Review of European Studies; Vol. 7, No. 10; 2015
ISSN 1918-7173 E-ISSN 1918-7181
Published by Canadian Center of Science and Education

Some Approaches to the Calibration of Internal Rating Models

Olga Stepanovna Rudakova[1] & Konstantin Ipatyev[1]

[1] Department of Banks & Bank Management, Financial University under the Government of the Russian Federation, School of Management, Moscow, Russian Federation

Correspondence: O. S. Rudakova, Department of Banks & Bank Management, Financial University under the Government of the Russian Federation, Moscow, Russian Federation. Tel: 7-495-615-7275. E-mail: bibm@fa.ru

Received: March 11, 2015 Accepted: April 10, 2015 Online Published: May 25, 2015

doi:10.5539/res.v7n10p25 URL: http://dx.doi.org/10.5539/res.v7n10p25

Abstract

This article covers the peculiarities of calibration of internal rating models which are the most popular approach to assessing credit risks. The authors address the most common approaches and methods used for rating models calibration, as well as propose their own algorithm for calibration, the main feature of which is taking into account the forecasted probability of default on the portfolio. Research methods include regression analysis, time series analysis (ARIMA models development). Reliability of the proposed approach has been verified on the basis of the portfolio, based on the debt obligations of Russia financial institutions. The advantage of the proposed approach towards determination of the average default probability of credit portfolio is that one gets a tool that allows you to construct a rating model that is "forward looking", respectively, appear to more quickly adapt to the changing patterns of rating environment.

Keywords: Basel II, credit risk, the probability of default, scoring points, correlation analysis

1. Introduction

Recent global crisis indicated high interrelation between both financial institutions and even regional financial systems with the global systematic risk. Though fiscal policy of regional regulators can reduce the impact of negative trends in the local market, absence of such regulating authority on the global scale stipulates that reduction of "regional systematic" risk does not mean proper mitigation of the "world systematic" risk. Accordingly, a crisis in one region or country can cause the "domino effect" and consume all other regions and countries, despite the futile efforts of local regulators. This effect is especially material for developing countries. The impact could be observed via the so-called "flight to quality" effect, e.g. the size of the foreign investment in a particular country. Being the economic growth driver of any country, credit institutions are at the same time are the most susceptible to systematic risk: hence only proper risk mitigation policies can be viewed as pillars for survival and successful doing business for credit institutions. Traditional banks are by default exposed to credit risks (which is the nature of their business) and hence proper evaluation of those risks is the cornerstone of their profitability and survival.

It should be noted that the main study of statistical approaches towards rating model development are associated with the work of Altman E., Englmanna B., Erlenmeyer W., Hayden E., Tasha D. and others. For the validation of rating models should be noted works Kohavi R., Cook D., Picard R., Rauhmaer R. and others. The Russian researchers on these issues should be noted works Ayvazian S. A., Bukhtin M. A., Halavan S. V., Karminsky A. M., Lobanov A. A., Peresetsky A. A, Pomazanov M. V., Putilovsky V. A. et al.

However, the works of these authors largely contain general description of the problem, or are considering the individual stages of the development of rating models. Little relates to the issues of development of rating models for sub-portfolios with low or no defaults. (Burakov, 2014b; 2014c) Most of existing research relates also to the allocation of economic capital and implication of the risk weighting calculation as proposed by the Basel Committee.

To remain a going concern a credit institution must have adequate methods and tools to differentiate borrowers by level of credit risk. In accordance with Basel II internal rating models can be viewed as such instrument, as providing for the best way to estimate the level of capital adequacy to cover credit risks because rating models providing representative result.

Usually the rating model set up includes the following steps:

1) Development of the scoring model,

2) Calibration of the rating model.

The 1st step is to setup the following equation:

$$\text{Score} = f\,(x_1,..,x_n;\, y_1,\ldots,\, y_n) \tag{1}$$

Where

Score is defined as value assigned to the borrower and reflecting its relative creditworthiness (Nurlybayeva & Balakayeva, 2013);

$x_1,..,x_n$—borrowers quantitative factors (for example, leverage, return on assets, current liquidity, etc.);

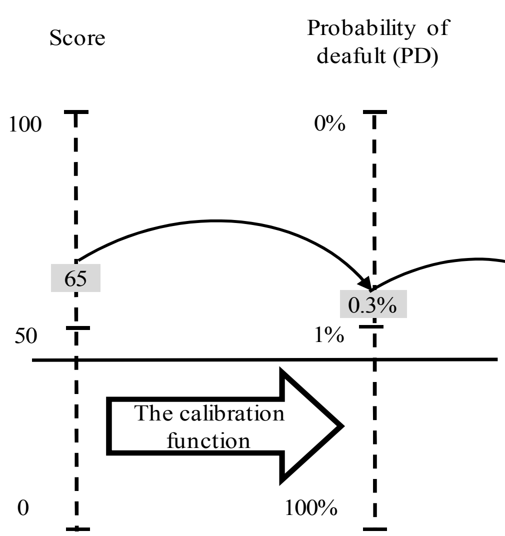
$y_1,\ldots,y_n$—borrowers qualitative factors (for example, quality of management, market position, credit history, etc.);

f (.)—the functional relationship that translates quantitative and qualitative factors in scoring points (usually a function of the logistic or normal distribution).

The scoring itself can only tell whether 1st Borrower is better than 2nd Borrower (for instance, scoring of the 1st Borrower is higher than that of the 2nd). This approach, however, does not give an answer to the question of the probability of default of the 1st Borrower relative to the 2nd Borrower. Another issue is lack of comparative basis of the scoring results of regular borrowers with borrowers having external ratings (assigned by global rating agencies—Moody's, Standard & Poor's, Fitch).

2. Calibration of the Rating Model

The above-mentioned issue can be solved via calibration of the model that is the process of determining a calibration function, which is used to adjust the scores by probability of default and internal ratings (see Figure 1). Practically, this would lead the average portfolio probability of default to reflect the actual (observed) value of probability of default.



Master-sclae

Rating class	Average PD	Lower PD	Upper PD
1	0,01%	0,00%	0,013%
2	0,015%	0,013%	0,02%
…	…	…	…
k	0,35%	0,29%	0,45%
k+1	0,705%	0,45%	0,95%
…	…	…	…
n-1	40%	25%	50%
n-1	100%	50%	100%

Figure 1. Correlation between the scoring figure and probability of default

Generally calibration is the most difficult step of the rating model setup.

Depending on the availability of default statistics one of the below calibration functions may be used for calculation purposes. (Table 1) (Ipatyev, 2012, 2013)

Table 1. Types of calibration function

Availability of default statistics	Function F (α, β)
Sufficient default statistics available	Binomial Logistic Regression (Note 1): $F(\alpha, \beta) = \frac{1}{1+e^{\alpha+\beta \cdot Score}}$
Absence of sufficient statistics of default, but there is sufficient number of observations with external ratings (ratings assigned by Moody's, S & P and/or Fitch))	(Log) linear regression: $\ln F(\alpha, \beta) = \alpha + \beta\ Score$
Absence of sufficient statistics of default and sufficient number of observations with external ratings (ratings assigned by Moody's, S & P and/or Fitch)), but there is available sufficient number observations with the expert rankings assigned by other rating agencies (not Moody's, S & P and/or Fitch) or business experts of the bank	Multinomial logistic regression (model of multiple (discrete) choice) (Note 1).

As observed from the calibration functions in Table 1, each of them is effectively a translation of the linear function α + β Score. The steeper the lien is, the lower is the differentiation model and hence less is the distribution of the ratings. (Figure 2) (Gurny, 2013)

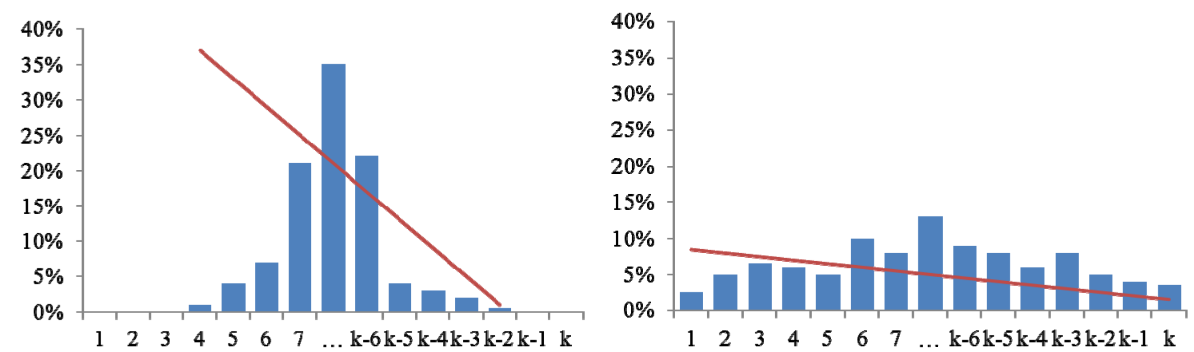


Figure 2. View of the calibration portfolio based on the angle of the calibration line

Multiplicative regression ratio β indicates the ratio by which the probability of default index increases while the value of scoring points changes by 1 rating notch. The additive ratio α is defined as the difference between scoring points and multiple of multiplicative factor and the probability of default values for a given rating. The ratios α and β of the calibration curve can be determined based on the following equation:

$$\frac{1}{N} \cdot \sum_{i=1}^{N} F_i(\alpha, \beta) = \overline{DR} \tag{2}$$

Where

N is the number of borrowers in a sub-portfolio for which a rating model is constructed;

$F_i\,(\alpha, \beta)$ is the calibration function;

$\overline{DR}$ is the observed default rate for the portfolio on the 1-year horizon.

There are several ways to determine the DR Index. The two most common approaches are the so-called "through the cycle" (TTC) and the "point in time" (PIT) (Loffler, 2012).

In accordance with the PIT approach DR can be determined on the basis of the number of default-approaching borrowers in the sub-portfolio for the next year. Thus, to determine DR_{PIT} we need to determine the number of non-defaulting borrowers into credit portfolio as of a year ago, and find out how many of them actually defaulted in the past 12 months.

In accordance with TCC approach it is necessary to determine the term of the credit cycle, i.e. the cycle, during which the banks are overoptimistic regarding the extension of credit facilities. During the next stage this "optimism" results in non-payments, non-performing loans and hence economic downturn, during which the banks (credit institutions) become risk averse and concentrate on bad loans write-offs rather than new loans. After some time the banks recover from the initial shock and resume loan extension, which promotes new economic growth that has a powerful impact on the banks' optimism thus leading to the new credit cycle. Therefore a borrower's probability of default determined by the TTC method resembles the long-term estimation of a borrower's financial stability.

Under Basel II criteria a 7-year cycle is proposed. Analysis of the dynamics of Russia's GDP over the last 20 years shows the validity of this approach for determination of the credit cycle length for the the Russian market. (see Figure 3).

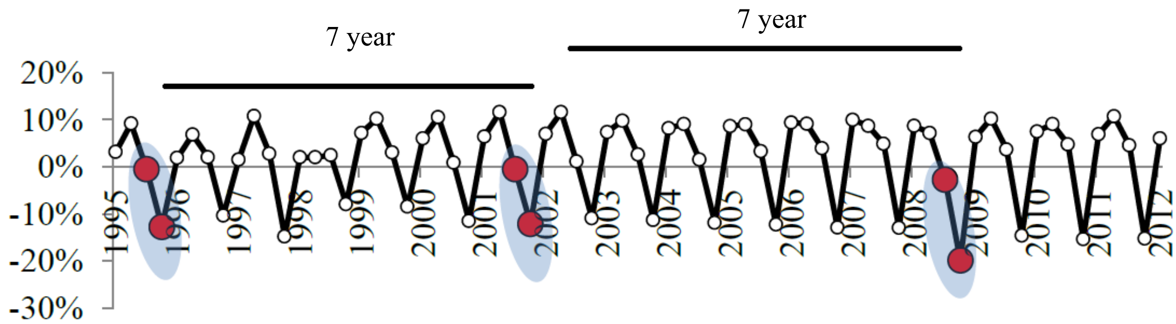


Figure 3. Russia's GDP dynamics 1995-2012 (Note 2)

Having determined the credit cycle one can switch to calculation of the DR indicator for the beginning of each of the respective periods. Level DR_{TTC} in this case will be equal to the average value of DR throughout each year in the credit cycle:

$$\overline{DR}TTC = \frac{1}{Years} \sum_{i=1}^{Years} DR_i \tag{3}$$

For calculation of the economic capital (used to cover unexpected losses) and provisions (used to cover expected losses) Basel II clearly identifies the need to consider the level of losses in the credit cycle, i.e one must use the default probability TTC (Basel, 2001). There is no firm criteria regarding the pricing setup. As already mentioned above, the only criteria explicitly required by the Basel II is the need to use the results of the rating systems for capital allocation, and business purposes, including pricing ("use test"). (Basel, 2006)

In the case of TTC approach is selected the cost of risk component in pricing will be too high in case of economic growth (leading to a bank losing its competitive edge over peers) while being too low in case of downturn, since expected pricing wouldn't cover the actual level of losses, having a negative impact on the actual profit figure.

Thus application of PIT approach deems more reasonable, however it has some demerits. For instance, in case the loan is provided for longer periods, the performance of the loan portfolio may render the price of the long-term credit too small compared with the market standard for a particular borrower. (Note 3) (Gobeljic, 2012)

3. Hybrid Approach towards Calibration of Rating Models

The alternative approach to both methods is use of a hybrid approach, i.e. weight-average between PIT and TTC approach. The longer the loan, the more it should be closer to the approach of TTC. At the same time, short-term loans require repayment of the loan within one year, or for the period during which, as the model assumes, the characteristics of the loan portfolio and external factors change is not significant. In this case, the approach should be close to the PIT. We suggest using the following example of such a function. (Ipatyev, 2012)

$$PD_{Hybrid} = PD_{PIT}\,(1-(0.1\,(D-1))) + PD_{TTC}\,(0.1\,(D-1)) \quad (4)$$

Where in:

D—the term of the requested loan in years (from 1 to N years, where N—the maximum number of years over which a bank issues a “standard” loans):

- If the loan period is less than a 1 year, then the value is 1;
- If the loan period is over N years, then the value is N;
- In other cases, if the loan period is not equal to a whole year, it is rounded according to the rules of mathematical rounding.

The obvious disadvantage of the PIT is that calibration is based on sub-portfolio data a year backwards, and not on the current sub-portfolio, making it impossible to determine the percentage of the current portfolio to be allocated to default within the next year.

This disadvantage could be avoided if one knew how many credit institutions currently existing would became defaults during the next 12 months. However, this information is not available. But on the other hand, a tool can be created that would allow estimation of this value and then this estimate could be used instead of the quantity itself. (Burakov, 2014a)

In this paper, we propose an approach to solve this problem, which is described by the following algorithm:

1) Correlation between DR and macroeconomic indicators is plotted (RTS / MICEX indices, interbank rates, inflation, etc.):

$$\overline{DR}_{TTC} = \frac{1}{Years}\sum_{i=1}^{Years} DR_i \quad (5)$$

2) For each of the macroeconomic indicators (x_i) included in the model (5), the time series is constructed (for example, on the methodology of ARIMA);

3) For each of the macroeconomic indicators included in the model (5) are built predictive values of the indicators for the year ahead;

4) Predicted macroeconomic indicators are substituted into the model (5), so we get forecast DR.

To illustrate the proposed approach, one needs to consider a hypothetical sub-portfolio consisting of all credit institutions in a country (Russia).

To construct the correlation between macroeconomic indicators and defaulted credit organizations we examined the dynamics of shift in the number of banks in the country and the number of banks with licenses recalled (see Figure 4).

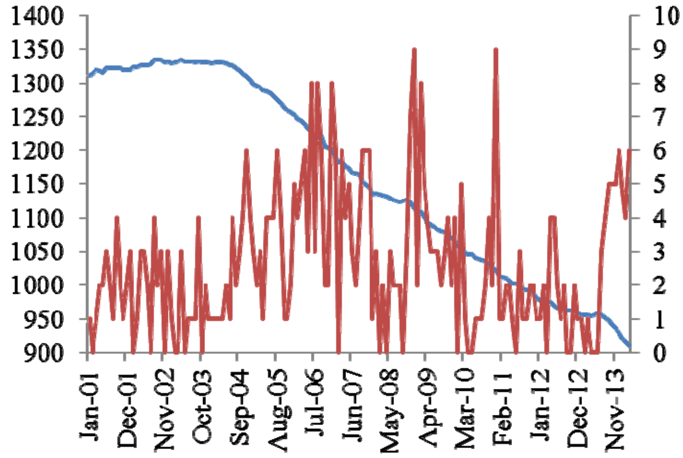


Figure 4. Dynamics of changes in the number of credit institutions in the banking system of Russia. The red curve—the dynamics of license recall

Also were analyzed the level of DR by month from January 2001 to March 2014 (see Figure 5).

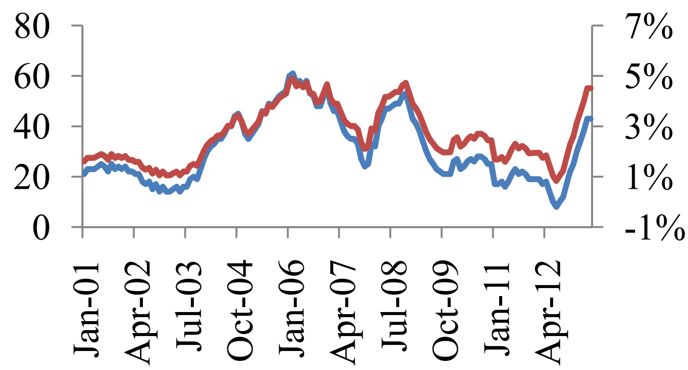


Figure 5. Dynamics of DR (%, red curve) and the amount of defaulted banks (blue) in the portfolio consisting of Russian credit institutions, for the period 2001-2013

Stage 1.

In the first stage of our algorithm were considered publicly available Russian macroeconomic indicators, including:

• MICEX index (total 40 indices) for the period 1997-2014. (Information taken from the official website of the MICEX—http://www.moex.com, see Table 2);

• Exchange rates (major currencies—USD, EUR) for the period 2004-201;

• Selected indicators of credit institutions (total 28 indicators, see Table 3);

• RTS indices (total 11 indices, information is taken from the official website of the MICEX http://www.moex.com).

Table 2. MICEX index

No.	Name of indicator
1	Second-Tier Index Moscow Exchange
2	Micex Corporate Bond Index
3	Micex Corporate Bond Index
4	Micex Corporate Bond Index—total revenue
5	Micex Corporate Bond Index (1-3 years, ranking ≥ B-)
6	Micex Corporate Bond Index (1-3 years, ranking ≥ B-)—total income
7	Micex Corporate Bond Index (3-5 years, ranking ≥ B-)
8	Micex Corporate Bond Index (3-5 years, ranking ≥ B-)—total income
9	Manufacturing index Moscow Exchange
10	Metals & Mining index Moscow Exchange
11	MICEX index
12	MICEX Innovation index
13	MICEX10 Index
14	Micex Municipal Bond index
15	Micex Municipal Bond index
16	Micex Municipal Bond index—total income
17	Oil & Gas index Moscow Exchange
18	Consumer goods and Services index Moscow Exchange
19	Telecoms index Moscow Exchange
20	Telecoms index Moscow Exchange
21	Transport index Moscow Exchange
22	Financials index Moscow Exchange
23	Chemicals index Moscow Exchange
24	Broad Market Moscow Exchange
25	Electric Utilities index Moscow Exchange
26	MICEX high cap
27	MICEX corporate bonds (currency)
28	MICEX corporate bonds (currency)1-3
29	MICEX corporate bonds (currency)3-5
30	MICEX municipal bonds (currency)
31	MICEX stock REPO 1 day
32	MICEX stock REPO 14 days
33	MICEX stock REPO 7 days
34	MICEX Bond REPO 1 day
35	MICEX Bond REPO 14 days
36	MICEX Bond REPO 7 days
37	MICEX standard cap
38	Technical stock index *
39	Technical bond index *
40	Technical index shares shares *

Table 3. Selected indicators of credit institutions

	Name of indicator
1	Net foreign assets
2	Non-residents obligations
3	Liabilities to non-residents
4	Resident obligations
5	Net government obligations
6	Federal government obligations
7	Regional and local government obligations
8	Liabilities to federal government
9	Liabilities to local government
10	Other financial companies obligations
11	Non-financial public sector entities obligations
12	Other non-financial entities obligations
13	Household obligations
14	Other segments' obligations
15	Broad money supply (BMS)
16	Money supply
17	Currency outside the banking system
18	Transferable deposits
19	Financial entities deposits
20	Non-financial public sector entities deposits
21	Other non-financial entities deposits
22	Household deposits
23	Other deposits
24	Deposits, which don't include in BMS
25	Securities (excluding shares and other equities), which don't include in BMS
26	Shares and other equities
27	Other liabilities
28	Other assets

Macroeconomic indicators published by ROSSTAT (inflation, unemployment, GDP, etc.) were not included in the analysis for the following reasons:

- Data are published with a delay of several months;
- Data is published quarterly (in our research we use monthly data);
- All macroeconomic indicators are highly correlated with the above indicators.

Thus, were considered 81 external indicators (explanatory variables).

In the first stage was constructed regression between the DR and the explanatory variables. Selected model, consisting of 5 explanatory indicators (R^2 is 97.57%). More detailed information is shown in Table 4.

Thus, we have found a certain correlation between DR (PIT) and external explanatory figures:

$$\begin{gathered} DR = 1.8bp^{*}\ RTSog—0.2bp^{*}MICEX10INDEX\ + \\ +\ 4.7bp^{*}MICEXCBITR\ +0.0001bp^{*}NFPSE—0.02bp^{*}EUR \end{gathered} \tag{6}$$

Table 4. Qualitative parameters of a regression model

Name of the explanatory variable	Code explanatory variable	Parameter estimation	Standard error	F—value	Pr > F
Sectoral index RTS (Oil and Gas)	RTSog	0.0180%	0.0000	83.20	<.0001
MICEX10 Index (Note 4)	MICEX10INDEX	-0.0017%	0.0000	134.60	<.0001
MICEX Corporate Bond Index	MICEXCBITR	0.0463%	0.0001	39.89	<.0001
Balances of non-financial state organizations	NFPSE	0.000012%	0.0000	25.91	<.0001
Exchange rates EUR/RUR	EUR	-0.0519%	0.0002	6.97	<.0001

Stage 2.

For each of the explanatory variables were constructed (by using ARIMA method) time series and with 1-year forecast. For example, RTS index RTSog by analyzing the autocorrelation function (ACF) and partial autocorrelation function (PACF) was defined as the time series form ARIMA (2, 0, 0). Statistical indicators and graphical representation of the time series RTSog and correlation analysis are presented in Table 5 and Figure 6, respectively.

Table 5. Statistical indicators of the time series RTsog

Variable = RTSog	
Average time series	199.26
Standard deviation	54.26
Number of observations	100

Checking the autocorrelation function for "white noise"

Lag	$\chi2$	Degree of freedom (number)	Pr > ChiSq	Autocorrelations					
6	268.92	6	<.0001	0.925	0.810	0.688	0.558	0.436	0.331
12	279.41	12	<.0001	0.236	0.150	0.076	0.011	-0.044	-0.097
18	290.39	18	<.0001	-0.131	-0.140	-0.137	-0.117	-0.100	-0.114
24	325.25	24	<.0001	-0.127	-0.148	-0.181	-0.217	-0.257	-0.289

Stage 3.

In the third stage were defined parameters of time series and constructed forecast values of the explanatory variables for 12 months ahead. The model was based on data for the period from January 2005 to March 2013 (the same period, we have a time series with DR values).

For example, according to the forecast, the rate RTSog within a year on 01.03.2014 and 01.04.2014 was to retain its value. As seen in Figure 7, despite the fact that the actual value indicator fluctuated around the prediction, the end of the period are substantially aligned.

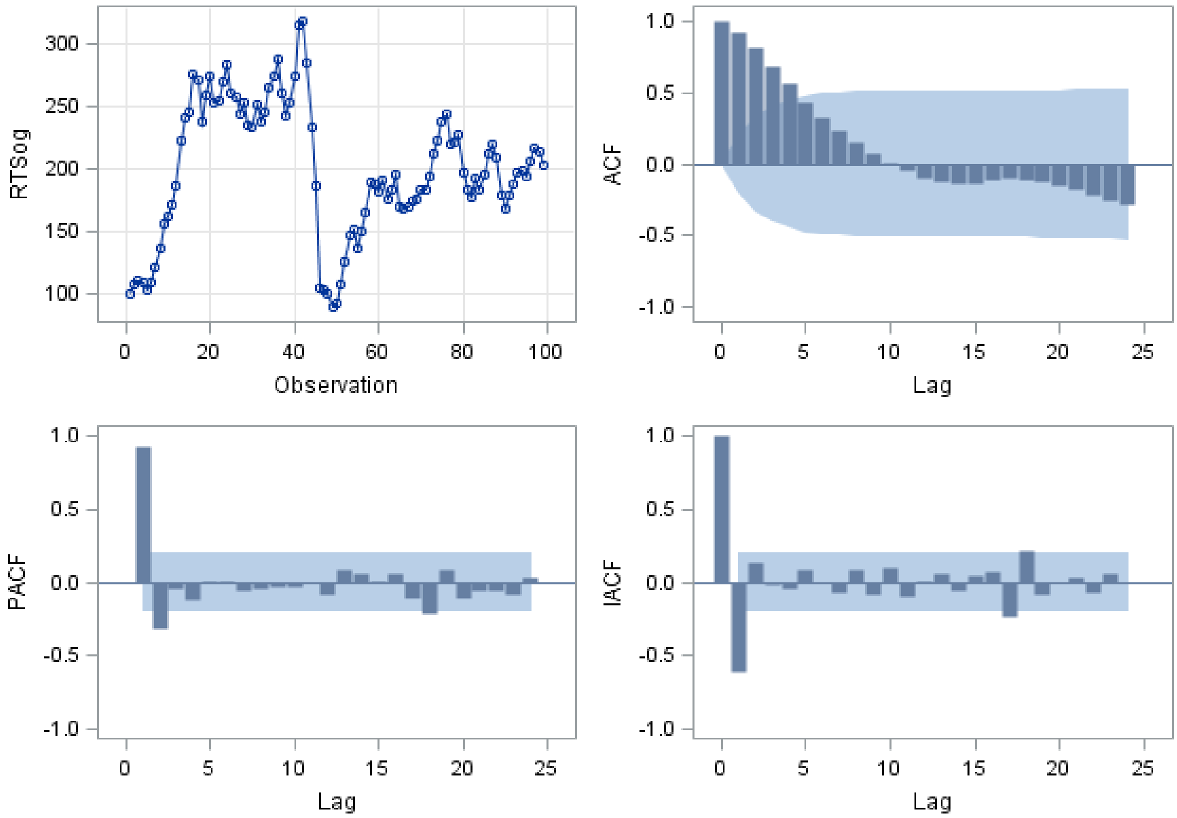


Figure 6. Graphical representation, ACF and PACF of a RTSog

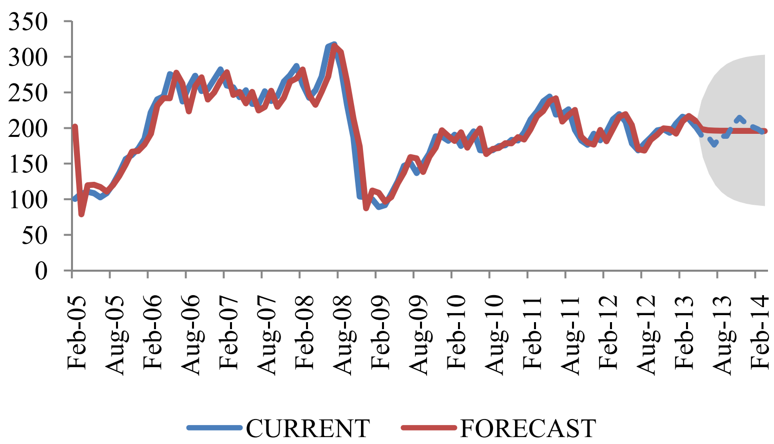


Figure 7. Forecast of RTSog time series

Thus, one can conclude that the constructed time series quite clearly predict the future value of the index. However, due to the other indicators the overall dynamics of the index is less positive, but the observed values of the series does not go beyond the boundaries of the level of confidence (95% level of confidence in the charts—a gray area), which, given the current unstable situation in the markets, is pointing to the sufficient quality of the given time series.

Stage 4.

After the substitution of predicted values in the regression equation we obtained forecast DR on 01.04.2014 equal to 2.3% (i.e., during the period 01.04.2014-01.04.2015), in accordance with our methodology assumes that will be recalled licenses (default) 2.3% of the credit institution of a total of 910 credit institutions, i.e., at 21 banks).

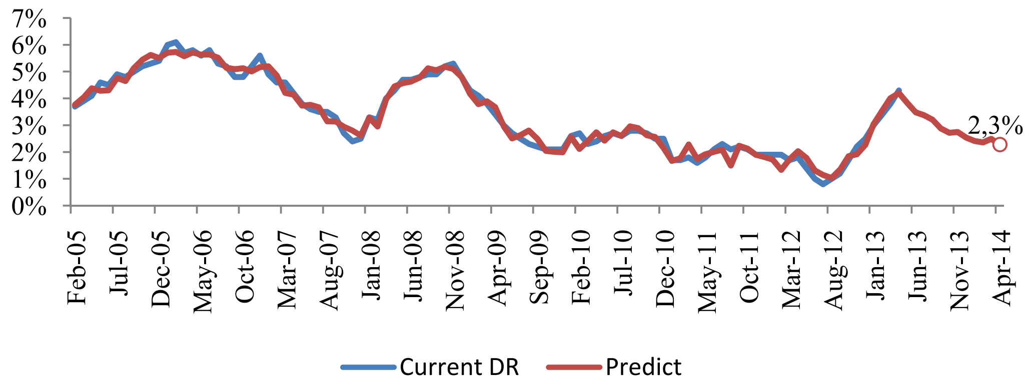


Figure 8. Predictive value of DR on March , 01, 2014

As the result we have the value of DR point-in-time on April 4, 2014 and can calibrate rating model for the current portfolio, but not for the portfolio a year earlier.

Similarly, we can calculate the predicted PD_{TTC} (7 years):

(3.80% + 3.11% + 2.29% + 1.59% + 1.85% + 4.51% + 2.30%) / 7 = 2.78%

The current value of PD_{TTC} (7 years) is 2.98%.

4. Conclusion

The advantage of the proposed approach towards determination of the average default probability of credit portfolio is that one gets a tool that allows you to construct a rating model that is "forward looking", respectively, appear to more quickly adapt to the changing patterns of rating environment.

In the example tackled in the article (based on PIT approach) average forecast probability of default on the portfolio will be equal to 2.3%. In case the model calibration is performed on the data a year earlier, the rate would be equal to 4.51%, which would have clearly reduced the competitive advantages of a credit institution, as products offered to them would cost too high. On the other hand, if we observed the opposite picture, and forecast the probability of default would be higher than the observed, the application of the proposed approach would allow preparing for a crisis, while the use of the observed values of DR could cause unexpected losses incurred by the credit institution.

References

Acharya, V. V. (2009). A Theory of Systemic Risk and Design of Prudential Bank Regulation. *Journal of Financial Stability, 5*(3), 224-255. http://dx.doi.org/10.2139/ssrn.236401

Basel Committee on Banking Supervision (BCBS). (2001). *The Internal Ratings-Based Approach*. Retrieved from http://www.bis.org/publ/bcbsca05.pdf

Basel Committee on Banking Supervision (BCBS). (2004). *International Convergence of Capital Measurement and Capital Standards* (A Revised Framework). Retrieved from http://www.bis.org/publ/bcbs128.pdf

Basel Committee on Banking Supervision (BCBS). (2006). The IRB Use Test: Background and Implementation. *Basel Committee Newsletter*, *9*. Retrieved from http://www.bis.org/publ/bcbs_nl9.pdf

Burakov, D. V. (2014a). Credit Cycles & Debt Restructuring: Refining Theoretical Flaws. *Review of European Studies*, *6*(4), 1-11. http://dx.doi.org/10.5539/res.v6n4p1

Burakov, D.V. (2014b). Do Methods of Estimation Affect Credit Risk Osccillations? *Mediterranean Journal of*

Social Sciences, *5*(20), 114-120. http://dx.doi.org/10.5901/mjss.2014.v5n20p114

Burakov, D.V. (2014c). How to Measure Herd Behavior on the Credit Market? *Mediterranean Journal of Social Sciences*, *5*(20), 516-521. http://dx.doi.org/10.5901/mjss.2014.v5n20p516

Chamalinsky, A. S., & Pomazanov, M. V. (2012). "Kalibrovka reytingovoy modeli dlya sektorov s nizkim kolichestvom defoltov" [Calibration of the rating model for sectors with low default occurrences]. *Financial risk management*, *2*, 82-94.

Gobeljic, P. (2012). *Classification of Probability of Default and Rating Philosophies*. Stockholm. Retrieved November 2, 2014, from http://www.math.kth.se/matstat/seminarier/reports/M-exjobb12/121203.pdf

Gurný, P., & Gurný, M. (2013). Comparison Of Credit Scoring Models On Probability Of Default Estimation For Us Banks. *Prague Economic Papers*, *2*, 163-181. Retrieved from https://ideas.repec.org/a/prg/jnlpep/v2013y2013i2id446p163-181.html (accessed 25 September 2014)

Ipatyev, K. N. (2012). "Tsenoobrazovaniye s uchetom kreditnogo riska" [Pricing based on the credit risk]. *Scientific Review: Economics and Management*, *3*, 102-104.

Ipatyev, K. N. (2013). Razrabotka vnutreney reitingovoy modeli v ramkach soglasheniya Bazel II v usloviyach otsutstvia statistiki po defoltam [Development of the internal rating model under Basel II criteria and absence of default statistics]. *Scientific Review*, *6*, 220-224.

Loffler, G. (2012). Can rating agencies look through the cycle? *Review of Quantitative Finance and Accounting*. Retrieved October 10, 2014, from http://link.springer.com/article/10.1007/s11156-012-0289-9

Nurlybayeva, K., & Balakayeva, G. (2013). Algorithmic Scoring Models. *Applied Mathematical Sciences*, *7*(12), 571-586. Retrieved October 20, 2014, from http://www.m-hikari.com/ams/ams-2013/ams-9-12-2013/nurlybayevaAMS9-12-2013.pdf

Notes

Note 1. Normal distribution may be also used.

Note 2. According to the Federal State Statistics Service (http://www.gks.ru/).

Note 3. We assume that the cost of funding remains constant.

Note 4. Index, calculated as the arithmetic mean of the price changes of the ten most liquid shares (basket index) traded on the Moscow Stock Exchange.