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Design of Innovative Web Structures Based on Spider Web Optimality Analysis

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Abstract

In this study the basic mechanics and engineering principles are applied find out possible reasons for the optimality of a typical spider web. Physical models are formulated connecting the external loads to internal loads and with geometry and materials. Then optimum design methodology using fuzzy goal of maximising the product of satisfactions on chosen decision variables is applied to design a web. The decision variables are three. The first is web material volume which is defined using the FSD principle, the second is catch area, and the third is desirability to maintain only tensile forces. The optimal analytical and FEM model results agree well with experimental data in large side frame forces and less well in inner guy forces. Some industrial application possibilities are discussed.

Keywords: web design, optimum fuzzy design, biomimetics, industrial applications

1. Introduction

Nature is full of hierarchical and symbiotic structures which are optimally designed in all design aspects. Ingenious solutions in nature are transmitted by genetic codes. Human information is not transferable genetically but has to be studied, stored to design codes and transmitted by education.

In this study the object is a deceptively simple spider web construction. The web material is extremely ingenious; it is intrinsically very strong, ductile with very high heat conductivity.

A general outline of spiders and biomechanics is discussed by (Wainwright, Biggs, Currey, & Gosline, 1976). The physical properties of spider's silk and their role in the design of orb-webs are analysed in by (Denny, 1976). (Cranford, Tarkova, Pugno, & Buehler, 2012) discuss the non-linear material behaviour of spider silk and reasons why it yields a robust web. They have observed that a nonlinear stress response results in superior resistance to structural defects in the web compared to linear elastic or elastic-plastic (softening) material behaviour. They have also shown (Cranford et al., 2012) that under distributed loads the stiff behaviour of silk under small deformation, before the yield point, is essential in maintaining the web's structural integrity. The superior performance of silk in webs is therefore not due merely to its exceptional ultimate strength and strain, but arises from the nonlinear response of silk threads to strain and their geometrical arrangement in a web (Cranford et al., 2012).

Optimum design of mechanical and biological machine structures can be done in many ways. A survey of engineering optimization is discussed by Rao (1996). Diaz (1988) has presents basics of fuzzy goals formulation in engineering tasks. This method has proved to be effective in several small highly non-linear design tasks. The method is applied to multi-objective and heuristic application optimum design by Martikka and Pöllänen (2009) and to optimum design of helical springs by Pöllänen and Martikka (2010) and to design of composite sandwich beams by Martikka and Taitokari (2011).

Engineering mechanics and heuristics rely on the prescient power of mathematics and natural laws which give finalistic guidance for inventing structures. Good theory guarantees good machines and webs too. Thus the spider must have a very ingenious theory programmed to its genes.

(Wood et al., 2003) show that uncertainty in engineering analysis usually pertains to stochastic uncertainty but other forma also exist. This shows that realistic engineering functions can be used in imprecision calculations, with reasonable computational performance

Wang and Terpenny (2003) consider the imprecision inherent in the early design stages. They combine an agent-based hierarchical design representation, set-based design generation; fuzzy design trade-off strategy and interactive design adaptations to reduce the search space while maintaining solution diversity. They use fitness function incorporating multi-criteria evaluation with constraint satisfaction.

In the present study many simplifications are made to test some design principles and bio mimicry.

The decisive properties of machines and creatures are determined by pre programmed instruction in machines and in genetics of the biological creatures. These determine their geometry, material selection and functioning and fitness for service.

The present approach of innovation and optimum design is based on basic mechanics with fuzzy goal formulations and heuristics. These models are combined synergically to formulate the desired properties of structures like the web.

Geometrically each web structure is conceptually triangular; it has a function, or its purpose, materials and geometry. These are defined by design variables, like dimensions and strength. These are combined to decision variables, like cost and reliability. The design goal of this re-invented web is to maximise the satisfaction on decision variables using a product or weakest-link link formulation. Still higher level decision variables can be defined like the basic needs of livelihood and reproduction. The geometry of the web is simplified and the principle of fully stressed design is too used to obtain the optimal solution by using this design methodology many optimal industrial web type structures can be designed and made. Geometrically and materially complex webs can be effectively analysed using FEM (Finite Element Method, NX Nastran).

2. Basic Mechanics of a Simplified Spider Web

The aim is to find out the basic principles of web design used by the spider. The approach is to use basic mechanics of engineering. The assumption is that the spider works as a sort of skilled engineering designer and manufacturer at the same time. According to studies (Cranford et al., 2012), the real strength of the web is not the silk but how its mechanical properties change as loads strain it, which is a very ingenious inbuilt feature which could be used in many areas of life to contain damage to a small area. They found the silk itself has an ability to soften or stiffen to withstand different types of loads - unlike any other natural or man-made fibres.

2.1 Model Geometry and Strength Assumptions

An idealisation of a typical spider's web is shown in Figure 1. According to (Denny, 1976) the spider Araneus spins strands of diameter d_s=3µm, with area A_s. The number of strands in threads is changed as needed.

The area of radius (r) has $N_r = 2$ strands, $A_r = 2A_s$, force $F_r = 0.11\ldots0.17$, stress $\sigma_r = F_r/A_r = 0.08/A_s$. The area of frame has N_f=10 strands, $A_f = N_f A_s = 10 A_s$, force $F_f \approx 1$, stress $\sigma_f = 0.1/A_s$,

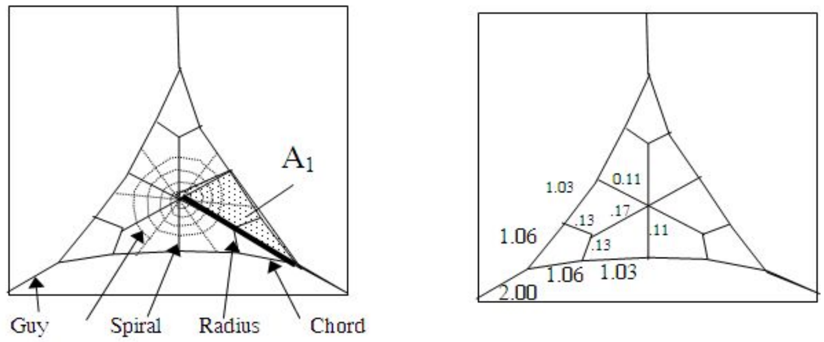
The area of guy has N_g=20 strands, $A_g = N_g A_s = 20A_s$, force $F_g = 2$, stress $\sigma_g = 0.1 /A_s$. The web is thus evenly stressed. For the web of A.Sericatus (Denny, 1976) gives the data

Strand diameter d_s=1.5µm, N_r= 2, $N_f = 4\ldots8$, $N_g = 8\ldots10$. Their topologies may differ.

Parkes (1965) has proposed the Maxwell lemma as criterion to check whether some plane truss is a minimum volume structure for resisting forces in the plane of the web. The lemma is

(1) All members are either in tension or compression

(2) All members are equally stressed near their breaking stress.



a) b)

Figure 1. Simplified model for the web of Araneus spider

a) Principal features of the web with notations radii, frame and guy threads contain about 4, 10 and 20 strands respectively. b) Simplification of the web showing only the main support threads and the distribution of the thread forces when the force of 2 units is pulling one guy

Model geometry is shown in Figure 2. This model is based on assumptions derived from observation of simple spider webs by (Denny, 1976).

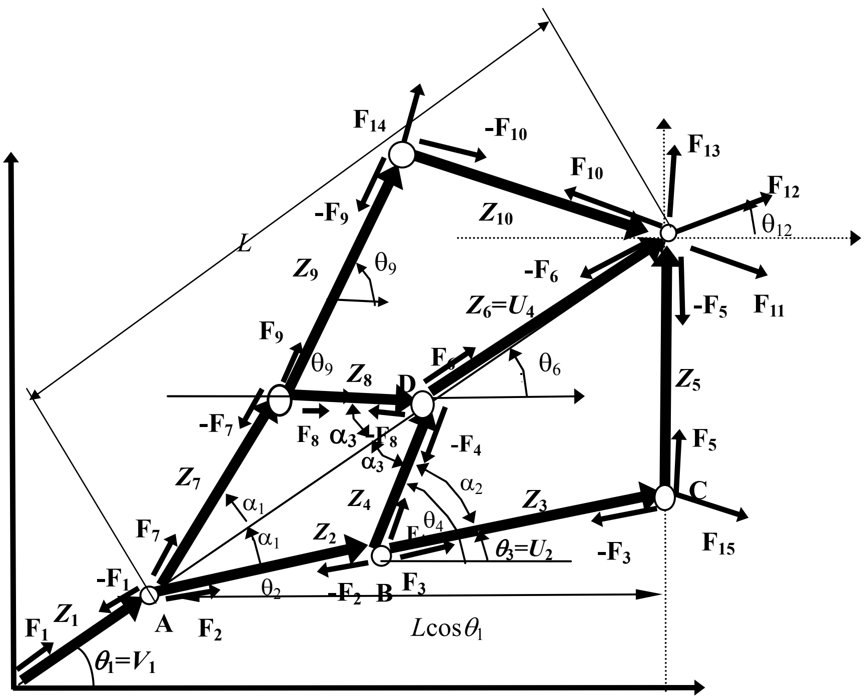


Figure 2. Part of a web with geometry and force vectors

Some simplifying assumptions are useful in this conceptual study.

First assumption is that external geometry is close to an ideal equilateral triangle.

Thus, by using symmetry only a sixth part of the whole structure needs to be analysed.

The second assumption is that the strength of all threads is constant. The cross section area for each thread is thus thread force divided by allowed stress assuming validity of FSD (Fully Stressed Design)

$$F_{\mathrm{k}} = A_{\mathrm{k}}\sigma_{\mathrm{all}}, \Rightarrow A_{\mathrm{k}} = \frac{F_{\mathrm{k}}}{\sigma_{\mathrm{all}}} \tag{1}$$

3. Forces in the Web

3.1 Model Geometry and Forces Constrained by Symmetry

Three design variables are sufficient. They can be chosen as the angles α_1, α_2 and α_3. Some angles are obtained directly by symmetry connections from Figure 2

$$\theta_1 = 30, \theta_2 = \theta_1 - \alpha_1, \theta_3 = \theta_1 + \alpha_3 - \alpha_2, \theta_4 = \theta_1 + \alpha_3 \tag{2}$$

$$\theta_5 = \tfrac{1}{2}\pi, \theta_6 = \theta_1, \theta_7 = \theta_1 + \alpha_1, \theta_8 = \theta_1 - \alpha_3, \theta_{15} = -\theta_3 \tag{3}$$

Some forces are obtained by symmetry. The force F_1=2 as by (Denny, 1976).

$$F_8 = F_4, F_{15} = F_3, F_9 = F_3, F_7 = F_2, F_1 = 2 \tag{4}$$

3.2 Force Equilibrium at Nodes

Force equilibrium at node A in Figure 2 gives

$$-\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_7 = 0 \tag{5}$$

$$-F_1\begin{bmatrix}\cos\theta_1\\ \sin\theta_1\end{bmatrix} + F_2\begin{bmatrix}\cos\theta_2\\ \sin\theta_2\end{bmatrix} + F_7\begin{bmatrix}\cos\theta_7\\ \sin\theta_7\end{bmatrix} = 0 \tag{6}$$

Substituting here the geometrical relationships from equations (2) and (3)

$$\begin{aligned} F_7 &= F_2 \\ \theta_7 &= \theta_1 + \alpha_1, \theta_2 = \theta_1 - \alpha_1 \end{aligned} \tag{7}$$

Gives

$$F_2\begin{bmatrix}\cos(\theta_1 - \alpha_1)\\ \sin(\theta_1 - \alpha_1)\end{bmatrix} + F_2\begin{bmatrix}\cos(\theta_1 + \alpha_1)\\ \sin(\theta_1 + \alpha_1)\end{bmatrix} = F_1\begin{bmatrix}\cos\theta_1\\ \sin\theta_1\end{bmatrix} \tag{8}$$

Or

$$F_2\begin{bmatrix}2\cos(\theta_1)\cos(\alpha_1)\\ 2\sin(\theta_1)\cos(\alpha_1)\end{bmatrix} = F_1\begin{bmatrix}\cos\theta_1\\ \sin\theta_1\end{bmatrix} \tag{9}$$

Whence

$$F_1 = 2, \quad F_2 = F_1\frac{1}{2\cos(\alpha_1)} \tag{10}$$

Force equilibrium at Node B gives

$$-\mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4 = 0 \tag{11}$$

Or

$$F_3\begin{bmatrix}\cos\theta_3\\ \sin\theta_3\end{bmatrix} + F_4\begin{bmatrix}\cos\theta_4\\ \sin\theta_4\end{bmatrix} = F_2\begin{bmatrix}\cos\theta_2\\ \sin\theta_2\end{bmatrix} \tag{12}$$

The thread vector direction angles depend on the three design variables as

$$\begin{aligned} \theta_4 &= \theta_1 + \alpha_3 \\ \theta_3 &= \theta_4 - \alpha_2 = (\theta_1 + \alpha_3) - \alpha_2 = \theta_1 + \alpha_3 - \alpha_2 \\ \theta_2 &= \theta_1 - \alpha_1 \end{aligned} \tag{13}$$

Substitution of these into the equilibrium equation gives in matrix form

$$F_3\begin{bmatrix}\cos(\theta_1 + \alpha_3 - \alpha_2)\\ \sin(\theta_1 + \alpha_3 - \alpha_2)\end{bmatrix} + F_4\begin{bmatrix}\cos(\theta_1 + \alpha_3)\\ \sin(\theta_1 + \alpha_3)\end{bmatrix} = F_1\frac{1}{2\cos(\alpha_1)}\begin{bmatrix}\cos(\theta_1 - \alpha_1)\\ \sin(\theta_1 - \alpha_1)\end{bmatrix} = \begin{bmatrix}h_1\\ h_2\end{bmatrix} \tag{14}$$

Here the abbreviations are

$$h_1 = F_1\frac{\cos(\theta_1 - \alpha_1)}{2\cos(\alpha_1)} = f_1F_1, h_2 = F_1\frac{\sin(\theta_1 - \alpha_1)}{2\cos(\alpha_1)} = f_2F_1 \tag{15}$$

Using these symbols the equilibrium equation becomes

$$\begin{bmatrix} \cos(\theta_1+\alpha_3-\alpha_2) & \cos(\theta_1+\alpha_3) \\ \sin(\theta_1+\alpha_3-\alpha_2) & \sin(\theta_1+\alpha_3) \end{bmatrix}\begin{bmatrix} F_3 \\ F_4 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}\begin{bmatrix} F_3 \\ F_4 \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \tag{16}$$

From this the two unknown forces can be solved using Cramer's rule

$$\begin{bmatrix} F_3 \\ F_4 \end{bmatrix} = \frac{1}{DetS}\begin{bmatrix} \sin(\theta_1+\alpha_3) & -\cos(\theta_1+\alpha_3) \\ -\sin(\theta_1+\alpha_3-\alpha_2) & \cos(\theta_1+\alpha_3-\alpha_2) \end{bmatrix}\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \tag{17}$$

The determinant of the equation system matrix is

$$\begin{aligned} DetS &= S_{11}S_{22} - S_{12}S_{21} \\ DetS &= \cos(\theta_1+\alpha_3-\alpha_2)\sin(\theta_1+\alpha_3) - \cos(\theta_1+\alpha_3)\sin(\theta_1+\alpha_3-\alpha_2) = \sin\alpha_2 \end{aligned} \tag{18}$$

The forces are thus

$$\begin{bmatrix} F_3 \\ F_4 \end{bmatrix} = \frac{1}{\sin\alpha_2}\begin{bmatrix} \sin(\theta_1+\alpha_3)h_1 - \cos(\theta_1+\alpha_3)h_2 \\ -\sin(\theta_1+\alpha_3-\alpha_2)h_1 + \cos(\theta_1+\alpha_3-\alpha_2)h_2 \end{bmatrix} \tag{19}$$

Whence the two forces are solved as

$$\begin{aligned} F_3 &= \frac{\sin(\theta_1+\alpha_3)f_1 - \cos(\theta_1+\alpha_3)f_2}{\sin\alpha_2} \bullet F_1 = f_{31}F_1 \\ F_4 &= \frac{-\sin(\theta_1+\alpha_3-\alpha_2)f_1 + \cos(\theta_1+\alpha_3-\alpha_2)f_2}{\sin\alpha_2} \bullet F_1 = f_{41}F_1 \end{aligned} \tag{20}$$

Force equilibrium at node C in Figure 2 gives

$$-\mathbf{F}_3 + \mathbf{F}_5 + \mathbf{F}_{15} = 0 \tag{21}$$

Or

$$-F_3\begin{bmatrix} \cos\theta_3 \\ \sin\theta_3 \end{bmatrix} + F_5\begin{bmatrix} \cos\theta_5 \\ \sin\theta_5 \end{bmatrix} + F_{15}\begin{bmatrix} \cos\theta_{15} \\ \sin\theta_{15} \end{bmatrix} = 0 \tag{22}$$

Substituting here from equation (4)

$$F_{15} = F_3, \theta_{15} = -\theta_3, \theta_5 = \tfrac{1}{2}\pi$$

Gives

$$-F_3\begin{bmatrix} \cos\theta_3 \\ \sin\theta_3 \end{bmatrix} + F_5\begin{bmatrix} 0 \\ 1 \end{bmatrix} + F_3\begin{bmatrix} \cos(\theta_3) \\ -\sin(\theta_3) \end{bmatrix} = 0 \tag{23}$$

From this one may solve

$$F_5 = 2\sin\theta_3 \cdot F_3, F_5 = 2\sin\theta_3 \cdot f_{31}F_1 \tag{24}$$

Force equilibrium at node D gives

$$-\mathbf{F}_4 + \mathbf{F}_6 - \mathbf{F}_8 = 0 \tag{25}$$

$$-F_4\begin{bmatrix} \cos\theta_4 \\ \sin\theta_4 \end{bmatrix} + F_6\begin{bmatrix} \cos\theta_6 \\ \sin\theta_6 \end{bmatrix} - F_8\begin{bmatrix} \cos\theta_8 \\ \sin\theta_8 \end{bmatrix} = 0 \tag{26}$$

Substituting here

$$F_8 = F_4, \theta_4 = \theta_1 + \alpha_3, \theta_6 = \theta_1, \theta_8 = \theta_1 - \alpha_3 \tag{27}$$

Gives

$$-F_4\begin{bmatrix} \cos(\theta_1+\alpha_3) \\ \sin(\theta_1+\alpha_3) \end{bmatrix} + F_6\begin{bmatrix} \cos\theta_1 \\ \sin\theta_1 \end{bmatrix} - F_4\begin{bmatrix} \cos(\theta_1-\alpha_3) \\ \sin(\theta_1-\alpha_3) \end{bmatrix} = 0 \tag{28}$$

This may be written in component form as

$$\begin{aligned} F_4[-\cos(\theta_1+\alpha_3) - \cos(\theta_1-\alpha_3)] + F_6\cos\theta_1 = 0 \\ F_4[-\sin(\theta_1+\alpha_3) - \sin(\theta_1-\alpha_3)] + F_6\sin\theta_1 = 0 \end{aligned} \tag{29}$$

Whence both give the same solution

$$F_4\left[2\cos(\theta_1)\cos(\alpha_3)\right]=F_6\cos\theta_1 \Rightarrow F_6=F_4\cdot 2\cos\alpha_3 \tag{30}$$

Summary of forces expressed as function of the known guy force F_1

$$\begin{aligned}
&F_1=2, F_2=\frac{1}{2\cos(\alpha_1)}F_1=f_{21}F_1, F_3=f_{31}F_1, F_4=f_{41}F_1\\
&F_5=f_{51}F_1=2\sin\theta_3\cdot F_3=2\sin\theta_3 f_{31}\bullet F_1\\
&F_6=F_4\cdot 2\cos\alpha_3=f_{61}F_1=2\cos\alpha_3\cdot f_{41}\bullet F_1\\
&F_7=F_2, F_8=F_4
\end{aligned} \tag{31}$$

4. Position Vectors from Loops

Position vectors are needed for the total solution. They can be found using the closures condition of position vector loops.

4.1 Web Loop Vector Equations

The first loop in Figure 1 gives the following equation in vector and component forms

$$\begin{aligned}
&\mathbf{L}-\mathbf{Z}_2-\mathbf{Z}_3=0\\
&L\cos\theta_1=Z_2\cos\theta_2+Z_3\cos\theta_3\\
&L\sin\theta_1=Z_2\sin\theta_2+Z_3\sin\theta_3+Z_5
\end{aligned} \tag{32}$$

The second loop equation in vector and component forms is

$$\begin{aligned}
&\mathbf{Z}_3+\mathbf{Z}_5-\mathbf{Z}_6-\mathbf{Z}_4=0\\
&Z_3\cos\theta_3-Z_6\cos\theta_6-Z_4\cos\theta_4=0, \theta_5=\tfrac{1}{2}\pi\\
&Z_3\sin\theta_3-Z_6\sin\theta_6-Z_4\sin\theta_4+Z_5=0
\end{aligned} \tag{33}$$

Substituting here the angles as function of known and independent design variables

$$\begin{aligned}
&\theta_4=\theta_1+\alpha_3\\
&\theta_3=\theta_4-\alpha_2=(\theta_1+\alpha_3)-\alpha_2=\theta_1+\alpha_3-\alpha_2, \theta_6=\theta_1
\end{aligned} \tag{34}$$

Using these one to transform equation (33) to

$$\begin{aligned}
&Z_3\cos(\theta_1+\alpha_3-\alpha_2)-Z_6\cos\theta_1-Z_4\cos(\theta_1+\alpha_3)=0\\
&Z_3\sin(\theta_1+\alpha_3-\alpha_2)-Z_6\sin\theta_1-Z_4\sin(\theta_1+\alpha_3)+Z_5=0
\end{aligned} \tag{35}$$

The goal is to obtain position angles and position vectors as function of the four chosen design variables

$$\theta_{\mathrm{k}}=\theta_{\mathrm{k}}(\alpha_1,\alpha_2,\alpha_3,Z_2), Z_{\mathrm{k}}=Z_{\mathrm{k}}(\alpha_1,\alpha_2,\alpha_3,Z_2) \tag{36}$$

4.2 Assembly of Loop Equations for Solution

Now there are 5 unknowns and 5 equations, considering angles as known

$$\begin{aligned}
&Z_2\cos\theta_2+Z_3\cos\theta_3=L\cos\theta_1\\
&Z_2\sin\theta_2+Z_3\sin\theta_3+Z_5=L\sin\theta_1\\
&Z_3\cos\theta_3-Z_6\cos\theta_6-Z_4\cos\theta_4=0\\
&Z_3\sin\theta_3-Z_6\sin\theta_6-Z_4\sin\theta_4+Z_5=0\\
&Z_2\cos\alpha_1+Z_4\cos\alpha_3+Z_6=L
\end{aligned} \tag{37}$$

This equation system may be written in matrix form

$$\begin{bmatrix}
\cos\theta_2 & \cos\theta_3 & 0 & 0 & 0\\
\sin\theta_2 & \sin\theta_3 & 0 & 1 & 0\\
0 & \cos\theta_3 & -\cos\theta_4 & 0 & -\cos\theta_6\\
0 & \sin\theta_3 & -\sin\theta_4 & 1 & -\sin\theta_6\\
\cos\alpha_1 & 0 & \cos\alpha_3 & 0 & 1
\end{bmatrix}
\begin{bmatrix} Z_2\\ Z_3\\ Z_4\\ Z_5\\ Z_6 \end{bmatrix}
=
\begin{bmatrix} L\cos\theta_1\\ L\sin\theta_1\\ 0\\ 0\\ L \end{bmatrix} \tag{38}$$

This may be written more compactly as

$$\begin{bmatrix} A_{11} & A_{12} & 0 & 0 & 0 \\ A_{21} & A_{22} & 0 & 1 & 0 \\ 0 & A_{32} & A_{33} & 0 & A_{35} \\ 0 & A_{42} & A_{43} & 1 & A_{45} \\ A_{51} & 0 & A_{53} & 0 & 1 \end{bmatrix} \begin{bmatrix} Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \\ Z_6 \end{bmatrix} = \begin{bmatrix} K_1 \\ K_2 \\ 0 \\ 0 \\ K_5 \end{bmatrix} \tag{39}$$

4.3 Solution of the Assembled Loop Equations for Loop Vector Lengths

From this linear system of equations one may solve the unknowns.

First the Z_2 is chosen a design variable which varied as independent design variable

The Z_3 is a dependent variable and may be solved as function of Z_2

$$Z_3 = \frac{K_1}{A_{12}} + \left[-\frac{A_{11}}{A_{12}} \right] \cdot Z_2 \tag{40}$$

Using this one obtains

$$Z_5 = K_2 - A_{21} \cdot Z_2 - A_{22} \cdot Z_3 \tag{41}$$

Using these models one obtains equation system for the next variables

$$\begin{bmatrix} A_{33} & A_{35} \\ A_{43} & A_{45} \end{bmatrix} \begin{bmatrix} Z_4 \\ Z_6 \end{bmatrix} = \begin{bmatrix} H_3 \\ H_4 \end{bmatrix} = \begin{bmatrix} -A_{32} \cdot Z_3 \\ -A_{42} \cdot Z_3 - Z_5 \end{bmatrix} \tag{42}$$

From this linear equation system the two variables may be solved

$$\begin{bmatrix} Z_4 \\ Z_6 \end{bmatrix} = \frac{1}{DetA} \begin{bmatrix} A_{45} & -A_{35} \\ -A_{43} & A_{33} \end{bmatrix} \begin{bmatrix} H_3 \\ H_4 \end{bmatrix}, Det[\mathrm{A}] = A_{33}A_{22} - A_{35}A_{43} \tag{43}$$

Whence

$$Z_4 = \frac{1}{DetA}\left[A_{45}H_3 - A_{35}H_4\right], Z_6 = \frac{1}{DetA}\left[-A_{43}H_3 + A_{33}H_4\right] \tag{44}$$

5. Decision Variables

The desired range for decision variable s_k is $R(s_k) = s_{kmin} < s_k < s_{kmax}$ and satisfaction on it, called $P(s_k)$.

The satisfaction function selection can be made as the customer wishes.

5.1 The Decision Variable of Minimising Volume of Thread Used

The first goal is to obtain maximum satisfaction on the magnitude of the volume of the web threads. Only one sixth parts is needed for modelling only

$$Vol = Z_2A_2 + Z_3A_3 + Z_4A_4 + \tfrac{1}{2}\left[Z_5A_5 + Z_6A_6\right] \tag{45}$$

Here the cross section areas are obtained as a function of thread forces and allowed stress using the fully stressed principle. The scaling stress is set to σ_{all} =1MPa.

$$F_k = A_k\sigma_{all} \Rightarrow A_k = \frac{F_k}{\sigma_{all}} = \frac{1}{\sigma_{all}} f_{k1}F_1 \tag{46}$$

Relative volume based to FSD is the first goal variable

$$Vol = \frac{1}{\sigma_{all}}\left\{Z_2F_2 + Z_3F_3 + Z_4F_4 + \tfrac{1}{2}\left[Z_5F_5 + Z_6F_6\right]\right\} \tag{47}$$

Thus

$$s(1) = \frac{Vol}{V_{max}}, P(1) = P(s_1), V_{max} = F_1 \cdot 4L\frac{1}{\sigma_{all}} \tag{48}$$

5.2 The Decision Variable of Catching Area

The second goal is to obtain maximal catching area. Now it is made dimensionless using a scaling area.

$$A = 6A_1 = 6\tfrac{1}{2}L \cdot \cos\theta_1 \cdot Z_5, \quad A_{\max} = L^2$$
$$s(2) = \frac{A}{A_{\max}}, \quad P(2) = P(s_2) \tag{49}$$

5.3 The Decision Variable of Obtaining a Reasonably Triangular Form for the Web

One may add all kinds of wishes into the algorithm. The spider need not worry about choose negative angles θ_3. But some algorithms are free to choose negative values unless restricted. This is redundant with s4. Calculations showed that same optimum was robustly achieved when it was set to constant value of full satisfaction to any θ_3 value, $P(3)=1$ all the time.

$$s(3) = \theta_3 > 0, P(3) = P(s_3), \tag{50}$$

5.4 The Decision Variable Requiring All Thread Forces to Be Tensile

When the largest of all thread forces is kept as positive then all other forces are also positive.

This means that the force F_6 should be positive, tensile and not negative. This requirement was justified since by the FSD approach the force F_6 is proportional to the volume of threads.

$$s(4) = F_6 > 0, P(4) = P(s_4), \tag{51}$$

5.5 Maximisation of Total Satisfaction as Goal

The total satisfaction PG is the product of partial goals

$$PG = P(1)\cdot P(2)\cdot P(3)\cdot P(4) \Rightarrow P(1)\cdot P(2)\cdot 1\cdot P(4) \tag{52}$$

The goal is maximization of total satisfaction. Now it is the product of satisfaction functions of each goal. The method of exhaustive search is used. Now only geometry is varied.

The advantage of this formulation is that the conventional goals and constraints are treated in a unified manner. Design variables are discrete and in the ranges α_1=10...33deg, α_2=45...60deg, α_1=16...30deg, Z_2=0.17...0.35deg.

6. Results of Analytical Optimum Design

Some results are shown in Tables 1 and 2. The goal was to maximise the total satisfaction.

Table 1. Results for parameters in Figure 2. Here P_i are satisfactions on decision variables s_i

Volume :	P1=0.848 P2=0.141	α1 = 28	Z1 θ1 F1= 0.01, 30, 2.0	Z4 θ4 F4 = 0.031, 43, 0.03
Catch area:	P3=0.888	α2 = 42	Z2 θ2 F2 = 0.015, 3, 1.13	Z5 θ5 F5 = 0.05, 90, 0.04
angle θ_3>0:	P4=0.98	α3 = 13	Z3 θ3 F3 = 0.072, 1, 1.11	Z6 θ6 F6 = 0.056, 30, 0.06
Force F_6>0:	PG=0.104			
Total sat:				

Table 2 shows the presently obtained optimal values and comparisons with experimental data and FEM results.

Table 2. Optimal values and comparisons with experimental data and FEM results

Length Z_k (m)	Angles (deg)	Forces F_k present opt.	Forces F_k in ref. (Denny, 1976)	Forces F_k FEM (Martikka & Pöllänen, 2009)
$Z_1 = 0.01$	$\theta_1 = 30$ (const)	$F_1 = 2.00$	2.00(const)	2.00(const)
$Z_2 = 0.01$(ind.var)	$\theta_2 = 3$ (dep var)	$F_2 = 1.13$	1.06	1.06
$Z_3 = 0.072$	$\theta_3 = 1$	$F_3 = 1.11$	1.03	1.06
$Z_4 = 0.031$	$\theta_4 = 43$	$F_4 = 0.031$	0.13	0.06
$Z_5 = 0.05$	$\theta_5 = 90$	$F_5 = 0.05$	0.11	0.24
$Z_6 = 0.056$	$\theta_6 = \theta_1 = 30$	$F_6 = 0.056$	0.17	0.13
$L = 0.1$ (const)	$a_1 = 28$ (ind.var.)	$Z_7 = F_2$		
	$a_2 = 42$ (ind.var.)			
	$a_9 = 13$ (ind.var.)			

In Table 2 the calculated results are shown using the optimum design and FEM methods. These agree fairly closely with the experimental data by (Denny, 1976). The three large side forces agree well and the inner guy forces agree less well. Figure 3 shows the measured forces and the optimised forces.

The chosen three decision variables were enough to give a good semblance of the spider web. But complex nonlinear hardening and softening behaviour at dynamical loads was not feasible to use in analytical models. But it can be done with FEM. Also use of the FSD design to get optimum may be only one part of the ingenuous design idea of the spider.

Most differences in forces occur at the radial force F_5. The analytical model emphasis more load bearing to the outer frame threads and less to the inner threads. The measured and FEM results agree better.

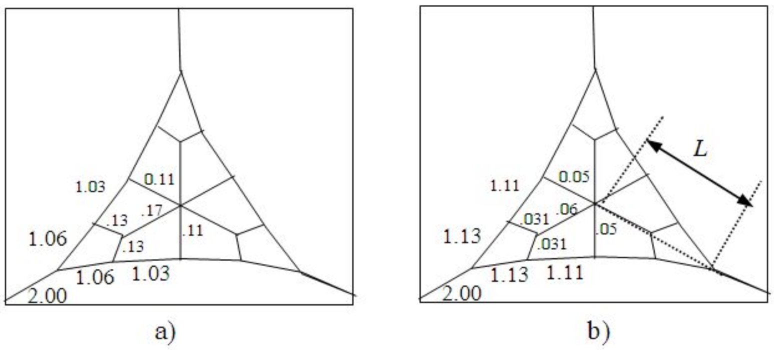


Figure 3. Comparison of thread forces. a) Experimental study of Araneus. b) Fuzzy optimum design results

7. Results of FEM Design of Web

FEM models were made using NX Nastran. Some results are shown in Figure 4a and 4b.

This is now a three-dimensional model. Elastic modulus E =4000MPa, Poisson's ratio ν=0.3.

The topology is the same as with the fuzzy optimum model. Non-linear solution method was used due to large deformations. An insect of mass 2g has impacted the web causing a mid normal displacement of 0.38L.

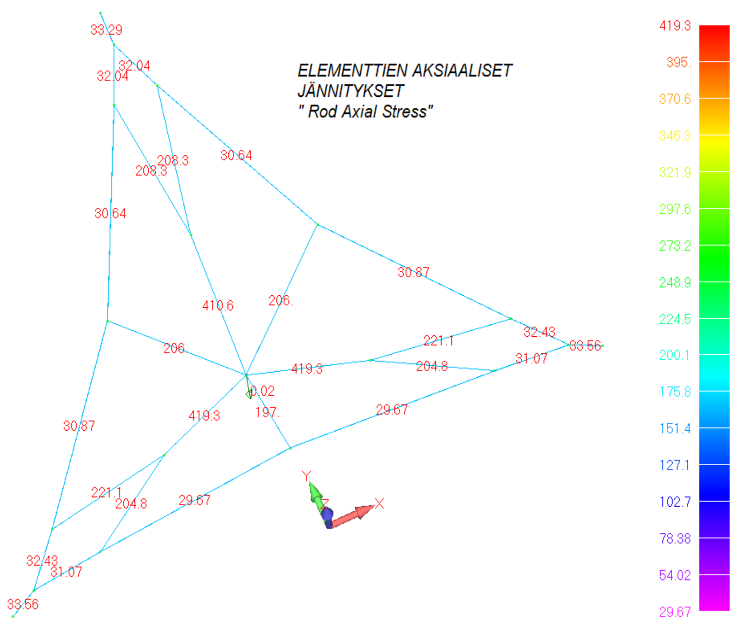


Figure 4a. FEM model results: Web with axial stresses

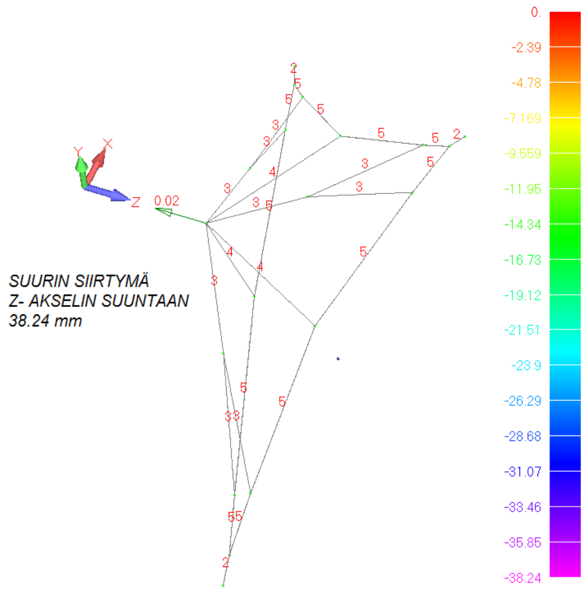


Figure 4b. FEM model results: Deformed form by an impact of an insect of mass 0.002kg at mid causing a normal displacement of 0.38 L

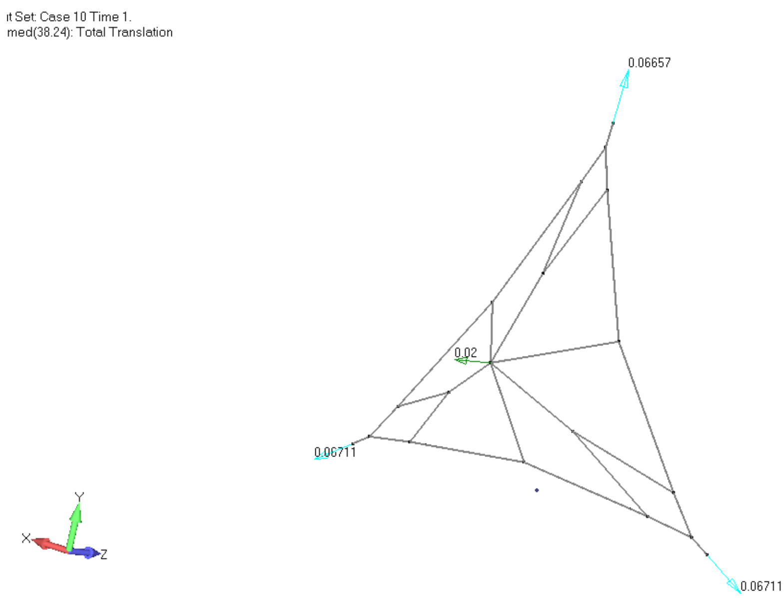


Figure 4c. FEM model results: Deformed model

8. Possibilities of Industrial Utilisation

Web like products are used in macro, micro and nano networks and also in reinforcements in composites and in safety textiles. The material of spider web would be ideal in strength to weight properties but the manufacturing is problematical. Possibilities of utilisation can be explored by using the basic design principles and materials science and creative optimum design methodology. First a basic idea for an innovation utilising the web ideas is needed. Then this concept may be optimised using a feasibility study. The fine-tuning and detail checking is done using FEM. Then prototypes may be manufactured.

9. Conclusions

- There is a rising a global needs to obtain new sustainable ecological product concepts.

- Nature is full of optimal sustainable products. They have innately programme mathematics to work optimally. We can obtain new product concepts by using biomimicry, provided we understand how to design and manufacture them.
- One example to test our design understanding of nature is the deceptively simple spider's web. The test goal was to find out reasons for its optimality. The other goals were to test the feasibility of optimum design with fuzzy satisfaction goals to find out reasons.
- What are the goals of the spider in web construction; we can guess them by considering how the spider constructs and uses the web. The design goals are probably web mass minimisation based on fully stressed design and to maximise the catch area.
- These design goals were expressed as maximisation of user satisfactions on these decisions variables resulting in a web resembling the actual spider's web.
- The behaviour of the web should be understood better to produce new technical innovations by biomimicry. It is rewarding to study its ingenious design to get industrial applications.
- There are two main engineering approaches to get new innovations. One is to start from basic principles and combine them innovatively and optimally. The second is to rely more on the use of accepted case study canon of examples and modify them somewhat to get predictable results. In the web case the real web was available and the method of 'back to basics' was also available. Some estimations show that the first approach leads more probably to unforeseen useful innovations than the second one.

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Appendix 1. Formulation of Goals in Fuzzy Form

In engineering tasks the optimal definition of goals and constraints is essential to get customer satisfaction on the result. In the concept stage the essential design variables are few, discrete and their relationships are highly non-linear. As humans see it, the main goal of a spider in designing and producing a web is to catch the prey. Thus the web functions as an essential survival means.

A fast enough search method is exhaustive learning search. Now all goals and constraints are formulated consistently by one flexible fuzzy function. This is illustrated in Figure A1.1.

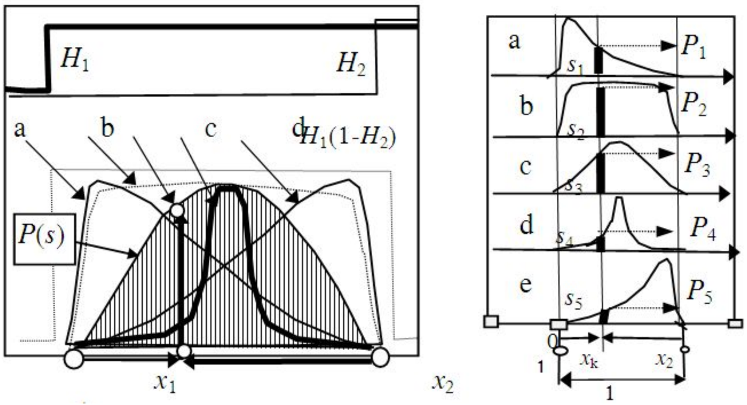

Figure A1.1. Principle of modelling of the general satisfaction functions. Its position and skewness can be varied

In the design algorithm the satisfaction function is defined for each decision variable *s* by inputting the left and right limits and two bias parameters *p*. The left skewed option *a* is useful to get low cost designs. Flat shape allows indifferent choice of *s*. The location of maximum can be shifted. The call is CALL pzz($s_{\min}$, $s_{\max}$, p_1, p_2, s, $P(s)$). The output is the satisfaction function $P(s)$. The decision variables *s* are changed to an internal dimensionless variable x_1

$$x_1 = \frac{s - s_{\min}}{s_{\max} - s_{\min}} \Rightarrow x_2 = 1 - x_1 \tag{A1.1}$$

The satisfaction function depends on one internal variable x_1 and two bias parameters

$$P(x_1) = (p_1 + p_2)^{p_1+p_2} \left(\frac{x_1}{p_1}\right)^{p_1} \left(\frac{1-x_1}{p_2}\right)^{p_2} H_{12} \tag{A1.2}$$

Here

$$H_{12} = H_1(s)(1 - H_2(s)) \tag{A1.3}$$

Two step functions are used to define the desired range of the decision variable

$$H_1(s) = \tfrac{1}{2}\left[1 + \operatorname{sgn}(s - s_{\min})\right], H_2(s) = \tfrac{1}{2}\left[1 + \operatorname{sgn}(s - s_{\max})\right] \tag{A1.4}$$

The total event s is intersection of separate events.

$$s = s_1 \cap s_2 \cap s_3 \cap s_4 \cap s_5 \cap s_6 \tag{A1.5}$$

Satisfaction on this event *s* is the probabilistic product intersection

$$P(s) \Rightarrow P(s) = P(s_1) \cdot P(s_2) \cdot \ldots\ldots \cdot P(s_n) \tag{A1.6}$$

The design goal is to maximise this product. The results are a trade-off between conflicting desires.

A Simple Example of a Tensile Web Thread Design to Clarify the Principles

The fuzzy design following approach is principle is illustrated using a single web thread.

Design variables (x(i) = DesV)

These are defined in geometry, materials and function.

- Geometrical DesV's are cross sectional area $A = x_1$ and length $L = x_2$
- Material DesV's are material classes im including strength R(im), unit cost c(im).

Now x_3 = im. Now im =1 for frame material and im = 2 for viscid material. Now im=1

- Functional DesV's are those which are related to load bearing function , now load force $x_4 = F$, The design variable vector is

$$\mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T = \begin{bmatrix} A & L & im & F \end{bmatrix}^T \tag{A1.7}$$

Decision variables, *DecV's*

These depend on the design variable, they are arrayed into a vector is **s** =f(**x**) = DecV's

$$\mathbf{s} = \begin{bmatrix} s_1 & s_2 \end{bmatrix}^T = \begin{bmatrix} K & N \end{bmatrix}^T \tag{A1.8}$$

Now for the sake of illustration only two are chosen. This decision is made in cooperation of the designer and the customer. In the case of the web the spider is designer, manufacturers and user of the web and some insect is customer.

Cost $K = s_1$ is the first DecV. Now it is only material cost.

$$s_1 = K = c(im)\rho(im)AL \tag{A1.9}$$

Factor of safety $N = s_2$ is the second DecV. Now it is based on mean values.

$$s_2 = N = \frac{R(im)}{\sigma}, \sigma = \frac{F}{A} \tag{A1.10}$$

Fuzzy satisfaction of the chosen decision variable is the third level

The goal of the spider is maximise its satisfaction on all thread parts of the web. Satisfaction on cost K is biased to small values which gives high satisfaction. The range is $s_1 = 0\ldots$ to some K value. Satisfaction on the N is largest within the safe range $N = 2\ldots4$ and small outside.

Now the conjunction I operation and probabilistic product intersection is chosen as appropriate in most design goal formulations.

$$P(\mathbf{s}) = I(s_1, s_2) = P_1 \cdot P_2 = P(s_k[x_i]) \tag{A1.11}$$

Fully stressed design

This FSD principle may be chosen if one is satisfied when all parts break simultaneously. But the FSD may a dominant goal but not the only one since the web is very tolerant to overloads and damage. Thus the spider may aim at desired reliability by some other approach. Using FSD one gets

$$s_2 = N \Rightarrow 1 = \frac{R(im)}{\sigma}, A = \frac{F}{R} = A_{\min} \Rightarrow s_1 = K_{\min} \tag{A1.12}$$

This choice gives minimal material volume but high web users' risk.

Conventional non fuzzy goal formulation

The conventional goal formulation with one goal and several constraints gives

$$\begin{gathered} \min(Q) = \frac{K}{K_0}, R(1) = \frac{N}{N_{\min}} - 1 > 0, R(2) = 1 - \frac{N}{N_{\max}} - 1 > 0 \\ N \Rightarrow N_{ave}, A = \frac{F}{R} N_{ave}, K = K_{\min} N_{ave} \end{gathered} \tag{A1.13}$$

This conventional approach gives more safe design by factor N_{ave}. With non-fuzzy formulation of one goal and two separate constraint formulations are needed and some more explanation. With fuzzy formulation one can easily define any kind of goals and any kind of constraints with then same standard formulation. The optimum is final and needs no more explaining.

Algorithm as Pseudocode

Total Satisfaction is First Initialised to a Low Value

P_{gbest} = .0000001,

Loop FOR each material class design variable indices

Loop FOR each geometrical design variable indices

Loop FOR each Functional design variable indices

Calculate each decision variable s_k .Calculate each satisfaction function $P(s)$ is obtained by a call

CALL pzz (s_1, s_2, p_1, p_2, s, $P(s)$). The output is and it varies in the range 0...

The total satisfaction is product of partial satisfactions.

P_s = 1, the initialisation first, before the loop

FOR i = 1 TO N, all decision variables are activated

Ps = Ps · Ps(i)

NEXT i

$P_g = P_s$ total satisfaction is obtains as product of partial ones

IF $P_g > P_{gbest}$ THEN 'estimation of progress

'new optimum is found better than previous

ELSE search is continued. END IF

NEXT design variable indices

Summary of the Present Approach

- Design variables x(i) (DesV's) are first defined
- Decision variables s_k (x(i)) (DecV's) are defined depending on the design variables.
- Satisfaction on each decision variable sk are calculated as satisfaction function P(sk(**x**) on sk
- The total goal is maximisation of their product $P = P(s_1(\mathbf{x}) \cdot P(s_2(\mathbf{x}) \cdot \ldots \cdot P(s_N(\mathbf{x})$

The goal value ranges in a psychological scale from 0 (no good) to 1 (good)

- The optimum is with sufficient probability in the near neighbourhood of the exact optimum since grids of the discrete design variables are dense enough and also since exhaustive search is used.

Other Approaches

Fuzzy definition is defined by Liang-Hsuan Chen& Ming-Chu Weng, (2004) in analogy with the present approach.

The Formulations of Liang et al. (2004)

They study Quality function deployment (QFD) as a product development process used to achieve higher customer satisfaction:

This comparison of these methodologies is presented in Table A1.1

Table A1.1. Comparison of a QFD methodology and the present fuzzy methodology

The QFD methodology by Liang-Hsuan & al (2004)	Present fuzzy methodology
the engineering characteristics (EC) affecting the product performance are designed to match the customer requirements	These EC's may correspond to the verbal selection of desires. Customer desires "safety" and engineers specify it as "factors of safety"
---- Design variable definition here---	The design variables x(i) are chosen
fuzzy approaches are applied to formulate customer requirements (CRs) and engineering design requirements (DRs),	The set CR's and DR's are in this method combined to decision variables (DecV = $s_k(x_i)$).
the cost (K=s_1) and technical difficulty (TD=N=s_2) of DR's are also considered as the other two goals	The cost $K = s_1$ and the factor of safety $N = s_2$. Now s_1 and s_2 are not goals but satisfaction on them are the partial goals are P_1 and P_2
The proposed approach can attain the maximal sum of **satisfaction degrees of all goals** (P_G) under each confidence degree.	The sum of satisfaction function means some ambivalence which is not as useful as "weakest link" model to get most optimal design
The P_P's seem to express use is union or disjunction which is measured fuzzily as $P(s) = P(s_1 \cap s_2 \cap (s_3 \cup s_4)))$ $P(s) = P_1 \cdot P_2 \cdot (P_3 + P_4 - P_3 \cdot P_4)$	The goal is defined as maximisation of satisfaction on event s as probabilistic product intersection of partial goals $P(\mathbf{s}) = I(s_1, s_2) = P_1 \cdot P_2 = P(s_k[x_i])$

Appendix 2. Fuzzy Design Background Theory

At the present design case fuzzy multiobjective optimization principles are used. This methodology is one of many similar ones but somewhat little known. Therefore some of its basic principles are reviewed. This methodology is based on results by Diaz (1988).

Definition of the Design Optimum

Generally an optimum may be defined as the best, but not unique, compromise, to fulfil a number of stated criteria under given constraints. In technical problems it is desired that the optima are robustly with high enough probability within the goal area.

The Total Design Event as a Set

The total design event is defined as a set s or the generalized goal set

$$s = H(s_1, s_2, s_3) \tag{A2.1}$$

Here s_1,s_2,.. are partial design event sets ,like cost or volume capacity. These are formulated as fuzzy sets. The symbol H indicates a known combination of operations on the argument sets s_1.

Operations on Sets

The two basic binary operations on the sets are utilized in design goal definitions.

First, if H is a non cooperative or intersection type binary operation rule to join two fuzzy sets, then its use gives the result

$$s = H(s_1, s_2) \rightarrow s_1 \text{ AND } s_2 = s_1 \cap s_2 \tag{A2.2}$$

Second, if H is a cooperative or union type binary operation rule to two join fuzzy sets, then its use gives the result

$$s = H(s_1, s_2) \rightarrow s_1 \text{ O}R\ s_2 = s_1 \cup s_2 \tag{A2.3}$$

Third, if H is a non symmetric operation rule to join three fuzzy sets, then

$$s = H(s_1, s_2, s_3) \rightarrow s_1 \text{O}R(s_2 \text{ AND } s_3) = s_1 \cup (s_2 \cap s_3) \tag{A2.4}$$

Satisfaction Measurements on Design Sets

Satisfaction on the fuzzy event set s is measured by some of the binary operations. The total satisfaction depends on partial satisfactions

$$P(s) = P(H(s_1, s_2)) \to F(P_1, P_2), \quad P(s_1) = P_1, \quad P(s_2) = P_2 \tag{A2.5}$$

Where P_1 and P_2 are membership functions or now partial satisfaction functions ranging from 0 to 1.

Conjunction I

$$I(P_1, P_2) \le \min(P_1, P_2) \forall P_1, P_2 \in [0,1] \tag{A2.6}$$

Examples of conjunction operators are Zadeh intersection

$$P(s) = I(P_1, P_2) = \min(P_1, P_2) \tag{A2.7}$$

and probabilistic product intersection

$$P(s) = I(P_1, P_2) = P_1 \cdot P_2 \tag{A2.8}$$

and

$$P(s) = I(P_1, P_2) = \max(0, P_1 + P_2 - 1) \tag{A2.9}$$

Now the goal is to maximise the probabilistic product intersection of equation (A2.8).

Appendix 3. Strength Models of Webs

A3.1 Strength Models

The silk threads of spider's web are made of two main material types frame silk and viscid silk according to Danny (1976). Threads are made of respective material by using integer number of strand to form a thread.

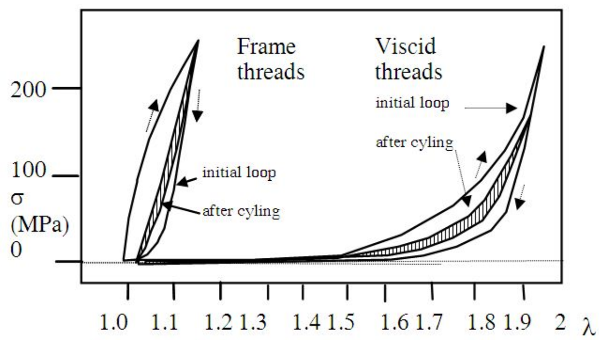

Figure A3.1. Cyclic loading of threads of A.seriacus according to Denny (1976) showing stress vs. stretch ratio for frame threads and for viscid threads

The spider apportions its material to its maximum advantage Viscid silk is used in the spiral catching orb. It has a low initial strength and allows large extension with only small elastic modulus. The prey can be catched softly with low impact. The frame silk is stiffer and takes main loads.

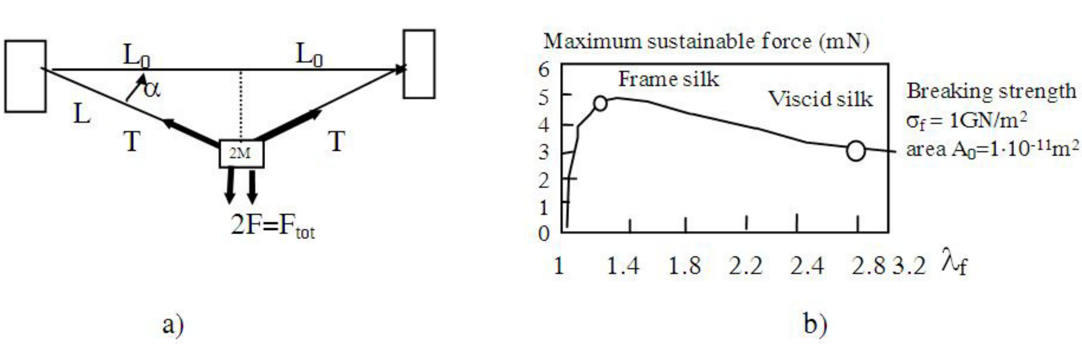

Figure A3.2. Web basics. a) Force equilibrium when a thread is stretched by a force. b) The theoretical sustainable force for a thread is product of breaking strength σ_f and cross sectional area A_0 as a function of stretch ratio λ_f. This model is based on data by (Denny, 1976)

A3.2 Thread Tension

Stretch ratio and deformation angle are

$$\lambda = \frac{L}{L_0} = 1 + \varepsilon_n \Rightarrow \lambda_f = \frac{L_f}{L_0} \Rightarrow \tag{A3.1}$$

Whence the strain is

$$\varepsilon = \frac{L}{L_0} - 1 = \frac{\Delta L}{L_0} = \frac{1}{\cos\alpha} - 1 = \sqrt{1+\tan^2\alpha} - 1 \approx 1 + \tfrac{1}{2}\tan^2\alpha - 1 \approx \tfrac{1}{2}\alpha^2 \tag{A3.2}$$

The deformation angle is

$$\cos\alpha_f = \frac{L_0}{L_f} = \frac{1}{\lambda_f} \Rightarrow \sin\alpha_f = \left[1 - \frac{1}{\lambda_f^{\,2}}\right] \tag{A3.4}$$

Thread specimen volume is constant under tension

$$V = V_0 \Rightarrow AL = A_0 L_0 = A_f L_f \tag{A3.5}$$

The tension force is

$$T_f = \sigma_f A_f = \sigma_f A_0 \frac{L_0}{L_f} = \sigma_f A_0 \frac{1}{\lambda_f} \tag{A3.6}$$

The maximum force is

$$F_{max} = T_f \sin\alpha_f = \tfrac{1}{2} F_{tot} = \sigma_f A_0 \frac{1}{\lambda_f^{\,2}} \cdot \left[\lambda_f^{\,2} - 1\right] \tag{A3.7}$$

This design idea can be illustrated by applying it to a thread system shown in Figure 5 modified from (Denny, 1976)

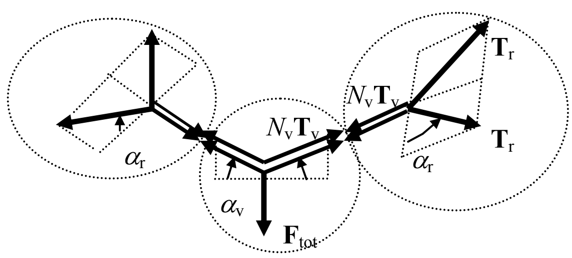

Figure 5. Tension free body system of four viscid threads connection two parallel radial threads

From these free body models one obtains for N_v viscid threads connecting two radial threads

$$F_{tot} = 2N_v T_v \sin\alpha_v \tag{A3.8}$$

The left radial thread force is sum of N_v parallel threads forces

$$N_v T_v = 2T_r \sin\alpha_r \tag{A3.9}$$

Whence the connection of the total force F_{tot} and the tension T_r at the radial thread is

$$F_{tot} = 4T_r \sin\alpha_r \sin\alpha_v \tag{A3.10}$$

The advantage of the design is that the sine factor increases with deformation of the threads. The ductility is large so that the tension does not grow excessively large.

A3.3 Use of Web for Catching Desired Projectiles

The projectiles are edible insects. One is the house fly musca domestica with mass $m = 1.2 \cdot 10^{-5}$ kg and impact velocity $v' = 2.61$ m/s, according to Denny (1976). Kinetic energy is

$$W_k = \tfrac{1}{2} m v^2 = \tfrac{1}{2}\left(1.2 \cdot 10^{-5}[kg]\right) \cdot \left(2.61\left[\tfrac{m}{s}\right]\right)^2 = 40 \cdot 10^{-6}[Nm] \tag{A3.11}$$

Volume of a typical basic strand is

$$v_s = \tfrac{\pi}{4} d_s{}^2 L_s = \tfrac{\pi}{4}\left(1.5 \cdot 10^{-6}[m]\right)^2 \left(0.03[m]\right) = 0.017 \cdot 10^{-12}\left[m^3\right] \tag{A3.12}$$

Fracture energy stored in the volume of one typical strand is

$$W_{\mathrm{f1}} = \tfrac{1}{2}\sigma_{\mathrm{f}}\varepsilon_{\mathrm{f}} v_s = \tfrac{1}{2}\left(1000 \cdot 10^6\left[\tfrac{N}{m^2}\right]\right)(0.3) \cdot v = 2.55 \cdot 10^{-6}[Nm] \tag{A3.13}$$

A3.4 Energy Storage before Breaking

The energy storage using a power law model for tensile stress strain relationship is

$$U = \frac{1}{n+1}\sigma_{\mathrm{f}}\varepsilon_{\mathrm{ft}} = \frac{1}{6}\sigma_{\mathrm{f}}\varepsilon_{\mathrm{ft}} \tag{A3.14}$$

For a viscid thread the power law exponent $n = n_{\mathrm{v}} = 5$

$$U_v = \frac{1}{6}1000MPa \cdot \ln(3) = \frac{1}{6}1000MPa \cdot 1.1 = 200 \cdot 10^6 \frac{Nm}{m^3}, n = 5 \tag{A3.15}$$

For a nearly elastic frame thread the power law exponent is $n_{\mathrm{frame}} = 1$. True fracture strain is

$$\varepsilon_{\mathrm{ft,frame}} = \ln\left(1 + \varepsilon_{\mathrm{f,frame}}\right) = \ln\left(\lambda_{\mathrm{f,frame}}\right) = \ln(1.25)$$

$$U_{\mathrm{f}} = \frac{1}{n_{frame}+1}\sigma_{\mathrm{f,frame}}\varepsilon_{\mathrm{ft,frame}} = \frac{1}{2}1000MPa \cdot \ln(1.25) = 100....200 \cdot 10^6 \frac{Nm}{m^3} \tag{A3.16}$$

The volumes can be related to viscid volume V_{v} consisting of N_{v} strands

$$V_v = N_{\mathrm{v}} \cdot \tfrac{\pi}{4} d_s{}^2 L_s = N_{\mathrm{v}} v_s \tag{A3.17}$$

Thus the fracture energy of one viscid thread is

$$W_v = U_v V_v \tag{A3.18}$$

Frame volume V_{f} with N_{f} strands in a frame bundle is

$$V_f = N_{\mathrm{f}} \cdot \tfrac{\pi}{4} d_s{}^2 L_s = N_{\mathrm{f}} \cdot v_s \quad , \quad Wf = UfVf \tag{A3.19}$$

The goal is that the energy absorbed without damage is large than the largest feasible projectile energy

$$W_v + W_f > W_{\mathrm{k}} \tag{A3.20}$$

Now experimental data supports the rough equality, Denny (1976)

$$U_f \approx U_v \tag{A3.21}$$

Thus

$$U \cdot v_s\left(N_v + N_{\mathrm{f}}\right) > W_{\mathrm{k}} \tag{A3.22}$$

Substituting typical values gives

$$150 \cdot 10^6\left[\frac{Nm}{m^3}\right] \cdot 0.017 \cdot 10^{-12}\left[m^3\right]\left(N_v + N_{\mathrm{f}} x_{\mathrm{d}}{}^2 x_{\mathrm{L}}\right) > 40 \cdot 10^{-6}[Nm] \tag{A3.23}$$

The number N_{v} of basic strand volumes v_{s} can be estimated roughly. Values for two spiders are:

For Araneous $N_{\mathrm{f}} = 10$, for A. Sericatus $N_{\mathrm{f}} = 4\ldots8$. One may assume an average $\overline{N}_{\mathrm{f}} = 8$. This gives

$$N_v + N_{\mathrm{f}} \approx 16 \Rightarrow N_{\mathrm{v}} \approx 16 - 8 = 8 \tag{A3.24}$$

A3.5 Tensile Test Data

Tensile tests of typical silk threads, based on data by Denny (1976) are shown in Figure 6.

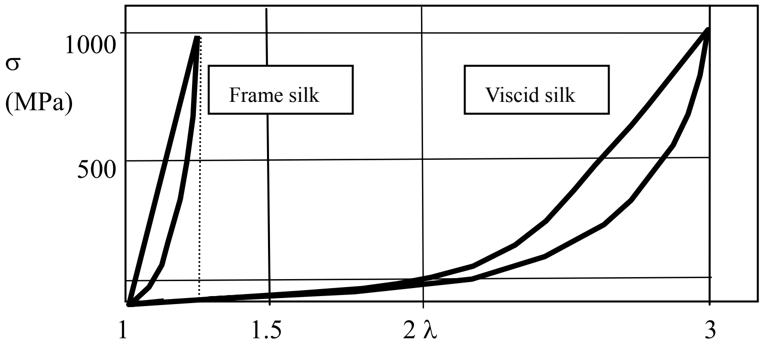

Figure 6. Tensile tests of typical silk threads, based on data by (Denny ,1976)

A3.5.1 Frame Silk

Frame silk is nearly linear elastic. The fracture (f) strain may be written as

$$\lambda_{\mathrm{f}} = \frac{L_{\mathrm{f}}}{L_0} = 1.25, \varepsilon_{\mathrm{f}} = \lambda_{\mathrm{f}} - 1 = 1.25 - 1 = 0.25 \tag{A3.25}$$

Thus the true (t) strain is

$$\varepsilon_{\mathrm{ft}} = \ln(1 + \varepsilon_{\mathrm{f}}) = \ln(\lambda_{\mathrm{f}}) = \ln(1.25) \tag{A3.26}$$

Linear elastic model is feasible

$$\sigma = \sigma_{\mathrm{f}}\left(\frac{\varepsilon}{\varepsilon_{\mathrm{f}}}\right) \tag{A3.27}$$

Some typical numerical values give

$$\sigma = 1000MPa\left(\frac{\varepsilon}{0.25}\right) = 4000MPa\varepsilon = E\varepsilon \tag{A3.28}$$

According to Denny (1976) the elastic modulus is in the range

$$E_{\mathrm{final}} = 2570....4630MPa$$

General simple model for nominal stress σ as function of true strain

$$\varepsilon_{\mathrm{t}} = \ln(\lambda), \sigma(\varepsilon_{\mathrm{t}}) = \sigma_{\mathrm{f}}\left(\frac{\varepsilon_{\mathrm{t}}}{\varepsilon_{\mathrm{ft}}}\right)^{\mathrm{n}} \tag{A3.29}$$

Energy stored in deformation per unit volume is

$$U = \int_0^{\varepsilon_{t1}} \sigma(\varepsilon_{\mathrm{t}}) d\varepsilon_{\mathrm{t}} = \int_0^{\varepsilon_{t1}} \sigma_{\mathrm{f}}\left(\frac{\varepsilon_{\mathrm{t}}}{\varepsilon_{\mathrm{ft}}}\right)^{\mathrm{n}} d\varepsilon_{\mathrm{t}} \tag{A3.30}$$

Or

$$U = \sigma_{\mathrm{f}}\left(\frac{\varepsilon_{\mathrm{t}}}{\varepsilon_{\mathrm{ft}}}\right)^{\mathrm{n}} \frac{\varepsilon_{\mathrm{t}}}{n+1}, U_{\mathrm{f}} = \sigma_{\mathrm{f}}\left(\frac{\varepsilon_{\mathrm{ft}}}{\varepsilon_{\mathrm{ft}}}\right)^{\mathrm{n}} \frac{\varepsilon_{\mathrm{ft}}}{n+1} = \frac{1}{n+1}\sigma_{\mathrm{f}}\varepsilon_{\mathrm{ft}} \tag{A3.31}$$

For frame silk, $n = 1$

$$U_{\mathrm{f,frame}} = \frac{1}{n+1}\sigma_{\mathrm{f}}\varepsilon_{\mathrm{ft}} \Rightarrow \frac{1}{2}\sigma_{\mathrm{f}}\varepsilon_{\mathrm{ft}} = \frac{1}{2}1000MPa \cdot \ln(1.25) = 120 \cdot 10^6 \frac{Nm}{m^3} \tag{A3.32}$$

A3.5.2 Viscid Silk

Nominal (n) and true (t) strain are

$$\varepsilon_{\mathrm{n}} = \lambda - 1 \Rightarrow \varepsilon_{\mathrm{t}} = \ln(1+\varepsilon_{\mathrm{n}}) = \ln(\lambda) \tag{A3.33}$$

Nominal stress vs. true strain is

$$\sigma(\varepsilon_{\mathrm{t}}) = \sigma_{\mathrm{f}}\left(\frac{\varepsilon_{\mathrm{t}}}{\varepsilon_{\mathrm{ft}}}\right)^{\mathrm{n}} \tag{A3.34}$$

Fracture strain is

$$\varepsilon_{\mathrm{f}} = \lambda_{\mathrm{f}} - 1 \Rightarrow \varepsilon_{\mathrm{ft}} = \ln(1+\varepsilon_{\mathrm{f}}) = \ln(\lambda_{\mathrm{f}}) = \ln 3 \tag{A3.35}$$

The two parameters for the nominal stress vs. true strain models are obtained by data fitting to two chosen points

$$\begin{aligned} &\varepsilon_{0\mathrm{t}} = \lambda_0 - 1 \Rightarrow \varepsilon_{0\mathrm{t}} = \ln(1+\varepsilon_0) = \ln(\lambda_0) \Rightarrow \ln(1+1) = \ln(2) \\ &\sigma(\varepsilon_{0\mathrm{t}}) = \sigma_{\mathrm{f}}\left(\frac{\varepsilon_{0\mathrm{t}}}{\varepsilon_{\mathrm{ft}}}\right)^{\mathrm{n}} \Rightarrow \sigma(\ln 2) = \sigma_{\mathrm{f}}\left(\frac{\ln 2}{\varepsilon_{\mathrm{ft}}}\right)^{\mathrm{n}} = 1000MPa\left(\frac{\ln 2}{\ln 3}\right)^{\mathrm{n}} = 100MPa \end{aligned} \tag{A3.36}$$

From this the exponent n may be solved

$$n = \ln\left(\frac{\sigma(\varepsilon_{0\mathrm{t}})}{\sigma_{\mathrm{f}}}\right)\frac{1}{\ln\left(\frac{\varepsilon_{0\mathrm{t}}}{\varepsilon_{\mathrm{ft}}}\right)} = \ln\left(\frac{100}{1000}\right)\frac{1}{\ln\left(\frac{\ln 2}{\ln 3}\right)} = 5 \tag{A3.37}$$

The unit energy stored in viscid thread is

$$\begin{aligned} &U_{\mathrm{f,viscid}} = \frac{1}{n+1}\sigma_{\mathrm{f}}\varepsilon_{\mathrm{ft}} = \frac{1}{6}\sigma_{\mathrm{f}}\varepsilon_{\mathrm{ft}} = \frac{1}{6}1000MPa \cdot \ln(3) = \frac{1}{6}1000MPa \cdot 1.1 = 200 \\ &U_{\mathrm{f,viscid}} = 200 \cdot 10^6 \frac{Nm}{m^3} \end{aligned} \tag{A3.38}$$

(Denny, 1976) proposes that the viscous energy $U_{\mathrm{f,\ viscid}}$ is about that same as with frame silk.