A Rank Based High Resolution Bearing Estimation Method in Passive Arrays

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Abstract

A rank based bearing estimation method of a passive array is proposed in this paper. Adding a random signal to an array data model to increase the rank and changing the location of the signal to reduce the rank, is the main action. To obtain the optimum estimation in noisy environments a criteria function is introduced. The peaks of the function give us the best candidate for location of the main targets. An array of three sensors with two independent targets is simulated to demonstrate the qualification.

Keywords: Correlation, Signal Space, Noise Space, Eigenvalue

1. Introduction

Processing the signals received on an array of sensors to find location of emitters is one of the greatest interesting problems in sonar and radar applications (Kumaresan, R. Trufts, D. W. 1983). A general case considers an arbitrary number and location of sensors and emitters in an arbitrary noise environment with arbitrary mean and covariance. As a real condition in most practical cases, we deal with independent sources and independent noise so that the correlation between emitters and noise and emitters is zero (Huang, Y. D. and Barkat, M. 1991). This property is the basis of many high resolution bearing estimation techniques under assumption of zero mean and independency of noise and emitters (Schmidt, 1979). Our proposed method is concentrated on a half wavelength ***M+1*** dimensional array with ***P*** independent emitters under the condition of $P \leq M$ and zero mean Gaussian normal noise. We begin our discussion with introducing a data model for an ***M+1*** dimensional array and one emitter. After explaining the basic concepts in this area we will present our method.

1.1 The data model

According to the figure-1 the received wavefront on first sensor is x_0 according to the following equation

$$x_0 = \frac{1}{r} a(t - r/v) e^{i\omega(t-r/v)} + n_0 \tag{1}$$

where ***a(t)*** is the time varying envelope of the emitter that is attended by ***1/r***, ***r*** is the distance, ***v*** is wave velocity ,***ω*** is working frequency and n_0 is the noise on this sensor(Harabi, F. Changuel, H. and Gharsallah, A. 2007). This signal is received on the second sensor with $\Delta r/v$ sec delay according to the next relation

$$x_1 = \frac{1}{r} a(t - (r + \Delta r)/v) e^{i\omega(t-(r+\Delta r)/v)} + n_1 \tag{2}$$

In this equation $v=\lambda/T$ is the wave velocity where λ and T are the length and period of the wave respectively. After some simplification and considering $d=\lambda/2$ and $\Delta r = d\sin(\theta)$ we have

$$\frac{1}{r} a(t - (r + \Delta r) /v) e^{i2\pi r/\lambda} \approx c(t) \tag{3}$$

$$x_1 = c(t)e^{i\omega t} e^{i\pi d\sin(\theta)} + n_1 \tag{4}$$

The term $\Delta r/v$ in exponential function creates $\psi = \pi \, d\sin(\theta)$ delay on the sensor signal. Computing the $M+1$ signals of the array we have the following data model

$$x_0 = c(t)e^{i\omega t} + n_0$$
$$x_1 = c(t)e^{i\omega t} e^{i\psi} + n_1$$
$$x_M = c(t)e^{i\omega t} e^{iM\psi} + n_M \tag{5}$$

Now as general model, we arrange the equation to consider P emitters for an array of $M+1$ sensor.

$$x_0 = c_1(t)e^{i\omega t} + c_2(t)e^{i\omega t} + \ldots c_p(t)e^{i\omega t} + n_0$$
$$x_1 = c_1(t)e^{i\omega t} e^{2i\psi 1} + c_2(t)e^{i\omega t} e^{2i\psi 2} + \ldots cp(t)e^{i\omega t} e^{2i\psi p} + n_1$$
$$x_M = c_1(t)e^{i\omega t} e^{Mi\psi 1} + c_2(t)e^{i\omega t} e^{Mi\psi 2} + \ldots c_p(t)e^{i\omega t} e^{Mi\psi p} + n_M \tag{6}$$

We should denote some important problems in the data model that is considered in our method. It is supposed that the location of the targets is constant during the process while the envelope signal is changing with time. This factor limits us to estimate the bearing of low speed targets. Continuing the investigation we consider matrix model of data as follow

$$X = \left(A1(\psi 1) \middle| A2(\psi 2)....\middle| A_p(\psi_p)\right) \begin{pmatrix} S_1 \\ S_2 \\ S_p \end{pmatrix} + \begin{pmatrix} n_0 \\ n_1 \\ n_M \end{pmatrix} = A_{M\times P} \, S_{Px1} + N_{Mx1} \tag{7}$$

1.2 Basic Concepts

The vector $A_1(\psi_1)$ which is the first column of the incident matrix A, is the incident vector of the first target at angle ψ_1. Each column of matrix A is an incident vector of the related target so this matrix can be interpreted as signal subspace of all the targets that is perpendicular to noise subspace. The rank of matrix A is the number of the independent columns that is equal to the number of independent targets (Nuttall, 1976). If there are two correlated targets in the system one of the columns in matrix A is linear combination of the others so the rank will be decreased consequently. The determinant of the matrix A is zero because the rank is lower than M and this matrix has at least one zero eigenvalue (Donelli, M., S. Caorsi, F. DeNatale, M. Pastorino, and A. Massa, A. 2004). The vector S is the vector of time dependent magnitude of target signals and N is the received noise vector. It should be denoted that this assumption that noise samples are independent of each other is reasonable in most practical applications. Most important is that we do not have A, S and N separately and we can only obtain the X vector in practice. The process to decompose the noise and signal subspaces as the basis of many high resolution algorithms is consisted of two *correlation* and *averaging* steps. So these two steps together are such transformation that distinct the signal and noise subspace from the correlation matrix. The mapping trend begins by taking the K frame of data under the satisfactory sampling rate to compute the K coloration matrices. Averaging then achieves the final correlation matrix with necessary specification that enables us to factorize the matrix to two separated signal and noise subspaces. To show the very interesting property of the process, we first consider the correlation matrix R before the averaging step. Considering the equation (7) the coloration is computed as

$$R = X X^T = (A_{M\times P} \, S_{Px1} + N_{M\times 1})(A_{M\times P} \, S_{P\times 1} + N_{M\times 1})^T \tag{8}$$

After the simplification we have

$$R = ASS^T A^T + ASN^T + NS^T A^T + NN^T \tag{9}$$

The determinant of matrix R is zero so the determinant of all term should be zero. As the conclusion of averaging process, some important changes are happened:

1- SS^T is the correlation matrix of emitters that its entries changes under the averaging process such that the multiplication terms of different targets tends to be zero because they are independent. This event changes its determinant to a non-zero value because the non-diagonal terms go to zero while the diagonal ones remain.

2-SN^T and NS^T are the correlation matrices between noise and signal and all the elements tend to be zero after the averaging process.

3- The interesting change is happened on the noise correlation matrix $\boldsymbol{NN^T}$. The diagonal entries tend to variance of the noise $\boldsymbol{\sigma^2}$, the others go to mean and determinant is non-zero after the averaging step so $\boldsymbol{NN^T}$ is $\boldsymbol{\sigma^2 I}$.

As matrix $\boldsymbol{A}$ is independent of the averaging process, its determinant doesn't change so the determinant of $\boldsymbol{ASS^TA^T}$ is remained zero while $\boldsymbol{NN^T}$ has found a non-zero determinant. The conclusion is that the averaged correlation matrix $\boldsymbol{R_f}$ will have a non-zero determinant. At this condition we obtain

$$\mathrm{Det}(\mathrm{ASS^T A^T}) = \mathrm{Det}(\mathrm{R_f} - \sigma^2 \mathrm{I}) = 0 \tag{10}$$

According to this relation $\boldsymbol{\sigma^2}$can be interpreted as the eigenvalue of - $\boldsymbol{R_f}$ or $\boldsymbol{\sigma^2}$ is minimum eigenvalue of $\boldsymbol{R_f}$. To find eigenvalues of $\boldsymbol{R_f}$ we have

$$\mathrm{Det}(\lambda \mathrm{I} - \mathrm{R_f}) = 0 \tag{11}$$

where $\boldsymbol{\lambda}$ is eigenvalue of $\boldsymbol{R_f}$. The above relation is equal to

$$\mathrm{Det}(\,(\lambda - \sigma^2 \mathrm{I}) - \mathrm{ASS^T A^T}\,) = 0 \tag{12}$$

This relation means that eigenvalues of the matrix $\boldsymbol{ASS^TA^T}$ are $\boldsymbol{\lambda}$- $\boldsymbol{\sigma^2}$ or the eigenvalues of $\boldsymbol{R_f}$ are the eigenvalues of matrix $\boldsymbol{ASS^TA^T}$ plus $\boldsymbol{\sigma^2}$. Therefore for $\boldsymbol{P}$ nonzero eigenvalues of $\boldsymbol{ASS^TA^T}$ as $\boldsymbol{\lambda_p}$, the correlation matrix $\boldsymbol{R_f}$ has $\boldsymbol{\lambda_p + \sigma^2}$ eigenvalues while the $\boldsymbol{Q=M+1-P}$ remained eigenvalues of $\boldsymbol{R_f}$, are $\boldsymbol{\sigma^2}$. The consequent of the above analysis is that the signal subspace can be recovered by $\boldsymbol{P}$ number of $\boldsymbol{\lambda_p + \sigma^2}$ eigenvalues and related eigenvectors while the noise subspace by $\boldsymbol{Q}$ number of $\boldsymbol{\sigma^2}$ eigenvalues and related eigenvectors as the following equation

$$\mathrm{R_f} = \Sigma(\lambda_{\mathrm{pi}} + \sigma^2)\mathrm{V_i V_i}^{\mathrm{T}} + \Sigma\sigma^2 \mathrm{U_j U_j}^{\mathrm{T}} \tag{13}$$

The first term of the equation above recovers the signal subspaces of the correlation matrix $\boldsymbol{R_f}$ in which the eigenvalues are the combination of targets and noise. As we deal with signal to noise relation, under strong noisy condition the eigenvalues that reconstruct signal subspace will be influenced and changed according to the equation (13). The second term that recovers noise subspace and is perpendicular to signal subspace means that the internal product of each vector in the signal subspace with the noise subspace is zero. MUSIC (Multiple Signal Classification) as a high resolution bearing estimation method uses this property to find the location of targets (Schmidt, 1979)(Liggett, 1973). As the incident vectors are perpendicular to the noise subspace this algorithm search to maximize the following cost function

$$J = \left(\begin{pmatrix} 1e^{i\psi} & e^{ip\psi} \end{pmatrix} \begin{pmatrix} \Sigma U_j U_j^{\,T} \end{pmatrix} \begin{pmatrix} 1 \\ e^{-i\psi} \\ e^{-ip\psi} \end{pmatrix} \right)^{-1} \tag{14}$$

The signal vectors that maximize $\boldsymbol{J}$ would be the best candidates for the bearing of the targets.

1.3 Rank of Correlation Matrix

The rank of a matrix is the number of independent columns or rows that can be detected by the number of non-zero eigenvalues under a singular value decomposition process. As practical point of view in the case that signals to noise ratio is weak, computing the rank would be difficult. This difficulty is appeared because the level of noise increases and some of the minimum eigenvalues are places on threshold band of decision. This problem decreases the capability of detecting the matrix rank. In the conditions that the rank plays important roles, using appropriate method is necessary. As in the proposed algorithm in this paper the rank of correlation matrix has the main role, a proper rank detecting method is essential (Saidi, Z. and Bourennane, S. 2007). An adaptive threshold level for decision can decrease the problem by introducing a variable level related to the variance of noise. Independent of the methodology, having reasonable and stable criteria to estimate rank is important. Our idea in this research is to consider a threshold approximately equal to minimum eigenvalue.

2. Methodology

Our proposed method in this article is in the base of the following items:

1- The targets are independent so their correlation is zero

2- The rank of the correlation matrix is equal to the independent targets

The center of concentration of our novel method is that when a target is placed in the direction of another for either independent or correlated condition, the rank of the correlation matrix decreases by one. This property gives the idea of adding an independent target to the data model so that with changing its location, the rank of new correlation matrix reduced by one when it is placed in direction of any targets. Adding an independent source to the data model to increase the rank of correlation matrix and changing its angle from zero to 180 degree to reduce the rank is the basis for finding the main targets. As the estimation of the rank of the correlation matrix is in the base of computing its eigenvalues, the difficulties of detecting problem are appeared especially when the variance of noise is high. In using this method the following factors are important:

1-Adding new target to the data model should not change the model so the amplitude should not be high.

2- Computing the number of eigenvalues that are approximately equal to minimum

2- Comparison with the original condition should be done under the appropriate criteria

To implement the method we first compute the eigenvalues of the original correlation matrix. Then we add an independent signal to model with proper amplitude and the angle of zero. To detect the rank we change the angle and search to maximize the following function

$$J = \left(\Sigma(\lambda_{inew} - \lambda_{iold})^2 / \lambda_{1old} \right)^{-1} \quad \text{for i = Q to M} \tag{15}$$

λ_{new} and λ_{old} are minimum eigenvalues of the new and original correlation matrix respectively. The new minimum eigenvalues λ_{new} will be computed for each angle to cover the 180 degree band. The result is a function J that is maximized at the angle of the main targets. The term λ_{old} is used to give relative sensibility. The amplitude of the added signal is considered to be about 1/10 of the first diagonal entry of the original correlation matrix to prevent of considerable change in data model. As the added signal should be independent of each target, the best candidate is a random signal. In conclusion we add a noise with variable location to the model and change the location from -90 to 90 to compute J. The peak locations of J give us the reduction of rank by one and consequently the location of the targets.

3. Implementation

To implement the algorithm, we first introduce the adding technique. A source at location zero degree means that its related signal is received on array by the same phase so when it is in location φ, we should add this signal to our data model as follow

$$x_{0new} = x_{0old} + a(t)e^{i\omega t}$$

$$x_{1new} = x_{1old} + a(t)e^{i\omega t}\, e^{i\varphi}$$

$$x_{Mnew} = x_{Mnew} + a(t)e^{i\omega t}\, e^{iM\varphi} \tag{16}$$

The added signal is a K sample of a sine wave at working frequency ω. This signal is modulated by a random signal $a(t)$ and shifted by phase φ. $a(t)$ is a random signal because it should be independent of any arbitrary target to satisfies the translation property as discussed in the data model section. We add this signal with one degree of resolution in a range of -90 to 90 degrees. In each step the new correlation matrix is constructed and decomposed to its subspaces to compute criteria function J as a function of φ. Reduction of the rank will happen in peak points of J so we can plot J to obtain the result. A data model for an array of three sensors and two targets is simulated to test the method. We locate our target at 21 and 30 degrees as the first test. The conclusion of implementation of the proposed method in 0 db signal to noise ratio is demonstrated in figure-2. The behavior of criteria function is very sharp because the rank is a discrete quantity so it cannot be changed gradually. In the second test according to figure-3, we locate two targets very close at 28 and 30 degrees. The figure clearly shows the capability of the method to detect very near targets in 0 db signal to noise ratio by a passive array of three sensors.

4. Conclusion

A high resolution rank based bearing estimation method in passive array is proposed in this paper. The method works in the base of adding a variable location random signal to the real data model and search to find the locations that reduces rank of the correlation matrix by changing the angle of the added signal. Rank estimation is done by decomposing correlation matrix to its signal and noise subspace and computing minimum eigenvalues of the matrix. A cost function is introduced to show the location of the main targets where it is maximized. An array of three sensors is simulated to show the capability of the method in detecting very near targets in 0 db signal to noise ratio. The graphs demonstrated in the figure-2 and figure-3 clearly show that this technique is enable to distinct two near targets by a simple array. In the first case, there are two targets at 21 and 30 degree respectively. Figure-2 clearly shows that the algorithm has detected two targets sharply. The result of the second case in figure-3 demonstrates that the resolution of the method in three element arrays is approximately 2 degrees in 0 db signals to noise condition. As the resolution can be increased by using more number of elements in an array it can be concluded that this method can achieve more accurate results in a larger array.

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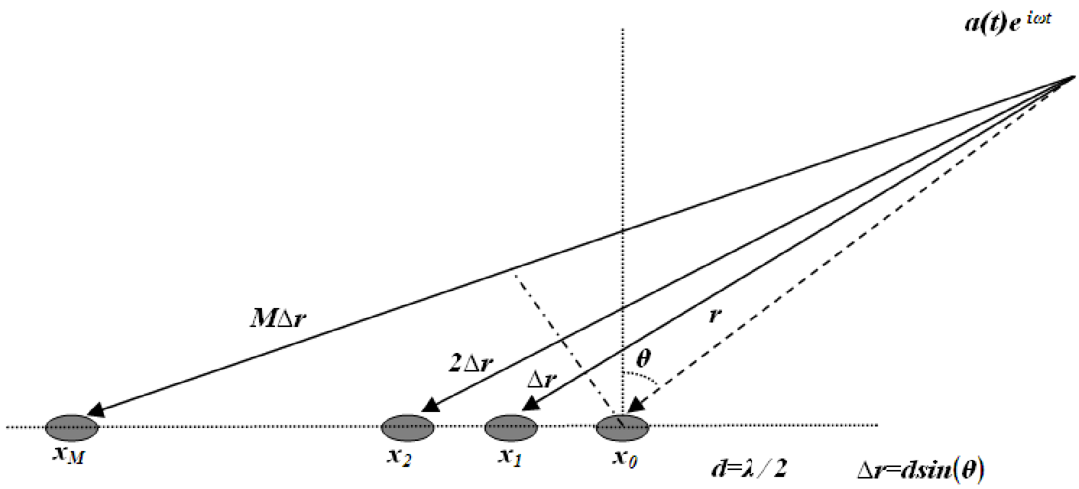

Figure 1. The Passive Array of Sensors

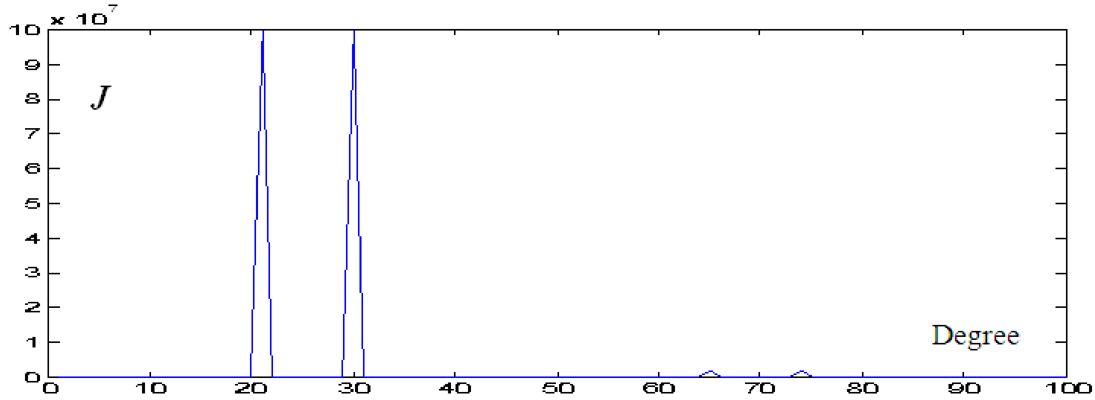

Figure 2. Rank Based Estimation for 21 and 30 Degrees in 0 db Signal to Noise Ratio

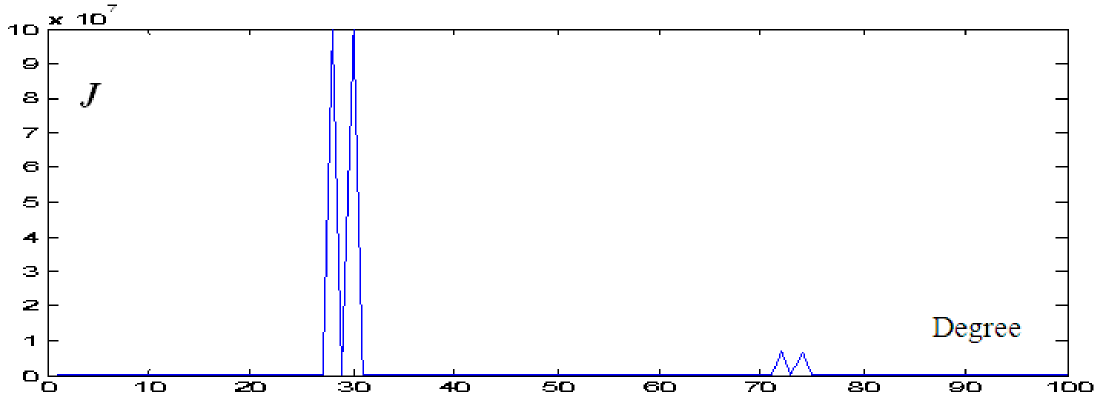

Figure 3. Rank Based Estimation for 28 and 30 degrees in 0 db Signal to noise Ratio