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Soil Probabilistic Slope Stability Analysis Using Stochastic Finite Difference Method

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Abstract

The paper contrasts results obtained by the partially factored limit state design method and a more advanced Random Finite Difference Method (RFDM) in a benchmark problem of slope stability analysis with variable undrained shear strength. Local Average Subdivision method was used to simulate the non-Gaussian random variables. The key difference between the methods is that RFDM takes into account spatial variability of soil parameters allowing slope failure to occur naturally along the path of least resistance. The probabilistic method leads to predictions of the "probability of slope failure" as opposed to the more traditional "factor of safety" measure of slope safety in the limit state design method; however, they give significant different results depending on the level of the variability. Analyses conducted using Monte Carlo Simulation show that the same partial factor can have very different levels of risk depending on the degree of uncertainty of the mean value of the soil shear strength. Calibration studies show the partial factor necessary to achieve target probability values.

Keywords: slope stability analysis, probabilistic analysis, limit state design, soil spatial variability, monte carlo simulation

1. Introduction

It is well recognized that the soil variability has a significant effect on the stability of slopes. However, in practice, the variability is not considered properly in routine slope stability analysis. This is due mainly to the fact that the effects of soil variability are complex and difficult to quantify. Furthermore, most of the available slope stability analysis computer programs, used in practice, are unable to consider the factors. Traditionally, the geotechnical engineering community deals with the uncertainties by applying a factor of safety or considering a relatively large permissible stability ratio (e.g., Terzaghi, 1996). However, using a global factor of safety, the variability of different sources of uncertainties is not accountable. Currently, the geotechnical community is preoccupied with the transition from an overall safety factor to limit state design method with partial factors (e.g., BS EN 1997, 2004). The design values of the material properties are obtained by applying factors to the characteristic values. The selection of appropriate partial factor values to achieve the required reliability level is introduced in Eurocode 7 through various design approaches.

Although the limit state design method has some advantages comparing to the traditional factor of safety method, however, the methods generally called 'deterministic' which cannot simulate the degree of soil variability. The limitation of the traditional methods is shown in Figure 1, where all the three cases have same factor of safety as a ratio of the load to resistance, but have different probability of failures (shown as hatched area in the figure).

An alternative method based directly on the probabilistic methods can be used to assess the reliability level of the design. Eurocode 7 does not provide any guidance on the direct use of fully probabilistic reliability methods. However, EN1990 states that the information provided in Annex C, which includes guidance on the reliability index values, may be used as a basis for probabilistic design methods.

The research presented in this paper focuses on investigation and quantifying the influence of soil variability on slope stability using probabilistic analysis. A random field generation technique using Local Average Subdivision (LAS) method (Fenton & Vanmarcke, 1990) in conjunction with finite difference method is used for reliability analysis of a slope in Monte Carlo framework.

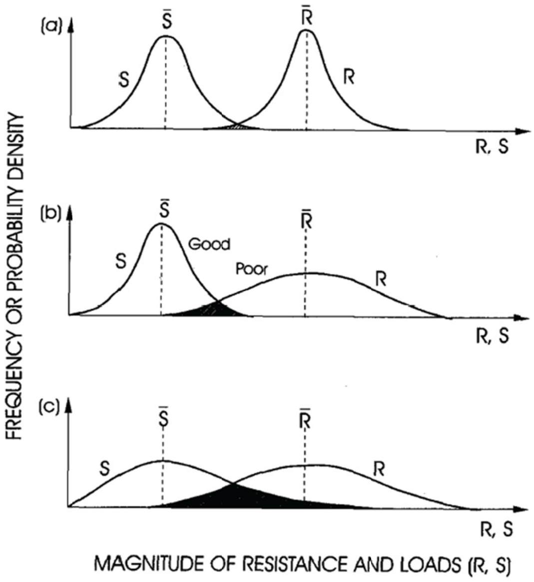

Figure 1. Possible load (S) and resistance (R) distributions (Green, 1989)

2. Modeling Soil Spatial Variability

The soil variability is quantified by its mean, coefficient of variation and scale of fluctuation (or correlation distance). Having statistically characterized a soil layer based on data obtained at discrete locations, it is possible to generate random field predictions of the spatial variability across the entire site.

The present strategy generates random fields using the Local Average Subdivision method. The algorithm is broken down into the following basic steps (Hicks & Spencer, 2010):

1. LAS generates a square 2D isotropic standard normal random field. The field is generated by uniformly subdividing the domain into square cells, maintaining the mean value of the subdivided cells through upward averaging, with each cell value spatially correlated with its neighbours based upon an exponential Markov covariance function. This is given by

$$\beta(\tau_v, \tau_h) = \sigma^2 \exp\left(-\sqrt{\left(\frac{2\tau_v}{\theta_v}\right)^2 + \left(\frac{2\tau_h}{\theta_h}\right)^2}\right) \tag{1}$$

   where β is the covariance, τ is the lag distances in the vertical and horizontal directions.

2. An anisotropic field is generated by squashing the isotropic field; i.e., preserving θ_h in the horizontal plane while compressing θ in the vertical direction to become θ_v.
3. The anisotropic field is transformed from a standard normal distribution to another distribution by using a suitable transformation, i.e., for lognormal conversion:

$$X(x) = \exp\{\mu_{\ln X} + \sigma_{\ln X} Z(x)\} \tag{2}$$

where $X(x)$ defines the centroid of the random field cell and $Z(x)$ is the local average for a random field cell at location x . An example of a random field for a two-dimensional soil domain is illustrated in Figure 2. The dark cells have higher values of c_u, whereas the light cells have lower values for the prescribed statistical characteristics.

3. Random Finite Difference Method (RFDM)

Although the Random Finite Element Method (RFEM) has been around in various guises since the 1980s, the technique was developed by (Fenton & Griffiths, 1993; Griffiths & Fenton, 1993) in the 1990s. It involves mapping a random field onto a finite element mesh and subsequent analysis of the problem by finite elements.

Similar approach is used here. The random field is generated using MATLAB (MATLAB, 2012). The mapping of the properties involves assigning each random field cell value to a finite difference, or sampling point within the element, thus mapping the spatial variation of the properties to the deterministic finite difference (FD) analysis. Due to the range of possible random fields, the analysis is carried out within a Monte Carlo framework, where, in each realization, the random field is mapped onto the FD domain, a deterministic FD analysis is undertaken, and the required measure of performance is recorded.

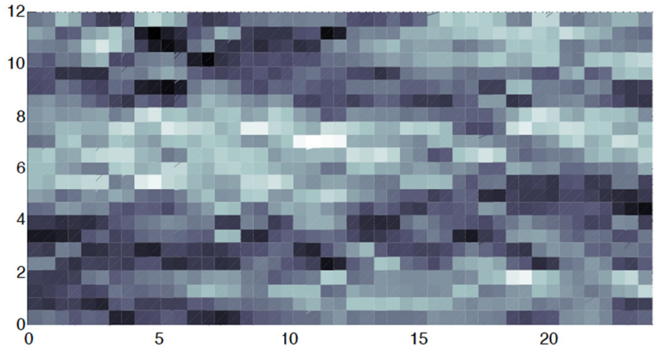

Figure 2. An example of random field simulation using Local Average Subdivisiob method.

Meanwhile, the spatial variability of undrained shear strength, c_u, was modeled by it coefficient of variation (COV) and scale of fluctuation θ. In the interest of generality, the scale of fluctuation was normalized by the slope height H, i.e., θ / H, while the mean value of c_u was expressed in terms of a dimensionless stability coefficient, Ns, similar to Taylor (1937) stability number as

$$N_s = \frac{\mu_{c_u}}{\gamma H} \tag{3}$$

where μ_{c_u} = the mean value of c_u, γ= unit weight of soil; and H =slope height.

Figure 3 shows a typical finite difference mesh for a 1:1 cohesive slope with $\beta = 45^\circ$ and D = 2. An element size of 1m is adopted for the finite difference mesh in the figure. The input parameters for the parametric study are summarized in Table 1. Probabilistic analysis of a cohesive slope problem using the RFEM were previously conducted by Griffiths and Fenton (2004). However, the current parametric studies extent their work by investigating slopes with different geometry. The current study also aim at developing a set of probabilistic stability charts, which can be used for a preliminary estimate of probability of failure P_f.

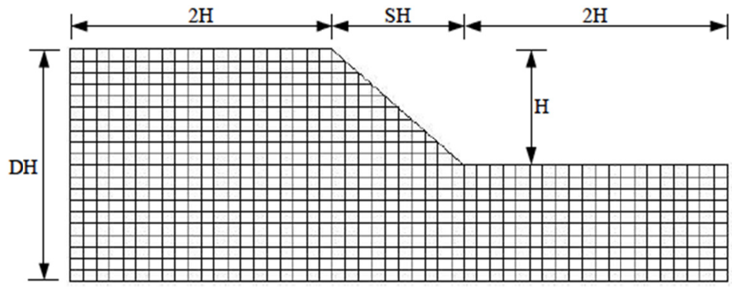

Figure 3. Typical finite difference mesh for a 1:1 cohesive slope ($\beta = 45^\circ$, $D = 2$)

Table 1. Input parameters for parametric studies

Parameters	Input values
β	14°, 18.4°, 26.6°, 45°
D	1, 2, 3
N_s	0.1, 0.2, 0.3, 0.4, 0.5
COV	0.1, 0.3, 0.5, 1.0
θ / H	0.1, 0.5, 1, 5, 10

4. Probabilistic Analysis Results

Figure 4 shows the effect of varying the number of realizations of Monte Carlo simulation on the probability of failure that starts to converge at 2,000 realizations. Further increment in the number of realizations causes only minor changes in the results.

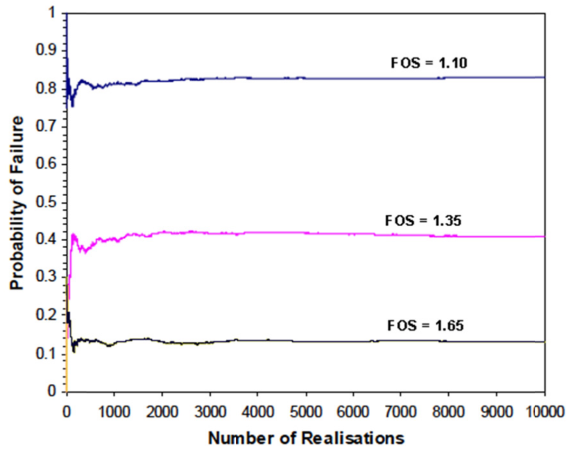

Figure 4. Effect of number of realizations on probability of failure (COV=0.5, $\theta / H = 1$)

Prior to the probabilistic analyses, deterministic finite difference analyses were conducted using limit state design method according to the material factors defined in Eurocode 7. The results are summarized in Table 2.

Table 2. Factor of safety for slope stability analysis using limit state design method in Eurocode 7

N_s	μ_c (kPa)	FOS (FDM)
0.1	20	0.55
0.2	40	1.10
0.3	60	1.65
0.4	80	2.15
0.5	100	2.70

Based on 2,000 realizations of MCS for each parametric group described in Table 1, the influence of each parameter on the estimated P_f is investigated. Figures 5 and 6 show the effects of varying COV on P_f for different values of θ / H with Ns fixed at 0.2 and 0.3, respectively.

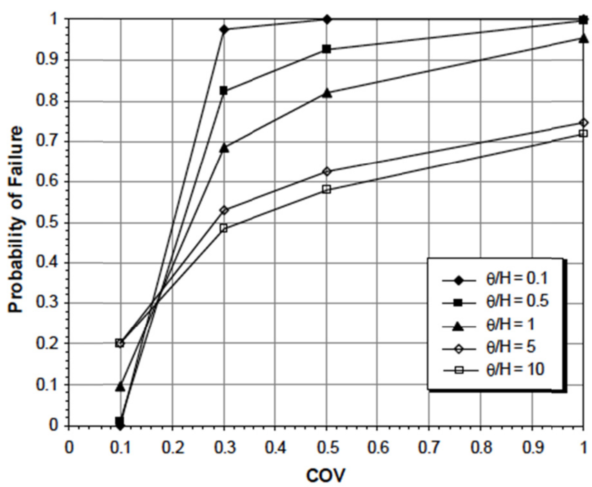

Figure 5. Effect of COV on probability of failure for different values of θ / H with N_s=0.2 ($\beta = 45^\circ$, $D = 2$)

In general, P_f increases as COV increases (i.e. increasing variability in c_u). When Ns=0.2 and FOS =1.10 in deterministic analysis (i.e. marginally stable slope), P_f increases significantly as COV increases from 0.1 to 0.3 for all values of θ/H. However, the increase in P_f becomes lesser as COV increases from 0.3 to 0.5, which is the upper bound value for COV suggested in the literature (Phoon & Kulhawy, 1996). On the other hand, for the slope with Ns=0.3 and FOS=1.65 (i.e. a stable slope), the rate of increase in P_f as COV increases from 0.1 to 0.3 is smaller than that observed for slope with Ns=0.2. However, P_f increases significantly as COV increases from 0.5 to 1.0. Figures 7 and 8 show the effect of varying θ/H on P_f for different values of COV with Ns fixed at 0.2 and 0.3, respectively. Two different trends are obvious, which are dependent on the values COV, i.e. P_f either increases or decreases as θ/H increases. For Ns = 0.2, P_f increases as θ/H increases when COV=0.1, but P_f decreases as θ/H increases when COV=0.3, 0.5 and 1.0. On the other hand, for Ns=0.3, P_f increases as θ/H increases when COV=0.1, 0.3 and 0.5, but P_f decreases as θ/H increases when COV=1.0.

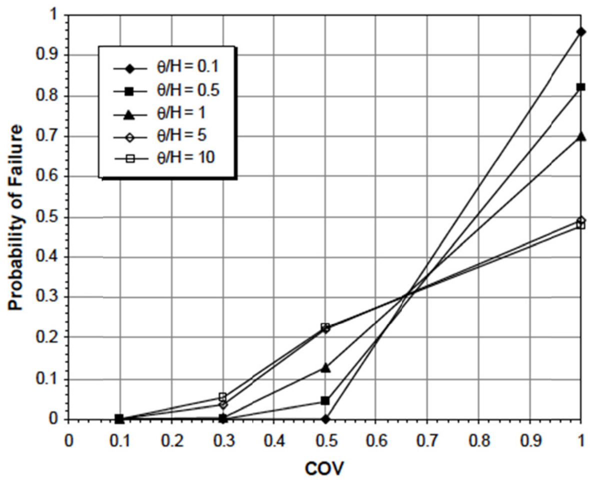

Figure 6. Effect of COV on probability of failure for different values of θ/H with Ns=0.3 ($\beta = 45^{\circ}$, $D = 2$)

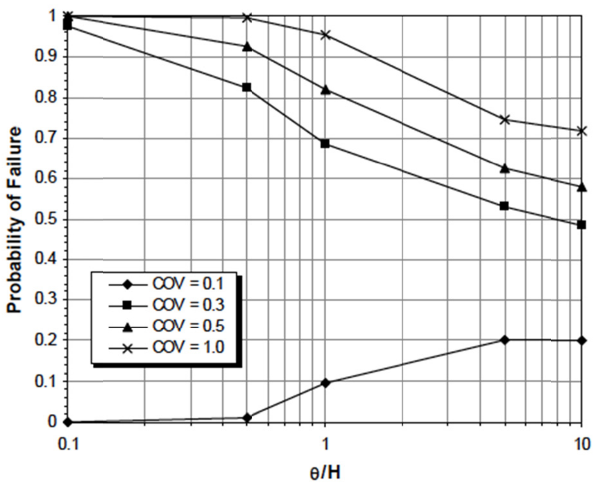

Figure 7. Effect of θ/H on probability of failure for different values of COV with Ns=0.2 ($\beta = 45^{\circ}$, $D = 2$)

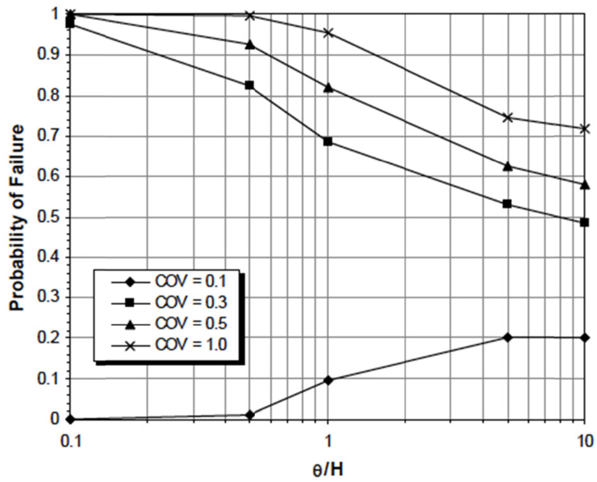

Figure 8. Effect of θ / H on probability of failure for different values of COV with Ns=0.3 ($\beta = 45^{\circ}$, $D = 2$)

Figure 9 shows the plots of P_f versus FOS for different degrees of anisotropy (i.e. θ_h / θ_v) with COV=0.5. It is observed that the curves intersect at FOS ~ 1.4. When FOS < 1.42, the isotropic assumption leads to higher estimate of P_f conservative. In contrast, when FOS>1.4, the isotropic case becomes unconservative, as a lower P_f is estimated. However, the effects are small compared to the effects of variation in the values of COV and θ / H.

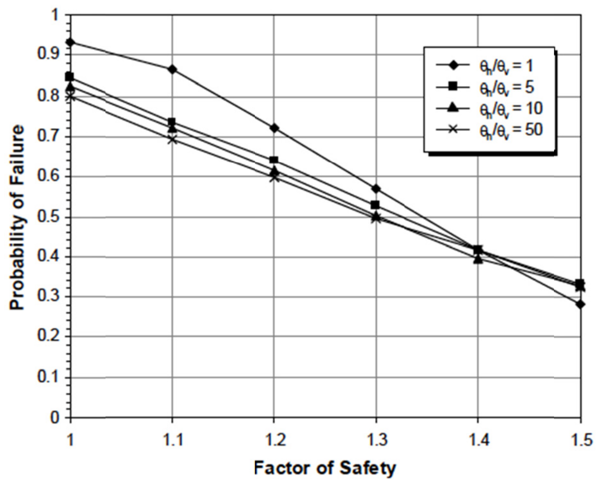

Figure 9. Probability of failure versus factor of safety for different degrees of anisotropy with COV=0.5 and Ns=0.3 ($\beta = 45^{\circ}$, $D = 2$)

5. Conclusion

The random finite difference method was used to investigate the influence of soil variability on the reliability of a cohesive soil slope. The spatial variability of the soil shear strength was modeled by the coefficient of variation and the scale of fluctuation using Local Average Subdivision method.

The results of analyses indicated that both COV and θ / H have a significant effect on the estimated P_f. It was generally found that, P_f increased as COV increased. However, as θ / H increases, P_f either decreased or increased, depending on the values of COV, Ns and slope geometry.

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