes and IP F(n)'s virtual pseudo-zeroes since both f(n)'s virtual zero gap (initial virtual zero minus next virtual zero) and F(n)'s virtual pseudo-zero gap (initial virtual pseudo-zero minus next virtual pseudo-zero) is given by 2π [denotes one Full cycle traversed by parameter t].

Cartesian Coordinates (x,y) is related to Polar Coordinates (r,θ) with $r = \sqrt{x^2 + y^2}$ and $\theta = tan^{-1}(\frac{y}{x})$. In anti-clockwise direction, it has four quadrants defined by the + or - of (x,y); viz, Quadrant I as (+,+), Quadrant II as (-,+), Quadrant III as (-,-), and Quadrant IV as (+,-). Figure 8 is the polar graph of $\zeta(\frac{1}{2} + \imath t)$ plotted along critical line for real values of t running from 0 to 34, horizontal axis: $Re\{\zeta(\frac{1}{2} + \imath t)\}$, and vertical axis: $Im\{\zeta(\frac{1}{2} + \imath t)\}$. NTZ are Origin intercept points or G[x=0,y=0]P. With 'gap' being synonymous with 'interval', NTZ gap is given by initial NTZ t-value minus next NTZ t-value. Running a Full cycle from 0π to 2π, size of each IP varying loop in Figure 8 is proportional to magnitude of its corresponding IP NTZ varying gap. We note the 2π here as observed in Figure 8 [on GP], Figure 9 [on virtual GP] and Figure 10 [on virtual GP] refers to IP varying loops transversed by parameter t with NTZ corresponding to t at Origin intercept (G[x=0,y=0]P); G[y=0]P and virtual G[y=0]P corresponding to t at x-axis intercept on x-axis' (+ve) 0π part and (-ve) 1π part; and G[x=0]P and virtual G[x=0]P corresponding to t at y-axis intercept on y-axis' (+ve) $\frac{\pi}{2}$ part and (-ve) $\frac{3\pi}{2}$ part. The virtual NTZ entity do not exist; viz, Origin intercept points do not occur in Figure 9 and Figure 10.

Sine and/or cosine f(n)'s IP (virtual) zeroes and F(n)'s IP (virtual) pseudo-zeroes:

Property I: At parameter $\sigma = \frac{1}{2}$, we denote zeroes to represent t-values for NTZ, G[y=0]P and G[x=0]P. $f(n)$ as DηF = 0 or sim-DηF = 0 gives rise to zeroes with varying zeroes gaps and $F(n)$ as DSPL = 0 gives rise to pseudo-zeroes with varying pseudo-zeroes gaps [whereby pseudo-zeroes can be converted to zeroes]. Since the corresponding zeroes gaps and pseudo-zeroes gaps are always identical in magnitude; NAV = 0 condition is validly preserved or conserved. [For simplicity, we will not discuss TAV ≠ 0 condition which is also validly preserved or conserved.] ***Property I*** **is usefully abbreviated as: NAV = 0 condition is validly preserved or conserved for f(n)'s IP zeroes and F(n)'s IP pseudo-zeroes.** ***Ditto*** **for f(n)'s IP virtual zeroes and F(n)'s IP virtual pseudo-zeroes at parameter** $\sigma \neq \frac{1}{2}$ **as previously explained above; viz, NAV = 0 condition is validly preserved or conserved for f(n)'s IP virtual zeroes and F(n)'s IP virtual pseudo-zeroes.**

Property II: At parameter $\sigma = \frac{1}{2}$, [IP t-values for NTZ, G[y=0]P and G[x=0]P obtained from $f(n)$] is equal to [IP (different) t-values for pseudo-NTZ, pseudo-G[y=0]P and pseudo-G[x=0]P obtained from $F(n)$] minus [Constant $\frac{\pi}{2}$]. ***Property II*** **is usefully abbreviated as: f(n)'s IP zeroes (t values) = F(n)'s IP pseudo-zeroes (t values) –** $\frac{\pi}{2}$**.** ***Ditto*** **for f(n)'s IP virtual zeroes and F(n)'s IP virtual pseudo-zeroes at parameter** $\sigma \neq \frac{1}{2}$ **as previously explained above; viz, f(n)'s IP virtual zeroes (t values) = F(n)'s IP virtual pseudo-zeroes (t values) –** $\frac{\pi}{2}$**.**

Deduction I: The x variable used in Riemann sum above in E-O Pairing is now replaced by the n variable used in sim-DηF. Conventionally, for a left [finite-interval] Riemann sum, i = 0, 1, 2, 3,..., n-1; and for a right [finite-interval] Riemann sum, i = 1, 2, 3, 4,..., n. Analogically, sim-DηF as a complex function with n = 1, 2, 3, 4,..., ∞ is now itself (*interpreted as*) a right [infinite-interval] Riemann sum given by $\sum_{i=1}^{n} \Delta n \cdot f(n_i) = \sum_{n=1}^{\infty} \Delta n \cdot f(n) = \sum_{n=1}^{\infty} f(n)$ since $\Delta n = 1$ in this function. As (i) $\int_{n=1}^{n=\infty} f(n)dn$ and (ii) $\sum_{n=1}^{\infty} f(n)$ are proportional; their f(n)'s zeroes derived from corresponding (i) DSPL [as pseudo-zeroes converted to zeroes] and (ii) sim-DηF [as zeroes] when interpreted as Riemann sum must agree whereby f(n) has parameters σ and t. Again, we note f(n)'s zeroes can be obtained from DSPL [viz, the antiderivative F(n)] because corresponding F(n)'s pseudo-zeroes (t values) = f(n)'s zeroes (t values) + $\frac{\pi}{2}$ whereby F(n) also has parameters σ and t.

Deduction II: The f(n)-in-isolation from sim-DηF is usefully perceived to give rise to one-dimensional lines whereas its corresponding full function with summation $\sum_{n=1}^{\infty}$ [which can be interpreted as Riemann sum] give rise to approximate two-dimensional NAV; viz, sim-DηF is validly and usefully regarded as Riemann sum. Precise two-dimensional NAV are obtained with integration application to sim-DηF [as zeroes] viz, definite integral $\int_{1}^{\infty}$(sim-DηF)dn = antiderivative DSPL [as pseudo-zeroes converted to zeroes]. As opposed to (a) x-axis intercept, (b) y-axis intercept and (c) Origin intercept which are usefully depicted with vertical axis: $Im\{\eta(\sigma + \imath t)\}$ and horizontal axis: $Re\{\eta(\sigma + \imath t)\}$; their corresponding two-dimensional NAVs can be usefully depicted with vertical axis: (a) $Im\{\eta(\sigma + \imath t)\}$, (b) $Re\{\eta(\sigma + \imath t)\}$ and (c) $ReIm\{\eta(\sigma + \imath t)\}$, and horizontal axis: t.

Information-Complexity conservation. Formulae can consist of (1) equations e.g. CP two 'non-varying' discrete-type and two continuous-type equations to independently calculate and incorporate all even and odd numbers, and IP 'varying' discrete-type equations DηF [as zeroes] (*proxy* for RζF [as zeroes]) and sim-DηF [as zeroes] or 'varying' continuous-type equation DSPL [as pseudo-zeroes converted to zeroes] to dependently calculate all NTZ and two types of GP; or (2) algorithms e.g. Sieve of Eratosthenes giving rise to IP two 'varying' discrete-type algorithms to dependently compute all prime and composite numbers. Thus, a given formula is simply a Black Box generating Output (having qualitative-like structural 'Complexity') when supplied with Input (having quantitative-like data 'Information').

A set of correct and complete ["formulae-laden"] mathematical arguments depicted as lemmas, corollaries and propositions must fully comply with Information-Complexity conservation. Intuitively, this is synonymous with Information-based complexity and can be considered as a unique all-purpose [quantitative and qualitative] analytic tool used with Mathematics for Completely Predictable problems and Mathematics for Incompletely Predictable problems. Respectively, these problems can literally be perceived as "simple systems" containing well-defined CP entities such as even and odd numbers; and "complex systems" containing well-defined IP entities such as prime and composite numbers, NTZ and two types of GP.

Complying with Information-Complexity conservation by DηF (*proxy* for RζF) in a qualitative-like manner will always result in maximum three types of axes-intercepts (viz, x-axis, y-axis and Origin intercepts as three types of GP) occurring at $\sigma = \frac{1}{2}$ and minimum two types of axes-intercepts (viz, x-axis and y-axis intercepts as two types of virtual GP) occurring at $\sigma \neq \frac{1}{2}$. Complying with Information-Complexity conservation, preservation or conservation of approximate NAV = 0 using sim-DηF [as (virtual) zeroes] in a quantitative-like manner occur at $\sigma = \frac{1}{2}$ as three types of GP, and occur at $\sigma \neq \frac{1}{2}$ as two types of virtual GP. Corresponding preservation or conservation of precise NAV = 0 using DSPL [as (virtual) pseudo-zeroes to (virtual) zeroes conversion] occur at $\sigma = \frac{1}{2}$ as three types of GP, and occur at $\sigma \neq \frac{1}{2}$ as two types of virtual GP.

The following are **two key concepts from Information-Complexity conservation**:
(I) Relevant end-product Laws [with obtained pseudo-zeroes converted to zeroes], equations or algorithms (i) will generate or incorporate CP entities such as even and odd numbers with simple properties consistently manifested by these entities; and (ii) will generate or incorporate IP entities such as prime and composite numbers, NTZ and two types of GP with complex properties consistently manifested by these entities.
(II) In principle, [A] CP 'non-varying' two discrete-type and two continuous-type equations to independently compute and incorporate all even and odd numbers, [B] IP two 'varying' discrete-type algorithms to dependently compute all prime and composite numbers, and [C] IP 'varying' discrete-type equations DηF [as zeroes] (*proxy* for RζF [as zeroes]) and sim-DηF [as zeroes] to dependently compute all NTZ and two types of GP could, respectively and correspondingly, be derived in a reverse-engineered manner from [A] two 'non-varying' discrete-type equations when language-expressed [in one combined table] using Dimension (2x - N) system, [B] two 'varying' discrete-type algorithms when language-expressed [in one combined table] using Dimension (2x - N) system, and [C] 'varying' continuous-type equation DSPL [as pseudo-zeroes converted to zeroes].

Our mathematical-formated and geometrical-formated treatises containing pure and applied mathematics in relevant "Mathematics for Incompletely Predictable problems" encompass new signatory ideas that will overcome insurmontable difficulties present in many previously attempted mathematical proofs for nominated open problem of Riemann hypothesis and explanations for its closely related two types of Gram points. *These difficulties are advocated to inevitably arise simply because of failure during previous attempts to specifically treat Riemann zeta function (or its proxy Dirichlet eta function) as unique mathematical object for full intrinsic analysis on its derived* ***de novo*** *complex properties*. As an intractable open problem in Number theory, Riemann hypothesis historically belongs to one of USD $1,000,000 Millennium Prize Problems in field of mathematics that were identified by Clay Mathematics Institute at turn of new millennium on May 24, 2000.

1.4 Open Problems in Number Theory with Incompletely Predictable Entities

["$\Delta x \longrightarrow 0$"] DSPL is the continuous format version of discrete format ["$\Delta x \longrightarrow 1$"] D$\eta$F (*proxy* for ["$\Delta x \longrightarrow 1$"] R$\zeta$F) and ["$\Delta x \longrightarrow 1$"] sim-D$\eta$F. Colloquially speaking, they enable either quantitative-like calculations on NAV or qualitative-like computations on axes-intercept points. As previously explained, these actions will result in the desired zeroes and pseudo-zeroes with Zeroes (t values) = Pseudo-zeroes (t values) – $\frac{\pi}{2}$. We take note, firstly, the unique situation of Origin intercept points (NTZ) obtained via approximate NAV = 0 giving zeroes t-values or via precise NAV = 0 giving pseudo-zeroes [which can be converted to zeroes] t-values will only occur when $\sigma = \frac{1}{2}$; and, secondly, the critical line of Riemann

Table 3. Two options to solve open problems in Number theory

Riemann zeta function	**Sieve of Eratosthenes**
↓ [*Path A option*] ↓	↓ [*Path A option*] ↓
Nontrivial zeros and two types of Gram points	Prime and Composite numbers
↑ [*Path B option*] ↑	↑ [*Path B option*] ↑
Dirichlet Sigma-Power Laws	**Dimension (2x - N), N = 2x - ΣPC$_x$-Gap**

zeta function (or its *proxy* Dirichlet eta function) is also uniquely denoted by $\sigma = \frac{1}{2}$ whereby, in Riemann hypothesis, all NTZ are conjectured to be located.

The Number '1' is neither prime nor composite. ["$\Delta x \longrightarrow 1$"] Sieve of Eratosthenes is a simple ancient algorithm for finding all prime numbers up to any given limit by iteratively marking as composite (i.e., not prime) the multiples of each prime, starting with first prime number 2. Multiples of a given prime are generated as a sequence of numbers starting from that prime, with constant difference between them equal to that prime. Dimension (2x - N) system can *dependently* generate all prime and composite numbers [and "extrapolated" Number '1' which is uniquely represented by Dimension (2x - 2)] whereas Sieve of Eratosthenes directly and indirectly give rise to prime and composite numbers (but not Number '1'). In using the unique Dimension (2x - N) system with N = 2x - ΣPC$_x$-Gap and x = all integers commencing from 1; Dimension (2x - N) when fully expanded is numerically just equal to ΣPC$_x$-Gap since Dimension (2x - N) = 2x - 2x + ΣPC$_x$-Gap = ΣPC$_x$-Gap.

Definition for above Dimension (2x - N) system is fully explained using two examples for position x = 31 and 32. For i and x $\in \mathbf{N}$ [as per data from Table 5]; ΣPC$_x$-Gap = ΣPC$_{x-1}$-Gap + Gap value at $\mathbf{P}_{i-1}$ or Gap value at $\mathbf{C}_{i-1}$ whereby (i) $\mathbf{P}_i$ or $\mathbf{C}_i$ at position x is determined by whether relevant x value belongs to a **P** or **C**, and (ii) both ΣPC$_1$-Gap and ΣPC$_2$-Gap = 0. Example, for position x = 31: 31 is **P** (**P11**). Desired Gap value at **P10** = 2. Thus ΣPC$_{31}$-Gap (55) = ΣPC$_{30}$-Gap (53) + Gap value at **P10** (2). Example, for position x = 32: 32 is **C** (**C20**). Desired Gap value at **C19** = 2. Thus ΣPC$_{32}$-Gap (57) = ΣPC$_{31}$-Gap (55) + Gap value at **C20** (2).

For i = natural numbers, ["$\Delta x \longrightarrow 1$"] equations E$_i$ = 2 X i and O$_i$ = (2 X i) - 1 *independently* give rise to all even and odd numbers (which are obtained from natural numbers thus excluding even number '0'). For i = real numbers ≥ 0, ["$\Delta x \longrightarrow 0$"] equations E = 2 X i and O = (2 X i) - 1 *independently* incorporates all even and odd numbers (including even number '0' at i = 0). Then, Origin intercept (0,0) will only occur in equation E = 2 X i when Output E = 0 is uniquely generated by Input i = 0. Dimension (2x - N) system can *independently* generate all even and odd numbers [including the ("zeroth") even number '0' which is uniquely represented by Dimension (2x - 0) obtained by incorporating x = 0]. In using the unique Dimension (2x - N) system with N = 2x - ΣEO$_x$-Gap] and x = all integers commencing from 1 [with even number '0' arbitrarily excluded]; Dimension (2x - N) when fully expanded is numerically just equal to ΣEO$_x$-Gap since Dimension (2x - N) = 2x - 2x + ΣEO$_x$-Gap = ΣEO$_x$-Gap.

Definition for above Dimension (2x - N) system is fully explained using two examples for position x = 31 and 32. For i and x $\in \mathbf{N}$ [as per data from Table 6]; ΣEO$_x$-Gap = ΣEO$_{x-1}$-Gap + Gap value at E$_{i-1}$ or Gap value at O$_{i-1}$ whereby (i) E$_i$ or O$_i$ at position x is determined by whether relevant x value belongs to **E** or **O**, and (ii) both ΣEO$_1$-Gap and ΣEO$_2$-Gap = 0. Example, for position x = 31: 31 is **O** (O16). Our desired Gap value at O15 = 2. Thus ΣEO$_{31}$-Gap (58) = ΣEO$_{30}$-Gap (56) + Gap value at O15 (2). Example, for position x = 32: 32 is **E** (E16). Our desired Gap value at E15 = 2. Thus ΣEO$_{32}$-Gap (60) = ΣEO$_{31}$-Gap (58) + Gap value at E15 (2).

To solve Riemann hypothesis, Polignac's and Twin prime conjectures (and explain two types of Gram points) while fully complying with Information-Complexity conservation; we could theoretically follow Path A or Path B as schematically depicted in Table 3. Both options require Mathematics for IP problems. Our utilized Path B option involves deriving DSPL and using Dimension (2x - N) system.

Riemann hypothesis (1859) proposed all NTZ in Riemann zeta function to be located on its critical line. Defined as IP problem is essential to correctly prove this hypothesis. All of infinite magnitude, NTZ when geometrically depicted as corresponding Origin intercept points together with two types of Gram points when geometrically depicted as corresponding x- and y-axes intercept points explicitly confirm they intrinsically form relevant component of point-intersections in these functions at $\sigma = \frac{1}{2}$. The equivalence of axes-intercept points are precise NAV = 0 [as pseudo-zeroes which can be converted to zeroes] calculated using DSPL and approximate NAV = 0 [as zeroes] calculated using Riemann sum when sim-DηF is interpreted as such. Defined as IP problems is essential for these explanations to be correct.

Remark 3 Mathematics for Incompletely Predictable problems equates to *sine qua non* (correctly) classifying problems involving Incompletely Predictable entities as Incompletely Predictable problems. This is achieved by incorporating certain identifiable (non-negotiable) mathematical steps with this procedure ultimately enabling us to rigorously prove or precisely explain our nominated open problems in Number theory.

Refined information on Incompletely Predictable entities of Gram and virtual Gram points: These entities all of infinite magnitude are *dependently* calculated using *complex equation* Riemann zeta function, $\zeta(s)$, or its *proxy* Dirichlet eta function, $\eta(s)$, in critical strip (denoted by $0 < \sigma < 1$) thus forming the relevant component of point-intersections. In Figure 8, Gram[y=0], Gram[x=0] and Gram[x=0,y=0] points are, respectively, geometrical x-axis, y-axis and Origin intercepts at critical line (denoted by $\sigma = \frac{1}{2}$). Gram[y=0] and Gram[x=0,y=0] points are, respectively, synonymous with traditional *'Gram points'* and *nontrivial zeros*. In Figures

Refined information on Incompletely Predictable entities of prime and composite numbers: These entities all of infinite magnitude are *dependently* computed (respectively) directly and indirectly using *complex algorithm* Sieve of Eratosthenes. Denote $\mathbb{C}$ to be uncountable complex numbers, **R** to be uncountable real numbers, **Q** to be countable rational numbers or roots [of non-zero polynomials], **R–Q** to be uncountable irrational numbers, **A** to be countable algebraic numbers, **R–A** to be uncountable transcendental numbers, **Z** to be countable integers, **W** to be countable whole numbers, **N** to be countable natural numbers, **E** to be countable even numbers, **O** to be countable odd numbers, **P** to be countable prime numbers, and **C** to be countable composite numbers. **A** are $\mathbb{C}$ (including **R**) that are countable rational or irrational roots. (i) Set **N** = Set **E** + Set **O**, (ii) Set **N** = Set **P** + Set **C** + Number '1', (iii) Set **A** = Set **Q** + Set **irrational roots**, and (iv) Set **N** ⊂ Set **W** ⊂ Set **Z** ⊂ Set **Q** ⊂ Set **R** ⊂ Set $\mathbb{C}$. Then Set **R–Q** = Set **irrational roots** + Set **R–A**. Note: With **E** and **O** obtained from **N**, we did not include '0' as **E** in above discussion.

Cardinality of a given set: With increasing size, arbitrary Set **X** can be countable finite set (CFS), countable infinite set (CIS) or uncountable infinite set (UIS). Cardinality of Set **X**, |**X**|, measures "number of elements" in Set **X**. E.g. Set **negative Gram[y=0] point** has CFS of negative Gram[y=0] point with |**negative Gram[y=0] point**| = 1, Set **even P** has CFS of even **P** with |**even P**| = 1, Set **N** has CIS of **N** with |**N**| = $\aleph_0$, and Set **R** has UIS of **R** with |**R**| = $\mathfrak{c}$ (cardinality of the continuum).

We compare and contrast CP entities (obeying Simple Elementary Fundamental Laws) against IP entities (obeying Complex Elementary Fundamental Laws) using examples:
(I) **E** are CP entities constituted by CIS of **Q** 2, 4, 6, 8, 10, 12.... Note: These are (positive) **E** derived from **N** which then do not include '0' as even number.
(II) **O** are CP entities constituted by CIS of **Q** 1, 3, 5, 7, 9, 11....
(III) **P** are IP entities constituted by CIS of **Q** 2, 3, 5, 7, 11, 13....
(IV) **C** are IP entities constituted by CIS of **Q** 4, 6, 8, 9, 10, 12....
(V) With values traditionally given by parameter t, nontrivial zeros in Riemann zeta function are IP entities constituted by CIS of **R–A** [rounded off to six decimal places]: 14.134725, 21.022040, 25.010858, 30.424876, 32.935062, 37.586178,....
(VI) Traditional 'Gram points' (or Gram[y=0] points) are x-axis intercepts with choice of index 'n' for 'Gram points' historically chosen such that first 'Gram point' [by convention at n = 0] corresponds to the t value which is larger than (first) nontrivial zero located at t = 14.134725. 'Gram points' are IP entities constituted by CIS of **R–A** [rounded off to six decimal places] with the first six given at n = -3, t = 0; at n = -2, t = 3.436218; at n = -1, t = 9.666908; at n = 0, t = 17.845599; at n = 1, t = 23.170282; at n = 2, t = 27.670182. We will not calculate any values for Gram[x=0] points.

Denoted by parameter t; nontrivial zeros, 'Gram points' and Gram[x=0] points all belong to well-defined CIS of **R–A** which will twice obey the relevant location definition [in CIS of **R–A** themselves and in CIS of numerical digits after decimal point of each **R–A**]. First and only negative 'Gram point' (at n = -3) is obtained by substituting CP t = 0 resulting in $\zeta(\frac{1}{2} + \imath t) = \zeta(\frac{1}{2})$ = -1.4603545, a **R–A** number [rounded off to seven decimal places] calculated as a limit similar to limit for Euler-Mascheroni constant or Euler gamma with its precise (1^{st}) position only determined by computing positions of all preceding (nil) 'Gram point' in this case. '0' and '1' are special numbers being neither **P** nor **C** as they represent nothingness (zero) and wholeness (one). In this setting, the idea of having factors for '0' and '1' is meaningless. All entities derived from well-defined simple/complex algorithms or equations are "dual numbers" as they can be simultaneously depicted as CP and IP numbers. For instance, **Q** '2' as **P** (and **E**), '97' as **P** (and **O**), '98' as **C** (and **E**), '99' as **C** (and **O**); CP '0' values in x=0, y=0 and simultaneous x=0, y=0 associated with various IP t values in ζ(s).

Algebraic Number Theory Versus Analytic Number Theory

Set **P** ⊂ Set **Z** ⊂ Set **Q**. Gaussian rationals, and Gaussian integers are complex numbers whose real and imaginary parts are (respectively) both rational numbers, and integer numbers. Gaussian primes are Gaussian integers z = a + bi satisfying one of the following properties.

1. If both a and b are nonzero, then a+bi is a Gaussian prime iff $a^2 + b^2$ is an ordinary prime [whereby iff is the written abbreviation for 'if and only if'].
2. If a = 0, then bi is a Gaussian prime iff |b| is an ordinary prime and |b| = 3 (mod 4).
3. If b = 0, then a is a Gaussian prime iff |a| is an ordinary prime and |a| = 3 (mod 4).

Prime numbers which are also Gaussian primes are 3, 7, 11, 19, 23, 31, 43,.... In Eq. (1), equivalent Euler product formula with product over prime numbers [instead of summation over natural numbers] faithfully represent Riemann zeta function, $\zeta(s)$. Eq. (2) is Riemann's functional equation involving transcendental number π (= 3.14159...). With denominators on left involving odd numbers and named after Gottfried Leibniz, Leibniz formula for π states that $\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots = \frac{\pi}{4}$. Expression $\zeta(2) = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6} \approx 1.6449340668482264364$7 involves π $\Longrightarrow$ division with two unrelated transcendental (irrational) numbers $\frac{\zeta(2)}{\pi^2}$ interestingly results in rational number $\frac{1}{6}$.

Algebraic number theory is loosely defined to deal with new number systems involving Completely Predictable or Incompletely Predictable entities such as even & odd numbers, prime & composite numbers, p-adic numbers, Gaussian primes, Gaussian rationals & integers, and complex numbers. A p-adic number is an extension of the field of rationals such that congruences modulo powers of a fixed prime number p are related to proximity in so-called "p-adic metric". The extension is achieved by an alternative interpretation of concept of "closeness" or absolute value viz. p-adic numbers are considered to be close when their difference is divisible by a high power of p: the higher the power, the closer they are. This property enables p-adic numbers to encode congruence information in a way that turns out to have powerful applications in number theory including, for example, attacking certain Diophantine equations and in famous proof of Fermat's Last Theorem by English mathematician Sir Andrew John Wiles in 1995.

Analytic number theory is loosely defined to deal with functions of a complex variable such as Riemann zeta function [containing nontrivial zeros and two types of Gram points] and other L-functions. Study of prime numbers, complex numbers and π being braided together in a pleasing trio is usefully visualized to be located at intersection of this two main branches of number theory. We separate our relatively elementary proof for Riemann hypothesis and relatively elementary explanations for two types of Gram points to belong to Analytic number theory, and our relatively elementary proofs for Polignac's and Twin prime conjectures [expectedly associated with paucity of functions] to belong to Algebraic number theory. Complex algorithms e.g. for generating prime and composite numbers are only defined at two end-points a,b (but not for interval [a,b] as they are not functions).

Public-key cryptography that is widely required for financial security in E-Commerce traditionally depend on solving difficult problem of factoring prime numbers for astronomically large numbers. The intrinsic "Incompletely Predictable" property present in prime numbers, composite numbers, nontrivial zeros and two types of Gram points can never be altered to "Completely Predictable" property. For this stated reason, it is a mathematical impossibility that providing rigorous proofs such as for Riemann hypothesis will, in principle, ever result in crypto-apocalypse. However, utilizing parallel computing (more than seriel computing), fast supercomputers and far-more-powerful quantum computers would theoretically allow solving difficult factorization problem in quick time, resulting in less secure encryption and decryption. Then using, for instance, quantum cryptography that relies on principles of quantum mechanics to encrypt and transmit data in a way that cannot be hacked will combat this issue.

Proposed by German mathematician Bernhard Riemann (September 17, 1826 – July 20, 1866) in 1859, Riemann hypothesis is mathematical statement on Riemann zeta function, $\zeta(s)$ [or its *proxy* Dirichlet eta function, $\eta(s)$] that critical line denoted by $\sigma = \frac{1}{2}$ contains complete Set **nontrivial zeros** with |**nontrivial zeros**| = $\aleph_0$. Alternatively, this hypothesis is geometrical statement on $\zeta(s)$ [or its *proxy* $\eta(s)$] that generated curves at $\sigma = \frac{1}{2}$ contain complete Set **Origin intercepts** with |**Origin intercepts**| = $\aleph_0$.

$$\zeta(s) = \frac{e^{(\ln(2\pi)-1-\frac{\gamma}{2})s)}}{2(s-1)\Gamma(1+\frac{s}{2})}\Pi_\rho\left(1-\frac{s}{\rho}\right)e^{\frac{s}{\rho}} = \pi^{\frac{s}{2}}\frac{\Pi_\rho\left(1-\frac{s}{\rho}\right)}{2(s-1)\Gamma\left(1+\frac{s}{2}\right)}$$

Depicted in full and abbreviated version, Hadamard product above is infinite product expansion of $\zeta(s)$ based on Weierstrass's factorization theorem displaying a simple pole at s = 1. It contains both trivial & nontrivial zeros indicating their common origin from $\zeta(s)$. Set **trivial zeros** occurs at σ = -2, -4, -6, -8, -10,…, ∞ with |**trivial zeros**| = $\aleph_0$ due to Γ function term in denominator. Nontrivial zeros occur at $s = \rho$ with γ denoting Euler-Mascheroni constant.

Remark 4 Confirming first 10,000,000,000,000 nontrivial zeros location on critical line implies but does not prove Riemann hypothesis to be true.

Locations of first 10,000,000,000,000 nontrivial zeros on critical line have previously been computed to be correct. Hardy (Hardy, 1914), and with Littlewood (Hardy & Littlewood, 1921), showed infinite nontrivial zeros on critical line (denoted by $\sigma = \frac{1}{2}$) by considering moments of certain functions related to $\zeta(s)$. This discovery cannot constitute rigorous proof for Riemann hypothesis because they have not exclude theoretical existence of nontrivial zeros located away from this line (when $\sigma \neq \frac{1}{2}$). Furthermore, it is literally a mathematical impossibility ("mathematical impasse") to be able to computationally check [in a complete and successful manner] the locations of all infinitely many nontrivial zeros to correctly lie on critical line.

2. Riemann Zeta Function and its *proxy* Dirichlet Eta Function

An L-function consists of a Dirichlet series with a functional equation and an Euler product. Examples of L-functions come from modular forms, elliptic curves, number fields, and Dirichlet characters, as well as more generally from automorphic forms, algebraic varieties, and Artin representations. They form an integrated component of 'L-functions and Modular Forms Database' (LMFDB) with far-reaching implications. In perspective, $\zeta(s)$, being the simplest example of an L-function, is a function of complex variable s ($= \sigma \pm \imath t$) that analytically continues sum of infinite series $\zeta(s) = \sum_{n=1}^{\infty}\frac{1}{n^s} = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \cdots$. The common convention is to write s as $\sigma + \imath t$ with $\imath = \sqrt{-1}$, and with σ and t real. Valid for $\sigma > 0$, we write $\zeta(s)$ as $Re\{\zeta(s)\} + \imath Im\{\zeta(s)\}$ and note that $\zeta(\sigma + \imath t)$ when $0 < t < +\infty$ is the complex conjugate of $\zeta(\sigma - \imath t)$ when $-\infty < t < 0$.

Also known as alternating zeta function, $\eta(s)$ must act as *proxy* for $\zeta(s)$ in critical strip (viz. $0 < \sigma < 1$) containing critical line (viz. $\sigma = \frac{1}{2}$) because $\zeta(s)$ only converges when $\sigma > 1$. This implies $\zeta(s)$ is undefined to left of this region in critical strip which then requires $\eta(s)$ representation instead. They are related to each other as $\zeta(s) = \gamma \cdot \eta(s)$ with proportionality factor $\gamma = \frac{1}{(1-2^{1-s})}$ and $\eta(s) = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^s} = \frac{1}{1^s} - \frac{1}{2^s} + \frac{1}{3^s} - \cdots$.

$$\begin{aligned}\zeta(s) &= \sum_{n=1}^{\infty}\frac{1}{n^s} \\ &= \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \cdots \\ &= \Pi_{p\ prime}\frac{1}{(1-p^{-s})} \\ &= \frac{1}{(1-2^{-s})}.\frac{1}{(1-3^{-s})}.\frac{1}{(1-5^{-s})}.\frac{1}{(1-7^{-s})}.\frac{1}{(1-11^{-s})}\cdots\frac{1}{(1-p^{-s})}\cdots\end{aligned} \tag{1}$$

Eq. (1) is defined for only $1 < \sigma < \infty$ region where $\zeta(s)$ is absolutely convergent with no zeros located here. In Eq. (1), equivalent Euler product formula with product over prime numbers [instead of summation over natural numbers] also represents $\zeta(s) \implies$ all prime and, by default, composite numbers are (intrinsically) "encoded" in $\zeta(s)$. This observation represents a strong reason to conveniently combine proofs for Riemann hypothesis, Polignac's & Twin prime conjectures.

$$\zeta(s) = 2^s\pi^{s-1}\sin\left(\frac{\pi s}{2}\right)\cdot\Gamma(1-s)\cdot\zeta(1-s) \tag{2}$$

With $\sigma = \frac{1}{2}$ as symmetry line of reflection, Eq. (2) is Riemann's functional equation valid for $-\infty < \sigma < \infty$. It can be used to find all trivial zeros on horizontal line at $\imath t = 0$ occurring when σ = -2, -4, -6, -8, -10,..., ∞ whereby $\zeta(s) = 0$ because factor $\sin(\frac{\pi s}{2})$ vanishes. Γ is gamma function, an extension of factorial function [a product function denoted by ! notation whereby n! $= n(n-1)(n-2)\ldots(n-(n-1))$] with its argument shifted down by 1, to real and complex numbers. That is, if n is a positive integer, $\Gamma(n) = (n-1)!$

$$\begin{aligned}\zeta(s) &= \frac{1}{(1-2^{1-s})}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^s} \\ &= \frac{1}{(1-2^{1-s})}\left(\frac{1}{1^s} - \frac{1}{2^s} + \frac{1}{3^s} - \cdots\right)\end{aligned} \tag{3}$$

Eq. (3) is defined for all $\sigma > 0$ values except for simple pole at $\sigma = 1$. As alluded to above, ζ(s) without $\frac{1}{(1-2^{1-s})}$ viz.

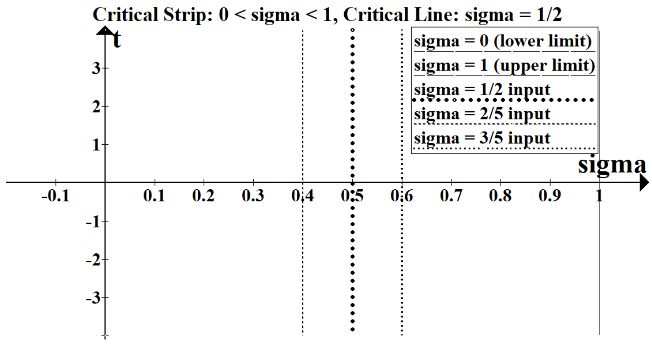

Figure 7. INPUT for $\sigma = \frac{1}{2}$, $\frac{2}{5}$, and $\frac{3}{5}$. $\zeta(s)$ has countable infinite set (CIS) of Completely Predictable trivial zeros at σ = all negative even numbers and [proposed] CIS of Incompletely Predictable nontrivial zeros at $\sigma = \frac{1}{2}$ for various t values.

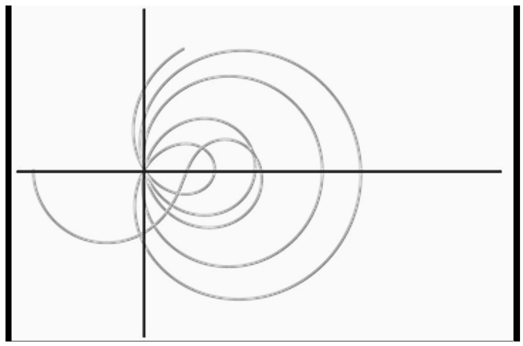

Figure 8. OUTPUT for $\sigma = \frac{1}{2}$ as Gram points. Figure 8 represents schematically depicted polar graph of $\zeta(\frac{1}{2} + \imath t)$ plotted along critical line for real values of t running from 0 to 34, horizontal axis: $Re\{\zeta(\frac{1}{2} + \imath t)\}$, and vertical axis: $Im\{\zeta(\frac{1}{2} + \imath t)\}$. There are presence of Origin intercept points which are totally absent in Figures 9 and 10 [with identical axes definitions but, respectively, adjusted to $\sigma = \frac{2}{5}$ and $\sigma = \frac{3}{5}$]

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^s}$ is η(s). It is a holomorphic function of s defined by analytic continuation and is mathematically defined at σ = 1 whereby analogous trivial zeros with presence for η(s) [but not for ζ(s)] on vertical straight line $\sigma = 1$ are found at $s = 1 \pm \imath \frac{2\pi k}{\ln(2)}$ where k = 1, 2, 3, 4,. . . , ∞.

Figure 7 depict complex variable s (= $\sigma \pm \imath t$) as INPUT with x-axis denoting real part Re{s} associated with σ, and y-axis denoting imaginary part Im{s} associated with t. Figures 8, 9 and 10 respectively depict $\zeta(s)$ as OUTPUT for real values of t running from 0 to 34 at $\sigma = \frac{1}{2}$ (critical line), $\sigma = \frac{2}{5}$ (non-critical line), and $\sigma = \frac{3}{5}$ (non-critical line) with x-axis denoting real part Re{$\zeta(s)$} and y-axis denoting imaginary part Im{$\zeta(s)$}. There are infinite types-of-spirals (loops) possibilities associated with each σ value arising from all infinite σ values in critical strip. Mathematically proving all nontrivial zeros location on critical line as denoted by solitary $\sigma = \frac{1}{2}$ value equates to geometrically proving all Origin intercepts occurrence at solitary $\sigma = \frac{1}{2}$ value. Both result in rigorous proof for Riemann hypothesis.

3. Equations for Riemann Hypothesis and Two Types of Gram Points

Calculations using Dirichlet eta function, $\eta(s)$ [*proxy* for Riemann zeta function, $\zeta(s)$] in discrete (summation) format for all σ values result in infinitely many equations **[and all Gram points entities at $\sigma = \frac{1}{2}$ as zeroes t-values axes-intercept points solutions]** for $0 < \sigma < 1$ critical strip region of interest with n = 1, 2, 3, 4, 5,. . . , ∞ as discrete integer number values. These equations geometrically represent entire plane of critical strip, thus (at least) allowing our proposed proof to be of a complete nature. We note sim-$\eta(s)$ in discrete (summation) format is obtained by applying Euler formula to $\eta(s)$ and when interpreted as Riemann sum, it gives rise to **approximate Net Area Value = 0 solutions to obtain all Gram points entities at $\sigma = \frac{1}{2}$ [as zeroes t-values]** with n = 1, 2, 3, 4, 5,. . . , ∞ as discrete integer number values. Dirichlet Sigma-Power Law is the antiderivative derived from solving the improper integrals of sim-$\eta(s)$ (lower limit a = 1 and upper limit b = ∞) whereby n = 1 to ∞ are continuous real number values. This Law will have its [multiple] +ve (above x-axis) and -ve (below x-axis) numerical **precise Net Area Value = 0 solutions to obtain all Gram points entities at $\sigma = \frac{1}{2}$ [as pseudo-zeroes t-values which are converted to zeroes t-values]** successfully computed.

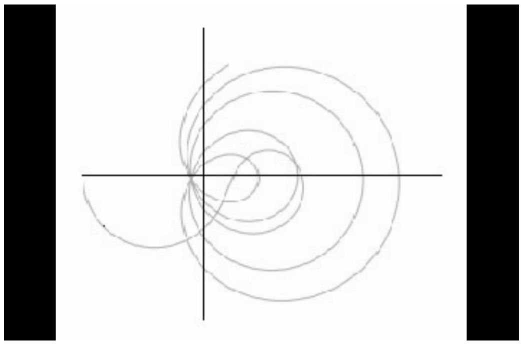

Figure 9. OUTPUT for $\sigma = \frac{2}{5}$ as virtual Gram points. Incompletely Predictable loops are shifted to the left of Origin with horizontal axis: $Re\{\zeta(\frac{2}{5} + \imath t)\}$, and vertical axis: $Im\{\zeta(\frac{2}{5} + \imath t)\}$. There are total absence of Origin intercept points.

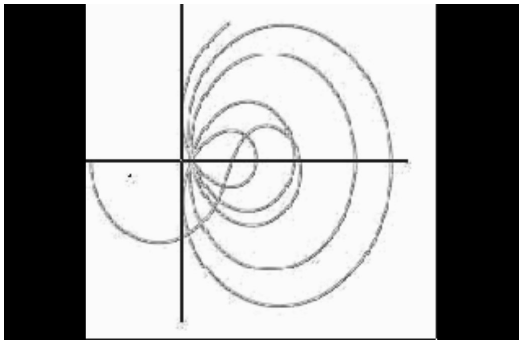

Figure 10. OUTPUT for $\sigma = \frac{3}{5}$ as virtual Gram points with horizontal axis: $Re\{\zeta(\frac{3}{5} + \imath t)\}$, and vertical axis: $Im\{\zeta(\frac{3}{5} + \imath t)\}$. Incompletely Predictable loops are shifted to the right of Origin. There are total absence of Origin intercept points.

Prerequisite lemmas, corollaries and propositions on $\eta(s)$ with associated extensive Calculus to derive necessary equations corresponding to relevant Dirichlet Sigma-Power Laws [as pseudo-zeroes to zeroes conversion] for nontrivial zeros (Riemann hypothesis) and two types of Gram points are outlined later on. These derived equations – same ones as outlined later on are given in full below by [additionally] incorporating constant of integration 'C':

I. Gram[x=0,y=0] points (or nontrivial zeros) for Riemann hypothesis obtained via pseudo-zeroes to zeroes conversion

$$\begin{aligned}\frac{1}{2\left(t^2+(\sigma-1)^2\right)} &\cdot \Big[(2n)^{1-\sigma}((t+\sigma-1)\sin(t\ln(2n))+(t-\sigma+1)\\ &\cdot\cos(t\ln(2n)))-(2n-1)^{1-\sigma}((t+\sigma-1)\\ &\cdot\sin(t\ln(2n-1))+(t-\sigma+1)\cos(t\ln(2n-1)))+C\Big]_1^\infty=0\end{aligned} \tag{4}$$

II. Gram[y=0] points (or 'usual' Gram points) obtained via pseudo-zeroes to zeroes conversion

$$-\frac{1}{2\left(t^2+(\sigma-1)^2\right)}\cdot\Big[(2n)^{1-\sigma}((\sigma-1)\sin(t\ln(2n))+t\cos(t\ln(2n)))-$$

$$(2n-1)^{1-\sigma}((\sigma-1)\sin(t\ln(2n-1))+t\cos(t\ln(2n-1)))+C\Big]_1^\infty=0 \tag{5}$$

III. Gram[x=0] points obtained via pseudo-zeroes to zeroes conversion

$$\frac{1}{2\left(t^2+(\sigma-1)^2\right)}\cdot\Big[(2n)^{1-\sigma}(t\sin(t\ln(2n))-(\sigma-1)\cos(t\ln(2n)))-$$

$$(2n-1)^{1-\sigma}(t\sin(t\ln(2n-1))-(\sigma-1)\cos(t\ln(2n-1)))+C\Big]_1^\infty=0 \tag{6}$$

Critical line is denoted by $\sigma = \frac{1}{2}$ and is conjectured in Riemannn hypothesis to be the unique location for all nontrivial zeros of Riemann zeta function. For nontrivial zeros, (1) its mathematical definition: ζ(s) = 0 or η(s) = 0 or sim-η(s) = 0, and (2) its geometrical definition: Origin intercept points (Gram[x=0,y=0] points) [as zeroes] or precise NAV = 0 [as pseudo-zeroes converted to zeroes] in Dirichlet Sigma-Power Law or approximate NAV = 0 [as zeroes] in sim-η(s) when interpreted as Riemann sum will only be uniquely valid when $\sigma = \frac{1}{2}$. Thus, presence of Origin intercept points only occur when $\sigma = \frac{1}{2}$; and absence of Origin intercept points only occur when $\sigma \neq \frac{1}{2}$. Also supported by arguments involving modulus of $\eta(s)$ and fully complying with Information-Complexity conservation, all nontrivial zeros when $\sigma = \frac{1}{2}$ are deduced to exist at (i) ζ(s) or η(s) or sim-η(s) = 0 as Origin intercept points [viz, zeroes], (ii) Dirichlet Sigma-Power Law = 0 as precise NAV = 0 [viz, pseudo-zeroes converted to zeroes], or (iii) Riemann sum interpretation of sim-η(s) = 0 as approximate NAV = 0 [viz, zeroes].

From subsection 3.1 below, exact Dimensional analysis (DA) homogeneity at $\sigma = \frac{1}{2}$ denotes $\sum$(all fractional exponents) as $2(1-\sigma)$ uniquely equates to ["exact"] whole number '1'; and inexact DA homogeneity at $\sigma \neq \frac{1}{2}$ denotes $\sum$(all fractional exponents) as $2(1-\sigma)$ uniquely equates to ["inexact"] fractional number '≠1'. As will also be subsequently accomplished below, we leave it as a simple exercise here for readers to confirm all above equations for substituted $\sigma = \frac{1}{2}$ and $\sigma \neq \frac{1}{2}$ [e.g. for $\sigma = \frac{2}{5}$ and $\frac{3}{5}$] values will, respectively, fully comply with exact and inexact DA homogeneity. We logically deduce this exercise will definitively equate to substantiating rigorous proof for Riemann hypothesis and providing precise explanations for two types of Gram points. In effect, original Dirichlet eta function [as zeroes], *proxy* for Riemann zeta function [as zeroes], is dependently treated as an unique mathematical object with direct application of Riemann integral to simplified Dirichlet eta function [as zeroes] to derive relevant Dirichlet Sigma-Power Laws [as pseudo-zeroes converted to zeroes]. Key complex properties or behaviors as exact and inexact Dimensional analysis homogeneity [that can uniquely represent mutually exclusive sets of Gram points and virtual Gram points] are elucidated from subsequent analysis of these Laws. We emphasize here that, strictly speaking, Dirichlet Sigma-Power Laws can ultimately only be directly derived from Dirichlet eta function and not Riemann zeta function with the later function not converging, and is thus undefined, in the critical strip ($0 < \sigma < 1$) of interest [whereby the critical line ($\sigma = \frac{1}{2}$) is located].

3.1 Exact and Inexact Dimensional Analysis Homogeneity for Equations

Respectively for 'base quantities' such as *length*, *mass* and *time*; their fundamental SI 'units of measurement' meter (m) is defined as distance travelled by light in vacuum for time interval 1/299 792 458 s with speed of light c = 299,792,458 ms^{-1}, kilogram (kg) is defined by taking fixed numerical value Planck constant h to be 6.626 070 15 X 10^{-34} Joules·second (Js) [whereby Js is equal to kgm^2s^{-1}] and second (s) is defined in terms of $\Delta vCs = \Delta(^{133}Cs)_{hfs}$ = 9,192,631,770 s^{-1}. Derived SI units such as J and ms^{-1} respectively represent 'base quantities' *energy* and *velocity*. The word 'dimension' is commonly used to indicate all those mentioned 'units of measurement' in well-defined equations.

Dimensional analysis (DA) is an analytic tool with DA homogeneity and non-homogeneity (respectively) denoting valid and invalid equation occurring when 'units of measurements' for 'base quantities' are "balanced" and "unbalanced" across both sides of the equation. E.g. equation 2 m + 3 m = 5 m is valid and equation 2 m + 3 kg = 5 'm·kg' is invalid (respectively) manifesting DA homogeneity and non-homogeneity.

Remark 5 We can validly apply exact and inexact Dimensional analysis homogeneity to certain well-defined equations.

Let (2n) and (2n-1) be 'base quantities' in our derived versions of [continuous-format] Dirichlet Sigma-Power Laws formatted in simplest forms as equations. E.g. DA on exponent $\frac{1}{2}$ in $(2n)^{\frac{1}{2}}$ when depicted in simplest form is desirable for our purpose but DA on exponent $\frac{1}{4}$ in equivalent $(2^2n^2)^{\frac{1}{4}}$ *not* depicted in simplest form is undesirable for our purpose. Fractional exponents as 'units of measurement' given by $(1-\sigma)$ for equations when $\sigma = \frac{1}{2}$ coincide with exact DA homogeneity[1]; and $(1-\sigma)$ for equations when $\sigma \neq \frac{1}{2}$ coincide with inexact DA homogeneity[2]. Respectively, exact DA homogeneity at $\sigma = \frac{1}{2}$ denotes $\sum$(all fractional exponents) as $2(1-\sigma)$ equates to ["exact"] whole number '1'; and inexact DA homogeneity at $\sigma \neq \frac{1}{2}$ denotes $\sum$(all fractional exponents) as $2(1-\sigma)$ equates to ["inexact"] fractional number '≠1' [Range: $0 < 2(1-\sigma) < 1$ and $1 < 2(1-\sigma) < 2$]. Note: *For calculations involving $2(1-\sigma)$ or $2(-\sigma)$ below, it is inconsequential whether σ values in these fractional exponents are depicted in simplest form or not in simplest form.*

Footnote 1, 2: (i) Exact and (ii) inexact DA homogeneity is applicable to Dirichlet Sigma-Power Laws as equations to calculate precise Net Area Values = 0 for (i) $\sigma = \frac{1}{2}$ (critical line) Gram points (given as pseudo-zeroes t-values which can be converted to zeroes t-values) and for (ii) $\sigma \neq \frac{1}{2}$ (non-critical lines) virtual Gram points (given as virtual pseudo-zeroes t-values which can be converted to virtual zeroes t-values). *Law of Continuity* is a heuristic principle *whatever succeed for the finite, also succeed for the infinite*. These Laws which inherently manifest themselves on finite and infinite time scale should "succeed for the finite, also succeed for the infinite".

Additional comments and deductions: Performing exact and inexact Dimensional analysis homogeneity on versions of [discrete-format] simplified Dirichlet eta functions is equally valid. Again, (2n) and (2n-1) are 'base quantities'. Fractional exponents as 'units of measurement' are now given by $(-\sigma)$. Respectively, exact DA homogeneity at $\sigma = \frac{1}{2}$ denotes $\sum$(all fractional exponents) as $2(-\sigma)$ equates to ["exact"] (negative) whole number '-1'; and inexact DA homogeneity at $\sigma \neq \frac{1}{2}$ denotes $\sum$(all fractional exponents) as $2(-\sigma)$ equates to ["inexact"] (negative) fractional number '≠-1' [Range: $-2 < 2(-\sigma) < -1$ and $-1 < 2(-\sigma) < 0$]. Geometrically, computation with at $\sigma = \frac{1}{2}$ (critical line) using simplified Dirichlet eta function [when interpreted as Riemann sum] will give rise to approximate Net Area Value = 0 condition. This condition enable obtaining results of relevant zeroes t-values and virtual zeroes t-values which, respectively, represent all Gram points and virtual Gram points.

Dirichlet eta function, $\eta(s)$, at s = $\sigma + \imath t$ with $\imath = \sqrt{-1}$, σ and t real is valid for $\sigma > 0$. Here, $\eta(s) = Re\{\eta(s)\}+\imath Im\{\eta(s)\}$. Then, $\eta(\sigma + \imath t)$ when $0 < t < +\infty$ is the complex conjugate of $\eta(\sigma - \imath t)$ when $-\infty < t < 0$ [which is also valid for $\sigma > 0$]. Given as [identical] ±t values; CIS nontrivial zeros or Gram[x=0,y=0] points (Origin intercepts) occurring when $\eta(s)$ [as zeroes] = $\zeta(s)$ [as zeroes] = simplified Dirichlet eta function = 0 [as zeroes] with calculated CIS of approximate Net Area Value = 0 is equivalent to Dirichlet Sigma-Power Law = 0 [as pseudo-zeroes which can be converted to zeroes] with calculated CIS of precise Net Area Value = 0. This situation which uniquely occur only when $\sigma = \frac{1}{2}$ can essentially represent Riemann hypothesis.

3.2 Summary of Rigorous Proof for Riemann Hypothesis

Outline of proof for Riemann hypothesis. To simultaneously satisfy two mutually inclusive conditions: I. *With rigid manifestation of exact DA homogeneity*, Set **nontrivial zeros** with |**nontrivial zeros**| = $\aleph_0$ is located on critical line (viz. $\sigma = \frac{1}{2}$) when $2(1 - \sigma)$ as $\sum$(all fractional exponents) = whole number '1' in Dirichlet Sigma-Power Law[3] [as pseudo-zeroes which are converted to zeroes]. II. *With rigid manifestation of inexact DA homogeneity*, Set **nontrivial zeros** with |**nontrivial zeros**| = $\aleph_0$ is not located on non-critical lines (viz. $\sigma \neq \frac{1}{2}$) when $2(1 - \sigma)$ as $\sum$(all fractional exponents) = fractional number '≠1' in Dirichlet Sigma-Power Law[3] [as virtual pseudo-zeroes which are converted to virtual zeroes].

Footnote 3: Ultimately derived from $\eta(s)$ [*proxy* for $\zeta(s)$], this Law [as 'Complex Elementary Fundamental Laws'-based solution results in (virtual) pseudo-zeroes which are converted to (virtual) zeroes] symbolizes end-product proof on Riemann hypothesis.

Riemann hypothesis mathematical foot-prints. Six identifiable steps to prove Riemann hypothesis: *Step 1* Use $\eta(s)$, *proxy* for $\zeta(s)$, in critical strip. *Step 2* Apply Euler formula to $\eta(s)$. *Step 3* Obtain simplified Dirichlet eta function which intrinsically incorporates *actual location [but not actual positions]* of all nontrivial zeros[4]. *Step 4* Apply Riemann integral to simplified Dirichlet eta function in discrete (summation) format results in continuous (integral) format. *Step 5* Obtain its antiderivative Dirichlet Sigma-Power Law [as pseudo-zeroes which are converted to zeroes] which also intrinsically incorporates *actual location [but not actual positions]* of all nontrivial zeros. *Step 6* Confirm exact DA homogeneity or inexact DA homogeneity for $\sum$(all fractional exponents) in this Law to, respectively, validate presence of nontrivial zeros or absence of nontrivial zeros.

Footnote 4: Respectively, Gram[y=0] points, Gram[x=0] points and nontrivial zeros are Incompletely Predictable entities with actual positions determined by setting $\sum Im\{\eta(s)\} = 0$, $\sum Re\{\eta(s)\} = 0$ and $\sum ReIm\{\eta(s)\} = Re\{\eta(s)\} + Im\{\eta(s)\} = 0$ to *dependently* calculate relevant positions of all preceding entities in neighborhood. Respectively, actual location of Gram[y=0] points, Gram[x=0] points and nontrivial zeros; and virtual Gram[y=0] points, virtual Gram[x=0] points and "absent" nontrivial zeros occur precisely at $\sigma = \frac{1}{2}$; and $\sigma \neq \frac{1}{2}$. Euler formula is commonly stated as $e^{\imath x} = \cos x + \imath \cdot \sin x$. *Step 2* is linked to *Step 3* since simplified Dirichlet eta function is obtained by applying Euler formula to $\eta(s)$ whereby $\zeta(s) = \gamma \cdot \eta(s) = \gamma \cdot [Re\{\eta(s)\} + \imath \cdot \mathrm{Im}\{\eta(s)\}]$. Proportionality factor $\gamma = \frac{1}{(1-2^{1-s})}$, $Re\{\eta(s)\} = \sum_{n=1}^{\infty}((2n-1)^{-\sigma}\cos(t\ln(2n-1)) - (2n)^{-\sigma}\cos(t\ln(2n)))$ and $Im\{\eta(s)\} = \sum_{n=1}^{\infty}((2n)^{-\sigma}\sin(t\ln(2n)) - (2n-1)^{-\sigma}\sin(t\ln(2n-1)))$. Complex number s in critical strip is designated by s = $\sigma + \imath t$ for $0 < t < +\infty$ and s = $\sigma - \imath t$ for $-\infty < t < 0$. *Step 4* is linked to *Step 5* since applying Riemann integral to simplified Dirichlet eta function will give rise to Dirichlet Sigma-Power Law.

Overall Proof for Riemann Hypothesis given by Theorem Riemann I – IV. Our elementary proof for Riemann hypothesis is now summarized in an overall manner by Theorem Riemann I – IV. For completeness and clarification of this proof, we supply the following underlying important (simple) mathematical arguments.

For $0 < \sigma < 1$, then $0 < 2(1 - \sigma) < 2$. The only whole number between 0 and 2 is '1' which coincide with $\sigma = \frac{1}{2}$. When $0 < \sigma < \frac{1}{2}$ and $\frac{1}{2} < \sigma < 1$, then [correspondingly] $0 < 2(1 - \sigma) < 1$ and $1 < 2(1 - \sigma) < 2$.

Legend: **R** = all real numbers. For $0 < \sigma < 1$, σ consist of $0 < \mathbf{R} < 1$. For $0 < 2(1 - \sigma) < 2$, $2(1 - \sigma)$ must (respectively) consist of $0 < \mathbf{R} < 2$. An important caveat is that previously used phrases such as "$\sum$(all fractional exponents) = whole number '1' / fractional number '≠1'", although not incorrect *per se*, should respectively be replaced by "$\sum$(all real exponents) = whole number '1' / real number '≠1'" for complete accurracy. We additionally note that as whole numbers ⊂ real numbers, we could also validly depict this phrase as "$\sum$(all real exponents) = real number '1' / real number '≠1'". We apply this caveat to Theorem Riemann I – IV.

Theorem Riemann I. Derived from Dirichlet eta function (*proxy* for Riemann zeta function), simplified Dirichlet eta function will exclusively contain *de novo* property for actual location [but not actual positions] of all nontrivial zeros.

Proof. We logically advocate the phrase "actual location [but not actual positions] of all nontrivial zeros" can validly be shortened to "actual location of all nontrivial zeros" which is also used in Theorem Riemann II, III and IV below. Theorem Riemann I essentially equates to Lemma 4.1 except without mentioning Euler formula application to Dirichlet eta function as required in the derivation of simplified Dirichlet eta function [which contains *de novo* property for elucidating "actual location of all nontrivial zeros"]. *The proof for Theorem Riemann I is now complete as it successfully incorporates proof for Lemma 4.1*□.

Theorem Riemann II. Dirichlet Sigma-Power Law (as pseudo-zeroes to zeroes conversion) [antiderivative in continuous (integral) format], which is derived from simplified Dirichlet eta function (as zeroes) [in discrete (summation) format], will exclusively manifest exact DA homogeneity only when real number exponent $\sigma = \frac{1}{2}$.

Proof. Proposition 4.2 refers to rigorous derivation of Dirichlet Sigma-Power Law (as pseudo-zeroes to zeroes conversion) [which contains *de novo* property for elucidating "actual location of all nontrivial zeros"] from simplified Dirichlet eta function (as zeroes). Proposition 4.3 refers to unique manifestation of exact DA homogeneity in Dirichlet Sigma-Power Law (as pseudo-zeroes to zeroes conversion) when real number exponent $\sigma = \frac{1}{2}$. Therefore Theorem Riemann II successfully incorporate the proofs from Propositions 4.2 and 4.3. *The proof for Theorem Riemann II is now complete*□.

Theorem Riemann III. Real number exponent σ [= $\frac{1}{2}$] parameter, being part of real number exponent (1 - σ) in Dirichlet Sigma-Power Law (as pseudo-zeroes to zeroes conversion) that satisfy exact DA homogeneity, is identical to real number exponent σ parameter mentioned in Riemann hypothesis which propose σ to also have exclusive value of $\frac{1}{2}$ (representing critical line) for "actual location of all nontrivial zeros", thus confirming this hypothesis to be true with full support and clarification provided by Theorem Riemann IV.

Proof. Since s = $\sigma \pm \imath t$, complete set of nontrivial zeros which is defined by $\eta(s) = 0$ is exclusively associated with one (and only one) particular $\eta(\sigma \pm \imath t) = 0$ value solution, and by default one (and only one) particular σ [conjecturally] = $\frac{1}{2}$ value solution. When performing exact DA homogeneity on Dirichlet Sigma-Power Law (as pseudo-zeroes to zeroes conversion) [which contains *de novo* property to elucidate "actual location of all nontrivial zeros"], the expression "If real number exponent σ parameter has exclusively $\frac{1}{2}$ value, only then will exact DA homogeneity be satisfied" implies one (and only one) possible mathematical solution. Theorem Riemann III reflect Theorem Riemann II on presence of exact DA homogeneity for $\sigma = \frac{1}{2}$ in Dirichlet Sigma-Power Law (as pseudo-zeroes to zeroes conversion). Consider three defining reasons: (i) σ parameter is intrinsically present in both Riemann zeta function and Dirichlet eta function, (ii) σ [= $\frac{1}{2}$] parameter is used to denote critical line of Riemann zeta function as part of the original Riemann hypothesis [whereby all nontrivial zeros are conjectured to be located on critical line], and (iii) Dirichlet Sigma-Power Law [with converting its pseudo-zeroes to zeroes to obtain nontrivial zeros] containing σ parameter is ultimately derived from Dirichlet eta function, which is *proxy* for Riemann zeta function. Then this Law definitely has identical σ parameter that is referred to by Riemann hypothesis. *The proof for Theorem Riemann III is now complete as we have simultaneous confirmation of (i) solitary $\sigma = \frac{1}{2}$ value in Dirichlet Sigma-Power Law [with converting its pseudo-zeroes to zeroes to obtain nontrivial zeros] satisfying exact DA homogeneity and (ii) critical line defined by solitary $\sigma = \frac{1}{2}$ value being the logically deduced "actual location [but with no request to determine actual positions] of all nontrivial zeros" as was proposed in original Riemann hypothesis*□.

Theorem Riemann IV. Condition 1. All $\sigma \neq \frac{1}{2}$ values representing (infinitely many) non-critical lines, viz. $0 < \sigma < \frac{1}{2}$ and $\frac{1}{2} < \sigma < 1$, will exclusively not contain "actual location of all nontrivial zeros" [and manifest *de novo* inexact DA homogeneity in Dirichlet Sigma-Power Law (as virtual pseudo-zeroes to virtual zeroes conversion with non-existent virtual nontrivial zeros)], together with Condition 2. One (and only one) $\sigma = \frac{1}{2}$ value representing (solitary) critical line will exclusively contain "actual location of all nontrivial zeros" [and manifest *de novo* exact DA homogeneity in Dirichlet Sigma-Power Law (as pseudo-zeroes to zeroes conversion to obtain all nontrivial zeros)], now confirm Riemann hypothesis to be true when these two mutually inclusive conditions are met.

Proof. Condition 2 Theorem Riemann IV simply reflect proof from Theorem Riemann III [which also incorporates Proposition 4.3 as alluded to by Theorem Riemann II] for "actual location of all nontrivial zeros" to exclusively be on critical line (given by $\sigma = \frac{1}{2}$ value) with manifesting *de novo* exact DA homogeneity $\sum$(all real number exponents) = real number '1' for this Law [with converting its pseudo-zeroes to zeroes to obtain all nontrivial zeros]. *The proof for Condition 2 Theorem Riemann IV is now complete*□. Corollary 4.4 confirms *de novo* inexact DA homogeneity in this Law (as virtual pseudo-zeroes to virtual zeroes conversion with non-existent virtual nontrivial zeros) [manifested as $\sum$(all real number exponents) = real number '≠1'] for all $\sigma \neq \frac{1}{2}$ values (non-critical lines) which are exclusively not associated with "actual location of all nontrivial zeros". This can also be rigorously confirmed by further applying inclusion-exclusion principle: Exclusive presence of nontrivial zeros on critical line for Condition 2 Theorem Riemann IV implies exclusive absence of nontrivial zeros on non-critical lines for Condition 1 Theorem Riemann IV. *The proof for Condition 1 Theorem*

Riemann IV is now complete□.

We logically deduce that explicit mathematical explanations why presence and absence of nontrivial zeros should (respectively) coincide precisely with $\sigma = \frac{1}{2}$ and $\sigma \neq \frac{1}{2}$ [which are literally the meta-properties ('overall' *complex properties*)] will require "complex" or convoluted mathematical arguments. Attempting to provide explicit mathematical explanation with "simple" mathematical arguments would intuitively mean nontrivial zeros have to be (incorrectly and impossibly) treated as Completely Predictable entities. These meta-properties are: Gram points equate to "Presence of three entities (i) nontrivial zeros, (ii) Gram[y=0] points and (iii) Gram[x=0] points that coincide precisely with $\sigma = \frac{1}{2}$"; and virtual Gram points equate to "Presence of two entities (i) virtual Gram[y=0] points and (ii) virtual Gram[x=0] points that coincide precisely with $\sigma \neq \frac{1}{2}$".

4. Prerequisite Lemma, Corollary and Propositions on Riemann Hypothesis

Lemma 4.1. Simplified Dirichlet eta function is derived directly from Dirichlet eta function with Euler formula application and it will intrinsically incorporate actual location [but not actual positions] of all nontrivial zeros.

Proof. Denote complex number ($\mathbb{C}$) as z = x + $\imath$·y. Then z = Re(z) + $\imath$·Im(z) with Re(z) = x and Im(z) = y; modulus of z, $|z| = \sqrt{Re(z)^2 + Im(z)^2} = \sqrt{x^2 + y^2}$; and $|z|^2 = x^2 + y^2$.

Euler formula is commonly stated as $e^{\imath x} = \cos x + \imath \cdot \sin x$. Euler identity (where $x = \pi$) is $e^{\imath\pi} = \cos\pi + \imath \cdot \sin\pi = -1 + 0$ [or stated as $e^{\imath\pi} + 1 = 0$]. The n^s of $\zeta(s)$ is expanded to $n^s = n^{(\sigma+\imath t)} = n^\sigma e^{t\ln(n)\cdot\imath}$ since $n^t = e^{t\ln(n)}$. Apply Euler formula to n^s result in $n^s = n^\sigma(\cos(t\ln(n)) + \imath \cdot \sin(t\ln(n)))$. This is written in trigonometric form [designated by short-hand notation $n^s(Euler)$] whereby n^σ is modulus and $t\ln(n)$ is polar angle (argument).

Apply $n^s(Euler)$ to Eq. (1). Then $\zeta(s)$ =Re$\{\zeta(s)\}$ + $\imath$·Im$\{\zeta(s)\}$ with $Re\{\zeta(s)\} = \sum_{n=1}^{\infty} n^{-\sigma}\cos(t\ln(n))$ and $Im\{\zeta(s)\} = -\sum_{n=1}^{\infty} n^{-\sigma}\sin(t\ln(n))$. As $\zeta(s)$ in Eq. (1) is absolutely convergent only when σ>1 where zeros never occur, we will not carry out further treatment here on this equation.

Apply $n^s(Euler)$ to Eq. (3). Then $\eta(s) = \gamma^{-1} \cdot \zeta(s) = Re\{\eta(s)\} + \imath\cdot$Im$\{\eta(s)\}$ whereby $Re\{\eta(s)\} =$
$\sum_{n=1}^{\infty}((2n-1)^{-\sigma}\cos(t\ln(2n-1)) - (2n)^{-\sigma}\cos(t\ln(2n)))$
and $Im\{\eta(s)\} = -\sum_{n=1}^{\infty}((2n-1)^{-\sigma}\sin(t\ln(2n-1)) - (2n)^{-\sigma}\sin(t\ln(2n))) =$
$\sum_{n=1}^{\infty}((2n)^{-\sigma}\sin(t\ln(2n)) - (2n-1)^{-\sigma}\sin(t\ln(2n-1)))$ with proportionality factor $\gamma = \frac{1}{(1-2^{1-s})}$.

Complex number s in critical strip is designated by s = $\sigma + \imath t$ for $0 < t < +\infty$ and s = $\sigma - \imath t$ for $-\infty < t < 0$. Nontrivial zeros equating to $\zeta(s) = 0$ give rise to our desired $\eta(s) = 0$. Modulus of $\eta(s)$, $|\eta(s)|$, is defined as $\sqrt{(Re\{\eta(s)\})^2 + (Im\{\eta(s)\})^2}$ with $|\eta(s)|^2 = (Re\{\eta(s)\})^2 + (Im\{\eta(s)\})^2$. Mathematically $|\eta(s)| = |\eta(s)|^2 = 0$ is an unique condition giving rise to $\eta(s) = 0$ occurring only when Re$\{\eta(s)\}$ = Im$\{\eta(s)\}$ = 0 as any non-zero values for Re$\{\eta(s)\}$ and/or Im$\{\eta(s)\}$ will always result in $|\eta(s)|$ and $|\eta(s)|^2$ having non-zero values. Important implication is that sum of $Re\{\eta(s)\}$ and $Im\{\eta(s)\}$ equating to zero [given by Eq. (4)] must always hold when $|\eta(s)| = |\eta(s)|^2 = 0$ and consequently $\eta(s) = 0$.

$$\sum ReIm\{\eta(s)\} = Re\{\eta(s)\} + Im\{\eta(s)\} = 0 \tag{7}$$

Advocating for existence of theoretical s values leading to non-zero values in Re$\{\eta(s)\}$ and Im$\{\eta(s)\}$ depicted as possibility +Re$\{\eta(s)\}$ = -Im$\{\eta(s)\}$ or -Re$\{\eta(s)\}$ = +Im$\{\eta(s)\}$ could, in principle, satisfy Eq. (7). In reality, the reverse implication is not necessarily true as these s values will not result in $|\eta(s)| = |\eta(s)|^2 = 0$. In any event, we need not consider these two possibilities since solving Riemann hypothesis involves nontrivial zeros [which are rigidly defined by $\eta(s) = 0$] with non-zero values in Re$\{\eta(s)\}$ and/or Im$\{\eta(s)\}$ not compatible with $\eta(s) = 0$. Note that $\eta(s) = 0$ (uniquely) occurring once when $\sigma = \frac{1}{2}$ as Gram points [viz, zeroes] and (non-uniquely) occurring infinitely often when $\sigma \neq \frac{1}{2}$ as virtual Gram points [viz, virtual zeroes] will always happen at appropriate times.

While fully complying with Information-Complexity conservation, preservation of quantitative NAV = 0 [(uniquely) occurring once when $\sigma = \frac{1}{2}$ as zeroes and (non-uniquely) occurring infinitely often when $\sigma \neq \frac{1}{2}$ as virtual zeroes] will always happen at appropriate times for Dirichlet Sigma-Power Law [as (virtual) pseudo-zeroes to (virtual) zeroes conversion] and simplified Dirichlet eta function [as (virtual) zeroes] when interpreted as Riemann sum. Again with direct connection to Riemann hypothesis through the common presence of parameter σ, critical line (denoted by $\sigma = \frac{1}{2}$)

is inevitably and logically conjectured to also be uniquely associated with presence of exact DA homogeneity (occurring only when $\sigma = \frac{1}{2}$) in this Law [as pseudo-zeroes to zeroes conversion obtained via precise NAV = 0] and Riemann sum [as zeroes obtained via approximate NAV = 0].

Eq. (7) is intrinsically incorporated into Dirichlet Sigma-Power Law [ultimately derived from Dirichlet eta function (*proxy* for Riemann zeta function)] since Eq. (7) can literally be taken to constitute an intermediate or common step for deriving this Law with this situation also simultaneously satisfying three conditions: I. The $\eta(s) = 0$ [as zeroes] definition for nontrivial zeros [conjectured to be located at $\sigma = \frac{1}{2}$ critical line] equates to Eq. (7), II. Precise NAV = 0 situation in Dirichlet Sigma-Power Law = 0 [as pseudo-zeroes to zeroes conversion] only occurs when $\sigma = \frac{1}{2}$, and III. [logically, as was originally proposed on the Riemann zeta function in Riemann hypothesis] "All nontrivial zeros must (consequently) be located on critical line of Riemann zeta function which is uniquely denoted only by $\sigma = \frac{1}{2}$".

Apply trigonometry identity $\cos(n) - \sin(n) = \sqrt{2}\sin\left(n + \frac{3}{4}\pi\right)$ to $Re\{\eta(s)\} + Im\{\eta(s)\}$ to get Eq. (8) with terms in last line built by mixture of terms from $Re\{\eta(s)\}$ and $Im\{\eta(s)\}$.

$$\sum ReIm\{\eta(s)\} = \sum_{n=1}^{\infty}[(2n-1)^{-\sigma}\cos(t\ln(2n-1)) - (2n-1)^{-\sigma}\sin(t\ln(2n-1))$$
$$-(2n)^{-\sigma}\cos(t\ln(2n)) + (2n)^{-\sigma}\sin(t\ln(2n))]$$
$$= \sum_{n=1}^{\infty}[(2n-1)^{-\sigma}\sqrt{2}\sin(t\ln(2n-1) + \frac{3}{4}\pi) - (2n)^{-\sigma}\sqrt{2}\sin(t\ln(2n) + \frac{3}{4}\pi)] \tag{8}$$

When depicted in terms of Eq. (7), Eq. (8) becomes

$$\sum_{n=1}^{\infty}(2n)^{-\sigma}\sqrt{2}\sin(t\ln(2n) + \frac{3}{4}\pi) = \sum_{n=1}^{\infty}(2n-1)^{-\sigma}\sqrt{2}\sin(t\ln(2n-1) + \frac{3}{4}\pi)$$
$$\sum_{n=1}^{\infty}(2n)^{-\sigma}\sqrt{2}\sin(t\ln(2n) + \frac{3}{4}\pi) - \sum_{n=1}^{\infty}(2n-1)^{-\sigma}\sqrt{2}\sin(t\ln(2n-1) + \frac{3}{4}\pi) = 0 \tag{9}$$

Eq. (9) can also be expanded as $\sum_{n=1}^{\infty} -(2n)^{-\sigma}(\sin(t\ln(2n)) - \cos(t\ln(2n)))$ $-\sum_{n=1}^{\infty} -(2n-1)^{-\sigma}(\sin(t\ln(2n-1)) - \cos(t\ln(2n-1))) = 0$ which contains both sine and cosine terms. $\eta(s)$ calculations for all σ values result in infinitely many of these type of equations for $0<\sigma<1$ critical strip region of interest with n = 1, 2, 3, 4, 5,..., ∞ as discrete integer number values. All these equations will geometrically represent entire plane of critical strip, thus (at least) allowing our proposed proof to be of a complete nature.

Eq. (9), which is our simplified Dirichlet eta function [with trigonometry identity $\cos(x) - \sin(x)$ being incorporated], is derived directly from Dirichlet eta function and it will intrinsically incorporate *actual location [but not actual positions]* of all nontrivial zeros. *The proof is now complete for Lemma 4.1*□.

Proposition 4.2. Dirichlet Sigma-Power Law [as pseudo-zeroes to zeroes conversion] representing continuous (integral) format and given as antiderivative can be derived directly from simplified Dirichlet eta function [as zeroes] in discrete (summation) format with Riemann integral application. Note: This Law [as pseudo-zeroes to zeroes conversion] representing continuous (integral) format refers to end-product obtained from "key step of converting Dirichlet eta function [as zeroes], *proxy* for Riemann zeta function [as zeroes], into its continuous format version".

Proof. In Calculus, integration is reverse process of differentiation viewed geometrically as the Area enclosed by curve of function and x-axis in a given interval. Apply definite integral I between limits (or points) a and b is to compute its value when $\Delta x \longrightarrow 0$, i.e. $I = \lim_{\Delta x \longrightarrow 0}\sum_{i=1}^{n} f(x_i)\Delta x_i = \int_a^b f(x)dx$. This is Riemann integral of function f(x) in interval [a, b] where a<b. Apply Riemann integral to simplified Dirichlet eta function [as zeroes] in ["$\Delta x \longrightarrow 1$"] discrete (summation) format which intrinsically incorporates *actual location [but not actual positions]* of all nontrivial zeros criterion in order to obtain Dirichlet Sigma-Power Law [as pseudo-zeroes to zeroes conversion] in ["$\Delta x \longrightarrow 0$"] continuous (integral) format with the later validly representing the former. Then Dirichlet Sigma-Power Law [as pseudo-zeroes to zeroes conversion] will also fullfil this criterion. Due to resemblance to power law functions in σ from s = $\sigma + \imath t$ being exponent of a power function n^{σ}, logarithm scale use, and harmonic ζ(s) series connection in Zipf's law; we elect to call this Law by its given name. A characteristic of this "discrete function" or "continuous Law" is the exact formula expression in usual mathematical

language $y = f(x_1)$ format description for a single-variable function. The variable is n obtained from $(2n)$ and $(2n-1)$ as 'base quantities' with parameters σ and t. Thus, $y = f(n)$ with discrete n = 1, 2, 3, 4, 5,. . . , ∞ or continuous n = 1 to ∞ whereby -∞ < t < +∞ and 0 < σ < 1 are the two parameters.

A proper integral is a definite integral which has neither limit a or b infinite and from which the integrand does not approach infinity at any point in the range of integration. An improper integral is a definite integral that has either or both limits a and b infinite or an integrand that approaches infinity at one or more points in the range of integration.

Involving sine and/or cosine functions, [multiple] +ve (above x-axis) and -ve (below x-axis) numerical Net Area Value = 0 solutions can be successfully computed for simplified Dirichlet eta function [as zeroes] when interpreted as Riemann sum and Dirichlet Sigma-Power Law [as pseudo-zeroes to zeroes conversion]. Here, Dirichlet Sigma-Power Law (antiderivative) is the solution to improper integral (with lower limit a = 1 and upper limit b = ∞) obtained from [validly] applying Riemann integral to simplified Dirichlet eta function. All relevant antiderivatives in this paper are derived from improper integrals with format $\int_1^\infty f(n)dn$ based on Eqs. (9), (16) and (18). Example for Eq. (9), involved improper integrals are from $\int_1^\infty (2n)^{-\sigma}\sqrt{2}\sin(t\ln(2n)+\frac{3}{4}\pi)dn$ - $\int_1^\infty (2n-1)^{-\sigma}\sqrt{2}\sin(t\ln(2n-1)+\frac{3}{4}\pi)dn = 0$. These improper integrals are seen to involve [periodic] sine and/or cosine function between limits 1 and ∞. Each improper integral can be validly expanded as $\int_{n=1}^{n=2} f(n)dn + \int_{n=2}^{n=3} f(n)dn + \int_{n=3}^{n=4} f(n)dn + ... + \int_{n=\infty-1}^{n=\infty} f(n)dn$ which, for all sufficiently large n as n⟶ ∞, will manifest *divergence by oscillation* (viz. for all sufficiently large n as n⟶ ∞, this cummulative total will not diverge in a particular direction to a solitary well-defined limit value such as sin $\pi/2 = 1$ or less well-defined limit value such as +∞).

With steps of manual integration shown using indefinite integrals [for simplicity], we solve definite integral based on T1 (first term) with $(2n)$ parameter in Eq. (9):

$$\int_1^\infty \frac{2^{\frac{1}{2}-\sigma}\sin\left(t\ln(2n)+\frac{3\pi}{4}\right)}{n^\sigma}dn = \int_1^\infty -\frac{\sin(t\ln(2n))-\cos(t\ln(2n))}{2^\sigma n^\sigma}dn.$$

We deduce the remaining two important integrals located in Proposition 5.2 to be "variations" of this particular integral here for nontrivial zero (with Right Hand Side term above involving both sine and cosine functions); viz, the integral with its term involving only sine function (for Gram[y=0] points) and the integral with its term involving only cosine function (for Gram[x=0] points). We check all derived antiderivatives to be correct using computer algebra system Maxima.

Simplifying and applying linearity, we obtain $2^{\frac{1}{2}-\sigma}\int \frac{\sin\left(t\ln(2n)+\frac{3\pi}{4}\right)}{n^\sigma}\mathrm{d}n.$

Now solving $\int \frac{\sin\left(t\ln(2n)+\frac{3\pi}{4}\right)}{n^\sigma}\mathrm{d}n$. Substitute $u = t\ln(2n)+\frac{3\pi}{4} \longrightarrow \mathrm{d}n = \frac{n}{t}\mathrm{d}u$,

use $n^{1-\sigma} = \mathrm{e}^{\frac{(1-\sigma)\left(u-t\ln(2)-\frac{3\pi}{4}\right)}{t}} = \frac{\mathrm{e}^{\frac{(\sigma-1)(4t\ln(2)+3\pi)}{4t}}}{t}\int \mathrm{e}^{\frac{(1-\sigma)u}{t}}\sin(u)\,\mathrm{d}u.$

Now solving $\int \mathrm{e}^{\frac{(1-\sigma)u}{t}}\sin(u)\,\mathrm{d}u$. We integrate by parts twice in a row: $\int \mathtt{f}\mathtt{g}' = \mathtt{f}\mathtt{g} - \int \mathtt{f}'\mathtt{g}$.

First time: $\mathtt{f} = \sin(u)$, $\mathtt{g}' = \mathrm{e}^{\frac{(1-\sigma)u}{t}}$

Then $\mathtt{f}' = \boxed{\cos(u)}$, $\mathtt{g} = \boxed{\frac{(1-\sigma)t\mathrm{e}^{\frac{(1-\sigma)u}{t}}}{\sigma^2-2\sigma+1}}$:

$$= \frac{(1-\sigma)t\mathrm{e}^{\frac{(1-\sigma)u}{t}}\sin(u)}{\sigma^2-2\sigma+1} - \int \frac{(1-\sigma)t\mathrm{e}^{\frac{(1-\sigma)u}{t}}\cos(u)}{\sigma^2-2\sigma+1}\,\mathrm{d}u$$

Second time: $\mathtt{f} = \boxed{\cos(u)}$, $\mathtt{g}' = \boxed{\frac{(1-\sigma)t\mathrm{e}^{\frac{(1-\sigma)u}{t}}}{\sigma^2-2\sigma+1}}$

Then $\mathtt{f}' = -\sin(u)$, $\mathtt{g} = \frac{t^2\mathrm{e}^{\frac{(1-\sigma)u}{t}}}{\sigma^2-2\sigma+1}$:

$$= \frac{(1-\sigma)t\mathrm{e}^{\frac{(1-\sigma)u}{t}}\sin(u)}{\sigma^2-2\sigma+1} - \left(\frac{t^2\mathrm{e}^{\frac{(1-\sigma)u}{t}}\cos(u)}{\sigma^2-2\sigma+1} - \int -\frac{t^2\mathrm{e}^{\frac{(1-\sigma)u}{t}}\sin(u)}{\sigma^2-2\sigma+1}\,\mathrm{d}u\right)$$

Apply linearity:

$$= \frac{(1-\sigma)t\mathrm{e}^{\frac{1-\sigma u}{t}}\sin(u)}{\sigma^2-2\sigma+1} - \left(\frac{t^2\mathrm{e}^{\frac{(1-\sigma)u}{t}}\cos(u)}{\sigma^2-2\sigma+1} + \frac{t^2}{\sigma^2-2\sigma+1}\int \mathrm{e}^{\frac{(1-\sigma)u}{t}}\sin(u)\,\mathrm{d}u\right)$$

As integral $\int \mathrm{e}^{\frac{(1-\sigma)u}{t}}\sin(u)\,\mathrm{d}u$ appears again on Right Hand Side, we solve for it:

$$= \frac{\frac{(1-\sigma)e^{\frac{(1-\sigma)u}{t}}\sin(u)}{t} - e^{\frac{(1-\sigma)u}{t}}\cos(u)}{\frac{\sigma^2-2\sigma+1}{t^2}+1}$$

Plug in solved integrals: $\dfrac{e^{\frac{(\sigma-1)(4t\ln(2)+3\pi)}{4t}}}{t}\displaystyle\int e^{\frac{(1-\sigma)u}{t}}\sin(u)\,du$

$$= \frac{e^{\frac{(\sigma-1)(4t\ln(2)+3\pi)}{4t}}\left(\frac{(1-\sigma)e^{\frac{(1-\sigma)u}{t}}\sin(u)}{t} - e^{\frac{(1-\sigma)u}{t}}\cos(u)\right)}{\left(\frac{\sigma^2-2\sigma+1}{t^2}+1\right)t}$$

Undo substitution $u = t\ln(2n) + \frac{3\pi}{4}$ and simplifying:

$$= \frac{e^{\frac{(\sigma-1)(4t\ln(2)+3\pi)}{4t}}\left(\frac{(1-\sigma)e^{\frac{(1-\sigma)\left(t\ln(2n)+\frac{3\pi}{4}\right)}{t}}\sin\left(t\ln(2n)+\frac{3\pi}{4}\right)}{t} - e^{\frac{(1-\sigma)\left(t\ln(2n)+\frac{3\pi}{4}\right)}{t}}\cos\left(t\ln(2n)+\frac{3\pi}{4}\right)\right)}{\left(\frac{\sigma^2-2\sigma+1}{t^2}+1\right)t}$$

Plug in solved integrals: $2^{\frac{1}{2}-\sigma}\displaystyle\int\frac{\sin\left(t\ln(2n)+\frac{3\pi}{4}\right)}{n^\sigma}\,dn$

$$= \frac{2^{\frac{1}{2}-\sigma}e^{\frac{(\sigma-1)(4t\ln(2)+3\pi)}{4t}}\left(\frac{(1-\sigma)e^{\frac{(1-\sigma)\left(t\ln(2n)+\frac{3\pi}{4}\right)}{t}}\sin\left(t\ln(2n)+\frac{3\pi}{4}\right)}{t} - e^{\frac{(1-\sigma)\left(t\ln(2n)+\frac{3\pi}{4}\right)}{t}}\cos\left(t\ln(2n)+\frac{3\pi}{4}\right)\right)}{\left(\frac{\sigma^2-2\sigma+1}{t^2}+1\right)t}$$

By rewriting and simplifying, $\displaystyle\int_1^\infty \frac{2^{\frac{1}{2}-\sigma}\sin\left(t\ln(2n)+\frac{3\pi}{4}\right)}{n^\sigma}dn$ is finally solved as

$$\left[\frac{(2n)^{1-\sigma}\left((t+\sigma-1)\sin(t\ln(2n)) + (t-\sigma+1)\cos(t\ln(2n))\right)}{2\left(t^2+(\sigma-1)^2\right)} + C\right]_1^\infty \tag{10}$$

For T2 (second term) with $(2n-1)$ parameter in Eq. (9), Eq. (10) equates to

$$\left[\frac{(2n-1)^{1-\sigma}\left((t+\sigma-1)\sin(t\ln(2n-1)) + (t-\sigma+1)\cos(t\ln(2n-1))\right)}{2\left(t^2+(\sigma-1)^2\right)} + C\right]_1^\infty \tag{11}$$

Without incorporating constant of integration 'C', Dirichlet Sigma-Power Law as equation derived from Eq. (9) is given by:

$$\begin{aligned}\frac{1}{2\left(t^2+(\sigma-1)^2\right)}\cdot\Big[&(2n)^{1-\sigma}((t+\sigma-1)\sin(t\ln(2n)) + (t-\sigma+1)\\ &\cdot\cos(t\ln(2n))) - (2n-1)^{1-\sigma}((t+\sigma-1)\\ &\cdot\sin(t\ln(2n-1)) + (t-\sigma+1)\cos(t\ln(2n-1)))\Big]_1^\infty = 0\end{aligned} \tag{12}$$

Intended derivation of Dirichlet Sigma-Power Law [with intrinsic ability for pseudo-zeroes to zeroes conversion] as equation has been successful. *The proof is now complete for Proposition 4.2*□.

Proposition 4.3. Exact Dimensional analysis homogeneity at $\sigma = \frac{1}{2}$ in Dirichlet Sigma-Power Law [pseudo-zeroes to zeroes conversion] as equation is indicated by $\sum$(all fractional exponents) = whole number '1'.

Proof. Without incorporating constant of integration 'C', Dirichlet Sigma-Power Law as equation for $\sigma = \frac{1}{2}$ value is given by:

$$\frac{1}{2t^2+\frac{1}{2}}\cdot\left[(2n)^{\frac{1}{2}}\left(\left(t-\frac{1}{2}\right)\sin(t\ln(2n)) + \left(t+\frac{1}{2}\right)\cos(t\ln(2n))\right) -\right.$$

$$\left.(2n-1)^{\frac{1}{2}}\left(\left(t-\frac{1}{2}\right)\sin(t\ln(2n-1)) + \left(t+\frac{1}{2}\right)\cos(t\ln(2n-1))\right)\right]_1^\infty = 0 \tag{13}$$

Evaluation of definite integrals Eq. (13), Eq. (21) and Eq. (22) using limit as n $\to +\infty$ for $0 < t < +\infty$ enable countless computations resulting in t values for (respectively) CIS nontrivial zeros, CIS Gram[y=0] and CIS Gram[x=0] points [as pseudo-zeroes to zeroes conversion]. Larger n values used for computations will correspond to increasing accuracy of these entities (which are all transcendental numbers). Complying with Information-Complexity conservation, preservation or conservation of quantitative Net Area Value = 0 when $\sigma = \frac{1}{2}$ will always happen at appropriate times for Eq. (13), Eq. (21) and Eq. (22). Otherwise, preservation or conservation of quantitative Net Area Value = 0 when $\sigma \neq \frac{1}{2}$ will always happen at appropriate times for Eq. (14), Eq. (23) and Eq. (24), respectively, enabling countless computations resulting in t values for CIS virtual Gram[y=0] and CIS virtual Gram[x=0] points [as virtual pseudo-zeroes to virtual zeroes conversion] with absent (virtual) nontrivial zeros.

$\sum$(all fractional exponents) as $2(1-\sigma)$ = whole number '1' for Eq. (13). This finding signify presence of complete set nontrivial zeros [as pseudo-zeroes to zeroes conversion] for Eq. (13). *The proof is now complete for Proposition 4.3*□.

Corollary 4.4. Inexact Dimensional analysis homogeneity at $\sigma \neq \frac{1}{2}$ [illustrated using $\sigma = \frac{2}{5}$] in Dirichlet Sigma-Power Law [virtual pseudo-zeroes to virtual zeroes conversion] as equation is indicated by $\sum$(all fractional exponents) = fractional number '≠1'.

Proof. Without incorporating constant of integration 'C', Dirichlet Sigma-Power Law as equation for $\sigma = \frac{2}{5}$ value is given by:

$$\frac{1}{2t^2+\frac{18}{25}} \cdot \left[(2n)^{\frac{3}{5}} \left((t-\tfrac{3}{5})\sin(t\ln(2n)) + (t+\tfrac{3}{5})\cos(t\ln(2n))\right) - (2n-1)^{\frac{3}{5}}\left(\left(t-\frac{3}{5}\right)\sin(t\ln(2n-1)) + \left(t+\frac{3}{5}\right)\cos(t\ln(2n-1))\right)\right]_1^{\infty} = 0 \tag{14}$$

$\sum$(all fractional exponents) as $2(1-\sigma)$ = fractional number '≠1' for Eq. (14). This finding signify absence of complete set nontrivial zeros [as virtual pseudo-zeroes to virtual zeroes conversion] for Eq. (14). *The proof is now complete for Corollary 4.4*□.

5. Prerequisite Lemma, Corollary and Propositions on Two Types of Gram Points

For Gram[y=0] & Gram[x=0] points (and corresponding virtual Gram[y=0] & virtual Gram[x=0] points with different values), we apply a parallel procedure conducted on nontrivial zeros depicting abbreviated treatments and discussions.

Lemma 5.1. Simplified Gram[y=0] and Gram[x=0] points-Dirichlet eta functions are derived directly from Dirichlet eta function with Euler formula application and (respectively) they will intrinsically incorporate actual location [but not actual positions] of all Gram[y=0] and Gram[x=0] points.

Proof. For Gram[y=0] points, the equivalent of Eq. (7) and Eq. (9) are respectively given by Eq. (15) and Eq. (16) below.

$$\sum ReIm\{\eta(s)\} = Re\{\eta(s)\} + 0, \text{ or simply } \mathrm{Im}\{\eta(s)\} = 0 \tag{15}$$

$$\sum_{n=1}^{\infty}(2n)^{-\sigma}\sin(t\ln(2n)) = \sum_{n=1}^{\infty}(2n-1)^{-\sigma}\sin(t\ln(2n-1))$$

$$\sum_{n=1}^{\infty}(2n)^{-\sigma}\sin(t\ln(2n)) - \sum_{n=1}^{\infty}(2n-1)^{-\sigma}\sin(t\ln(2n-1)) = 0 \tag{16}$$

For Gram[x=0] points, the equivalent of Eq. (7) and Eq. (9) are respectively given by Eq. (17) and Eq. (18) below.

$$\sum ReIm\{\eta(s)\} = 0 + Im\{\eta(s)\}, \text{ or simply } \mathrm{Re}\{\eta(s)\} = 0 \tag{17}$$

$$\sum_{n=1}^{\infty}(2n)^{-\sigma}\cos(t\ln(2n)) = \sum_{n=1}^{\infty}(2n-1)^{-\sigma}\cos(t\ln(2n-1))$$

$$\sum_{n=1}^{\infty}(2n)^{-\sigma}\cos(t\ln(2n)) - \sum_{n=1}^{\infty}(2n-1)^{-\sigma}\cos(t\ln(2n-1)) = 0 \tag{18}$$

Eq. (16) and Eq. (18) being the simplified Gram[y=0] and Gram[x=0] points-Dirichlet eta functions derived directly from $\eta(s)$ will intrinsically incorporate *actual location [but not actual positions]* of (respectively) all Gram[y=0] and Gram[x=0] points. *The proof is now complete for Lemma 5.1*□.

Proposition 5.2. Gram[y=0] and Gram[x=0] points-Dirichlet Sigma-Power Laws [as pseudo-zeroes to zeroes conversion] representing continuous (integral) format and given as antiderivatives are derived directly from simplified Gram[y=0] and Gram[x=0] points-Dirichlet eta functions [as zeroes] in discrete (summation) format with Riemann integral application. Note: This Law [as pseudo-zeroes to zeroes conversion] representing continuous (integral) format refers to end-product obtained from "key step of converting Dirichlet eta function [as zeroes], *proxy* for Riemann zeta function [as zeroes], into its continuous format version".

Proof. Antiderivatives below using (2n) parameter help obtain all subsequent equations: first one for Gram[y=0] points and second one for Gram[x=0] points.

$\int_1^{\infty}(2n)^{-\sigma}\sin(t\ln(2n))dn = \left[-\frac{(2n)^{1-\sigma}((\sigma-1)\sin(t\ln(2n))+t\cos(t\ln(2n)))}{2\left(t^2+(\sigma-1)^2\right)}+C\right]_1^{\infty}$

$\int_1^{\infty}(2n)^{-\sigma}\cos(t\ln(2n))dn = \left[\frac{(2n)^{1-\sigma}(t\sin(t\ln(2n))-(\sigma-1)\cos(t\ln(2n)))}{2\left(t^2+(\sigma-1)^2\right)}+C\right]_1^{\infty}$

For Gram[y=0] points-Dirichlet Sigma-Power Law as equation [which is the equivalent of Eq. (12)], it is given by Eq. (19) [without incorporating constant of integration 'C'].

$$-\frac{1}{2\left(t^2+(\sigma-1)^2\right)}\cdot\Big[(2n)^{1-\sigma}((\sigma-1)\sin(t\ln(2n))+t\cos(t\ln(2n)))-$$
$$(2n-1)^{1-\sigma}((\sigma-1)\sin(t\ln(2n-1))+t\cos(t\ln(2n-1)))\Big]_1^{\infty}=0 \tag{19}$$

For Gram[x=0] points-Dirichlet Sigma-Power Law as equation [which is the equivalent of Eq. (12)], it is given by Eq. (20) [without incorporating constant of integration 'C'].

$$\frac{1}{2\left(t^2+(\sigma-1)^2\right)}\cdot\Big[(2n)^{1-\sigma}(t\sin(t\ln(2n))-(\sigma-1)\cos(t\ln(2n)))-$$
$$(2n-1)^{1-\sigma}(t\sin(t\ln(2n-1))-(\sigma-1)\cos(t\ln(2n-1)))\Big]_1^{\infty}=0 \tag{20}$$

Intended derivation of Gram[y=0] and Gram[x=0] points-Dirichlet Sigma-Power Laws [both with intrinsic ability for pseudo-zeroes to zeroes conversion] as equations is successful. *The proof is now complete for Proposition 5.2*□.

Proposition 5.3. Exact Dimensional analysis homogeneity at $\sigma=\frac{1}{2}$ in Gram[y=0] and Gram[x=0] points-Dirichlet Sigma-Power Laws [pseudo-zeroes to zeroes conversion] as equations are indicated by $\sum$(all fractional exponents) = whole number '1'.

Proof. Without incorporating constant of integration 'C', Gram[y=0] points-Dirichlet Sigma-Power Law as equation for $\sigma=\frac{1}{2}$ value is given by:

$$-\frac{1}{2t^2+\frac{1}{2}}\cdot[(2n)^{\frac{1}{2}}\left(t\cos(t\ln(2n))-\frac{1}{2}\sin(t\ln(2n))\right)-$$
$$(2n-1)^{\frac{1}{2}}\left(t\cos(t\ln(2n-1))-\frac{1}{2}\sin(t\ln(2n-1))\right)\Big]_1^{\infty}=0 \tag{21}$$

Without incorporating constant of integration 'C', Gram[x=0] points-Dirichlet Sigma-Power Law as equation for $\sigma=\frac{1}{2}$ value is given by:

$$\frac{1}{2t^2+\frac{1}{2}}\cdot[(2n)^{\frac{1}{2}}\left(t\sin(t\ln(2n))+\frac{1}{2}\cos(t\ln(2n))\right)-$$
$$(2n-1)^{\frac{1}{2}}\left(t\sin(t\ln(2n-1))+\frac{1}{2}\cos(t\ln(2n-1))\right)\Big]_1^{\infty}=0 \tag{22}$$

$\sum$(all fractional exponents) as $2(1-\sigma)$ = whole number '1' for Eqs. (21) and (22). These findings signify presence of complete sets Gram[y=0] points for Eq. (21) and Gram[x=0] points for Eq. (22) [both as pseudo-zeroes to zeroes conversion]. *The proof is now complete for Proposition 5.3*□.

Corollary 5.4. Inexact Dimensional analysis homogeneity at $\sigma\neq\frac{1}{2}$ [illustrated using $\sigma=\frac{2}{5}$] in Gram[y=0] and Gram[x=0] points-Dirichlet Sigma-Power Laws [virtual pseudo-zeroes to virtual zeroes conversion] as equations are indicated by $\sum$(all fractional exponents) = fractional number '≠1'.

Proof. Without incorporating constant of integration 'C', Gram[y=0] points-Dirichlet Sigma-Power Law as equation for $\sigma=\frac{2}{5}$ value is given by:

$$-\frac{1}{2t^2+\frac{18}{25}}\cdot[(2n)^{\frac{3}{5}}\left(t\cos(t\ln(2n))-\frac{3}{5}\sin(t\ln(2n))\right)-$$
$$(2n-1)^{\frac{3}{5}}\left(t\cos(t\ln(2n-1))-\frac{3}{5}\sin(t\ln(2n-1))\right)\Big]_1^{\infty}=0 \tag{23}$$

Without incorporating constant of integration 'C', Gram[x=0] points-Dirichlet Sigma-Power Law as equation for $\sigma = \frac{2}{5}$ value is given by:

$$\frac{1}{2t^2 + \frac{18}{25}} \cdot [(2n)^{\frac{3}{5}} \left(t \sin(t \ln(2n)) + \frac{3}{5} \cos(t \ln(2n))\right) -$$

$$(2n-1)^{\frac{3}{5}} \left(t \sin(t \ln(2n-1)) + \frac{3}{5} \cos(t \ln(2n-1))\right)\Big]_1^{\infty} = 0 \quad (24)$$

$\sum$(all fractional exponents) as $2(1-\sigma)$ = fractional number '$\neq$1' for Eqs. (23) and (24). These findings signify presence of complete sets virtual Gram[y=0] points for Eq. (23) and virtual Gram[x=0] points for Eq. (24) [both as virtual pseudo-zeroes to virtual zeroes conversion]. *The proof is now complete for Corollary 5.4*$\square$.

6. Prime and Composite Numbers

Prime and Composite numbers are Incompletely Predictable entities dependently linked together in a sequential, cummulative and eternal manner since relationship Number '1' + Prime numbers (**P**) + Composite numbers (**C**) = Natural numbers (**N**) is always valid.

6.1 Dimensional Analysis on Dimension (2x - N) and Cardinality of Relevant Sets

We use the word "Dimensions" to denote well-defined Incompletely Predictable entities obtained from using our unique Dimension (2x - N) system. Relevant "Dimensions" *dependently* represent Number '1', **P** and **C**. Then *by default* any (sub)sets of **P** and **C** in well-defined equations can also be represented by their corresponding "Dimensions".

Remark 6 We can apply Dimensional analysis to "Dimensions" from Dimension (2x - N) system and cardinality of relevant sets in certain well-defined equations.

Let **X** denote **E**, **O**, **N** [which are classified as Completely Predictable numbers], **P** and **C** [which are classified as Incompletely Predictable numbers]. For x = 1, 2, 3, 4, 5,..., ∞; consider all **X** $\leq$ x. Then this "all **X** $\leq$ x" is definition for **X**-$\pi(x)$ [denoting "**X** counting function"] resulting in following two types of equations coined as (I) 'Exact' equation **N**-$\pi(x)$ = **E**-$\pi(x)$ + **O**-$\pi(x)$ with "non-varying" relationships **E**-$\pi(x)$ = **O**-$\pi(x)$ for all x = **E** and **E**-$\pi(x)$ = **O**-$\pi(x)$ - 1 for all x = **O**, and (II) 'Inexact' equation **N**-$\pi(x)$ = 1 + **P**-$\pi(x)$ + **C**-$\pi(x)$ with "varying" relationships **P**-$\pi(x)$ > **C**-$\pi(x)$ for all x $\leq$ 8; **P**-$\pi(x)$ = **C**-$\pi(x)$ for x = 9, 11, and 13; and **P**-$\pi(x)$ < **C**-$\pi(x)$ for x = 10, 12, and all x $\geq$ 14.

Let "Dimensions" and different (sub)sets of **E**, **O**, **N**, **P** and **C** be 'base quantities'. Then exponent '1' of "Dimensions" and cardinality of these (sub)sets in well-defined equations are corresponding 'units of measurement'. Performing DA on "Dimensions" for **PC** pairing is depicted later on. Performing DA on cardinality is depicted next.

For Set **N** = Set **E** + Set **O**, then |**N**| = |**E**| + |**O**| $\implies \aleph_0 = \aleph_0 + \aleph_0$ thus conforming with DA homogeneity.

For Set **N** = Set **P** + Set **C** + Number '1', then Set **N** - Number '1' = Set **P** + Set **C** and |**N** - Number '1'| = |**P**| + |**C**| $\implies \aleph_0 = \aleph_0 + \aleph_0$ thus conforming with DA homogeneity.

For Set **N** - Set **even P** - Number '1'= Set **odd P** + Set **even C** + Set **odd C**, then |**N** - **even P** - Number '1'| = |**odd P**| + |**even C**| + |**odd C**| $\implies \aleph_0 = \aleph_0 + \aleph_0 + \aleph_0$ thus conforming with DA homogeneity. Symbolically represented by all available **O** prime gap = 1 and **E** prime gaps = 2, 4, 6, 8, 10,...; **O** composite gap = 1 and **E** composite gap = 2; and **O** natural gap = 1; then |**Gap 1 N** - **Gap 1 P** - Number '1'| = |**Gap 2 P**| + |**Gap 4 P**| + |**Gap 6 P**| + |**Gap 8 P**| + |**Gap 10 P**| + ... + |**Gap 1 C**| + |**Gap 2 C**| $\implies \aleph_0 = \aleph_0 + \aleph_0 + \aleph_0 + \aleph_0 + \aleph_0 + ... \aleph_0 + \aleph_0$ thus conforming with DA homogeneity. It is known that |**Gap 1 P**| = |Number '2'| = 1 and |**Gap 1 N**| = |**Gap 1 C**| = |**Gap 2 C**| = $\aleph_0$. Then solving Polignac's and Twin prime conjectures translate to successfully proving |**Gap 2 P**| = |**Gap 4 P**| = |**Gap 6 P**| = |**Gap 8 P**| = |**Gap 10 P**| = ... = $\aleph_0$ with |**E prime gaps**| = $\aleph_0$.

Outline of proof for Polignac's and Twin prime conjectures. Requires simultaneously satisfying two mutually inclusive conditions: I. *With rigid manifestation of DA homogeneity*, quantitive[5] fulfillment by considering i $\in$ **E** for each Subset **odd P**$_i$ generated by **E** prime gap = i from Set **E prime gaps** occurs only if solitary cardinality value is present in equation Set **odd P** = $\sum_{i=2}^{\infty}$Subset **odd P**$_i$ with |**odd P**| = |**odd P**$_i$| = |**E prime gaps**| =$\aleph_0$, and II. *With rigid manifestation of DA non-homogeneity*, quantitive[5] fulfillment by considering i $\in$ **E** for each Subset **odd P**$_i$ generated by **E** prime gap = i from Set **E prime gaps** does not occur if more than one cardinality values are present in equation Set **odd P** > $\sum_{i=2}^{\infty}$Subset **odd**

$\mathbf{P}_i$ with |**E prime gaps**| $=\aleph_0$ having incorrect |Subset(s) **odd P**| = N (finite value) &/or Set **odd P** $> \sum_{i=2}^{N}$ Subset **odd** $\mathbf{P}_i$ with |**odd** $\mathbf{P}_i$| $= \aleph_0$ having incorrect |**E prime gaps**| = N (finite value).

Footnote 5: Qualitative fulfillment of |**odd P**| = |**odd** $\mathbf{P}_i$| = |**all E prime gaps**| $= \aleph_0$ equates to Plus-Minus Gap 2 Composite Number Alternating Law being precisely obeyed by all **E** prime gaps apart from first **E** prime gap precisely obeying Plus Gap 2 Composite Number Continuous Law. Derived using Dimension (2x - N) system, these Laws symbolize "end-result" proof on Polignac's and Twin prime conjectures. *Law of Continuity* is a heuristic principle *whatever succeed for the finite, also succeed for the infinite*. Then these Laws which inherently manifest 'Gap 2 Composite Number' on finite and infinite time scale should in principle "succeed for the finite, also succeed for the infinite".

Polignac's and Twin prime conjectures mathematical foot-prints. Six identifiable steps to prove these conjectures: *Step 1* Let N = 2x - ΣPC_x-Gap. Define Dimension (2x - N) to validly represent **P** & **C**. Considering x ∈ **N**, obtain Dimensions (2x - 2), (2x - 4), (2x - 5), (2x - 7), (2x - 8), (2x - 9), ..., (2x - ∞) with specific groupings to constitute all elements of Set **P** [culminating in obtaining all prime gaps (= **E** prime gaps + Solitary **O** prime gap) with |**all prime gaps**| $= \aleph_0$]. Note Dimension (2x - 2) represents x = 1 (Number '1') which is neither **P** nor **C**. Confirm all the obtained Dimension (2x - N) will comply with Information-complexity conservation. *Step 2* Considering i ∈ **E**, confirm perpetual recurrences of individual **E** prime gap = i (associated with its unique odd $\mathbf{P}_i$) occur only when depicted as specific groupings of Dimension $(2x - N)^1$ now endowed with exponent '1' for all ranges of x. *Step 3* Perform DA on exponent '1' in these Dimensions. *Step 4* Perform DA on equation Set **odd P** $= \sum_{i=2}^{\infty}$ Subset **odd** $\mathbf{P}_i$ to obtain |**odd P**| = |**odd** $\mathbf{P}_i$| $= \aleph_0$ whereby Subset **odd** $\mathbf{P}_i$ is derived from its associated unique **E** prime gap = i with |**E prime gaps**| $= \aleph_0$. *Step 5* Confirm 'Prime number' variable and 'Prime gap' variable complex algorithm "containing" all **P** with knowing their overall actual location [but not actual positions][6]. *Step 6* Derive Plus-Minus Gap 2 Composite Number Alternating Law & Plus Gap 2 Composite Number Continuous Law with Dimension $(2x - N)^1$.

Footnote 6: This phrase implies all **P** (and **C**) are Incompletely Predictable numbers. Actual positions require using complex algorithm Sieve of Eratosthenes to *dependently* calculate positions of all preceding **P** (and **C**) in neighborhood whereby $\mathbf{P}_{i+1} = P_i + G_{Pi}$ with $\mathbf{P}_1 = 2$ & $\mathbf{C}_{i+1} = C_i + G_{Ci}$ with $\mathbf{C}_1 = 4$. Here i = 1, 2, 3, 4, 5, ..., ∞.

'Complex Elementary Fundamental Laws'-based solutions of Plus-Minus Gap 2 Composite Number Alternating Law and Plus Gap 2 Composite Number Continuous Law are obtained by undertaking the non-negotiable mathematical steps outlined above. These Laws are literally Completely Predictable meta-properties ('overall' *complex properties*) arising from "interactions" between **P** and **C** producing relevant patterns of Gap 2 Composite Number perpetual appearances [albeit with Incompletely Predictable timing]. We logically deduce explicit mathematical explanation for these meta-properties requires "complex" mathematical arguments. Attempts to give explicit mathematical explanation with "simple" mathematical arguments intuitively meant Incompletely Predictable numbers **P** and **C** are (incorrectly & impossibly) treated as Completely Predictable numbers.

6.2 Brief Overview of Polignac's and Twin Prime Conjectures

Occurring over 2000 years ago (c. 300 BC), ancient Euclid's proof on infinitude of **P** in totality [viz. |**P**| $= \aleph_0$ for Set **P**] predominantly by *reductio ad absurdum* (proof by contradiction) is earliest known but not the only proof for this simple problem in Number theory. Since then dozens of proofs have been devised such as three chronologically listed: Goldbach's Proof using Fermat numbers (written in a letter to Swiss mathematician Leonhard Euler, July 1730), Furstenberg's Topological Proof in 1955 (Furstenberg, 1955), and Filip Saidak's Proof in 2006 (Saidak, 2006). The strangest candidate is likely to be Furstenberg's Topological Proof.

In 2013, Yitang Zhang proved a landmark result showing some unknown even number 'N' < 70 million such that there are infinitely many pairs of **P** that differ by 'N' (Zhang, 2014). By optimizing Zhang's bound, subsequent Polymath Project collaborative efforts using a new refinement of GPY sieve in 2013 lowered 'N' to 246; and assuming Elliott-Halberstam conjecture and its generalized form have further lower 'N' to 12 and 6, respectively. Intuitively, 'N' should have more than one valid values such that there are infinitely many pairs of **P** that differ by each of those 'N' values [thus suggesting existence of more than one Subset **odd** $\mathbf{P}_i$ with |**odd** $\mathbf{P}_i$| $= \aleph_0$]. We can only theoretically lower 'N' to 2 in regards to **P** with 'small gaps' but we anticipate there are an infinite number of **E** prime gaps in regards to **P** with 'large gaps' (which can be arbitrarily large in magnitude) that require "proof that each will generate its unique set of infinite **P**".

7. Supportive Role of Maximal and Non-Maximal Prime Gaps

Remark 7 Existence of maximal and non-maximal prime gaps supply crucial indirect evidence to intuitively support but does not prove "Each even prime gap will generate an infinite magnitude of odd prime numbers on its own accord".

Table 4. First 17 prime gaps depicted in format using maximal prime gaps & non-maximal prime gaps

Prime gap	Following prime number	Prime gap	Following prime number
1*	2	18*	523
2*	3	20*	887
4*	7	22*	1129
6*	23	24	1669
8*	89	26	2477
10	139	28	2971
12	199	30	4297
14*	113	32	5591
16	1831	..	....

Legend: maximal prime gaps is depicted with asterisk symbol (*) and non-maximal prime gaps is depicted without asterisk symbol.

We analyze data of all **P** obtained when extrapolated out over a wide range of x ≥ 2 integer values. As sequence of **P** carries on, **P** with ever larger prime gaps appears. For given range of x integer values, prime gap = n_2 is a 'maximal prime gap' if prime gap = n_1 < prime gap = n_2 for all $n_1 < n_2$. In other words, largest such prime gaps in this range are called maximal prime gaps. The term 'first occurrence prime gaps' refers to first occurrences of maximal prime gaps whereby maximal prime gaps are prime gaps of "at least of this length". We use maximal prime gaps to denote 'first occurrence prime gaps'. CIS non-maximal prime gaps (endorsed with nickname 'slow jumpers') always lag behind CIS maximal prime gaps for onset appearances in **P** sequence. These are shown for first 17 prime gaps in Table 4. Apart from **O** prime gap = 1 representing solitary even **P** '2', remaining **P** in Table 4 consist of representative single odd **P** for each **E** prime gap. These odd **P** individually make one-off appearance in **P** sequence in a *perpetual albeit Incompletely Predictable manner*. Initial seven of [majority] "missing" odd **P** are 5, 11, 13, 17, 19, 29, 31,... belonging to Subset **P** with 'residual' prime gaps are potential source of odd **P** in relation to proposal that each **E** prime gap from Set **E prime gaps** will generate its specific Subset **odd P**. Set **all P** from all prime gaps = Subset **P** from maximal prime gaps + Subset **P** from non-maximal prime gaps + Subset **P** from 'residual' prime gaps. Subset **P** from 'residual' prime gaps with representation from all **E** prime gaps includes all correctly selected "missing" odd **P**. These observations support but does not prove the proposition that each **E** prime gap will generate its own Subset **odd P** with |**odd P**| = $\aleph_0$.

For i ∈ **N**; primordial $P_i\#$ is analog of usual factorial for **P** = 2, 3, 5, 7, 11, 13,.... Then $P_1\#$ = 2, $P_2\#$ = 2 X 3 = 6, $P_3\#$ = 2 X 3 X 5 = 30, $P_4\#$ = 2 X 3 X 5 X 7 = 210, $P_5\#$ = 2 X 3 X 5 X 7 X 11 = 2310, $P_6\#$ = 2 X 3 X 5 X 7 X 11 X 13 = 30030, etc. English mathematician John Horton Conway coined the term 'jumping champion' in 1993. An integer n is a 'jumping champion' if n is the most frequently occurring difference (prime gap) between consecutive **P**<x for some x integer values. Example: for any x with 7<x<131, n = 2 (indicating twin **P**) is the 'jumping champion'. It has been conjectured that (i) the only 'jumping champions' are 1, 4 and primorials 2, 6, 30, 210, 2310, 30030,... and (ii) 'jumping champions' tend to infinity. Their required proofs will likely need proof of k-tuple conjecture. **P** from 'jumping champion' prime gaps have their onset appearances in **P** sequence in a *perpetual albeit Incompletely Predictable manner* [as another example to that outlined in previous paragraph].

8. Prerequisite Lemma, Corollary and Propositions on Prime and Composite Numbers

P and **C** numbers are traditionally "analyzed separately". The key definition behind Dimension (2x - N) is used to abstractly represent *dependent* **P** and **C** numbers (and Number '1') in a combined manner whereby N = 2x - ΣPC_x-Gap. This will lead to required mathematical arguments based on patterns in Gap 2 Composite Number to obtain Plus-Minus Gap 2 Composite Number Alternating Law & Plus Gap 2 Composite Number Continuous Law which will [respectively] solve Polignac's & Twin prime conjectures.

N (CIS): 1, 2, 3,..., +∞. Let x be from Set **X** such that x ∈ **N**. Consider x for upper boundary of interest in Set **X** whereby **X** is chosen from **N**, **E**, **O**, **P** or **C**.

Lemma 8.1. Natural counting function **N**-$\pi(x)$, defined as |**N** ≤ x|, is Completely Predictable by independently using simple algorithm to be equal to x.

Proof Formula to generate **N** with 100% certainty is $\mathbf{N}_i$ = i whereby $\mathbf{N}_i$ is the i^{th} **N** and i = 1, 2, 3,..., ∞. For a given $\mathbf{N}_i$, its i^{th} position is simply i. Natural gap (G_{Ni}) = $\mathbf{N}_{i+1}$ - $\mathbf{N}_i$, with G_{Ni} always = 1. There are x **N** ≤ x. Thus **N**-$\pi(x)$ = |**N** ≤ x| = x. *The proof is now complete for Lemma 8.1*□.

Lemma 8.2. Even counting function **E**-$\pi(x)$, defined as |**E** ≤ x|, is Completely Predictable by independently using simple algorithm to be equal to floor(x/2).

Proof. Formula to generate **E** with 100% certainty is $\mathbf{E}_i$ = iX2 whereby $\mathbf{E}_i$ is the i^{th} **E** and i = 1, 2, 3,..., ∞ abiding to mathematical label "All **N** always ending with a digit 0, 2, 4, 6 or 8". For a given $\mathbf{E}_i$, its i^{th} position is calculated as i = $\mathbf{E}_i$/2. Even gap (G_{Ei}) = $\mathbf{E}_{i+1}$ - $\mathbf{E}_i$, with G_{Ei} always = 2. There are $\lfloor \frac{x}{2} \rfloor$ **E** ≤ x. Thus **E**-$\pi(x)$ = |**E** ≤ x| = floor(x/2). *The proof is now complete for Lemma 8.2*□.

Lemma 8.3. Odd counting function **O**-$\pi(x)$, defined as |**O** ≤ x|, is Completely Predictable by independently using simple algorithm to be equal to ceiling(x/2).

Proof. Formula to generate **O** with 100% certainty is $\mathbf{O}_i$ = (iX2) - 1 whereby $\mathbf{O}_i$ is the i^{th} odd number and i = 1, 2, 3,..., ∞ abiding to mathematical label "All **N** always ending with a digit 1, 3, 5, 7, or 9". For a given $\mathbf{O}_i$ number, its i^{th} position is calculated as i = ($\mathbf{O}_i$ + 1)/2. Odd gap (G_{Oi}) = $\mathbf{O}_{i+1}$ - $\mathbf{O}_i$, with G_{Oi} always = 2. There are $\lceil \frac{x}{2} \rceil$**O**≤x. Thus **O**-$\pi(x)$ = |**O**≤x| = ceiling(x/2). *The proof is now complete for Lemma 8.3*□.

Lemma 8.4. Prime counting function **P**-$\pi(x)$, defined as |**P** ≤ x|, is Incompletely Predictable with Set **P** dependently obtained using complex algorithm Sieve of Eratosthenes.

Proof. Algorithm to generate $\mathbf{P}_i$ whereby $\mathbf{P}_1$ (= 2), $\mathbf{P}_2$ (= 3), $\mathbf{P}_3$ (= 5), $\mathbf{P}_4$ (= 7),..., ∞ with 100% certainty is based on Sieve of Eratosthenes abiding to mathematical label "All **N** apart from 1 that are evenly divisible by itself and by 1". Although we can check primality of a given **O** by trial division, we can never determine its position without knowing positions of preceding **P**. Prime gap (G_{Pi}) = $\mathbf{P}_{i+1}$ - $\mathbf{P}_i$, with G_{Pi} constituted by all **E** except 1^{st} G_{P1} = 3 - 2 = 1. **P**-$\pi(x)$ = |**P** ≤ x|. This is Incompletely Predictable and is calculated via mentioned algorithm. Using definition of prime gap, every **P** [represented here with aid of 'n' notation instead of usual 'i' notation] is written as $\mathbf{P}_{n+1}$ = P_n + G_{Pi} with $\mathbf{P}_1$ = 2. Here i & n = 1, 2, 3, 4, 5, ..., ∞. *The proof is now complete for Lemma 8.4*□.

Lemma 8.5. Composite counting function **C**-$\pi(x)$, defined as |**C**≤x|, is Incompletely Predictable with Set **C** derived as Set **N**-Set **P** [dependently obtained using complex algorithm Sieve of Eratosthenes]-Number '1'.

Proof. Composite numbers abide to mathematical label "All **N** apart from 1 that are evenly divisible by numbers other than itself and 1". Algorithm to generate $\mathbf{C}_i$ whereby $\mathbf{C}_1$ (= 4), $\mathbf{C}_2$ (= 6), $\mathbf{C}_3$ (= 8), $\mathbf{C}_4$ (= 9),..., ∞ with 100% certainty is based [indirectly] on Sieve of Eratosthenes via selecting non-prime **N** to be **C**. We define Composite gap G_{Ci} as $\mathbf{C}_{i+1}$ - $\mathbf{C}_i$ with G_{Ci} constituted by 1 & 2. **C**-$\pi(x)$ = |**C** ≤ x|. This is Incompletely Predictable and need to be calculated indirectly via the mentioned algorithm. Using definition of composite gap, every **C** [represented here with aid of 'n' notation instead usual 'i' notation] is written as $\mathbf{C}_{n+1}$ = C_n + G_{Ci} with $\mathbf{C}_1$ = 4. Here i & n = 1, 2, 3, 4, 5, ..., ∞. *The proof is now complete for Lemma 8.5*□.

Denote **X** to be **N**, **E**, **O**, **P** or **C**. X-$\pi(x)$ = |**X** ≤ x| with x ∈ **N**. We define and compute entity 'Grand-Total Gaps for **X** at x' (Grand-Total ΣX_x-Gaps).

Proposition 8.6. For any given x≥1 values in Set **N**, designated Complexity is represented by $\Sigma\mathbf{N}_x$-Gaps = x-N with N=1.

Proof. Set **N** (for x = 1 to 12): 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. **N**-$\pi(x)$ = 12. There are x - 1 = 11 **N**-Gaps each of '1' magnitude: 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1. $\Sigma\mathbf{N}_x$-Gaps = 11 X 1 = 11. This equates to "x - 1" – regarded as Complexity for solitary **N**. *The proof is now complete for Proposition 8.6*□.

Proposition 8.7. For any given x ≥ 1 values in constituent Set **E** and Set **O**, designated Complexity is represented by $\Sigma\mathbf{EO}_x$-Gaps = 2x - N with N = 4 being baseline minimal.

Proof. Set **E** and Set **O** (for x = 1 to 12): 2, 4, 6, 8, 10, 12 and 1, 3, 5, 7, 9, 11. **E**-$\pi(x)$ = 6 and **O**-$\pi(x)$ = 6. There are $\lfloor \frac{x}{2} \rfloor$-1 = 5 **E**-Gaps each of '2' magnitude: 2, 2, 2, 2, 2. $\Sigma\mathbf{E}_x$-Gaps = 5X2 = 10, and $\lceil \frac{x}{2} \rceil$-1 = 5 **O**-Gaps each of '2' magnitude: 2, 2, 2, 2, 2. $\Sigma\mathbf{O}_x$-Gaps = 5X2 = 10. Grand-Total $\Sigma\mathbf{EO}_x$-Gaps = 10+10=20. Depicted by Table 6 and Figure 12 in Appendix D, 2x - N = "2x - 4" [perpetual constant appearances of "N = 4 being baseline minimal"] is Complexity for **E** & **O** pairing. *The proof is now complete for Proposition 8.7*□.

Proposition 8.8. For selected x ≥ 2 values in constituent Set **P** and Set **C**, designated Complexity is cyclically represented by $\Sigma\mathbf{PC}_x$-Gaps = 2x - N with N = 7 being baseline maximal.

Proof. Set **P** and Set **C** (for x = 2 to 12): 2, 3, 5, 7, 11 and 4, 6, 8, 9, 10, 12. **P**-$\pi(x)$ = 5 and **C**-$\pi(x)$ = 6. There are four **P**-Gaps of 1, 2, 2, 4 magnitude and five **C**-Gaps of 2, 2, 1, 1, 2 magnitude. $\Sigma\mathbf{P}_x$-Gaps = 1 + 2 + 2 + 4 = 9. $\Sigma\mathbf{C}_x$-Gaps = 2 + 2 + 1 + 1 + 2 = 8. Grand-Total $\Sigma\mathbf{PC}_x$-Gaps = 9 + 8 = 17. Depicted by Table 5 and Figure 11, 2x - N = "2x - 7" [perpetual intermittent and cyclical appearances of "N = 7 being baseline maximal"] is Complexity for **P** and **C** pairing. *The proof is now complete for Proposition 8.8*□.

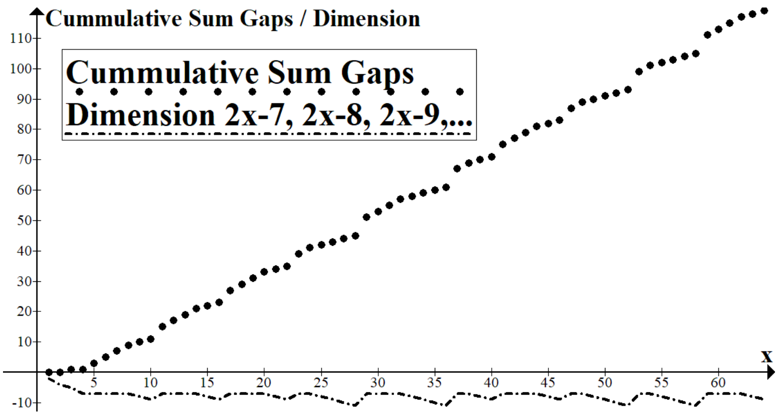

Figure 11. Prime-Composite finite scale mathematical (graphed) landscape. Data for x = 2 to 64

Designated Complexity is (i) x - N with N = 1 for Completely Predictable **N**, (ii) 2x - N with N = 7 (baseline maximal) for Incompletely Predictable **P** & **C**, and (iii) 2x - N with N = 4 (baseline minimal) for Completely Predictable **E** & **O**. Interpretations: **N** has "nil" Complexity, **E** & **O** have minimal Complexity, and **P** & **C** have maximal [varying] Complexity since "2x - 4" Grand-Total Gaps [with N = 4 as defacto baseline] occurring in **E-O** pairing is less than "2x - ≥7" Grand-Total Gaps [with N = 7 as defacto baseline] occurring in **P-C** pairing.

Let both x & N ∈ **N**. We tabulate in Table 5 and graph in Figure 11 [Incompletely Predictable] **P-C** mathematical landscape for a relatively larger x = 2 to 64 here (and ditto for [Completely Predictable] **E-O** mathematical landscape for relatively larger x = 1 to 64 in Appendix D). The term "mathematical landscape" denotes specific mathematical patterns in tabulated and graphed data. "Dimension" contextually denotes Dimension 2x - N whereby (i) allocated [infinite] N values result in Dimensions 2x - 7, 2x - 8, 2x - 9, ..., 2x - ∞ for **P-C** finite scale mathematical landscape and (ii) allocated [finite] N values for **E-O** finite scale mathematical landscape result in Dimension 2x - 4. For **P-C** pairing, initial one-off Dimensions 2x - 2, 2x - 4 and 2x - 5 (in consecutive order) are exceptions [with Dimension 2x - 2 validly representing Number '1' which is neither **P** nor **C**]. For **E-O** pairing, initial one-off Dimension 2x - 2 is an exception. **P-C** mathematical landscape consisting of Dimensions will intrinsically incorporate **P** and **C** in an integrated manner and there are infinite times whereby relevant Dimensions deviate away from 'baseline' Dimension 2x - 7 simply because **P** [and, by default, **C**] in totality are rigorously proven to be infinite in magnitude. In contrast, there is a complete lack of deviation away from 'baseline' Dimension 2x - 4 apart from one-off deviation caused by the initial Dimension 2x - 2 in Appendix D.

Bottom graph in Figure 11 symbolically represent "Dimensions" using ever larger negative integers. Dimensions 2x - 7, 2x - 8, 2x - 9, ..., 2x - ∞ are symbolically represented by -7, -8, -9, ..., ∞ with 2x - 7 displayed as 'baseline' Dimension whereby Dimension trend (Cumulative Sum Gaps) must repeatedly reset itself onto this 'baseline' Dimension on a perpetual basis. Dimensions represented by ever larger negative integers will correspond to **P** associated with ever larger prime gaps and this phenomenon will generally happen at ever larger x values (with complete presence of Chaos and Fractals being manifested in our graph). At ever larger x values, **P**-$\pi(x)$ will overall become larger but with a *decelerating* trend whereas **C**-$\pi(x)$ will overall become larger but with an *accelerating* trend. This support ever larger prime gaps appearing at ever larger x values. 'Overall magnitude of **C** will always be greater than that of **P**' will hold true from x = 14 onwards. For instance, position x = 61 corresponds to **P** 61 which is 18^{th} **P**, whereas [one lower] position x = 60 corresponding to **C** 60 is [much higher] 42^{nd} **C**.

9. Polignac's and Twin Prime Conjectures

Previous section alludes to **P-C** finite scale mathematical landscape. This section alludes to **P-C** infinite scale mathematical landscape. Let 'Y' symbolizes (baseline) Dimension 2x - 7. Let prime gap at $\mathbf{P}_i = \mathbf{P}_{i+1} - \mathbf{P}_i$ with $\mathbf{P}_i$ & $\mathbf{P}_{i+1}$ respectively symbolizes consecutive "first" & "second" **P** in any $\mathbf{P}_i$-$\mathbf{P}_{i+1}$ pairings. We denote (i) Dimensions YY grouping [depicted by 2x - 7 initially appearing twice in (iii)] to represent signal for appearances of **P** pairings other than twin **P** such as cousin **P**, sexy **P**, etc; (ii) Dimension YYYY grouping to represent signal for appearances of **P** pairings as twin **P**; and (iii) Dimension (2x - ≥7)-Progressive-Grouping allocated to 2x - 7, 2x - 7, 2x - 8, 2x - 9, 2x - 10, 2x - 11,..., 2x - ∞ as elements of *precise* and *proportionate* CFS Dimensions representation of an individual $\mathbf{P}_i$ with its associated prime gap namely, Dimensions 2x - 7 & 2x - 7 pairing = twin **P** (with both its prime gap & CFS cardinality = 2); 2x - 7, 2x - 7, 2x - 8 & 2x - 9 pairing = cousin **P** (with both its prime gap & CFS cardinality = 4); 2x - 7, 2x - 7, 2x - 8, 2x - 9, 2x - 10 & 2x - 11 pairing = sexy **P** (with both its prime gap & CFS cardinality = 6); and so on. The higher order [traditionally defined as closest possible] prime groupings of three **P** as **P** triplets, of four **P** numbers as prime quadruplets, of five **P**

x	**P_i or C_i Gaps**	**ΣPC_x-Gaps**	Dim	**x**	**P_i or C_i Gaps**	**ΣPC_x-Gaps**	Dim
1	N/A	0	2x-2	33	C21, 1	58	2x-8
2	P1, 1	0	2x-4	34	C22, 1	59	2x-9
3	P2, 2	1	2x-5	35	C23, 1	60	2x-10
4	C1, 2	1	Y	36	C24, 2	61	2x-11
5	P3, 2	3	Y	37	P12, 4	67	Y
6	C2, 2	5	Y	38	C25, 1	69	Y
7	P4, 4	7	Y	39	C26, 1	70	2x-8
8	C3, 1	9	Y	40	C27, 1	71	2x-9
9	C4, 1	10	2x-8	41	P13, 2	75	Y
10	C5, 2	11	2x-9	42	C28, 2	77	Y
11	P5, 2	15	Y	43	P14, 4	79	Y
12	C6, 2	17	Y	44	C29, 1	81	Y
13	P6, 4	19	Y	45	C30, 1	82	2x-8
14	C7, 1	21	Y	46	C31, 2	83	2x-9
15	C8, 1	22	2x-8	47	P15, 6	87	Y
16	C9, 1	23	2x-9	48	C32, 1	89	Y
17	P7, 2	27	Y	49	C33, 1	90	2x-8
18	C10, 2	29	Y	50	C34, 1	91	2x-9
19	P8, 4	31	Y	51	C35, 1	92	2x-10
20	C11, 1	33	Y	52	C36, 1	93	2x-11
21	C12, 1	34	2x-8	53	P16, 6	99	Y
22	C13, 2	35	2x-9	54	C37, 1	101	Y
23	P9, 6	39	Y	55	C38, 1	102	2x-8
24	C14, 1	41	Y	56	C39, 1	103	2x-9
25	C15, 1	42	2x-8	57	C40, 1	104	2x-10
26	C16, 1	43	2x-9	58	C41, 1	105	2x-11
27	C17, 1	44	2x-10	59	P17, 2	111	Y
28	C18, 2	45	2x-11	60	C42, 2	113	Y
29	P10, 2	51	Y	61	P18, 6	115	Y
30	C19, 2	53	Y	62	C43, 1	117	Y
31	P11, 6	55	Y	63	C44, 1	118	2x-8
32	C20, 1	57	Y	64	C45, 1	119	2x-9

Legend: **C** = composite, **P** = prime, Dim = Dimension, Y = 2x - 7 (for visual clarity), N/A = Not Applicable.

Table 5 Prime-Composite finite scale mathematical (tabulated) landscape. Data for x = 2 to 64.

numbers as prime quintuplets, etc consist of serendipitous groupings abiding to mathematical rule: With exception of three 'outlier' **P** 3, 5, & 7; groupings of any three **P** as **P**, **P**+2, **P**+4 combination (viz. manifesting two consecutive twin **P**) is a mathematical impossibility. The 'anomaly' one of every three consecutive **O** is a multiple of three, and hence this number cannot be **P**, explains this impossibility. Then closest possible **P** grouping for prime triplets must be either **P**, **P**+2, **P**+6 or **P**, **P**+4, **P**+6 format. We will not discuss the other groupings of prime quadruplets, prime quintuplets, etc.

P groupings not respecting traditional closest-possible-prime groupings are also the norm occurring infinitely often, indicating continual presence of prime gaps ≥ 6. As **P** become sparser at larger range, perpetual presence of (i) prime gaps ≥ 6 [proposed to arbitrarily represent 'large gaps'] and (ii) prime gaps 2 & 4 [proposed to arbitrarily represent 'small gaps'] with progressive greater magnitude will cummulatively occur for each prime gap but always in a decelerating manner. With permanent requirement at larger range of intermittently resetting to baseline Dimension 2x - 7 occurring [either two or] four times in a row, nature seems to dictate perpetual twin **P** or one other non-twin **P** occurrences is inevitable.

We dissect Dimension YYYY unique signal for twin **P** appearances: Initial two CFS Dimensions YY components of YYYY represent "first" **P** component of twin **P** pairing. Last two Dimensions YY components of YYYY signifying appearance of "second" **P** component of twin **P** pairing is also the initial first-two-element component of full CFS Dimensions representation for "first" **P** component of following non-twin **P** pairing. Twin **P** are uniquely represented by repeating *single* type Dimension 2x - 7. In all other 'higher order' **P** pairings (with prime gaps ≥ 4), they require *multiple* types Dimension representation. There is qualitative aspect association of *single* type Dimension representation for twin **P** resulting in "less colorful" Plus Gap 2 Composite Number *Continuous Law* as opposed to *multiple* types Dimension representation for all other 'higher order' **P** pairings resulting in "more colorful" Plus-Minus Gap 2 Composite Number *Alternating Law*. 'Gap 2 Composite Number' occurrences in both Laws on finite scale are (directly) observed in Figure 11 & Table 5 for x = 2 to 64, and on infinite scale are (indirectly) deduced using logical arguments for all x values.

We endow all "Dimensions" with exponent of '1' for perusal in on-going mathematical arguments. $\mathbf{P}_1 = 2$ is represented by CFS as Dimension $(2x - 4)^1$ (with both prime gap & CFS cardinality = 1); $\mathbf{P}_2 = 3$ is represented by CFS as Dimensions $(2x - 5)^1$ & $(2x - 7)^1$ (with both prime gap & CFS cardinality = 2); $\mathbf{P}_3 = 5$ is represented by CFS Dimension $(2x - 7)^1$ & $(2x - 7)^1$ (with both prime gap & CFS cardinality = 2), etc.

Proposition 9.1. Let Case 1 be Completely Predictable **E** & **O** pairing and Case 2 be Incompletely Predictable **P** & **C** pairing. Furthermore, let Case 1 and Case 2 be independent of each other. Then for any given x value, there exist grand total number of Dimensions such that it exactly equal to either two combined subtotal number of Dimensions to precisely represent **E** & **O** in Case 1, or three combined subtotal number of Dimensions to precisely represent **P** & **C** & Number '1' in Case 2.

Proof. **N** is directly constituted from either combined **E** & **O** in Case 1 or combined **P** & **C** & Number '1' in Case 2 – Number '1' is neither **P** nor **C**. Correctly designated infinitely many CFS of Dimensions used to represent combined **E** & **O** in Case 1 and combined **P** & **C** & Number '1' in Case 2 must also directly and proportionately be representative of relevant **N** arising from combined subtotal of **E** & **O** in Case 1 and from combined subtotal of **P** & **C** & Number '1' in Case 2. *The proof is now complete for Proposition 9.1*□.

Proposition 9.2. Let Case 1 be Completely Predictable **E** & **O** pairing and Case 2 be Incompletely Predictable **P** & **C** pairing. Furthermore, let Case 1 and Case 2 be independent of each other. Part I: For any given x value apart from x = 1 value in Case 1 and x = 1, 2, and 3 values in Case 2; Dimension $(2x - N)^1$ representations of all Completely Predictable **E** & **O** in Case 1 and all Incompletely Predictable **P** & **C** & Number '1' in Case 2 are given by N = 4 in Case 1 and by N ≥ 7 in Case 2. Part II: Complying with Information-complexity conservation, odd **P** obeys Plus-Minus Composite Gap 2 Number Alternating Law for prime gaps ≥ 4 and Plus Composite Gap 2 Number Continuous Law for prime gap = 2.

Proof. Apart from first Dimension $(2x - 2)^1$ representation in **E** & **O** pairing in Case 1 and first three Dimension $(2x - 2)^1$, Dimension $(2x - 4)^1$ and Dimension $(2x - 5)^1$ representations in **P** & **C** pairing in Case 2; possible N value in Dimension $(2x - N)^1$ representation are shown to be (constantly) baseline minimum 4 for Case 1 and (variably) baseline maximal 7 for Case 2. For Case 2, we again note Dimension $(2x - 2)^1$ to (validly) represent Number '1' which is neither **P** nor **C**. These nominated Dimensions represent possible (constant) baseline "2x - 4" Grand-Total Gaps as per Proposition 8.7 for Case 1 & (variable) baseline "2x - 7" Grand-Total Gaps as per Proposition 8.8 for Case 2. All CFS of Dimensions that can be used to precisely represent combined **E** & **O** in Case 1 will persistently consist of same Dimension $(2x - 4)^1$ after first Dimension $(2x - 2)^1$. Perpetual repeated deviation of N values away from N = 7 (baseline maximum) in Case 2 is simply representing infinite magnitude of **P** & **C**. *The proof is now complete for Part I of Proposition 9.2*□.

With exception of Number '1', all natural numbers must comply with Information-complexity conservation in the sense that they can always be represented by unique CFS Dimensions (2x - N) in a dual manner as **E** or **P** (for solitary Number '2'), **E** or **C**, **O** or **P**, and **O** or **C**. Dimension (2x - 2) validly represents Number '1' which is **O** but is neither **P** nor **C**.

Derived Dimensions will comply with Incompletely Predictable property as explained using **P** '61'. At Position x = 61 equating to $\mathbf{P}_{18}$ = 61, it is represented by CFS Dimensions $(2x - 7)^1$, $(2x - 7)^1$, $(2x - 8)^1$, $(2x - 9)^1$, $(2x - 10)^1$ & $(2x - 11)^1$ (with both prime gap & CFS cardinality = 6). This representation indicates an "unknown but correct" **P** with prime gap = 6 when we intentionally conceal full information '61' = 31^{st} **O** = 18^{th} **P** with prime gap = 6. But to arrive at this unique representation requires complex calculations of all preceding CFS Dimensions thus manifesting hallmark Incompletely Predictable property of CFS Dimensions. This is not so when '61' is treated as Completely Predictable **O** with its position simply calculated as (61 + 1) / 2 = 31^{st} **O** and uniquely represented by CFS Dimension (2x - 4).

Overall sum total of individual CFS Dimensions required to represent every **P** is infinite in magnitude as |**all P**| = $\aleph_0$. Standalone Dimensions YY groupings [representing signals for "higher order" non-twin **P** appearances] &/or as front Dimensions YY (sub)groupings [which by itself is fully representative of twin **P** as Dimensions YYYY appearances] need to recur on an indefinite basis. Then twin **P** and "higher order" cousin **P**, sexy **P**, etc should aesthetically all be infinite in magnitude because (respectively) they regularly and universally arise as part of Dimension YYYY and Dimension YY appearances. An isolated **P** is defined as a **P** such that neither **P** - 2 nor **P** + 2 is **P**. In other words, isolated **P** is not part of a twin **P** pair. E.g., 23 is an isolated **P** since 21 & 25 are both **C**. Repeated inevitable presence of Dimension YY grouping is nothing more than indicating repeated occurrences of isolated **P**. This constitutes another view on Dimension YY.

CIS of Gap 1 Composite Numbers are fully associated with non-twin **P** as they eternally occur in between any two consecutive non-twin **P**. CIS of Gap 2 Composite Numbers are (i) fully associated with twin **P** as they are eternally present in between any twin **P** pair, and (ii) partially associated with non-twin **P** as they are eternally present alternatingly or intermittently in between any two consecutive non-twin **P**. Then (i) Gap 1 Composite Numbers do not have valid representation by **E** prime gap = 2, and (ii) Gap 2 Composite Numbers have valid representations by all **E** prime gaps = ["consistently" only for] 2, ["inconsistently" for each of] 4, 6, 8, 10,.... This is an alternative view on **P** from perspective of CFS composite gaps [instead of CIS prime gaps] with intrinsic patterns having *alternating presence* and *absence* of Gap 2 Composite Numbers associated with every CFS Dimensions representations of **P** with prime gaps ≥ 4, viz. 'Plus-Minus Gap 2 Composite Number Alternating Law'. CFS Dimensions representations of Twin **P** are associated with Gap 2 Composite Numbers, viz. 'Plus Gap 2 Composite Number Continuous Law'.

Yitang Zhang rigorously proved in 2013 there is an unknown even number prime gap 'N' < 70 million that will generate an infinite number of **P**. We will outline the convoluted mathematical arguments [involving *"self-perpetuating" interactions* amongst and between different prime gaps] that totally justify **P-C** infinite scale mathematical landscape when these two [mutually exclusive & dependent] Laws are also fully valid. The crucial implication is there is one 'N' [if not more than one 'N'] < 70 million which could be represented by 2 or 4 or 6 or 8 or...or (70 million - 2) that will generate an infinite number of **P**. In Table 5, twin **P** (prime gap = 2) in its unique CFS Dimensions format always has Gap 2 Composite Numbers in a [constant] pattern thus complying with Plus Gap 2 Composite Number Continuous Law. Assume 'N' = 4. This corresponds to cousin **P** (prime gap = 4) which in its unique CFS Dimensions format has two Gap 1 Composite Numbers & then one Gap 2 Composite Number [combined] pattern alternating with three consecutive Gap 1 Composite Numbers [non-combined] pattern. From this simple observation alone, we deduce we can generate an infinite magnitude of **C** from each composite gaps 1 & 2. Gap 2 Composite Numbers alternating pattern behavior in cousin **P** will not hold true unless all other non-cousin **P** are infinite in magnitude and integratedly supplying essential "driving mechanism" to eternally sustain this Gap 2 Composite Numbers alternating pattern behavior in cousin **P**. At least a non-cousin **P** [such as twin **P**] and cousin **P** in their CFS Dimensions formats must both be CIS intertwined together when depicted using **C** [having composite gaps = 1 & 2] with each supplying their own peculiar (infinite) share of associated Gap 2 Composite Numbers [contributing to overall pool of Gap 2 Composite Numbers].

An inevitable statement in relation to "Gap 2 Composite Numbers pool contribution" based on above reasoning: At the bare minimum, twin **P** and cousin **P** must be infinite in magnitude. An inevitable impression: All generated subsets of **P** from 'small gaps' [of 2 & 4] and 'large gaps' [of ≥ 6] alike should each be CIS thus allowing true uniformity in **P** distribution. Again we see in Table 5 depicting **P-C** data for x = 2 to 64 that, for instance, **P** with prime gap = 6 must also persistently have this 'last-place' Gap 2 Composite Numbers intermittently appearing in certain rhythmic alternating patterns, thus complying with Plus-Minus Gap 2 Composite Number Alternating Law. The above-mentioned "driving mechanism" now supplied by all other **P** with prime gaps ≠ 6 [e.g. (again) from twin **P**] should all be infinite in magnitude. This CFS Dimensions representation for **P** with prime gap = 6 will again generate their infinite share of associated Gap 2 Composite Numbers to contribute to this pool. Thus, presence of this *eternally repeating* last-place Gap 2 Composite Numbers in various alternating pattern in appearances & non-appearances for **P** from prime gaps = 4 & 6 must *self-generatingly* be similarly extended in a mathematically consistent fashion *ad infinitum* to all other remaining infinite number of prime gaps. *The proof is now complete for Part II of Proposition 9.2*□.

10. Rigorous Proofs for the Now-Named as Polignac's and Twin Prime Hypotheses

We outline Theorem Polignac-Twin prime I to IV depicting proofs for Polignac's and Twin prime conjectures in a rigorous manner. Gap 1 Composite Numbers do not have valid representation by **E** prime gap = 2, and Gap 2 Composite Numbers have valid representations by all **E** prime gaps = ["consistently" only for] 2, ["inconsistently" for each of] 4, 6, 8, 10,.... Plus-Minus Gap 2 Composite Number Alternating Law confirms that Gap 2 Composite Numbers present in each **P** with prime gaps ≥ 4 situation must appear as some sort of "rhythmic patterns of alternating presence and absence" for Gap 2 Composite Numbers. Twin **P** with prime gap = 2 obeying Plus Gap 2 Composite Number Continuous Law is understood as special situation of "(non-)rhythmic patterns with continual presence" for relevant Gap 2 Composite Numbers.

In 1849 when French mathematician Alphonse de Polignac (1826 - 1863) was admitted to Polytechnique, he created Polignac's conjecture which relates complete set of odd **P** to all **E** prime gaps. Made earlier by de Polignac in 1846, Twin prime conjecture relating twin prime numbers to prime gap = 2, is then simply a subset of Polignac's conjecture.

Theorem Polignac-Twin prime I. Incompletely Predictable prime numbers $\mathbf{P}_n$ = 2, 3, 5, 7, 11, ..., ∞ or composite numbers $\mathbf{C}_n$ = 4, 6, 8, 9, 10, ..., ∞ are CIS with overall actual location [but not actual positions] of all prime or composite numbers accurately represented by complex algorithm involving prime gaps G_{Pi} viz. $\mathbf{P}_{n+1} = P_n + G_{Pi}$ or involving composite gaps G_{Ci} viz. $\mathbf{C}_{n+1} = C_n + G_{Ci}$ whereby prime & composite numbers are symbolically represented here with aid of 'n' notation instead of usual 'i' notation; and i & n = 1, 2, 3, 4, 5, ..., ∞. $\mathbf{P}_1 = 2$ in first algorithm represents the very first (and only even) **P**. $\mathbf{C}_1 = 4$ in second algorithm represent the very first (and even) **C**.

Proof. We treat above algorithms as unique mathematical objects looking for key intrinsic properties and behaviors. Each **P** or **C** is assigned a unique prime or composite gap. Absolute number of **P** or **C** and (thus) prime or composite gaps are infinite in magnitude. As original formulae containing all **P** or **C** by themselves (viz. without supplying prime or composite gaps as "input information" to generate **P** or **C** as "output complexity"), these algorithms intrinsically incorporate overall actual location [but not actual positions] of all **P** or **C**. *The proof is now complete for Theorem Polignac-Twin prime I*□.

Theorem Polignac-Twin prime II. Set of prime gaps G_{Pi} = 2, 4, 6, 8, 10, ..., ∞ is infinite in magnitude whereby these prime gaps accurately and completely represented by Dimensions $(2x - 7)^1$, $(2x - 8)^1$, $(2x - 9)^1$, ..., $(2x - \infty)^1$ must satisfy Information-complexity conservation in a consistent manner.

Proof. Part I of Proposition 9.2 proved all **P** are represented by Dimension $(2x - N)^1$ with N ≥ 7 for any given x value (except for x = 2 & 3 values). Although x = 1 is neither **P** nor **C**, it is validly represented by Dimension $(2x - 2)^1$. If each **P** is endowed with a specific prime gap value, then each such prime gap must [via logical mathematical deduction] be represented by Dimension $(2x - N)^1$. Complete argument to support this nominated method of prime gap representation using Dimensions will fully comply with Information-complexity conservation was given in Part II of Proposition 9.2. The preceding mathematical statements are correct as there is a unique prime gap value associated with each **P**. Proposition 10.1 below based on principles from Set theory provides further supporting materials that prime gaps are infinite in magnitude. *The proof is now complete for Theorem Polignac-Twin prime II*□.

Theorem Polignac-Twin prime III. To maintain Dimensional analysis (DA) homogeneity, those Dimensions $(2x - N)^1$ from Theorem Polignac-Twin prime II must contain eternal repetitions of well-ordered sets constituted by Dimensions $(2x-7)^1$, $(2x-8)^1$, $(2x-9)^1$, $(2x-10)^1$, $(2x-11)^1$, ..., $(2x-\infty)^1$.

Proof. This Theorem is stated in greater details as To maintain DA homogeneity, those aforementioned [endowed with exponent 1] Dimensions $(2x - N)^1$ from Theorem Polignac-Twin prime II must repeat themselves indefinitely in following specific combinations – (i) Dimension $(2x - 7)^1$ only appearing as twin [two-times-in-a-row] and quadruplet [four-times-in-a-row] sequences, and (ii) Dimensions $(2x - 8)^1$, $(2x - 9)^1$, $(2x - 10)^1$, $(2x - 11)^1$,..., $(2x - \infty)^1$ appearing as progressive groupings of **E** 2, 4, 6, 8, 10,..., ∞." To accommodate the only even **P** '2', exceptions to this DA homogeneity compliance will expectedly occur right at beginning of **P** sequence – (i) one-off appearance of Dimensions $(2x - 2)^1$, $(2x - 4)^1$ and $(2x - 5)^1$ and (ii) one-off appearance of Dimension $(2x - 7)^1$ as a quintuplet [five-times-in-a-row] sequence which is equivalent to (eternal) non-appearance of Dimension $(2x - 6)^1$ at x = 4. [We again note Dimension $(2x - 2)^1$ validly represent Number '1' which is neither **P** nor **C**.] These sequentially arranged sets are CFS whereby from x = 11 onwards, each set always commence initially as 'baseline' Dimension $(2x - 7)^1$ at x = **O** values and always end with its last Dimension at x = **E** values. Each set also have varying cardinality with values derived from all **E**; and correctly combined sets always intrinsically generate two infinite sets of **P** and, by default, **C** in an integrated manner. Our Theorem Polignac-Twin prime III simply represent a mathematical summary derived from Sections 8 & 9 of all expressed characteristics of Dimension $(2x - N)^1$ when used to represent **P** with intrinsic display of DA homogeneity. See Proposition 10.2 for more details on DA aspect. *The proof is now complete for Theorem Polignac-Twin prime III*□.

Theorem Polignac-Twin prime IV. Aspect 1. The "quantitive" aspect to existence of both prime gaps and their associated prime numbers as sets of infinite magnitude will be shown to be correct by utilizing principles from Set theory. Aspect 2. The "qualitative" aspect to existence of both prime gaps and their associated prime numbers as sets of infinite magnitude will be shown to be correct by 'Plus-Minus Gap 2 Composite Number Alternating Law' and 'Plus Gap 2 Composite Number Continuous Law'.

Proof. Required concepts from Set theory involve cardinality of a set with its 'well-ordering principle' application. Supporting materials for these concepts based on 'pigeonhole principle' in relation to Aspect 1 are outlined in Proposition 10.1 below. 'Plus-Minus Gap 2 Composite Number Alternating Law' is applicable to all **E** prime gaps [apart from first **E** prime gap = 2 for twin primes]. The prime gap = 2 situation will obey 'Plus Gap 2 Composite Number Continuous Law'. These Laws are in essence Laws of Continuity inferring underlying intrinsic driving mechanisms that enables infinity magnitude association for both prime gaps & prime numbers to co-exist. By the same token, these Laws have important implication that they must be applicable to the relevant prime gaps on an perpetual time scale. Supporting materials in relation to Aspect 2 are found in Proposition 9.2. *The proof is now complete for Theorem Polignac-Twin prime IV*□.

Two mutually inclusive conditions: Condition 1. Presence of all Dimensions that repeat themselves on an indefinite basis and with exponent of '1' give rise to complete sets of **P** & **C** ["DA-wise one & only one mathematical possibility argument" associated with inevitable *de novo* DA homogeneity], and Condition 2. Presence of any Dimension(s) that do not repeat itself (themselves) on an indefinite basis or with exponent other than '1' give rise to incomplete set of **P** & **C** or incorrect set of non-**P** & non-**C** ["DA-wise mathematical impossibility argument" associated with inevitable *de novo* DA non-homogeneity]. When met, these two conditions fully support the point CFS Dimensions representations of **P** & **C** [with respective prime & composite gaps] are totally accurate. Condition 1 reflect proof from Theorem Polignac-Twin prime III as all **P** & **C** are associated with DA homogeneity when their Dimensions are endowed with exponent of '1'. Condition 2 invoke corollary on inevitable appearance of incomplete **P** or **C** or non-**P** or non-**C** [associated with DA non-homogeneity] being tightly incorporated into this mathematical framework. See Propositions 10.1 & 10.2, and Corollary 10.3 for supporting materials on DA homogeneity & non-homogeneity.

We analyze **P** (& **C**) in terms of (i) measurements based on cardinality of CIS and (ii) pigeonhole principle which states that if n items are put into m containers, with n>m, then at least one container must contain more than one item. We note that ordinality of all infinite **P** (& **C**) is "fixed" implying that each one of the infinite well-ordered Dimension sets conforming to CFS type as constituted by Dimensions $(2x - 7)^1$, $(2x - 8)^1$, $(2x - 9)^1$, $(2x - 10)^1$, $(2x - 11)^1$, ..., $(2x - \infty)^1$ on respective gaps for **P** (& **C**) must also be "fixed".

Proposition 10.1. "Even number prime gaps are infinite in magnitude with each even number prime gap generating odd prime numbers which are again infinite in magnitude" is supported by principles from Set theory and two Laws based on Gap 2 Composite Number.

Proof. We validly exclude even **P** '2' here. Let (i) cardinality $T = \aleph_0$ for Set **all odd P** derived from **E** prime gaps 2, 4, 6,..., ∞, (ii) cardinality $T_2 = \aleph_0$ for Subset **odd P** derived from **E** prime gap 2, cardinality $T_4 = \aleph_0$ for Subset **odd P** derived from **E** prime gap 4, cardinality $T_6 = \aleph_0$ for Subset **odd P** derived from **E** prime gap 6, etc. Paradoxically, (as sets) $T = T_2 + T_4 + T_6 +... + T_\infty$ equation is valid despite (their cardinality) $T = T_2 = T_4 = T_6 =... = T_\infty$ [with well-ordering principle "stating that every non-empty set of positive integers contains a least element" fulfilled by each (sub)set]; and **E** prime gaps are 'infinite in magnitude' can justifiably be perceived instead as 'arbitrarily large in magnitude' since cumulative sum total of **E** prime gaps is relatively much slower to attain the 'infinite in magnitude' status when compared to cumulative sum total of **P** which rapidly attain this status. But if Subset **odd P** derived from one or more **E** prime gap(s) are finite in magnitude, this will breach the $\aleph_0$ cardinality 'uniformity' resulting in (i) DA non-homogeneity and (ii) inequality (as sets) $T > T_2 + T_4 + T_6 +... + T_\infty$. In language of pigeonhole principle "stating that if n items are put into m containers with n > m, then at least one container must contain more than one item", residual **odd P** (still CIS in magnitude) not accounted for by CFS-type **E** prime gap(s) will have to be [incorrectly] contained in one (or more) of composite gap(s). These arguments using cardinality constitute proof that **E** prime gaps & odd **P** generated from each **E** prime gap, are all CIS. *The proof [on "quantitative" aspect] is now complete for Proposition 10.1*□.

Complete set of **P** is represented by Dimensions $(2x - N)^1$. Table 5 & Figure 11 on **PC** finite scale mathematical landscape depict perpetual repeating features used in "qualitative" statements supporting (i) Plus-Minus Gap 2 Composite Number Alternating Law (stated as **C** with composite gaps = 2 present in each of **P** with prime gaps ≥ 4 situation must be observed to appear as some sort of rhythmic patterns of alternating presence and absence of this type of **C**), and (ii) Plus Gap 2 Composite Number Continuous Law (stated as **C** with composite gaps = 2 continual appearances in each of (twin) **P** with prime gap = 2 situation). Plus-Minus Gap 2 Composite Number Alternating Law has built-in intrinsic mechanism to automatically generate all prime gaps ≥ 4 in a mathematically consistent *ad infinitum* manner. Plus Gap 2 Composite Number Continuous Law has built-in intrinsic mechanism to automatically generate prime gap = 2 appearances in a mathematically consistent *ad infinitum* manner. These two Laws refer to end-products obtained from "the second key step of using our unique Dimension (2x - N) system instead of Sieve of Eratosthenes". *The proof [on "qualitative" aspect] is now complete for Proposition 10.1*□.

Proposition 10.2. The presence of Dimensional analysis homogeneity always result in correct and complete set of prime (and composite) numbers.

Proof. DA homogeneity is completely dependent on all Dimensions being consistently endowed with exponent '1'. As all **P** (& **C**) are "fixed", we deduce from Figure 11 & Table 5 that there is one (& only one) way to represent Information-Complexity conservation using our defined Dimensions. Thus, there is one (& only one) way to depict all **P** (& **C**) using these Dimensions in a self-consistent manner and this is achieved with the one (& only one) DA homogeneity possibility. *The proof is now complete for Proposition 10.2*□.

Corollary 10.3. The presence of Dimensional analysis non-homogeneity always result in incorrect and/or incomplete set of prime (and composite) numbers.

Proof. For optimal clarity, we endow all Dimensions with exponent '1' depicted as $(2x - 7)^1$, $(2x - 8)^1$, $(2x - 9)^1$, $(2x - 10)^1$, $(2x - 11)^1$,..., $(2x - \infty)^1$. Proposition 10.2 equates DA homogeneity with correct & complete set of **P** (& **C**). There will be "more than one" DA possibilities when, for instance, a particular [first] term from $(2x - 7)^0$, $(2x - 8)^1$, $(2x - 9)^1$,..., $(2x - \infty)^1$ "terminates" prematurely and does not perpetually repeat [with loss of continuity]. There are intuitively two 'broad' DA possibilities here; namely, (one) DA homogeneity possibility and (one) DA non-homogeneity possibility – Dimension $(2x - 7)^0$ [= 1] with its exponent arbitrarily set as '0' against-all-trend in this case. Thus, Dimension $(2x - 7)^1$ that stop recurring at some point in **P** (or **C**) sequence may cause well-ordered sets (as CFS) from progressive groupings of [**E**] 2, 4, 6, 8, 10,..., ∞ for Dimensions $(2x - 8)^1$, $(2x - 9)^1$, $(2x - 10)^1$, $(2x - 11)^1$,..., $(2x - \infty)^1$ to stop existing (and ultimately for sequential **P** (or **C**) to stop appearing) at that point with ensuing outcome that **P** (or **C**) may overall be incorrectly finite or incomplete in magnitude. Manifesting DA non-homogeneity, particular Dimension(s) endowed with fractional exponent values other than '1' such as '$\frac{2}{5}$' or '$\frac{3}{5}$' will also result in non-**P** (or non-**C**) [fractional] numbers.

Each [fixed] finite scale mathematical landscape "page" as part of [fixed] infinite scale mathematical landscape "pages" for **P** & **C** display Chaos [sensitivity to initial conditions viz. positions of subsequent **P** & **C** are "sensitive" to positions of initial **P** & **C**] and Fractals [manifesting fractal dimensions with self-similarity viz. those aforementioned Dimensions for **P** & **C** are always present, albeit in non-identical manner, for all ranges of $x \geq 2$]. Advocated in another manner, Chaos and Fractals phenomena of those Dimensions for **P** & **C** are always present signifying accurate composition of **P** & **C** in different [predetermined] finite scale mathematical landscape "(snapshot) pages" for **P** & **C** that are self-similar but never identical – and there are an infinite number of these finite scale mathematical landscape "(snapshot) pages". The crucial mathematical step in representing all **P** (& **C**) and prime (& composite) gaps with "Dimensions" based on Information-Complexity conservation allows us to obtain the two Laws based on Gap 2 Composite Numbers and perform DA on these entities. The 'strong' principle argument is [full] presence of DA homogeneity in Dimension(s) equates to complete set of **P** (& **C**) whereas [partial] presence of DA non-homogeneity in Dimension(s) does not equate to complete set of **P** (& **C**). We also advocate for a 'weak' principle argument supporting DA homogeneity for **P** (& **C**) in that nature should not "favor" any particular Dimension(s) to terminate and therefore DA non-homogeneity cannot exist for **P** (& **C**). Abiding to an advocated convention that 'conjecture' be termed 'hypothesis' once proven; we can now label these conjectures as Polignac's and Twin prime hypotheses.

11. Conclusions

This original research paper is advocated to be a novel achievement as we manage to simultaneously model COVID-19 from Medicine as well as solve [unconnected] intractable open problems from Number theory using our versatile Fic-Fac Ratio. In other words, we successfully relate open problems from Number theory when considered as a frontier branch of Mathematics to COVID-19 from Medicine when considered as other science, technology and biology.

Transmitted between animals and people, zoonotic virus SARS-CoV-2 which originated from Wuhan, China causing COVID-19 has been clearly shown not to be a laboratory construct or a purposefully manipulated virus (Andersen et al, 2020). Some overall goals of publishing this paper are to promote Mathematics as the 'Universal Language of Science', and foster global cooperation between all nations on planet Earth to effectively combat and better understand the deadly 2020 Coronavirus pandemic. **Note:** The contextural use of supramaximal elevation or fall of cytokines is based on phenomenon and proposed homeostatic mechanism of *supramaximal elevation* in B-type natriuretic peptide and its N-terminal fragment levels in anephric patients with heart failure [previously introduced by us in 2012 (Ting & Pussell, 2012)]. This mechanism consists of analyzing the permutations with repetition formula: $n^r = n^2$ from combinatorics involving 'n' individual factors that tend to have **non-linear** elevating or lowering properties viz. 'r' = 2. Antibody-directed therapy such as convalescent plasma, hyperimmune-globulin and monoclonal antibodies may also play an important role in more rapid control and clearance of SARS-CoV-2.

From our August 12, 2020 14-page paper entitled "Showing role of Angiotensin-converting enzyme 2 in COVID-19 using novel Fic-Fac Ratio" (J. Y. C. Ting) located at URL https://vixra.org/abs/2008.0082 Science Category: Physics of Biology, we also provide a Case Report for medically-oriented readers of a 43 year-old man with acute respiratory distress syndrome (ARDS) on August 28, 2003 from viral pneumonia together with applications from Fic-Fac Ratio to creatively explain COVID-19's drug and vaccine developments, and mitigation measures to combat the resulting pandemic. This patient had initial severe Type 1 Respiratory Failure viz. decreased PaO2 < 60 mmHg (8.0 kPa) with normal or subnormal PaCO2 < 50 mmHg (6.7 kPa) which rapidly deteriorated to severe Type 2 Respiratory Failure viz. decreased PaO2 < 60 mmHg (8.0 kPa) and increased PaCO2 > 50 mmHg (6.7 kPa) requiring intubation and ventilation.

We envisage two *mutually exclusive* groups of entities: [totally] Unpredictable entities and [totally] Predictable entities. The first group dubbed Type I entities or Completely Unpredictable entities can arise as [totally] random physical processes in nature e.g. radioactive decay is a stochastic (random) process occurring at level of single atoms. According to Quantum theory, it is impossible to predict when a particular atom will decay regardless of how long the atom has existed. For a collection of atoms, expected decay rate is characterized in terms of their measured decay constants or half-lives. The second group is constituted by two subgroups: dubbed Type II entities or Completely Predictable entities e.g. Even-Odd number pairing and dubbed Type III entities or Incompletely Predictable entities e.g. Prime-Composite number pairing. Intuitively, every single mathematical argument from complete set of mathematical arguments required to fully solve a given Incompletely Predictable problem (containing *dependent* types of Incompletely Predictable entities) must be correct obeying Mathematics for Completely Predictable problems. Then Mathematics for Incompletely Predictable problems is literally the mathematical framework for describing complex properties present in these entities.

CIS of [Completely Predictable] natural numbers 1, 2, 3, 4, 5, 6, 7,... having CIS of [Completely Predictable] natural gaps 1, 1, 1, 1, 1, 1,... are constituted by three dependent sets of numbers: (i) CIS of [Incompletely Predictable] odd prime numbers 3, 5, 7, 11, 13, 17,... having CIS of [Incompletely Predictable] prime gaps 2, 2, 4, 2, 4,... plus CFS of solitary [Incompletely Predictable] even prime number 2 having CFS of [Incompletely Predictable] prime gap 1 (ii) CIS of [Incompletely Predictable] even and odd composite numbers 4, 6, 8, 9, 10, 12,... having CIS of [Incompletely Predictable] composite gaps 2, 2, 1, 1, 2, 2,.... and (iii) CFS of solitary odd number '1' [neither prime nor composite]. Treated as Incompletely Predictable problems endowed with "meta-properties", we gave relatively elementary proofs on Polignac's & Twin prime conjectures by (1) employing our unique Dimension (2x - N) system instead of Sieve of Eratosthenes to obtain prime & composite numbers [and Number '1'] and then self-consistently derive 'Plus Gap 2 Composite Number Continuous Law' for prime gap equal to 2 & 'Plus-Minus Gap 2 Composite Number Alternating Law' for prime gaps greater than 2; and (2) demonstating DA homogeneity with presence of [solitary] cardinality value $\aleph_0$ occurring in all [even number prime gap] subsets of prime numbers and in set of even number prime gaps. **Note:** By virtue of wordings used in these two mentioned Laws; then apart from first prime number '2', all other prime numbers [represented by prime gap = 2 and prime gaps > 2] are *dependently* linked to composite numbers [represented by Gap 2 Composite Number].

In effect, for i = 1, 2, 3, 4, 5, ..., ∞; original algorithms $\mathbf{P}_{i+1} = \mathrm{P}_i + \mathrm{G}_{Pi}$ with $\mathbf{P}_1 = 2$ and $\mathbf{C}_{i+1} = \mathrm{C}_i + \mathrm{G}_{Ci}$ with $\mathbf{C}_1 = 4$ are dependently treated as unique mathematical objects with key properties and behaviors to obtain these two Laws. In reverse-engineered manner, all **P**, **C** and Number '1' can be successfully generated from the relevant Dimension (2x - N) system. Future ongoing research on prime numbers are fundamentally challenging and seemingly endless. For instance, when prime gap sizes are tallied, the frequency distribution of prime gaps for primes obtained between 2 to 1,600,000,000 depicts peaks occurring at multiple-of-6 prime gaps = 6, 12, 18, 24, etc [with the largest prime gap found in this range being 292]. It is not known whether this particular "macro-property" with peaks occurring at multiple-of-6 prime gaps will still recur when additional obtained primes much larger than 1,600,000,000 are included in the analysis. Do these peaks persistently manifest themselves at multiple-of-6 prime gaps or will these peaks now manifest themselves at multiple of another even number? Do manifestation of these peaks if they eventually disappear altogether when studied on a much larger scale indirectly support the intuitive notion that "primes arising from each of the even number prime gaps are all infinite [or, even perhaps, all finite] in magnitude"? To rigorously answer these simple questions using correct and complete mathematical arguments will inevitably require novel mathematical directions and/or research techniques.

An interesting 'thought' experiment: In a (imaginary) statistical plot on frequency distribution of prime gaps at extremely large scale, only the [solitary] peak of prime gap = 2 "remains" with its observed height progressively and slowly increasing when this scale is being expanded [by further incorporating additional obtained primes]. Although an unlikely scenario [whereby overall total number of primes will, in principle, numerically still be infinite in magnitude], this observation would theoretically support, but not prove, (i) Twin prime conjecture to be true [viz, there are infinitely many primes arising from prime gap = 2] and (ii) primes arising from each of the prime gaps = 4, 6, 8, 10,... to likely all be finite in magnitude. We deduce another [seemingly] logical but unlikely scenario complying with "overall total number of primes will, in principle, numerically still be infinite in magnitude" is for (i) primes arising from each of prime gaps = 2, 4, 6, 8, 10,... to all be finite in magnitude and (ii) prime gaps to consist of infinitely many (arbitrarily large) even numbers.

Harnassed properties: (1) Nontrivial zeros and two types of Gram points are [*dependently*] derived from "Axes intercept relationship interface" using Riemann zeta function, or its *proxy* Dirichlet eta function; and (2) Prime and composite numbers are [*dependently*] derived from "Numerical relationship interface" using Sieve of Eratosthenes. Using prime gaps as analogy, there are (for instance) "nontrivial zeros gaps" between two consecutive nontrivial zeros with these gaps being Incompletely Predictable entities. Prime number theorem describes asymptotic distribution of prime numbers among positive integers by formalizing intuitive idea that prime numbers become less common as they become larger through precisely quantifying rate at which this occurs using probability. An important secondary spin-off arising out of solving Riemann hypothesis result in absolute and full delineation of prime number theorem. This theorem relates to prime counting function which is usually denoted by $\pi(x)$ with $\pi(x)$ = number of prime numbers ≤ x. In other words, solving Riemann hypothesis is instrumental in proving efficacy of techniques that estimate $\pi(x)$ efficiently. This confirm "best possible" bound for error ("smallest possible" error) of prime number theorem.

In mathematics, logarithmic integral function or integral logarithm li(x) is a special function. Relevant to problems of physics with number theoretic significance, it occurs in prime number theorem as an estimate of $\pi(x)$ whereby its form is defined so that li(2) = 0; viz. li(x) $\equiv \int_2^x \frac{du}{\ln u}$ = li(x) - li(2). There are less accurate ways of estimating $\pi(x)$ such as conjectured by Gauss and Legendre at end of 18th century. This is approximately x/$\ln x$ in the sense $\lim_{x\to\infty} \frac{\pi(x)}{x/\ln x} = 1$. Skewes' number is any of several extremely large numbers used by South African mathematician Stanley Skewes as upper bounds for smallest natural number x for which li(x)<$\pi(x)$. These bounds have since been improved by others: there is a crossing near $e^{727.95133}$ but it is not known whether this is the smallest. John Edensor Littlewood who was Skewes' research supervisor proved in 1914 (Littlewood, 1914) that there is such a [first] number; and found that sign of difference $\pi(x)$ - li(x) changes infinitely often. This refute all prior numerical evidence that seem to suggest li(x) was always > $\pi(x)$. The key point is [100% accurate] perfect $\pi(x)$ mathematical tool being "wrapped around" by [less-than-100% accurate] approximate li(x) mathematical tool infinitely often via this 'sign of difference' changes meant that li(x) is the most efficient approximate mathematical tool. Contrast this with "crude" x/$\ln x$ approximate mathematical tool where studied values diverge away from $\pi(x)$ at increasingly greater rate for larger range of prime numbers.

Using classification system in Appendix C, a formula is either non-Hybrid or Hybrid integer sequence. Inequation with two 'necessary' Ratio (R) or equation with one 'unnecessary' R contains non-Hybrid integer sequence. Equation with one 'necessary' R contains Hybrid integer sequence. "In the limit" Hybrid integer sequence approach unique Position X, it becomes non-Hybrid integer sequence for all Positions ≥ Position X. Kinetic energy (KE) has its endowed units in MJ when m_0 = rest mass in kg and v = velocity in ms^{-1}. In classical mechanics concerning low velocity with v<<c, Newtonian KE = $\frac{1}{2}m_0v^2$. In relativistic mechanics concerning high velocity with v≥0.01c, Relativistic KE = $\frac{m_0c^2}{\sqrt{1-(v^2/c^2)}} - m_0c^2$. Obtained from the later by binomial approximation or by taking first two terms of Taylor expansion for reciprocal square root, the former approximates the later well at low speed. We can arbitrarily and metaphorically assign Pseudo-SIR model to represent '<100% accurracy' Newtonian KE and $SEIR$ model to represent '100% accurracy' Relativistic KE. "In the limit" Newtonian KE at low speed approach Relativistic KE at high speed, we achieve *perfection*.

Critical line of Riemann zeta function is denoted by $\sigma = \frac{1}{2}$ whereby all nontrivial zeros are proposed to be located in the 1859 Riemann hypothesis. Treated as Incompletely Predictable problems, we gave a relatively elementary proof on Riemann hypothesis while also explaining the existence of three types of Gram points and two types of virtual Gram points by analyzing the complex (meta-) properties of relevant Dirichlet Sigma-Power Laws viz. (1) exact DA homogeneity [occurring only once when $\sigma = \frac{1}{2}$] in these Laws with ability to convert their obtained pseudo-zeroes to zeroes in order to obtain nontrivial zeros (Origin intercept points or Gram[x=0,y=0] points) as one type of Gram points and two other closely related types of Gram points [since f(n)'s IP zeroes (t values) = F(n)'s IP pseudo-zeroes (t values) – $\frac{\pi}{2}$]; and (2) inexact DA homogeneity [occurring infinitely many times when $\sigma \neq \frac{1}{2}$] in these Laws with ability to convert their obtained virtual pseudo-zeroes to virtual zeroes in order to obtain two types of virtual Gram points [since f(n)'s IP virtual zeroes (t values) = F(n)'s IP virtual pseudo-zeroes (t values) – $\frac{\pi}{2}$].

Supplementary materials Completely Predictable even numbers with regular intervals of 'non-varying' even gaps of 2 as worked example for E-O Pairing: Approximate area (given as Riemann sum) from $\lim_{n\to\infty}\sum_{i=0}^{n} \Delta x \cdot (2i) = \lim_{n\to\infty}\sum_{i=0}^{n}(2i)$ [as Δx = 1] = Precise area from $\int_0^n (2i)di = [\mathrm{i}^2 + C]_0^n$ = (n^2 - 0^2). The zero at i = 0 [which mathematically equates to i = 0 to 0] can always be identically given by (i) the Riemann sum $\sum_{i=0}^{0}(2i) = (2X0) + (2X0) = 0$ [zero area] and (ii) the solved integral (antiderivative) $\int_0^0 (2i)di$ = (0^2 - 0^2) = 0 [zero area]. We further note for i = 0 to (say) 3 viz, first four [whole] even numbers E_0 = 0, E_1 = 2, E_2 = 4 and E_3 = 6 [with sum total 0 + 2 + 4 + 6 = 12]; then the perpetual phenomenon (Approximate area) $\sum_{i=0}^{3}(2i) = \sum_{i=0}^{1}(2i) + \sum_{i=2}^{3}(2i) = (2X0) + (2X1) + (2X2) + (2X3) = 12$ that overestimate and is non-varyingly related to (Precise area) $\int_0^3 (2i)di = \int_0^1 (2i)di + \int_1^2 (2i)di + \int_2^3 (2i)di$ = $[\mathrm{i}^2+C]_0^3$ = (3^2 - 0^2) = 9 will be validly observed. Incompletely Predictable prime numbers with irregular intervals of 'varying' prime gaps as worked example for P-C Pairing: Approximate area (given as Riemann sum) from $\lim_{n\to\infty}\sum_{i=1}^{n}\Delta x\cdot(P_i + \mathrm{pGap}_i) = \lim_{n\to\infty}\sum_{i=1}^{n}(P_i + \mathrm{pGap}_i)$[as Δx = 1] = Precise area from $\int_1^n (P_i + \mathrm{pGap}_i)di$. We further note for i = 1 to (say) 4 viz, first four prime numbers 2, 3, 5 and 7 [with sum total 2 + 3 + 5 + 7 = 17]; then the perpetual phenomenon (Approximate area) $\sum_{i=1}^{4}(P_i + \mathrm{pGap}_i) = \sum_{i=1}^{2}(P_i + \mathrm{pGap}_i) + \sum_{i=3}^{4}(P_i + \mathrm{pGap}_i)$ = (2+1) + (3+2) + (5+2) + (7+4) = 26 that [arbitrarily] underestimate and is varyingly related to (Precise area) $\int_1^4 (P_i + \mathrm{pGap}_i)\mathrm{d}i = \int_1^2 (P_i + \mathrm{pGap}_i)\mathrm{d}i + \int_2^3 (P_i + \mathrm{pGap}_i)\mathrm{d}i + \int_3^4 (P_i + \mathrm{pGap}_i)\mathrm{d}i = \int_1^2 (3 + 2) + (2 + 1)\mathrm{d}i + \int_2^3 (5 + 2) + (3 + 2)\mathrm{d}i + \int_3^4 (7 + 4) + (5 + 2)\mathrm{d}i = [8i + C]_1^2 + [12i + C]_2^3 + [18\mathrm{i} + \mathrm{C}]_3^4 = 8 + 12 + 18 = 38$ can only be metaphorically, but without true mathematical validity, observed. This is because our algorithm, using variable i and given as P_{i+1} = P_i + pGap_i, is only defined at end-points a, b. It is not a function and is thus not defined in interval [a,b].

Eq. (9) is the only version of simplified Dirichlet eta function (as zeroes) that can uniquely and non-negotiably incorporate trigonometry identity $\cos(n) - \sin(n) = \sqrt{2}\sin\left(n + \frac{3}{4}\pi\right)$ **application and mathematically allow / explain Origin intercept points (nontrivial zeros or Gram[x=0,y=0] points) to exist when** $\sigma = \frac{1}{2}$ **[with sum exponents =** $2(-\sigma)$ **= -1 being a (negative) whole number].** This trigonometry identity can only involve sine and cosine terms with exponent being whole number 1 [and not fractional number]. Thus, we can only comply with exact Dimensional analysis homogeneity for this Incompletely Predictable problem with sum exponents = $2(1-\sigma)$ = 1 being also a whole number [and not fractional number] for derived Dirichlet Sigma-Power Law (as pseudo-zeroes to zeroes conversion). Eq. (9) is equivalently written as $\sum_{n=1}^{\infty} -(2n)^{-\sigma}(\sin(t\ln(2n)) - \cos(t\ln(2n))) - \sum_{n=1}^{\infty} -(2n-1)^{-\sigma}(\sin(t\ln(2n-1)) - \cos(t\ln(2n-1))) = 0$ that contains both sine and cosine terms. This will stop unsuspecting readers from incorrectly treating this function [without $\sqrt{2}$ and $\frac{3}{4}\pi$ constants] as $\sum_{n=1}^{\infty}(2n)^{-\sigma}\sin(t\ln(2n) - \sum_{n=1}^{\infty}(2n-1)^{-\sigma}\sin(t\ln(2n-1) = 0$. Serendipitously, this last equation is precisely the simplified Dirichlet eta function for Gram[y=0] points.

Just as dE/di = d(2i)/di = (constant) 2 mentioned in E-O Pairing for even numbers has its perpetually valid [intrinsic] simple property to precisely indicate even number gaps = 2; so are presence of exact and inexact Dimensional analysis (DA) homogeneity in simplified Dirichlet eta function [as zeroes] or Dirichlet Sigma-Power Law [as pseudo-zeroes to zeroes conversion] has its perpetually valid [intrinsic] complex (meta-) property to precisely differentiate between Gram points and virtual Gram points. The word 'Dimension' or 'Dimensional' in DA is traditionally, conveniently and arbitrarily used to indicate [but not used to define] analysis on 'units of measurement' [such as kg, m and, advocated by us, exponents involving σ and '1'] for corresponding 'base quantities' [such as mass, length and, advocated by us, (2n), (2n-1) and Dimension (2x - N) = Dimension $(2x - N)^1$]. Here, the two possible scenarios are [mathematically valid] DA homogeneity and [mathematically invalid] DA non-homogeneity. But the word Dimension is an English word with nil acceptance by anyone that it can even remotely indicate or resemble 'units of measurement' such as kg or m instead of exponents. We advocate use of [mathematically valid] exact and inexact DA homogeneity which are mathematically fully defined in the very specific context of Dirichlet Sigma-Power Law [as pseudo-zeroes to zeroes conversion] and simplified Dirichlet eta function [as zeroes] is also conveniently and arbitrarily correct; viz. mathematically, this action is justifiably correct.

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Appendix

A. Gram's Law and traditional 'Gram points'

Named after Danish mathematician Jørgen Pedersen Gram (June 27, 1850 – April 29, 1916), traditional 'Gram points' (Gram[y=0] points) are other conjugate pairs values on critical line defined by $Im\{\zeta(\frac{1}{2} \pm \imath t)\} = 0$. Belonging to Incompletely Predictable entities, they obey Gram's Rule and Rosser's Rule with some characteristic properties outlined by our brief exposition below: Z function is used to study Riemann zeta function on critical line. Defined in terms of Riemann-Siegel theta function & Riemann zeta function by $Z(t) = e^{\imath\theta(t)}\zeta(\frac{1}{2} + \imath t)$ whereby $\theta(t) = \arg(\Gamma(\frac{(2\imath t+1)}{4})) - \frac{\ln \pi}{2}t$; it is also called Riemann-Siegel Z function, Riemann-Siegel zeta function, Hardy function, Hardy Z function, & Hardy zeta function.

The algorithm to compute Z(t) is called Riemann-Siegel formula. Riemann zeta function on critical line, $\zeta(\frac{1}{2} + \imath t)$, will be real when $\sin(\theta(t)) = 0$. Positive real values of t where this occurs are called 'Gram points' and can also be described as points where $\frac{\theta(t)}{\pi}$ is an integer. Real part of this function on critical line tends to be positive, while imaginary part alternates more regularly between positive & negative values. That means sign of Z(t) must be opposite to that of sine function most of the time, so one would expect nontrivial zeros of Z(t) to alternate with zeros of sine term, i.e. when θ takes on integer multiples of π. This turns out to hold most of the time and is known as Gram's Rule (Law) – a law which is violated infinitely often though. Thus Gram's Law is statement [on the manifested property] that nontrivial zeros of Z(t) alternate with 'Gram points'. 'Gram points' which satisfy Gram's Law are called 'good', while those that do not are called 'bad'. A Gram block is an interval such that its first & last points are good 'Gram points' and all 'Gram points' inside this interval are bad. Counting nontrivial zeros then reduces to counting all 'Gram points' where Gram's Law is satisfied and adding the count of nontrivial zeros inside each Gram block. With this process we need not locate nontrivial zeros but just have to accurately compute Z(t) to show that it changes sign.

B. Ratio Study and Inequations

A mathematical equation, containing ≥ one variables, is a statement that values of two [' left-hand side' (LHS) and 'right-hand side' (RHS)] mathematical expressions is related as equality: LHS = RHS; or as inequalities: LHS < RHS, LHS > RHS, LHS ≤ RHS, or LHS ≥ RHS. A ratio is one mathematical expression divided by another. The term 'unnecessary' Ratio (R) for any given equation is explained by two examples: (1) LHS = RHS and with rearrangement, 'unnecessary' R is given by $\frac{LHS}{RHS} = 1$ or $\frac{RHS}{LHS} = 1$; and (2) LHS > RHS and with rearrangement, ' unnecessary' R is given by $\frac{LHS}{RHS} > 1$ or $\frac{RHS}{LHS} < 1$. Consider exponent y ∈ all **R** values and base x ∈ **R**≥0 values for mathematical expression x^y. Equations such as $x^1 = x$, $x^0 = 1$ and $0^y = 0$ are all valid. Simultaneously letting both x and y = 0 is an incorrect mathematical action because x^y as function of two-variables is not continuous and is undefined at Origin. If we elect to carry out this "balanced" action [equally] on x and y, we obtain (simple) inequation $0^0 \neq 1$ with associated perpetual obeyance of '=' equality symbol in x^y for all applicable **R** values except when both x and y = 0. The Number '1' value in this inequation is justified by two arguments: I. Limit of x^y value as both x and y tend to zero (from right) is 1 [thus fully satisfying criterion "x^y is right continuous at the Origin"]; and II. Expression x^y is product of x with itself y times [and thus x^0, the "empty product", should be 1 (no matter what value is given to x)].

Mathematical operator 'summation' obey the law: We can break up a summation across a sum or difference but not across a product or quotient viz, factoring a sum of quotients into a corresponding quotient of sums is an incorrect mathematical action. But if we elect to carry out this action equally on LHS and RHS products or quotients in a suitable equation, we obtain two (unique) 'necessary' R denoted by R1 for LHS and R2 for RHS whereby R1 ≠ R2 relationship always hold. We define 'Ratio Study' as intentionally performing this incorrect [but "balanced"] mathematical action on suitable equation [equivalent to one (non-unique) 'unnecessary' R] to obtain its inequation [equivalent to two (unique) 'necessary' R]. We note that performing Ratio Study to obtain inequations involving $\mathbb{C}$ does not involve defining a relation between two $\mathbb{C}$. Given Set $\mathbb{C}$ is a field (but not an ordered field), it is also not possible to define a relation between two given (z_1 and z_2) $\mathbb{C}$ as $z_1 < z_2$ since inequality operation is not compatible with addition and multiplication.

C. Hybrid method of Integer Sequence classification

Let $a_k(n)$ denote an arbitrary list of integer sequence whereby k = 1, 2, 3,... and all integer sequence are of infinite length. Consider two integer sequence $a_1(n)$ and $a_2(n)$ which are (1) specifically given by their respective type of inequality (or equality) "mathematical operators"; and (2) based on one nominated type of "mathematical function". Integers from $a_1(n)$ and $a_2(n)$ are identical to each other except for the *interspersed* finite number of 'exceptional' terms located in either $a_1(n)$ or $a_2(n)$. In other words, this special phenomenon will allow definition for a subsequence with finitely many altered elements known as 'exceptional' terms. The integer sequence having these 'exceptional' terms is Hybrid integer sequence, and the other is its [corresponding] non-Hybrid integer sequence. Then these two unique integer sequences when grouped together belongs to "Hybrid method of Integer Sequence classification".

Thus, this novel classification enables meaningful pairing of two unique integer sequences. Involving the factorial (!) function, this is exampled by our exotic A228186 Hybrid integer sequence (Ting, 2013) with its corresponding A100967 non-Hybrid integer sequence (Noe, 2004). It is currently unclear whether (1) there are more than one existing pair of these extraordinary integer sequences based on ! function, and (2) whether they could involve other mathematical functions apart from ! function. With challenge to discover more, A228186 is the first ever known Hybrid integer sequence synthesized from a "Combinatorics Ratio". For our 'Position i' notation, we let 'i' as belonging to the complete set of natural numbers. We conventionally assign 'n' to denote 'Position i' viz., n = 0, 1, 2, 3, 4, 5,..., ∞. We now succinctly explain below the complete and correct mathematical arguments that rigorously substantiate A228186 and A100967 when grouped together as belonging to "Hybrid method of Integer Sequence classification".

Precisely defined as "Smallest natural number k > n such that (k+n+1)!(k-n-2)! < 2k!(k-1)!" or alternatively defined as "Greatest natural number k > n such that calculated peak values for ratio R = $\frac{CombinationsWithRepetition}{CombinationsWithoutRepetition} = \frac{(k+n-1)!(n-k)!}{n!(n-1)!}$ belong to maximal rational numbers < 2"; A228186 is equal to [infinite length] non-Hybrid (usual

x	E_i or O_i Gaps	ΣEO_x-Gaps	Dim	x	E_i or O_i Gaps	ΣEO_x-Gaps	Dim
1	O1, 2	0	2x-2	33	O17, 2	62	Y
2	E1, 2	0	Y	34	O17, 2	64	Y
3	O2, 2	2	Y	35	O17, 2	66	Y
4	E2, 2	4	Y	36	O17, 2	68	Y
5	O3, 2	6	Y	37	O17, 2	70	Y
6	E3, 2	8	Y	38	O17, 2	72	Y
7	O4, 2	10	Y	39	O17, 2	74	Y
8	E4, 2	12	Y	40	O17, 2	76	Y
9	O5, 2	14	Y	41	O17, 2	78	Y
10	E5, 2	16	Y	42	O17, 2	80	Y
11	O6, 2	18	Y	43	O17, 2	82	Y
12	E6, 2	20	Y	44	O17, 2	84	Y
13	O7, 2	22	Y	45	O17, 2	86	Y
14	E7, 2	24	Y	46	O17, 2	88	Y
15	O8, 2	26	Y	47	O17, 2	90	Y
16	E8, 2	28	Y	48	O17, 2	92	Y
17	O9, 2	30	Y	49	O17, 2	94	Y
18	E9, 2	32	Y	50	O17, 2	96	Y
19	O10, 2	34	Y	51	O17, 2	98	Y
20	E10, 2	36	Y	52	O17, 2	100	Y
21	O11, 2	38	Y	53	O17, 2	102	Y
22	E11, 2	40	Y	54	O17, 2	104	Y
23	O12, 2	42	Y	55	O17, 2	106	Y
24	E12, 2	44	Y	56	O17, 2	108	Y
25	O13, 2	46	Y	57	O17, 2	110	Y
26	E13, 2	48	Y	58	O17, 2	112	Y
27	O14, 2	50	Y	59	O17, 2	114	Y
28	E14, 2	52	Y	60	O17, 2	116	Y
29	O15, 2	54	Y	61	O17, 2	118	Y
30	E15, 2	56	Y	62	O17, 2	120	Y
31	O16, 2	58	Y	63	O17, 2	122	Y
32	E16, 2	60	Y	64	O17, 2	124	Y

Legend: E = even, **O** = odd, Dim = Dimension, Y = 2x - 4 (for visual clarity).
Table 6 Even-Odd mathematical (tabulated) landscape. Data for x = 1 to 64.

garden-variety) integer sequence A100967 except for the interspersed finite 21 'exceptional' terms at Positions 0, 11, 13, 19, 21, 28, 30, 37, 39, 45, 50, 51, 52, 55, 57, 62, 66, 70, 73, 77, and 81 with their values given by relevant A100967 terms plus 1. The first 49 terms [from Position 0 to Position 48] of A100967 "Least k such that binomial(2k+1, k-n-1) ≥ binomial(2k, k) viz. (2k+1)!k!k! ≥ (2k)!(k-n-1)!(k+n+2)!" are listed below: 3, 9, 18, 29, 44, 61, 81, 104, 130, 159, 191, 225, 263, 303, 347, 393, 442, 494, 549, 606, 667, 730, 797, 866, 938, 1013, 1091, 1172, 1255, 1342, 1431, 1524, 1619, 1717, 1818, 1922, 2029, 2138, 2251, 2366, 2485, 2606, 2730, 2857, 2987, 3119, 3255, 3394, and 3535. For those 21 'exceptional' terms: at Position 0, A228186 (= 4) is given by A100967 (= 3) + 1; at Position 11, A228186 (= 226) is given by A100967 (= 225) + 1; at Position 13, A228186 (= 304) is given by A100967 (= 303) + 1; at Position 19, A228186 (= 607) is given by A100967 (= 606) + 1; etc. A useful concept: Commencing from Position 0 onwards "in the limit" this Position approaches 82, A228186 Hybrid integer sequence is identical to and becomes A100967 non-Hybrid integer sequence for all Position i ≥ 82.

D. Tabulated and graphical data on Even-Odd mathematical landscape

We tabulate in Table 6 and graph in Figure 12 [Completely Predictable] **E-O** mathematical landscape for x = 1 to 64. Involved Dimensions are 2x - 2 & 2x - 4 with Y denoting Dimension 2x - 4 for visual clarity. Mathematical landscape of Dimension 2x - 4 (except for first and only Dimension 2x - 2) intrinsically incorporates **E** & **O** in an integrated manner. Except for first **O**, all Completely Predictable **E** & **O** and their associated gaps are represented by countable finite set of [single] Dimension 2x - 4. Dimensions 2x - 2 & 2x - 4 are symbolically represented by -2 & -4 with

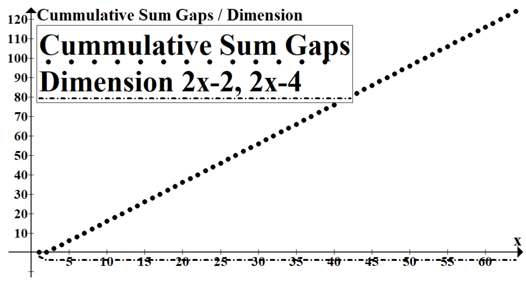

Figure 12. Even-Odd mathematical (graphed) landscape. Data for x = 1 to 64

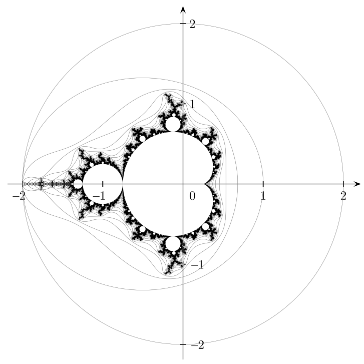

Figure 13. The Mandelbrot set can be faithfully reproduced when iterated using identical initial conditions. It displays quasi-self-similarity

2x - 4 displayed as 'baseline' Dimension whereby Dimension trend (Cumulative Sum Gaps) must reset itself onto this (Grand-Total Gaps) 'baseline' Dimension after initial Dimension 2x - 2 on a permanent basis. Graphical appearances of Dimensions symbolically represented by two negative integers are Completely Predictable with both Even-$\pi(x)$ and Odd-$\pi(x)$ becoming larger at a constant rate. There is a complete absence of Chaos and Fractals phenomena.

E. Complexity arising from Life at the Edge of Chaos-Fractal

The most famous fractal equation is the 2D Mandelbrot set (Figure 13), named after the mathematician Benoit Mandelbrot of Yale University, who coined the name "fractals" for the resulting shapes in 1975. The Mandelbrot set is the set of complex numbers c for which the function $f_c(z) = z^2 + c$ does not diverge when iterated from $z = 0$, i.e., for which the sequence $f_c(0)$, $f_c(f_c(0))$, etc, remains bounded in absolute value. Three common techniques for generating fractals are:
Escape-time fractals – These are defined by a (deterministic) recurrence relation at each point in a space (such as the complex plane). Examples: Mandelbrot set, Julia set, Burning Ship fractal and Lyapunov fractal.
Iterated function systems – These have a (deterministic) fixed geometric replacement rule. Examples: Cantor set, Sierpinski carpet, Sierpinski gasket (Figure 14), Peano curve, Koch snowflake, Harter-Heighway dragon curve, T-Square and Menger sponge.

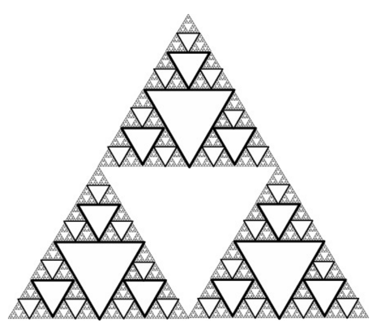

Figure 14. The Sierpinski gasket, also called Sierpinski triangle, is a fractal attractive fixed set with overall shape of an equilateral triangle subdivided recursively into smaller equilateral triangles. It displays exact self-similarity

Random fractals – Generated by stochastic rather than deterministic processes. Examples: trajectories of the Brownian motion, Levy flight, fractal landscapes and the Brownian tree. The latter yields so-called mass- or dendritic fractals, for example, diffusion-limited aggregation or reaction-limited aggregation clusters.

Fractals can be classified according to the three types of self-similarity:
Exact self-similarity – the fractal appears identical at different scales. Fractals defined by iterated function systems often display exact self-similarity.
Quasi-self-similarity – the fractal appears approximately (but not exactly) identical at different scales. Quasi-self-similar fractals contain small copies of the entire fractal in distorted and degenerate forms. Fractals defined by recurrence relations are usually quasi-self-similar but not exactly self-similar.
Statistical self-similarity – the fractal has numerical or statistical measures which are preserved across scales. Most reasonable definitions of ”fractal” trivially imply some form of statistical self-similarity. (Fractal dimension itself is a numerical measure which is preserved across scales.) Random fractals are examples of fractals which are statistically self-similar, but neither exactly nor quasi-self-similar.

In mathematics, ’sensitivity to initial conditions’ means that each point in a chaotic system is arbitrarily closely approximated by other points, with significantly different future paths or trajectories. Thus, an arbitrarily small change or perturbation of the current trajectory may lead to significantly different future behavior. A ’self-similar’ object is exactly or approximately similar to a part of itself (i.e., the whole has the same shape as one or more of the parts). Useful overview of deterministic [*not* stochastic] processes: ’Chaos’ is [mathematically] synonymous with chaotic nonlinear dynamical systems which are ”complex systems” described by discrete or continuous nonlinear (deterministic) equations or algorithms and manifesting the key feature of *sensitivity to initial conditions*. ’Fractal’ is [geometrically] synonymous with fractional geometry which deals with ”geometrical objects” (graphs) having fractional (fractal) dimensions and manifesting the key feature of *self-similarity*. **Each unique geometrical objects when deterministically computed from a given Chaos is precisely its Fractal**. The mentioned ’complex systems” in this paper contain well-defined Incompletely Predictable entities such as nontrivial zeros and two types of Gram points specified by Riemann zeta function (or its proxy Dirichlet eta function) together with prime and composite numbers specified by Sieve of Eratosthenes. We observe complete presence of Chaos and Fractals phenomena manifested in Figure 11 that involve the relevant Incompletely Predictable entities whereby it manifests **self-similarity or, more precisely, Quasi-self-similarity**. By the same token, we will observe complete absence of Chaos and Fractals phenomena in Figure 12 that involves the Completely Predictable entities of even and odd numbers. Counterintuitively, one could technically consider in a strict geometrical sense that the graphed ”straight line” [which is clearly identical at different scales] in Figure 12 containing Completely Predictable entities of even and odd numbers does manifest Chaos and Fractals phenomena that display Exact self-similarity.

The inspiring idiom ”Complexity arising from Life at the Edge of Chaos-Fractal” led us to provide a Hierarchical Classification for Elementary-Emergent Fundamental Laws (EEFL). Implied by the definition for ’Fundamental Laws’, then EEFL must by default be perfectly applicable to Terrestrial human beings on planet Earth (endowed with advanced civilization) and also Extraterrestrial alien beings on some hypothetical remote planet (endowed with super-advanced civilization). Thus one could also appropriately coin our Fundamental Laws as the Extraterrestrial-Terrestrial EEFL.

Human heart can simplistically be thought of having a ”plumbing system” consisting of heart muscle pump, coronary arteries and cardiac valves; and an ”electrical system” consisting of specialized heart muscle cells giving rise to pacemakers and electrical conduction pathways & networks. Human brain is the most complex organ in human body. It produces our every thought, action, memory, feeling and experience of the world. It consists of jelly-like mass of tissue weighing around 1.4 kilograms, and contains a staggering one hundred billion nerve cells (neurons). The complexity of the connectivity between these cells is mind-blowing with each neuron making contact with thousands or even tens of thousands of others, via tiny structures called synapses. Our brains form about a million new connections per second. Our conscious mind commands and our subconscious mind obeys. Thus, our subconscious mind is an unquestioning servant that works day and night to make our behavior fits a pattern consistent with our emotionalized thoughts, hopes, and desires. The pattern and strength of the connections is constantly changing and no two brains are alike. It is in these changing connections that memories are stored, subconscious mind operate, habits learned and personalities shaped by reinforcing certain patterns of brain activity, and losing others. Structurally, the human brain contains ”grey matter” and ”white matter”. The grey matter is the cell bodies of the neurons, while the white matter is the branching network of thread-like tendrils called dendrites and axons that spread out from the cell bodies to connect to other neurons. However, the human brain also has another even more numerous type of cell called glial cells. These outnumber neurons about ten times over. Once thought to be support cells, they are now known to amplify neural signals and to be as important as neurons in mental calculations.

The three types of entities:

Type I Entities	Completely Unpredictable entities
Type II Entities	Completely Predictable entities
Type III Entities	Incompletely Predictable entities

The location-based definitions for Type II Entities and Type III Entities are:

Completely Predictable (Type II) Entities: Locationally defined as entities whose position is independently determined by simple calculations using simple equation or algorithm without needing to know related positions of all preceding entities in neighborhood.
Incompletely Predictable (Type III) Entities: Locationally defined as entities whose position is dependently determined by complex calculations using complex equation or algorithm with needing to know related positions of all preceding entities in neighborhood.

In order of increasing complexity, we have the following Laws:

Law I: Simple Elementary Fundamental Law for ”simple” Nonliving Things with simple properties
Law II: Complex Elementary Fundamental Law for ”complex” Nonliving Things with complex properties
Law III: Simple Emergent Fundamental Law for ”simple” Living Things with simple properties
Law IV: Complex Emergent Fundamental Law for ”complex” Living Things with complex properties

Solving Completely Predictable and Inompletely Predictable problems:

Solving Completely Predictable problems in both Simple 'Nonliving' Elementary and 'Living' Emergent cases: Many Simple properties ⟶ [Simple Elementary and Emergent Solutions]
Solving Incompletely Predictable problems in both Complex 'Nonliving' Elementary and 'Living' Emergent cases: Many Simple properties ⟶ Few Complex properties ⟶ [Complex Elementary and Emergent Solutions]

Postulated association between Entities and Laws:

Law I is obeyed by Type II Entities e.g. (simple Nonliving Thing) even and odd numbers with even number and odd number gaps.
Law II is obeyed by Type III Entities e.g. (complex Nonliving Thing) nontrivial zeros related to Riemann hypothesis, and prime numbers related to Polignac's & Twin prime conjectures.
Law III is obeyed by Type II + Type III Entities e.g. (simple Living Thing) human heart as an organ manifesting hemodynamic and electrical properties.
Law IV is obeyed by Type I + Type II + Type III Entities e.g. (complex Living Thing) human brain which is often dubbed ”the most complex structure in the universe” manifesting a whole range of neuro-psychological and neuro-psychiatric properties.

Creationism versus Evolution debate for Nonliving Things (obeying Law I and Law II) giving rise to Living Things (obeying Law III and Law IV) is compared and contrasted below:

Process of Creationism: Associated with major religions e.g. Islam and Christianity. From the Bible, Adam and Eve was estimated to be created just over 6,000 years ago by world's leading young-earth creationist organizations.
Process of Evolution: Atheists usually believe the Big Bang (when our Universe was created) occur about 13.8 billion years ago. The first true man appeared 13.7998 billion years after the beginning (or about 200,000 years ago).

In short summary, the human brain manifest Natural Intelligence, consciousness, self-awareness, memory; mental illness such as anxiety, depression, schizophrenia; ”dark triad” of personality consisting of three negative traits [viz. the tendency to manipulate others (Machiavellianism), seek admiration and special treatment (narcissism), and to be callous and insensitive (psychopathy)]; and ”light triad” of personality consisting of three positive traits [viz. the opposite of Machiavellianism (Kantianism), valuing dignity and worth of each individual person (humanism), and believing that people are fundamentally good (Faith in humanity)]. Artificial Intelligence (AI) in Nonliving Things can be regarded as human endeavor to simulate Natural Intelligence in Living Things using powerful computers such as super-computers or quantum computers. DNA is a double helix, while RNA is a single helix. Both have sets of nucleotides that contain genetic information. DNA is a molecule that contains instructions for Living Things to be born, mature, reproduce, and died. One would commonly concur that there are 'Simple' Living Things such as bacteria without brain and 'Complex' Living Things such as intelligent human with highly developed brain. The dividing line between Living Things and Nonliving Things is that the former is ”powered” by DNA with an important implication that Natural Intelligence, consciousness and self-awareness can only be ”powered” by DNA [which are organic]. Then by reasonable assumption, properties such as consciousness and self-awareness can never be present in AI created using computers [which are inorganic].