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Direct Conversation of Generalized Parameters of Multicomponent Two-Terminal Networks Using Capacitive Differentiator RC-Chains

Vladimir I. Ivanov[1] & Vitalij S. Titov[1]

[1] South-West State University, Department of Computer Science, Kursk, Russia

Correspondence: Vladimir I. Ivanov, South-West State University, Department of Computer Science, Kursk, Russia. E-mail: viva37@mail.ru

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Abstract

We present a device for direct conversion of generalized parameters of passive multicomponent two-terminal networks (TTN) with excitation of measuring circuit (MC), comprising a model component and the two-terminal network being tested, on applying to the measuring circuit voltage pulses which vary as the *n*-th power of the time and *n*-fold signal differentiation at measuring circuit input and output with differentiators being executed on RC-chains.

Keywords: multicomponent two-terminal network, generalized parameters, conversion of the parameters

1. Introduction

Transformation of the parameters of bipolar electrical circuits is an important branch of modern information and measuring equipment. Areas for application of converters: transformation parameters of physical processes using parametric sensors, control elements and components electronic equipment. Converters parameters multielement passive multicomponent two-terminal networks (TTN) can be constructed using the direct conversion and equilibration. Converters with balancing voltage or current TTN compensating signal are highly accurate, but have a low speed. When monitoring processes with rapidly changing properties appropriate to use direct object parameters being tested. To convert the parameters of measuring circuit (MC) into electrical signals as MC, it is advisable to use a voltage divider into one arm which includes the measured two-terminal, and in another-defensive chain. If the impact on the use voltage pulse, the shape of which has the form of a power function

$$u_{10}(t) = \frac{U_\mathrm{m} t^n}{t_\mathrm{imp}^n}\,, \tag{1}$$

where U_m is the test impulse amplitude, t_imp is their duration, and in a steady-set mode on completion of the transient process, the divider output response represents a total of impulses in the form of power functions with indices from n to zero:

$$u_{20}(t) = \frac{n!H_0 U_\mathrm{m} t^n}{n!t_\mathrm{imp}^n} + \frac{n!H_1 U_\mathrm{m} t^{n-1}}{(n-1)!t_\mathrm{imp}^n} + \ldots + \frac{n!H_{n-1} U_\mathrm{m} t}{1!t_\mathrm{imp}^n} + \frac{n!H_n U_\mathrm{m}}{0!t_\mathrm{imp}^n}\,. \tag{2}$$

Each component of tension (2) contains information on the corresponding generalized MC parameter. Where H_0, H_1, H_2,…, H_n are generalized parameters of measuring circuit transfer function, with its operator image being generally expressed as

$$H(p) = \frac{b_0 + b_1 p + b_2 p^2 + \ldots}{a_0 + a_1 p + a_2 p^2 + \ldots}\,. \tag{3}$$

When $a_0 \neq 0$, generalized parameters are defined by recurrence formula (Ivanov, Titov, & Golubov, 2010)

$$H_0 = \frac{b_0}{a_0};\quad H_1 = \frac{b_1 - H_0 a_1}{a_0};\quad H_2 = \frac{b_2 - H_0 a_2 - H_1 a_1}{a_0};\quad H_3 = \frac{b_3 - H_0 a_3 - H_1 a_2 - H_2 a_1}{a_0}; \tag{4}$$

The value of each output signal component includes information on one or several electrical parameters of the two-terminal network. The exponent of n of the power impulse must correspond to the number of N-components in the two-terminal network being tested: $n \geq N{-}1$. Thus, for example, at excitation of the measuring circuit (MC) by cubic impulses of voltage, the MC output signal becomes:

$$u_{20}(t) = \frac{H_0 U_\mathrm{m} t^3}{t_\mathrm{imp}^3} + \frac{3H_1 U_\mathrm{m} t^2}{t_\mathrm{imp}^3} + \frac{6H_2 U_\mathrm{m} t}{t_\mathrm{imp}^3} + \frac{6H_3 U_\mathrm{m}}{t_\mathrm{imp}^3}. \tag{5}$$

Triple differentiation of signal (5) allows to define every H-parameter

$$\frac{du_{20}}{dt} = \frac{3\tau H_0 U_\mathrm{m} t^2}{t_\mathrm{imp}^3} + \frac{6\tau H_1 U_\mathrm{m} t}{t_\mathrm{imp}^3} + \frac{6\tau H_2 U_\mathrm{m}}{t_\mathrm{imp}^3}; \tag{6}$$

$$\frac{d^2 u_{20}}{dt^2} = \frac{6\tau^2 H_0 U_\mathrm{m} t}{t_\mathrm{imp}^3} + \frac{6\tau^2 H_1 U_\mathrm{m}}{t_\mathrm{imp}^3}; \tag{7}$$

$$\frac{d^3 u_{20}}{dt^3} = \frac{6\tau^3 H_0 U_\mathrm{m}}{t_\mathrm{imp}^3}. \tag{8}$$

Results of determination of H-parameters directly on formulas (5)–(8) depend on amplitude of feed-in impulse. For the removal of this factor it is necessary to add the second channel of differentiators:

$$\frac{du_{10}}{dt} = \frac{3\tau U_\mathrm{m} t^2}{t_\mathrm{imp}^3}; \tag{9}$$

$$\frac{d^2 u_{10}}{dt^2} = \frac{6\tau^2 U_\mathrm{m} t}{t_\mathrm{imp}^3}; \tag{10}$$

$$\frac{d^3 u_{10}}{dt^3} = \frac{6\tau^3 U_\mathrm{m}}{t_\mathrm{imp}^3} \tag{11}$$

and to ration the values of signals (5), (6), (7), (8) by signals (1), (9), (10), (11) accordingly. Thus, formulas for the calculation of H-parameters will look like:

$$\frac{d^3 u_{20}}{dt^3} \bigg/ \frac{d^3 u_{10}}{dt^3} = H_0; \tag{12}$$

$$\frac{d^2 u_{20}}{dt^2} \bigg/ \frac{d^2 u_{10}}{dt^2} = H_0 + \frac{H_1}{t}; \tag{13}$$

$$\frac{du_{20}}{dt} \bigg/ \frac{du_{10}}{dt} = H_0 + \frac{2H_1}{t} + \frac{2H_2}{t^2}: \tag{14}$$

$$\frac{u_{20}}{u_{10}} = H_0 + \frac{3H_1}{t} + \frac{6H_2}{t^2} + \frac{6H_3}{t^3}. \tag{15}$$

In publications (Ivanov, Titov, & Petrov, 2011, 2012) it was justified the method of direct conversion of H-parameters of the two-terminal network using operations of n-fold differentiation of the test impulse signal and divider output voltage comprising a single-component network Z_0 and a two-terminal network (TTN) of the object measured. When the transient process in the measuring circuit is completed, sampling of transient values of divider input and output signals is performed, as well as all differentiator outputs. The scheme of the converter of parameters of the four-element two-pole network with differentiation of signals is given on Figure 1.

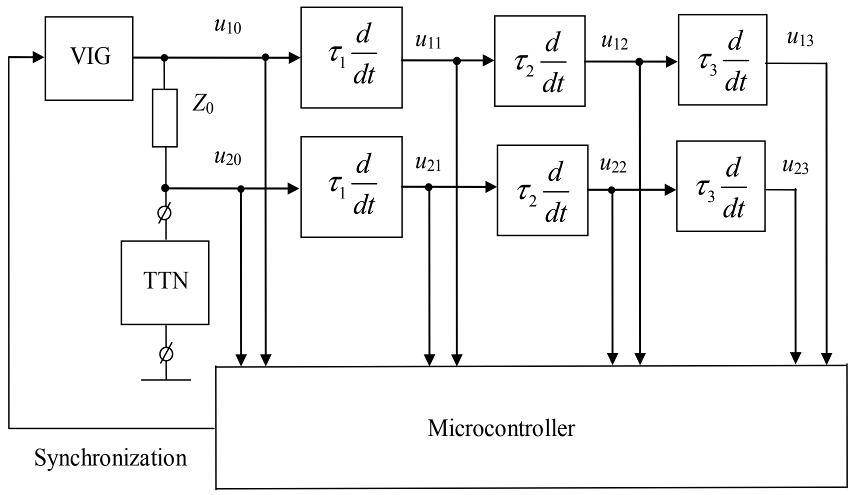

Figure 1. Parameter converter scheme with differentiators

In the article (Ivanov, Titov, & Petrov, 2012) a method and device of direct transformation are considered with differentiators on operational amplifiers (OpAmp). For steady work of multistage differentiators the correction of frequency description of OpAmp is applied in area of high-frequencies.

2. Using Passive *RC*-Section for Differentiation Signal

Differentiating cascades on active components and operating amplifiers in particular has drawbacks, such as the tendency to stability loss, complexity of provision of cascade identity, stability, and drift elimination. In this article it is suggested to apply differentiators on the passive *RC*-chains consisting of condensers and resistors. Use of passive circuits is proposed as multi-cascade differentiators comprising in-series differentiating *RC*-chains. The scheme of converter with differentiating *RC*-chains is presented on Figure 2 (Ivanov, Emelyanov, Titov, & Sohan, 2011).

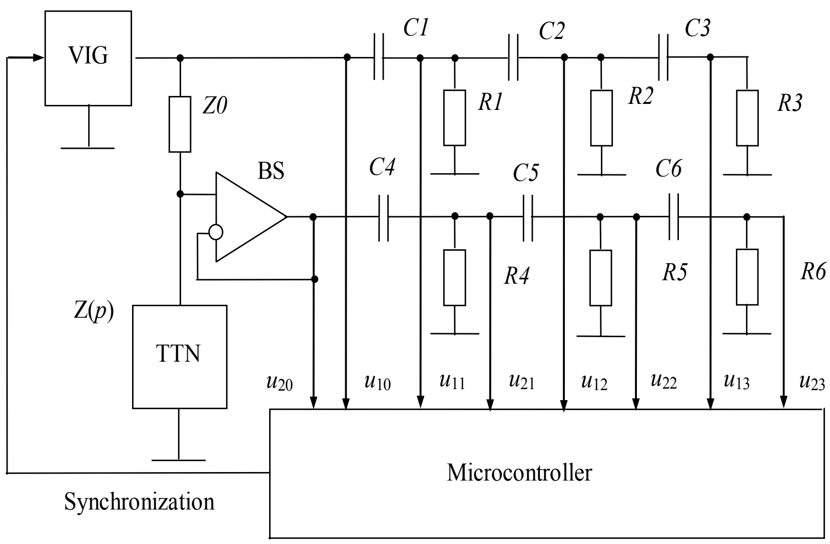

Figure 2. Scheme of parameter converter of two-terminal network with differentiating *RC*-chains

Each differentiator has three differentiating *RC*-chains: R_1C_1, R_2C_2, and R_3C_3 in the first one and R_4C_4, R_5C_5, and R_6C_6 in the second. The first differentiator input receives a signal from the voltage impulse generator (VIG) output powering the measuring circuit, and the second differentiator input receives voltage from the two-terminal network (TTN). Buffer stage (BS) eliminates the influence of the second differentiator input circuit on impedance of the two-terminal network being tested. Transfer function of the only one differentiating *RC*-chains is expressed as:

$$K_{1RC}(p)=\frac{pRC}{1+pRC},$$

To obtain a n-cascade differentiator having transfer function

$$K_{nRC}(p)=\frac{p^n(RC)^n}{(1+pRC)^n},$$

it would be good to include buffer cascades between RC-chains complicating the device scheme. However, a transfer function close to the one described above can be provided using only passive RC circuits. To simplify analytical expressions, it is reasonable to set each cascade's time constants to the same values: $R_1C_1 = R_2C_2 = R_3C_3 = \tau$, using different capacitance and resistance values in each cascade. For example, if one takes $R_2C_2 = RC = \tau$ in the second chain, then resistance is to be reduced and capacitance increased as much as in the first chain, and, conversely, resistance is to be increased and capacitance reduced by as much as in the third chain:

$$C_1=\frac{C}{m};\quad R_1=mR;\quad C_3=mC;\quad R_3=\frac{R}{m},$$

where $m < 1$. Let us define transfer functions for the output of the first, second and third cascades of differentiators and their generalized parameters. Transfer function for the first RC-chain output is as follows

$$K_{1\mathrm{RC}}(p)=\frac{p\tau+p^2(2+m)\tau^2+p^3\tau^3}{1+p(3+2m)\tau+p^2\left(3+2m+m^2\right)\tau^2+p^3\tau^3}. \qquad (16)$$

Generalized parameters of transfer function (16) equal the following:

$$K_{10}=0;\ K_{11}=\tau;\ K_{12}=-(1+m)\tau^2;\ K_{13}=\left(1+3m+m^2\right)\tau^3. \qquad (17)$$

Transfer function for the second RC-chain output is expressed as

$$K_{2\mathrm{RC}}(p)=\frac{p^2\tau^2+p^3\tau^3}{1+p(3+2m)\tau+p^2\left(3+2m+m^2\right)\tau^2+p^3\tau^3}, \qquad (18)$$

and its generalized parameters equal to the following:

$$K_{20}=0;\quad K_{21}=0;\quad K_{22}=\tau^2;\quad K_{23}=-2(1+m)\tau^3. \qquad (19)$$

Finally, the transfer function for the third RC-chain output can be presented with the formula

$$K_{3\mathrm{RC}}(p)=\frac{p^3\tau^3}{1+p(3+2m)\tau+p^2\left(3+2m+m^2\right)\tau^2+p^3\tau^3}, \qquad (20)$$

and its K-parameters equal to

$$K_{30}=0;\quad K_{31}=0;\quad K_{32}=0;\quad K_{33}=\tau^3. \qquad (21)$$

Using expressions for operator image of cubic-form power impulse (1) and generalized parameters (17), (19), (21) of transfer functions $K_{1\mathrm{RC}}(p)$, $K_{2\mathrm{RC}}(p)$ and $K_{3\mathrm{RC}}(p)$, we find signals at outputs of the first differentiator's three cascades:

$$u_{11}(t)=\frac{3\tau U_m\left(t^2-2(1+m)\tau t+2\left(1+3m+m^2\right)\tau^2\right)}{t_{\mathrm{imp}}^3}; \qquad (22)$$

$$u_{12}(t) = \frac{6\tau^2 U_m \left(t - 2(1+m)\tau\right)}{t_{\text{imp}}^3}; \tag{23}$$

$$u_{13}(t) = \frac{6\tau^3 U_m}{t_{\text{imp}}^3}. \tag{24}$$

Likewise, using operator image of voltage on the two-terminal network being tested (5), transfer functions $K_{1RC}(p)$, $K_{2RC}(p)$ and $K_{3RC}(p)$ and their generalized parameters (17), (19), (21), we find signals at outputs of each of the second differentiator's three cascades on completion of the transient process in the measuring circuit:

$$u_{21}(t) = \frac{3\tau H_0 U_m \left(t^2 - 2(1+m)\tau t + 2\left(1+3m+m^2\right)\tau^2\right)}{t_{\text{imp}}^3} + \frac{6\tau H_1 U_m \left(t - (1+m)\tau\right)}{t_{\text{imp}}^3} + \frac{6\tau H_2 U_m}{t_{\text{imp}}^3}; \tag{25}$$

$$u_{22}(t) = \frac{6\tau^2 H_0 U_m \left(t - 2(1+m)\tau\right)}{t_{\text{imp}}^3} + \frac{6\tau^2 H_1 U_m}{t_{\text{imp}}^3}; \tag{26}$$

$$u_{23}(t) = \frac{6\tau^3 H_0 U_m}{t_{\text{imp}}^3}. \tag{27}$$

The output signal ratios of the third differentiating cascades (27) and (24), second differentiating cascades (26) and (23), first differentiating cascades (25) and (22), as well as voltages on the two-terminal network and power impulse at time t, respectively, equal the following:

$$\frac{u_{23}}{u_{13}} = H_0; \tag{28}$$

$$\frac{u_{22}}{u_{12}} = H_0 + \frac{H_1}{t - 2(1+m)\tau}; \tag{29}$$

$$\frac{u_{21}}{u_{11}} = H_0 + \frac{2H_1\left(t - (1+m)\tau\right) + 2H_2}{t^2 - 2(1+m)\tau t + 2\left(1+3m+m^2\right)\tau^2}; \tag{30}$$

$$\frac{u_{20}}{u_{10}} = H_0 + \frac{3H_1}{t} + \frac{6H_2}{t^2} + \frac{6H_3}{t^3}. \tag{31}$$

Using Equations (28)–(31), one can calculate the values of H-parameters of the measuring circuit:

$$H_0 = \frac{u_{23}}{u_{13}}; \tag{32}$$

$$H_1 = \left(\frac{u_{22}}{u_{12}} - \frac{u_{23}}{u_{13}}\right) \cdot \left(t - 2(1+m)\tau\right); \tag{33}$$

$$H_2 = \left(\frac{u_{21}}{u_{11}} - 2\frac{u_{22}}{u_{12}} + \frac{u_{23}}{u_{13}}\right) \cdot \left(\frac{t^2}{2} - (1+m)\tau t + (1+m)^2\tau^2\right) + \left(\frac{u_{22}}{u_{12}} - \frac{u_{23}}{u_{13}}\right) \cdot (1+m)\tau t; \tag{34}$$

$$H_3 = \left(\frac{u_{20}}{u_{10}} - 3\frac{u_{21}}{u_{11}} + 3\frac{u_{22}}{u_{12}} - \frac{u_{23}}{u_{13}}\right) \cdot \frac{t^3}{6} + \left(\frac{u_{21}}{u_{11}} - 2\frac{u_{22}}{u_{12}} + \frac{u_{23}}{u_{13}}\right) \cdot \left((1+m)\tau t^2 - (1+m)^2\tau^2 t\right). \tag{35}$$

3. Numerical Simulation

To check an adequacy for the converter's theoretical model parameters, the modeling of devices' generalized parameters measurement for four-element *RLC* two-pole networks was executed. The scheme of one of them is represented by R_1, C_1, R_2, L_1 on Figure 3. Modeling is executed by means of the MultiSim program, and calculation – MathCad.

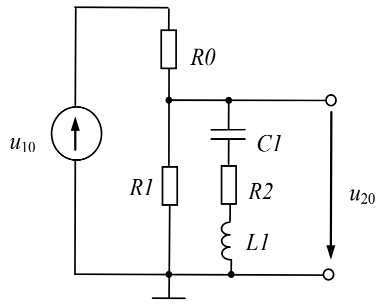

Figure 3. The scheme of four-element *RCL* two-pole network

Parameters of elements of the measuring scheme: R_0 = 1 kOm, R_1 = 1.5 kOm, R_2 = 2.5 kOm, C_1 = 5 nF, L_1 = 4.5 mH. The test signal had a form of a cubic impulse of tension with amplitude U_m = 10 V. Duration of an impulse of time 250 μs. Constant of time of *RC* of a chain τ = 25 μs, parameter *m* = 0.1.

Transfer function of a divider of R_0-*Z* has an appearance

$$H_{\mathrm{RCL}}(p)=\frac{R_1+pR_1R_2C_1+p^2R_1L_1C_1}{R_1+R_0+p\left[(R_1+R_0)R_2+R_1R_0\right]C_1+p^2(R_1+R_0)L_1C_1}.$$

Its generalized parameters are equal

$$H_0=\frac{R_1}{R_1+R_0};\quad H_1=-H_0^2R_0C_1;\quad H_2=H_0^2R_0C_1^2\left(R_2+H_0R_0\right);\quad H_3=H_0^2R_0C_1^2\left[L_1-\left(R_2+H_0R_0\right)^2C_1\right].$$

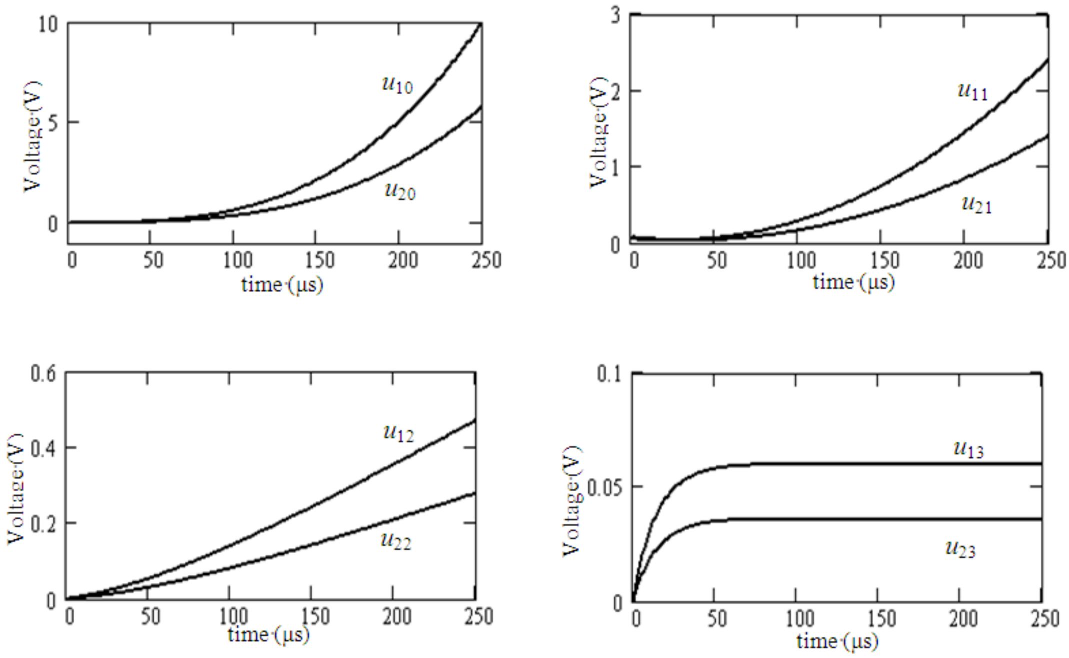

Figure 4. Diagrams of voltages in converter parameters

Figure 4 shows time diagrams of voltages on the inputs and outputs of each differentiating cascade.

Voltage measurement timepoint in all control points t = 200 μs. Results of measurements: u_{10} = 5.12 V; u_{20} = 2.953682 V; u_{11} = 1.4646 V; u_{21} = 0.85163 V; u_{12} = 0.348 V; u_{22} = 0.20448 V; u_{13} = 0.06 V; u_{23} = 0.036 V.

Results of calculations of the generalized parameters in formulas (32)–(35): H_0 = 0.6; H_1 = –1.8 μs; H_2 = 27.9 μs^2; H_3 = –391.95 μs^3. The received values of H-parameters coincide with the parameters of a measuring chain found in formulas (4).

4. The Conclusions

It is shown that in the device for definition of generalized parameters of multicomponent passive two-terminal networks with differentiating signals, the measuring system can be used differentiators, consisting of series-connected passive *RC*-circuits. Simple scheme on stable and adjustment-free components (capacitors and resistors) allows you to measure the generalized parameters of a wide class of objects that have a schema of replacement *RC*-, *RL*- and *RLC*-two-terminal device. Received the analytical relations for calculation of the generalized parameters for the totality of the measured values of input and output voltages of the measuring circuit and the outputs of both channels differentiation.

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