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Altering the Principle of Relativity

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Abstract

A unique hyperbolic geometry paradigm requires altering the Relativistic principle that absolute velocity is unmeasurable. There is no absolute velocity, but in the case where a constant velocity is made from a half-angle velocity, a variable velocity is the same as (absolute) acceleration. Relativity is based on local Lorentz geometry. Our mathematical geometry constructs circle and hyperbola vectors with hyperbolic terms in an original formulation of complex numbers. We use a point on a hyperbola as a frame of reference. A theory is given that time and our velocity are inversely related. The physical laws of motion by Galileo, Newton and Einstein are forged using the half-angle velocity to electromagnetic velocity. The field of kinetic, potential and gravitational force accelerations is established. An experiment exemplifies the math from the Earth's frame of reference. We discover a possible dark energy and gravitational accelerations and a geometry of gravitational collapse.

Keywords: acceleration, angle, coordinate, time, velocity

1. Introduction

Conventional trigonometry defines coordinates on a circle as:

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x} \tag{1}$$
$$\csc\theta = \frac{r}{y} \qquad \sec\theta = \frac{r}{x} \qquad \cot\theta = \frac{x}{y}$$

where $r = (x^2 + y^2)^{1/2}$, of which we will call angle $\theta_c = \tan^{-1}\frac{y_c}{x}$. H.S.M. Coxeter, F.R.S. (1907–2003), explains (Coxeter, 1998, pp. 238, 277, 295, 306) how in a right triangle ABC parallel lines $BA = c = \infty$ and CA meet in infinity (Coxeter, 1978) with the angle of parallelism $\angle ABC$ geodesic $B = \theta_h = \tan^{-1}\frac{1}{x}$ as follows:

$$\begin{aligned} \tan a &= \tan c \cos B & \cos A &= \cos a \sin B & \tan b &= \sin a \tan B \\ \tanh a &= \tanh\infty \cos B & \cos 0 &= \cosh a \sin B & \tanh b &= \sinh a \tan B \\ \tanh a &= 1 \cos B & 1 &= \cosh a \sin B & 1 &= \sinh a \tan B \\ \tanh a &= \cos B & \operatorname{sech} a &= \sin B & \sinh a &= \cot B. \end{aligned}$$

Let us beforehand consider the Fig. 1(a) angle ψ lying on a spherical circle with a horizontal hyperbola $x^2 - y^2 = 1$. (Chrystal, 1931, pp. 312-313) We say $\tan\psi = \sinh a = \sinh\sinh^{-1} y = y$ because $\tan 2\frac{\psi}{2} = \frac{2\tan\psi/2}{1-\tan^2\psi/2} = \frac{2\tanh a/2}{1-\tanh^2 a/2} = 2\sinh\frac{a}{2}\cosh\frac{a}{2} = \sinh 2\frac{a}{2}$. But rather than saying $\tan\psi = \tanh a$, we have $\tan\frac{1}{2}\psi = \tanh\frac{1}{2}a$. Now, Coxeter's (Coxeter, 1989, pp. 92–94, 267, 291, 315, 372, 376–377) angle of parallelism $B = \theta_h = \tan^{-1}\frac{1}{x}$ in Fig. 1(b) displays the classical vertical hyperbola $y^2 - x^2 = 1$ coordinates (Martin, 1972, pp. 318–319, 414–434) partially used (Bolyai, 1987, p. 207) by

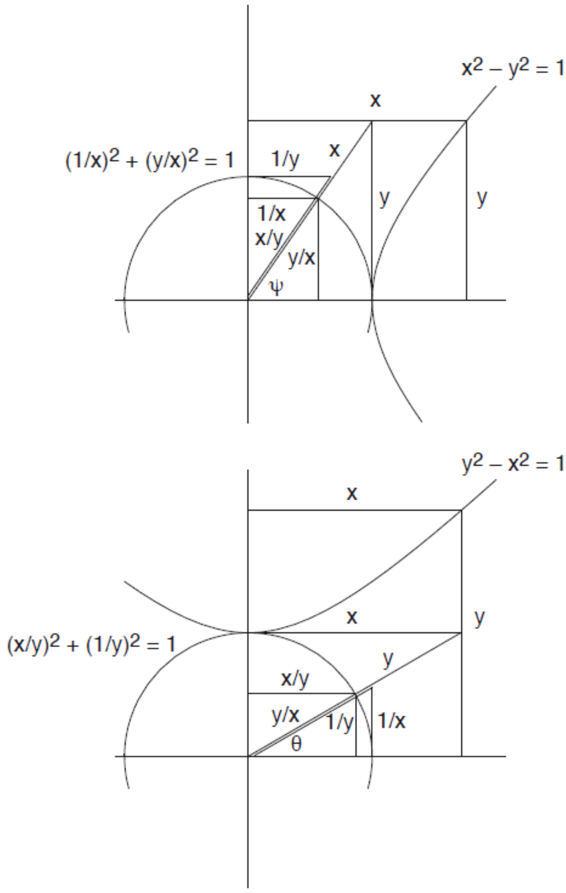

Figure 1: (a) Above, horizontal, and (b) below, vertical hyperbola and circle coordinate angles, as defined in hyperbolic trigonometry. Reproduced with kind permission from Spec in Sci and Tech **21**, 214, 219 (1999) Russell Eskew Copyright 1999 Springer, Springer Science and Business Media

Nikolai Ivanovic Lobachevskii (1792–1856) (Lobachevskii, 1840, p. 41) as shown below:

$$\begin{aligned}
x &= \cot\theta = \sinh\sinh^{-1}x = k(\sinh^{-1}\cot\Pi(x)) = k(\sinh^{-1}\sinh\frac{x}{k}) = \operatorname{csch}\alpha \\
\frac{1}{x} &= \tan\theta = \operatorname{csch}\sinh^{-1}x = k(\sinh^{-1}\tan\Pi(x)) = k(\sinh^{-1}\operatorname{csch}\frac{x}{k}) = \sinh\alpha \\
\frac{x}{y} &= \cos\theta = \tanh\sinh^{-1}x = k(\sinh^{-1}\cos\Pi(x)) = k(\sinh^{-1}\tanh\frac{x}{k}) = \operatorname{sech}\alpha \\
y &= \csc\theta = \cosh\sinh^{-1}x = k(\sinh^{-1}\csc\Pi(x)) = k(\sinh^{-1}\cosh\frac{x}{k}) = \coth\alpha \\
\frac{1}{y} &= \sin\theta = \operatorname{sech}\sinh^{-1}x = k(\sinh^{-1}\sin\Pi(x)) = k(\sinh^{-1}\operatorname{sech}\frac{x}{k}) = \tanh\alpha \\
\frac{y}{x} &= \sec\theta = \coth\sinh^{-1}x = k(\sinh^{-1}\sec\Pi(x)) = k(\sinh^{-1}\coth\frac{x}{k}) = \cosh\alpha,
\end{aligned} \tag{2}$$

when $\sinh^{-1}x = \ln\frac{1}{v} = a$ and $\alpha = \ln\frac{1+v}{1-v}$ (see page 27). The hyperbolic functions defined by Johann Heinrich Lambert (1728–1777) stem from $\sinh x = (e^x - e^{-x})/2$. By $\sinh^{-1}\sinh x = \ln(\sinh x + \sqrt{(\sinh x)^2 + 1}) = x$, we also have $\sinh\sinh^{-1}x = \sinh a = (e^{\sinh^{-1}x} - e^{-\sinh^{-1}x})/2 = x = \cot\theta$. We have $\cosh\sinh^{-1}x = \cosh a = (e^{\sinh^{-1}x} + e^{-\sinh^{-1}x})/2 = y = \sqrt{x^2+1} = \csc\theta$ as well. George Martin (Coxeter, 1998, pp. 300–302) creates the *distance scale* $k = \frac{x}{a}$ for any concentric horocircles of distance x in the Bolyai-Lobachevskii plane (Coxeter, 1989, p. 301). The angle of parallelism (Eskew, 1999, 2011) $\angle ABC$ is of Lobachevskii's $\theta = 2\tan^{-1}e^{-a} = 2\tan^{-1}e^{-\sinh^{-1}x} = \tan^{-1}\frac{1}{x}$ and of Martin's critical function $\Pi(x) = 2\tan^{-1}e^{-x/k} = 2\tan^{-1}e^{-(\sinh a)/k} = 2\tan^{-1}e^{-(x)/(x/a)}$. We do not have $B = \theta = \Pi(a)$. (Coxeter, 1998, p. 312)

A point (x, y) on the vertical hyperbola $y^2 - x^2 = 1$ is understood as an inertial frame of reference, or observer. Within electromagnetic velocity (Misner, Thorne, & Wheeler, 1970, pp. 20, 67) $\tan\theta = \tanh\alpha = \beta = v/c$, when a first observer (Schutz, 2009, pp. 1-2). sees a Galilean velocity $\tilde{v} = v' + v$ and a second observer sees $v' = \tilde{v} - v$ his velocity relative to the first is a constant $v = \tan\frac{1}{2}\theta = \tanh\frac{1}{2}\alpha = 1/(x+y)$. A Galilean velocity $\tilde{v} = v' + v = 1 = xu$ is made from an Einsteinian velocity $u = \frac{1}{x} = \tan\theta = (\tanh\alpha)(\cosh\alpha) = \sinh\alpha = \beta/(1-\beta^2)^{1/2}$, because $\tan 2\frac{\theta}{2} = \frac{2\tan\theta/2}{1-\tan^2\theta/2} = \frac{2\tanh\alpha/2}{1-\tanh^2\alpha/2} = (2v)/(1-v^2) = 2\sinh\frac{\alpha}{2}\cosh\frac{\alpha}{2} = \sinh 2\frac{\alpha}{2}$. We also have relative velocities $v' = \tilde{v} - v = 1 - \frac{1}{x+y}$ and $u' = u - v = \frac{1}{x} - \frac{1}{x+y}$. Absolute acceleration $dv/d\theta$, a.k.a. kinetic energy, relative acceleration $dv/d\alpha$, a.k.a. potential energy, and frame acceleration dv/dt, a.k.a. gravity are made from a half-angle velocity $v = \tan\frac{1}{2}\theta = \tanh\frac{1}{2}\alpha = 1/(x+y) = 1/t$ distance/time, where $0 \le v \le 1$. Time can have an analytic quantity $t = e^{\sinh^{-1}x} = x + (x^2+1)^{1/2} = x + y$ seconds, which we are saying is about an event point $P(t)$. Our math utilizes the complex plane rather than the spacetime diagram (Taylor & Wheeler, 1966, pp. 1–58). We advocate stating $vt = [1/(x+y)][x+y] = 1$ distance and acceleration $a_{\text{frame}} = dv/dt = \frac{d}{dt}t^{-1} = -v^2 = -[1/(x+y)]^2$ distance/time^2 when, say, the Earth's gravitational acceleration is $g = -9.81$ m/s^2.

2. Complex numbers

2.1 Vectors

Now we will show how hyperbolic coordinates make the vectors of complex numbers. By Eq. (1) the point on a circle is $(x_c, y_c) = (r\cos\theta_c, r\sin\theta_c)$. Let (x, y) be a frame of reference point on the vertical hyperbola. Then the hyperbola or circle vectors can be made of the hyperbola's x and y rather than the circle's x_c and y_c.

Proof. If a circle vector $z_1 = re^{i\theta_h} = r(\cos\theta_h + i\sin\theta_h) = r(\frac{x}{y} + i\frac{1}{y})$ where (x, y) is on the Eq. (2) vertical hyperbola $y^2 - x^2 = 1$ and $\theta_h = \tan^{-1}\frac{1}{x}$ then we can write:

$$\begin{aligned} z_1 = re^{i\theta_h} = r(\cos\theta_h + i\sin\theta_h) &= r(\frac{x}{y} + i\frac{1}{y}) \\ |z_1| = ((\cos\theta_h)^2 + (\sin\theta_h)^2)^{1/2} &= \left(\left(\frac{x}{y}\right)^2 + \left(\frac{1}{y}\right)^2\right)^{1/2} = 1 \\ = ((\csc\theta_h)^2 - (\cot\theta_h)^2)^{1/2} &= (y^2 - x^2)^{1/2} = 1, \end{aligned}$$

rather than a Eq. (1) polar coordinate $re^{i\theta_c}$. Any point with $0 \le \theta_h \le \frac{\pi}{4}$ on the complex plane can be reached at $re^{i\theta_h}$ and translated into the frame of reference (x, y). If a hyperbolic vector $z_2 = \cot\theta_h + i\csc\theta_h = x + iy$ where (x, y) is on the Eq. (2) vertical hyperbola $y^2 - x^2 = 1$ then we denote our unconventional *complex number* (Marsden, 1999, pp. 3, 7, 12, 16) thusly:

$$\begin{aligned} z_2 = (a, b) = a + ib &= \cot\theta_h + i\csc\theta_h = x + iy \\ |z_2| = ((a+ib)(a-ib))^{1/2} &= (a^2 + b^2)^{1/2} \\ &= ((\csc\theta_h)^2 + (\cot\theta_h)^2)^{1/2} = (y^2 + x^2)^{1/2}, \end{aligned}$$

when lengths $r = ((r\cos\theta_c)^2 + (r\sin\theta_c)^2)^{1/2}$ and $|z_2|$ are different. Equation (1) uses a circular angle $\theta_c = \tan^{-1}\frac{y_c}{x}$, but hyperbolic trigonometry Eq. (2) employs an angle $\theta_h = 2\tan^{-1} e^{-\sinh^{-1}x} = \tan^{-1}\frac{1}{x} = \tan^{-1}\frac{1}{\cot\theta_h}$. Uniting conventional (Palka, 1991, pp. 5–15) with our unconventional complex numbers, we obtain $x = r\cos\theta_c = r\cos\tan^{-1}\frac{y_c}{x} = \cot\theta_h$ and $y = (x^2+1)^{1/2} = \csc\theta_h$. So we have a vector-valued function $f(z) = f(x, y) = f(\cot\theta, \csc\theta) = u(x, y) + iv(x, y)$ for a $Df(x, y)$. Note that $\sin\theta_c$ and $\cos\theta_c$ have a range of $[-1, 1]$. But $\cot\theta_h$ has a range of $(-\infty, \infty)$, with similar $\csc\theta_h$. We do not have $re^{i\theta} = |z|e^{i(\arg z)}$. (Marsden, 1999, p. 27). □

Example: When $\theta_c = \tan^{-1}\frac{y_c}{x} = \tan^{-1}\frac{3\times 1/2}{3\times\sqrt{3}/2} = \tan^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{6}$, the polar coordinate $re^{i\theta_c}$ of length $r = ((r\cos\theta_c)^2 + (r\sin\theta_c)^2)^{1/2} = ((3\times\frac{\sqrt{3}}{2})^2 + (3\times\frac{1}{2})^2)^{1/2} = 3$ goes through the point $(\frac{\sqrt{3}}{2}, \frac{1}{2})$ to the point $(x_c, y_c) = (3\times\frac{\sqrt{3}}{2}, 3\times\frac{1}{2})$. When $\theta_h = \tan^{-1}\frac{1}{x} = \tan^{-1}\frac{1}{3\times\sqrt{3}/2}$, the circle vector is $z_1 = re^{i\theta_h} = r(\cos\theta_h + i\sin\theta_h) = r(\frac{x}{y} + i\frac{1}{y}) = 3((3\times\frac{\sqrt{3}}{2})/((3\times\frac{\sqrt{3}}{2})^2 + 1)^{1/2}) + i(1/((3\times\frac{\sqrt{3}}{2})^2+1)^{1/2})$. The hyperbolic vector $z_2 = x + iy = r\cos\theta_c + i((r\cos\theta_c)^2 + 1)^{1/2} = \cot\theta_h + i\csc\theta_h$ of length $|z_2| = (x^2+y^2)^{1/2} = ((\cot\theta_h)^2 + (\csc\theta_h)^2)^{1/2} = ((3\times\frac{\sqrt{3}}{2})^2 + (((3\times\frac{\sqrt{3}}{2})^2+1)^{1/2})^2)^{1/2} \doteq 3.8078$ goes to the point $(x, y) = (x, (x^2+1)^{1/2}) = (r\cos\theta_c, ((r\cos\theta_c)^2+1)^{1/2}) = (\cot\theta_h, \csc\theta_h) = (\cot\tan^{-1}(1/(3\times\frac{\sqrt{3}}{2})), (\cot\tan^{-1}(1/((3\times\frac{\sqrt{3}}{2}))^2+1))^{1/2})$. We have $x = r\cos\theta_c = \cot\theta_h = 3\times\frac{\sqrt{3}}{2}$. Table 1 uses $x = e^{n\pi/2}$. When $n = 1, 2, 3\ldots$ starting at $m = n$, we obtain $\pi = \frac{m}{n}\cos^{-1}(\cos\frac{n\pi}{m})$, for the real counterpart of $e^{in\pi/2} = \cos\frac{n\pi}{2} + i\sin\frac{n\pi}{2}$.

2.2 Applying velocity in complex numbers

Given a half-angle velocity $v = \tan\frac{1}{2}\theta = \tanh\frac{1}{2}\alpha = \frac{1}{x+y}$ and any two complex numbers $z = a + ib$ and $u = x + iy$ where (x, y) is on the hyperbola $y^2 - x^2 = 1$, if $u^2 = z$ then by taking the square roots of negative numbers (Marsden, 1999, pp. 3, 7, 12, 16) we acquire:

$$\begin{aligned} u^2 &= a + ib \\ &= (x + iy)(x + iy) = (x^2 - y^2) + i2xy \\ &= ((\sinh\ln\frac{1}{v})^2 - (\cosh\ln\frac{1}{v})^2) + i2(\sinh\ln\frac{1}{v})(\cosh\ln\frac{1}{v}) \\ &= ((\cot\theta_h)^2 - (\csc\theta_h)^2) + i2(\cot\theta_h)(\csc\theta_h) \\ &= -1 + i2(\sinh\ln\frac{1}{v})(\cosh\ln\frac{1}{v}) = -1 + i\sinh 2\ln\frac{1}{v}. \end{aligned}$$

We also have (Ahlfors, 1970, pp. 3-13) the double-angle in $z = a + ib$ and circular vector $w = re^{i\theta_h} = r(\cos\theta_h + i\sin\theta_h)$ for $w^2 = z = r^2(\cos 2\theta_h + i\sin 2\theta_h)$, shown as:

$$\begin{aligned} [1]^2 &= \left[\left(\frac{x}{y}\right)^2 + \left(\frac{1}{y}\right)^2\right]^2 = [(\cos\theta_h)^2 + (\sin\theta_h)^2]^2 \\ &= \left[\left(\frac{x}{y}\right)^2 - \left(\frac{1}{y}\right)^2\right]^2 + \left[2\left(\frac{x}{y}\right)\left(\frac{1}{y}\right)\right]^2 = [\cos 2\theta_h]^2 + [\sin 2\theta_h]^2 \\ &= [y^2 - x^2]^2 = [(\csc\theta_h)^2 - (\cot\theta_h)^2]^2 \\ a^2 + b^2 &= [y^2 + x^2]^2 = [1]^2 + (2xy)^2 = [-1]^2 + (2\cot\theta_h\csc\theta_h)^2 \\ (a^2 + b^2)^{1/2} &= y^2 + x^2 = [(-a + (a^2 + b^2)^{1/2})/2] + [(a + (a^2 + b^2)^{1/2})/2], \end{aligned}$$

where $a = -1$ and $b = 2xy = 2\cot\theta\csc\theta = \sinh 2\ln\frac{1}{v}$. The equation $u^2 = z$ solves to $\pm((x^2)^{1/2} + i(y^2)^{1/2}) = ((a + (a^2 + b^2)^{1/2})/2)^{1/2} + i((-a + (a^2 + b^2)^{1/2})/2)^{1/2}$, depending on $\pm b$. We have $z^{1/2} = (x^2)^{1/2} + 0i$ if and only if $z = (x^2) + 0i > 0$. We have $z^{1/2} = 0 + (y^2)^{1/2}i$ if and only if $z = -((-x)^2 + 1) + 0i < 0$. The two $z^{1/2}$ coincide if and only if $z = 0 + 0i$. Use $x = \cot\theta_h$, $y = \csc\theta_h$ rather than $x = r\cos\theta_c$, $y = r\sin\theta_c$.

The vertical hyperbola vector has a power of:

$$z^n = (x + iy)^n = (\cot\theta + i\csc\theta)^n = (\sinh\ln\frac{1}{v} + i\cosh\ln\frac{1}{v})^n.$$

The logarithmic, exponential, and nth root functions appear as:

$$\begin{aligned} \ln z &= \ln|z| + i\theta = \ln(y^2 + x^2)^{1/2} + i\theta \\ &= \ln((\csc\theta)^2 + (\cot\theta)^2)^{1/2} + i\theta \\ &= \ln((\cosh\ln\frac{1}{v})^2 + (\sinh\ln\frac{1}{v})^2)^{1/2} + i\theta; \end{aligned}$$

$$\begin{aligned} e^z &= e^x e^{iy} = e^{\cot\theta} e^{i\csc\theta} = e^{\cot\theta}(\cos\csc\theta + i\sin\csc\theta) \\ &= e^{\sinh\ln\frac{1}{v}} e^{i\cosh\ln\frac{1}{v}} = e^{\sinh\ln\frac{1}{v}}(\cos\cosh\ln\frac{1}{v} + i\sin\cosh\ln\frac{1}{v}) = |e^z|\frac{e^z}{|e^z|}; \end{aligned}$$

$$\begin{aligned} z^{1/n} &= e^{(\ln z)/n} = e^{(\ln|z| + i\theta)/n} = e^{(\ln((y^2+x^2)^{1/2}) + i\theta)/n} \\ &= e^{(\ln(y^2+x^2)^{1/2})/n} e^{i\theta/n} = (y^2 + x^2)^{1/2n} e^{i(\theta+2\pi k)/n}; \end{aligned}$$

$$\begin{aligned} z &= e^{\ln z} = e^{\ln|z|} e^{i\theta} = |z|\frac{z}{|z|} \\ &= |z|e^{i\theta} = (y^2 + x^2)^{1/2}(\cos\theta + i\sin\theta) \\ &= \ln e^z = \ln|e^z| + i\arg(e^z) = \ln e^x + iy \\ &= x + iy = \cot\theta + i\csc\theta, \end{aligned}$$

where $\theta = 2\tan^{-1} e^{-\sinh^{-1} x} = \tan^{-1}\frac{1}{x}$ with $0 \le \theta \le \frac{\pi}{4}, k = 0, 1, \ldots, n - 1$.

Table 1. Values of a few coordinates at some points (x, y) on $y^2 - x^2 = 1$.

x	$t = 1/v$	$\theta = \Pi(x)$	k
$\cot\theta$	$x + \sqrt{x^2+1}$	$2\tan^{-1} e^{\ln v}$	x/a
0	1.0	$\pi/2 = 1.5707$	0
e^0	2.414213562	$\pi/4 = 0.7853$	1.134592657
$e^{\pi/2}$	9.723795265	0.204960468	2.11488971
e^{π}	46.30298215	0.043187049	6.033754226
$e^{3\pi/2}$	222.6400485	0.008983049	20.59321402
$e^{2\pi}$	1070.984245	0.001867441	76.75832388
$e^{5\pi/2}$	5151.942712	0.000388203	301.3843059
$e^{3\pi}$	24783.29566	0.000080700	1224.722226
$e^{7\pi/2}$	119219.483	0.000016776	5099.765758
$e^{4\pi}$	573502.6263	0.000003487	21626.07401
$e^{9\pi/2}$	2758821.412	0.000000725	93012.91224
$e^{5\pi}$	13271248	0.000000151	404583.825
$e^{11\pi/2}$	63841038.32	0.000000031	1776134.25
$e^{6\pi}$	307105870.8	0.000000007	7857302.779
$e^{13\pi/2}$	1477325845	0.000000001	34985338.41
$e^{7\pi}$	7106642561	2.81×10^{-10}	156642344.9
$e^{15\pi/2}$	3.41×10^{10}	5.85×10^{-11}	704725077.5
$e^{8\pi}$	1.64×10^{11}	1.21×10^{-11}	3183871713
$e^{17\pi/2}$	7.91×10^{11}	2.52×10^{-12}	1.44×10^{10}
$e^{9\pi}$	3.80×10^{12}	5.25×10^{-13}	6.56×10^{10}
$e^{19\pi/2}$	1.83×10^{13}	1.09×10^{-13}	2.99×10^{11}
$e^{10\pi}$	8.80×10^{13}	2.27×10^{-14}	1.37×10^{12}
$e^{21\pi/2}$	4.23×10^{14}	4.72×10^{-15}	6.28×10^{12}
$e^{11\pi}$	2.03×10^{15}	9.81×10^{-16}	2.89×10^{13}
$e^{23\pi/2}$	9.80×10^{15}	2.04×10^{-16}	1.33×10^{14}
$e^{12\pi}$	4.71×10^{16}	4.24×10^{-17}	6.14×10^{14}
$e^{25\pi/2}$	2.26×10^{17}	8.81×10^{-18}	2.83×10^{15}
$e^{13\pi}$	1.09×10^{18}	1.83×10^{-18}	1.31×10^{16}
$e^{27\pi/2}$	5.24×10^{18}	3.80×10^{-19}	6.08×10^{16}
$e^{14\pi}$	2.52×10^{19}	7.92×10^{-20}	2.82×10^{17}
$e^{29\pi/2}$	1.21×10^{20}	1.64×10^{-20}	1.31×10^{18}
$e^{15\pi}$	5.84×10^{20}	3.42×10^{-21}	6.11×10^{18}
$e^{31\pi/2}$	2.81×10^{21}	7.11×10^{-22}	2.84×10^{19}
$e^{16\pi}$	1.35×10^{22}	1.47×10^{-22}	1.32×10^{20}
$e^{33\pi/2}$	6.50×10^{22}	3.07×10^{-23}	6.19×10^{20}
$e^{17\pi}$	3.12×10^{23}	6.39×10^{-24}	2.89×10^{21}
$e^{35\pi/2}$	1.50×10^{24}	1.32×10^{-24}	1.35×10^{22}
$e^{25\pi}$	2.57×10^{34}	7.77×10^{-35}	1.62×10^{32}
∞	2∞	0	∞

$$x = \cot\theta_h = r\cos\theta_c = e^{n\pi/2}, \pi = \frac{m}{n}\cos^{-1}(\cos\frac{n\pi}{m})$$

$$\text{time} = t = 1/v = x + (x^2+1)^{1/2} = x + y \quad \text{seconds}$$

$$a = \sinh^{-1} x = \frac{x}{k} = \ln\frac{1}{v}$$

$$k = \frac{x}{a}$$

$$\angle ABC = \theta_h = 2\tan^{-1} e^{-a} = \cos^{-1}\frac{x}{y} = \tan^{-1}\frac{1}{x}$$

$$= \Pi(x) = 2\tan^{-1} e^{-x/k} = 2\tan^{-1} e^{-(x)/(x/a)}$$

Table 2. Velocity and field of accelerations.

velocity v 1/time	kinetic energy $a_{\text{absolute}} = dv/d\theta$	potential energy $a_{\text{relative}} = dv/d\alpha$	-gravity $-a_{\text{frm}} = v^2$
1.0	1.0	0	1.0
0.414213562	0.5 + 0.08578643	0.5 − 0.08578643	0.1715728
0.102840503	0.5 + 0.00528808	0.5 − 0.00528808	0.0105761
0.021596881	0.5 + 0.00023321	0.5 − 0.00023321	0.0004664
0.004491555	0.5 + 0.00001008	0.5 − 0.00001008	0.0000201
0.001867443	0.5 + 0.00000174	0.5 − 0.00000174	0.0000034
0.000194102	0.5 + 0.00000001	0.5 − 0.00000001	3×10^{-8}
0.000040350	$0.5 + (8 \times 10^{-10})$	$0.5 - (8 \times 10^{-10})$	2×10^{-9}
0.000008388	$0.5 + (3 \times 10^{-11})$	$0.5 - (3 \times 10^{-11})$	7×10^{-11}
0.000001744	$0.5 + (1 \times 10^{-12})$	$0.5 - (1 \times 10^{-12})$	3×10^{-12}
0.000000362	$0.5 + (6 \times 10^{-14})$	$0.5 - (6 \times 10^{-14})$	1×10^{-13}
0.000000151	$0.5 + (1 \times 10^{-14})$	$0.5 - (1 \times 10^{-14})$	2×10^{-14}
0.000000016	$0.5 + (1 \times 10^{-16})$	$0.5 - (1 \times 10^{-16})$	2×10^{-16}
0.000000003	$0.5 + (5 \times 10^{-18})$	$0.5 - (5 \times 10^{-18})$	1×10^{-17}
6.76×10^{-10}	$0.5 + (2 \times 10^{-19})$	$0.5 - (2 \times 10^{-19})$	4×10^{-19}
1.40×10^{-10}	$0.5 + (9 \times 10^{-21})$	$0.5 - (9 \times 10^{-21})$	1×10^{-20}
2.92×10^{-11}	$0.5 + (4 \times 10^{-22})$	$0.5 - (4 \times 10^{-22})$	8×10^{-22}
6.08×10^{-12}	$0.5 + (1 \times 10^{-23})$	$0.5 - (1 \times 10^{-23})$	3×10^{-23}
1.26×10^{-12}	$0.5 + (7 \times 10^{-25})$	$0.5 - (7 \times 10^{-25})$	1×10^{-24}
2.62×10^{-13}	$0.5 + (3 \times 10^{-26})$	$0.5 - (3 \times 10^{-26})$	6×10^{-26}
5.46×10^{-14}	$0.5 + (1 \times 10^{-27})$	$0.5 - (1 \times 10^{-27})$	2×10^{-27}
1.13×10^{-14}	$0.5 + (6 \times 10^{-29})$	$0.5 - (6 \times 10^{-29})$	1×10^{-28}
2.36×10^{-15}	$0.5 + (2 \times 10^{-30})$	$0.5 - (2 \times 10^{-30})$	5×10^{-30}
4.90×10^{-16}	$0.5 + (1 \times 10^{-31})$	$0.5 - (1 \times 10^{-31})$	2×10^{-31}
1.02×10^{-16}	$0.5 + (5 \times 10^{-33})$	$0.5 - (5 \times 10^{-33})$	1×10^{-32}
2.12×10^{-17}	$0.5 + (2 \times 10^{-34})$	$0.5 - (2 \times 10^{-34})$	4×10^{-34}
4.40×10^{-18}	$0.5 + (9 \times 10^{-36})$	$0.5 - (9 \times 10^{-36})$	1×10^{-35}
9.16×10^{-19}	$0.5 + (4 \times 10^{-37})$	$0.5 - (4 \times 10^{-37})$	8×10^{-37}
1.90×10^{-19}	$0.5 + (1 \times 10^{-38})$	$0.5 - (1 \times 10^{-38})$	3×10^{-38}
3.96×10^{-20}	$0.5 + (7 \times 10^{-40})$	$0.5 - (7 \times 10^{-40})$	1×10^{-39}
8.23×10^{-21}	$0.5 + (3 \times 10^{-41})$	$0.5 - (3 \times 10^{-41})$	6×10^{-41}
1.71×10^{-21}	$0.5 + (1 \times 10^{-42})$	$0.5 - (1 \times 10^{-42})$	2×10^{-42}
3.55×10^{-22}	$0.5 + (6 \times 10^{-44})$	$0.5 - (6 \times 10^{-44})$	1×10^{-43}
7.39×10^{-23}	$0.5 + (2 \times 10^{-45})$	$0.5 - (2 \times 10^{-45})$	5×10^{-45}
1.53×10^{-23}	$0.5 + (1 \times 10^{-46})$	$0.5 - (1 \times 10^{-46})$	2×10^{-46}
3.19×10^{-24}	$0.5 + (5 \times 10^{-48})$	$0.5 - (5 \times 10^{-48})$	1×10^{-47}
6.64×10^{-25}	$0.5 + (2 \times 10^{-49})$	$0.5 - (2 \times 10^{-49})$	4×10^{-49}
3.88×10^{-35}	$0.5 + (7 \times 10^{-70})$	$0.5 - (7 \times 10^{-70})$	1×10^{-69}
0	0.5	0.5	0

$$\text{velocity} = v = \tan\frac{1}{2}\theta = \tanh\frac{1}{2}\alpha = \left(\frac{1}{x+y}\right)\frac{1}{\text{time}}$$

$$\text{kinetic}\quad \text{absoluteaccel} = a_{\text{absolute}} = \frac{dv}{d\theta} = \left(\frac{1}{2}+\frac{v^2}{2}\right)\frac{1}{\text{time}^2}$$

$$\text{potential}\quad \text{relativeaccel} = a_{\text{relative}} = \frac{dv}{d\alpha} = \left(\frac{1}{2}-\frac{v^2}{2}\right)\frac{1}{\text{time}^2}$$

$$\text{gravity}\quad \text{frameaccel} = a_{\text{frame}} = \frac{dv}{dt} = -v^2\frac{1}{\text{time}^2}.$$

3. Discussion

3.1 Accelerated frames of reference

Choosing an arbitrary frame of reference point, say $(x, y) = (5, \sqrt{26})$, can quantify physical laws of motion. Consider Isaac Newton's (1642–1727) second law of motion,

$$F = ma = mg = G\frac{mM}{r^2},$$

about a force F that for gravitation force is exerted on a particle of mass m by one of large mass M, producing an acceleration a at a distance r (as opposed to $|z_2|$) on a gravitational field g, computed using the gravitational constant G. It applies to the addition of velocities equation (Gleeson, 2010) of Galileo Galilei (1564–1642):

$$\tilde{v} = v' + v = 1 = xu = x(u' + \frac{1}{x}v) = xu' + v, \tag{3}$$

(i.e., a moving particle's velocity with respect to a fixed reference frame). The quantities are the (Jackson, 1999, pp. 531–532) "Einsteinian velocity":

$$u = \frac{1}{x} = \tan\theta = \sinh\alpha = \frac{2v}{1 - v^2} = u' + \frac{1}{x}v, \tag{4}$$

the "relative velocity" $v' = \tilde{v} - v = 1 - \frac{1}{x+y} = xu'$, (i.e., the particle's velocity with respect to a moving reference frame), and a constant "transport velocity" v made from a half-angle velocity $v = \frac{1}{x+y} = \tan\frac{1}{2}\theta = \tanh\frac{1}{2}\alpha$, (i.e., the velocity of the moving reference frame). Like Newton's first law, we say that a body moving at a constant velocity by an inertial observer is unaffected by the replacement of $\tilde{v}$ by one observer and of v' by a second observer whose velocity relative to the first is v.

Albert Einstein (1879–1955) described a high-energy Eq. (3) with Eq. (5):

$$u = \frac{u' + v}{1 + u'v/c^2}. \tag{5}$$

Like Einstein (Lorentz, 1923, pp. 41, 46, 50) we hold when $v = \frac{1}{x+y} = 1$ distance/time is $c = 299792458$ meters/second, that upon the photon $u' = u - \frac{1}{x}v = \frac{1}{x} - \frac{1}{x}v = c$, we have (Einstein, 1957, pp. 26–30) $u = u' + \frac{1}{x}v = \frac{1}{x} = c$. If u' has the magnitude c, then so does u, regardless of v. (Eskew, 2016).

Galileo showed that different freely falling bodies experience exactly the same acceleration at a given point in space. Table 2 shows that at the highest inverse velocity and time, $1/v = t = \infty$, we have (French, 1971, Chapter 12) accelerations $a = a' = \frac{1}{2}$ when $a = a' - a_{\text{frame}} = \frac{dv}{d\theta} = \frac{dv}{d\alpha} - \frac{dv}{dt}$.

Let $v = \tan\frac{1}{2}\theta_h = \tanh\tanh^{-1} v = \tanh\frac{1}{2}\ln\frac{1+v}{1-v} = \tanh\frac{1}{2}\alpha = 1/(x + y) = 1/t$ distance/time be the half-angle velocity (Anderson, 2005, Chapter 3, p. 124) to $\beta = \tan\theta = \tanh\alpha = v/c$. Then the field of accelerations is shown as:

$$\begin{gathered}
\frac{dv}{d\theta} = \frac{dv}{d\alpha} - \frac{dv}{dt} \\
\left(\frac{1}{2} + \frac{v^2}{2}\right) = \left(\frac{1}{2} - \frac{v^2}{2}\right) - \left(-v^2\right) \\
a = a' - a_{\text{frame}} \\
\text{accel}_{\text{absolute}} = \text{accel}_{\text{relative}} - \text{accel}_{\text{frame}}.
\end{gathered}$$

The accelerating, noninertial frame of reference becomes:

$$\begin{gathered}
F_\theta = F_\alpha - F_{\text{frame}} \\
m\frac{dv}{d\theta} = m\frac{dv}{d\alpha} - m\frac{dv}{dt}.
\end{gathered}$$

Proof. The half-angle velocity is $v = \tan\frac{1}{2}\theta = \tanh\tanh^{-1} v = \tanh\frac{1}{2}\ln\frac{1+v}{1-v} = \tanh\frac{1}{2}\alpha = 1/(x + y) = 1/t$ distance/time. With the calculus notation u the accelerations are the derivatives resulting in trigonometry and algebra:

$$\frac{dv}{d\theta} = \frac{dv}{dt}\frac{dt}{d\theta} = \frac{d}{d\theta}\tan\frac{1}{2}\theta = \frac{d}{d\theta}\tan u = (\sec^2 u)\frac{du}{d\theta} = \frac{1}{2}\sec^2\frac{1}{2}\theta = \frac{1}{2} + \frac{v^2}{2}$$
$$\frac{dv}{d\alpha} = \frac{dv}{dt}\frac{dt}{d\alpha} = \frac{d}{d\alpha}\tanh\frac{1}{2}\alpha = \frac{d}{d\alpha}\tanh u = (\operatorname{sech}^2 u)\frac{du}{d\alpha} = \frac{1}{2}\operatorname{sech}^2\frac{1}{2}\alpha = \frac{1}{2} - \frac{v^2}{2}$$
$$\frac{dv}{dt} = \frac{dv}{d\alpha}\frac{d\alpha}{dt} = \frac{d}{dt}t^{-1} = -\frac{1}{(1/v)^2} = -v^2$$

$$\frac{d\alpha}{dt} = \frac{d\alpha}{dv}\frac{dv}{dt} = \frac{d}{dt}\ln\frac{1+(1/t)}{1-(1/t)} = \frac{1}{u}\frac{du}{dt} = \left(\frac{1-(1/t)}{1+(1/t)}\right)\left(\frac{2(1/t^2)}{(1-(1/t))^2}\right) = \frac{2}{t^2-1}$$
$$\frac{d\theta}{dt} = \frac{d\theta}{dv}\frac{dv}{dt} = \frac{d}{dt}\sin^{-1}\operatorname{sech}\ln t = \frac{d}{dt}\sin^{-1}u = \frac{u'}{\sqrt{1-u^2}}$$
$$= \frac{-((\operatorname{sech}\ln t)(\tanh\ln t))1/t}{\sqrt{1-(\operatorname{sech}\ln t)^2}} = -\frac{2}{t^2+1}$$

□

This occurrence means that the field $a = a' - a_{\text{frame}}$ is equivalent to an acceleration of the coordinate frame at that event point (Dainton, 2001, p. 263) in space $P(t) = P(\frac{1}{v}) = 1/(e^{\alpha/2}) = 1/[(1+v)/(1-v)]^{1/2} = P(x+y)$. As the gravitational fictitious force moving reference frame rolls down from an inertial $a_{\text{frame}} = dv/dt = -v^2 = -1/(1/v)^2 = -1/\infty^2 = 0\ \frac{1}{t^2}$ to $a_{\text{frame}} = -v^2 = -1/(1/v)^2 = -1/1^2 = -1\ \frac{1}{t^2}$, the particle relative to the moving reference frame declines from the potential $a_{\text{relative}} = dv/d\alpha = (1/2 - v^2/2) = 1/2\ \frac{1}{t^2}$ to 0, and the fixed observer sees the particle's kinetic real force acceleration rise from $a_{\text{absolute}} = dv/d\theta = (1/2 + v^2/2) = 1/2\ \frac{1}{t^2}$, of a rest frame velocity $v = 0\ \frac{1}{t}$ to $a_{\text{absolute}} = (1/2 + v^2/2) = 1\ \frac{1}{t^2}$, for a light velocity of $v = 1\ \frac{1}{t}$. Tables 1 and 2 illustrate our coordinates.

3.2 Half-angle velocity and absolute acceleration

By Galilean Invariance there is no experiment that can be performed that can measure the velocity of a moving observer (Gleeson, 2010). However we can measure $u = (2v)/(1-v^2)$ with the half-angle velocity $v = \tan\frac{1}{2}\theta_h = \tanh\frac{1}{2}\alpha = 1/(x+y) = 1/t$ of a moving observer. We can detect the presence of accelerations and measure the relative velocity $v' = \tilde{v} - v = 1 - 1/(x+y) = xu'$ between two bodies because constant velocity made from half-angle velocity measures the Galilean velocity $\tilde{v} = v' + v = 1 = xu = x(u' + \frac{1}{x}v)$.

Proof. Velocity can only be measured in relation to some specified point of rest, therefore, it is said, absolute velocity does not exist. Equation (4) Einsteinian velocity $u = 1/x = \tan\theta = \sinh\alpha = (2v)/(1-v^2) = u' + \frac{1}{x}v$ is measured with a constant $v = \frac{1}{x+y}$ made from a half-angle velocity where $0 \le v \le 1$ is one-dimensional and measurable. In Einstein's Special Relativity the velocity of light $\tilde{v} = v = 1$ is invariant; as we said about Eq. (5), Einsteinian velocity is absolute motion $u = 1/x = 1/0 = \infty = c$ and $u' = u - \frac{1}{x}v = c$. Absolute acceleration $\left[\frac{dv}{d\theta} = \frac{d}{d\theta}\tan\frac{1}{2}\theta = \frac{1}{2}\sec^2\frac{1}{2}\theta = \frac{1}{2} + \frac{v^2}{2}\right] = \left[\frac{dv}{d\alpha} = \frac{d}{d\alpha}\tanh\frac{1}{2}\alpha = \frac{1}{2}\operatorname{sech}^2\frac{1}{2}\alpha = \frac{1}{2} - \frac{v^2}{2}\right] - \left[\frac{dv}{dt} = \frac{d}{dt}t^{-1} = -v^2\right]$ is variable velocity $\frac{1}{2} + \frac{v^2}{2} = (\frac{1}{2} - \frac{v^2}{2}) - (-v^2)$. □

To quote Einstein's (1911) principle (Lorentz, 1923, p. 100) of the local equivalence between a "gravitational field" and an acceleration:

> "We arrive at a very satisfactory interpretation of this law of experience, if we assume that the systems K and K' are physically exactly equivalent, that is, if we assume that we may just as well regard the system K as being in a space free from gravitational fields, if we then regard K as uniformly accelerated. This assumption of exact physical equivalence makes it impossible for us to speak of the absolute acceleration of the system of reference, just as the usual theory of relativity forbids us to talk of the absolute velocity of a system; and it makes the equal falling of all bodies in a gravitational field seem a matter of course."

Newton argued that there is an absolute acceleration (Schutz, 2009, pp. 1–2). The accepted theory of gravity is Einstein's theory of General Relativity (Einstein, 1950). The Einstein field equation is $G+\Lambda g = 8\pi T$, where T is the Riemann stress-energy tensor and Λ is the cosmological constant (Misner, Thorne, & Wheeler, 1970, pp. 410, 707). We will conclude that Relativity is based on local Lorentz geometry. This essay, however, can express half-angle velocity and absolute acceleration because of our unique projective geometry Eq. (2) which does not include Riemannian geometry nor topology (Coxeter, 1989, p. 230). We conjecture an accelerated coordinate frame of a half-angle velocity $v = \tan\frac{1}{2}\theta_h = \tanh\frac{1}{2}\alpha = 1/t$ for a field of accelerations. The vector $z_2 = x + iy = \cot\theta_h + i\csc\theta_h$ is basic science.

3.3 An experiment

When 1 distance/time is 299792458 meters/second = c, the speed of light, we advocate stating a fictitious force acceleration $a_{\text{frame}} = dv/dt = \frac{d}{dt}t^{-1} = -1/(1/v)^2 = -v^2 = -[1/(x+y)]^2$ distance/time^2 when, say, the Earth's gravitational acceleration is $g = (-v^2)(299792458) = -299792458/30570323 = -9.80665$ m/s^2 at $x = 2764.521704$. (Use $(x+y)^2 = 30570323$ and $y^2 = x^2+1$.) Let $(x,y) = (x, \sqrt{x^2+1}) = (2764.521704, 2764.521885)$ be the frame of reference of the Earth. Earth's orbital velocity is 29.8 km/s. We say the slice of time is $t = 1/v = x+y = 2764.521704 + 2764.521885 = 5529.043588$ seconds. The Earth's particle wave is moving at a half-angle velocity $v = 1/(x+y) = 1/t = 1/(2764.521704 + 2764.521885) = 1/5529.043588$ distance/time = 54221.3953 meters/second. As the Earth passes a fixed observer, the observer sees the particle's Galilean velocity at $\tilde{v} = v' + v = 1 = 299792458$ meters/second. A particle on Earth moves forward at a relative velocity $v' = \tilde{v} - v = 1 - 1/(x+y) = 1 - 1/5529.043588 = 0.999819137$ distance/time = 299738236.6 meters/second. The Earth's particle real force absolute acceleration is $a_{\text{absolute}} = \frac{1}{2} + \frac{v^2}{2}$ distance/time^2 = $(\frac{1}{2} + \frac{v^2}{2})(299792458) = 149896233.9$ m/s^2. The relative acceleration is $a_{\text{relative}} = \frac{1}{2} - \frac{v^2}{2}$distance/time^2 = $(\frac{1}{2} - \frac{v^2}{2})(299792458) = 149896224.1$ m/s^2. The field of accelerations becomes $149896233.9 = 149896224.1 - (-9.80665)$ m/s^2.

4. Conclusion

This work contends that there is an underlying theory of v within velocity $\beta = \tan\theta = \tanh\alpha = v/c$ for a Galilean velocity $\tilde{v} = v' - v = 1$. A slice of time can have an analytic quantity $t = e^{\sinh^{-1}x} = x + (x^2+1)^{1/2} = x+y$ seconds. We are claiming a half-angle velocity $v = \tan\frac{1}{2}\theta = \tanh\frac{1}{2}\alpha = 1/(x+y) = 1/t$ distance/time. We advocate stating $vt = [1/(x+y)][x+y] = 1$ distance and a particle's absolute acceleration $a_{\text{absolute}} = dv/d\theta = 1/2 + v^2/2$ distance/time^2 for a field of accelerations. All of the coordinates in Fig. 1(b) move together. The angle of parallelism θ_h graphs both the circle and the $y^2 - x^2 = 1$ vertical hyperbola in hyperbolic terms. The complex plane point (x,y) makes the hyperbolic angle $\theta_h = 2\tan^{-1} e^{-\sinh^{-1}x} = \tan^{-1}\frac{1}{x}$, rather than the circular $\theta_c = \tan^{-1}\frac{y_c}{x}$. Our circular vector $z_1 = re^{i\theta_h} = r(\cos\theta_h + i\sin\theta_h) = ((r\cos\theta_c)^2 + (r\sin\theta_c)^2)^{1/2}\left(\frac{x}{y} + i\frac{1}{y}\right)$ and hyperbolic vector $z_2 = r\cos\theta_c + i((r\cos\theta_c)^2+1)^{1/2} = \cot\theta_h + i\csc\theta_h = x + iy$ are made with the hyperbola's x and y, rather than the circle's x_c and y_c. The hyperbolic *modulus* is $|z_2| = (x^2+y^2)^{1/2} = ((\cot\theta_h)^2 + (\csc\theta_h)^2)^{1/2}$. Any point with $0 \le \theta_h \le \frac{\pi}{4}$ on the complex plane can be reached at $re^{i\theta_h}$ and translated into the frame of reference (x,y). We have $x = e^{n\pi/2}$ with $\pi = \frac{m}{n}\cos^{-1}(\cos\frac{n\pi}{m})$ starting from $m = n$ as the real counterpart to complex numbers $e^{in\pi/2} = \cos\frac{n\pi}{2} + i\sin\frac{n\pi}{2}$.

In the local Lorentz frame of reference every particle moves in a straight line with uniform velocity β. In hyperbolic geometry the "straight line" is due to the angle of parallelism geodesic θ_h or $\Pi(x)$. The Lorentz frame of reference ("inertial frame of reference") is rectified as $\beta = v/c = \tan\theta = \tanh\alpha$ by Galilean velocity $\tilde{v} = v' + v = xu = 1$ with Einsteinian velocity $u = \frac{1}{x} = \tan\theta_h = (\tanh\alpha)(\cosh\alpha) = \sinh\alpha = \beta/(1-\beta^2)^{1/2} = (2v)/(1-v^2) = u' + \frac{1}{x}v$, relative velocities $v' = \tilde{v} - v = 1 - 1/(x+y) = xu'$ and $u' = u - v = \frac{1}{x} - \frac{1}{x+y}$, and vector velocity $\frac{dt}{d\tau} = 1/(1-\beta^2)^{1/2} = \frac{1}{\tau} = \frac{y}{x} = \sec\theta_h = \cosh\alpha = \gamma$. Event points in space (Misner, Thorne, & Wheeler, 1970, pp. 20, 49–50) happen $P(t) = P(\frac{1}{v}) = 1/(e^{\alpha/2}) = 1/[(1+v)/(1-v)]^{1/2} = P(x+y)$. We have τ is *proper time* and s is *proper distance* with *time dilation* Δt and *length contraction* s' thusly:

$$\tau = \Delta t\frac{1}{\gamma} = (y-x)\frac{x}{y} \qquad \Delta t = \tau\gamma = \frac{s}{v} = y - x$$

$$s = v\Delta t = \frac{1}{x+y}(y-x) \qquad s' = s\frac{1}{\gamma} = v\tau = (\frac{y-x}{x+y})(\frac{x}{y}).$$

Hyperbolic geometry has $(\cos\theta)^2 + (\sin\theta)^2 = (x/y)^2 + (1/y)^2 = 1$ with $x = \cot\theta$ and $y = \csc\theta$. Local Lorentz geometry holds the interval $-\tau^2 = s^2 = (AB)^2 = -(\cos\theta)^2 = (AC)^2 + (BC)^2 = (CZ)^2 + (BC)^2 = -1^2 + (\sin\theta)^2 = -(AQ)(A\grave{P}) = (\sin\theta+1)(\sin\theta-1) = -\tau\tau$ between vector events $B-A$ (or AB). A light ray calculated from events $\grave{P}B$ to events BQ lies with B off and with $ACZ = A\grave{P}QZ$ on the particle's *world line* $P(\tau)$. Vectors $z_2 = x + iy$ are alternately made with $t = \frac{1}{v} = x + y$ than spacetime events $P(\tau) = P(\frac{\Delta t}{\gamma})$. We may have Lorentzian $B - A = P(\tau) - P(0) = P(\cos\theta) - P(0) = P(1) - P(0)$ at one second but hyperbolic $P(t) = P(x+y) = P(0+1)$ events happen on the complex plane. Combining hyperbolic geometry, distance scale k and local Lorentz geometry might be called *pregeometry*, gravitational collapse (Misner, Thorne, & Wheeler, 1970, p. 1203). We assert that half-angle velocity and absolute acceleration exist in mathematics and are measurable. A particle's real force acceleration is $a_{\text{absolute}} = (1/2 + v^2/2)(299792458)$ m/s^2 with a gravitational fictitious force acceleration $g = (-v^2)(299792458)$ m/s^2. Can this expanding real force be dark energy?

Joshua Frieman, director of the Dark Energy Survey, Frieman (2015) describes the particle, with my brackets, as follows:

> "Dark energy takes the form of a so far undetected ['quintessence'] particle that could be a distant cousin

of the recently discovered Higgs boson . . . a particle acting like a ball rolling down a hill at each point in space. The rolling ball carries both kinetic energy (because of its motion) [like our absolute acceleration] and potential energy (because of the height of the hill it is rolling down) [like our relative acceleration]; the higher an object is, the greater its potential energy is. As it rolls down, its potential energy declines, and its kinetic energy rises . . . . If the quintessence particle is extremely light, . . . then it would be rolling very slowly today, with relatively little kinetic versus potential energy. In that case, its effect on cosmic expansion would be similar but not identical to that of vacuum energy and would lead to acceleration [of the universe]."

If dark energy (Riess & Livio, 2016) $a_{\text{absolute}} = \frac{dv}{d\theta} = 1/2 + v^2/2 = 1/2$ is undetected due to the energy of the vacuum (General Relativity's cosmological constant Λ), (Misner, Thorne, & Wheeler, 1970, pp. 410, 707). then we have the kinetic versus potential energy $a_{\text{frame}} = \frac{dv}{dt} = -v^2 = 0 = 1/2 - 1/2$ at $v = 0$, with a total energy $a_{\text{frame}} = -1 = 0 - 1$ and a dark energy of 1 at $v = 1$. General Relativity is a theory determined by relative acceleration $a_{\text{relative}} = \frac{dv}{d\alpha} = 1/2 - v^2/2$. The Einstein field equation is $G+\Lambda g = 8\pi T$, where T is the Riemann stress-energy tensor. Equation (2) is a projective geometry, which does not include Riemannian geometry, nor topology (Coxeter, 1989). With Riess and Livio (2016) we alternatively hypothesize dark energy may be an energy field that pervades the universe, imbuing every point in space $P(t) = P(\frac{1}{v}) = 1/(e^{\alpha/2}) = 1/[(1 + v)/(1 - v)]^{1/2} = P(x + y)$ with a property $a_{\text{absolute}} = \frac{dv}{d\theta} = 1/2 + v^2/2$ that counteracts the pull of gravity $g = -v^2$.

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