A Design Method of BTT Missile Autopilot with the Impact of Feedforward

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Abstract

In order to solve the feedforward problem of traditional back-to-turn (BTT) missile autopilot, this paper proposed a controller design method considering the forward effect. Firstly, according to the three-channel mathematic model of BTT missile, we built a mathematic model of autopilot control system. Secondly, by employing the Nussbaum-type gain technique as well as the adding a power integrator design, and based on the design needs of tracking and controlling overload steadily, we proposed a global nonlinear control strategy, and then designed a continuous nonlinear autopilot, which solved the feedforward problem of BTT missile on pitching channel. Thirdly, we strictly proved the stability of the control system in finite time by applying the method of Lyapunov stability theory. Finally, we gave a simulation example to show that the designed control system not only overcome the influence of uncertain factors and the problem of the stable error, but also improved the tracking precision.

Keywords: BTT, Feedforward controller, Nonlinear systems, Adaptive control, Output tracking

1. Introduction

No matter whether it is back-to-turn (BTT) or slide-to-turn (STT) missile, the change of the vertical or lateral overload of rudder is through autopilot by the deflection. When the rudder is deflected, the missile's postures are adjusted accordingly, so as to change the vertical or lateral aerodynamics of missile body and missile wings; meanwhile, the rudder yields another vertical or lateral aerodynamics, which influences the loading of missile. These influences are the missile's forward effect from the missile surface deflection to the missile body.

The existence of the forward effect actually adds a zero-point to the system, which makes the design of the missile autopilot (especially the design based on the modern control theory) more difficult; thus most the previous missile autopilots always ignored the forward effect, which makes the system design easier but brings large errors to the system. Meanwhile, when the forward effect is strong, these errors are always leading to a failure of the design(Zhou Jun, Chen Xin-hai, Zhou Feng-qi, 1994)(Xing Li-dan, Chen Wan-chun, Yin Xing-liang, 2008)(Tong Chun-xia, Wang Zheng-jie, Zhang Tian-qiao, 2006).

In this paper, in view of the pitching channel of BTT missile, we convert the mathematic model of missile body into a state equation, apply Nussbaum-type gain technology and back-stepping design method, propose a global nonlinear control strategy based on the design requirement of stable trace controlling, and design a continuous nonlinear autopilot to make sure the missile body vertical n_y traces homing order n_{yc} , thereby we resolve the problem of BTT missile's pitching channel forward effect. Finally, we use Lyapunov stable theory to strictly prove that our control system can achieve the overall situation's stability in finite time.

2. Dynamic model of missile body

For advanced BTT missile, since the rolling channel is not stable anymore, there exist many strong couplings between each channel, among these couplings, the inertia coupling and the dynamic coupling are especially

important. After considering the effect of inertia coupling and dynamic coupling, the three channels body mathematic model for BTT missile can be written as follows:

$$\begin{aligned} \dot{\alpha} &= w_{z} - w_{x}\beta - a_{4}\alpha - a_{5}\delta_{z} \\ \dot{\beta} &= w_{y} + w_{x}\alpha - b_{4}\beta - b_{5}\delta_{y} \\ \dot{w}_{z} &= -a_{1}w_{z} - a_{1}'\dot{\alpha} - a_{2}\alpha - a_{3}\delta_{z} + \frac{J_{x} - J_{y}}{J_{z}}w_{x}w_{y} \\ \dot{w}_{y} &= -b_{1}w_{y} - b_{1}'\dot{\beta} - b_{2}\beta - b_{3}\delta_{y} + \frac{J_{z} - J_{x}}{J_{y}}w_{x}w_{z} \end{aligned}$$
(1)

where α and β are the missile's attack angle and sideslip angle respectively; w_x , w_y and w_z are the rotational angular velocities in missile body coordinate; δ_y and δ_z are the rudder's deflexion angles on the yaw channel and pitching channel; J_x , J_y and J_z are the moments of inertia in missile body coordinate; a_1 , a'_1 , a_2 , a_3 , a_4 , a_5 , b_1 , b'_1 , b_2 , b_3 , b_4 and b_5 are the aerodynamic parameters on the pitching channel and yaw channel.

Here, the vertical overload of missile body can be expressed as:

$$n_y = V(a_4\alpha + a_5\delta_z)/g \tag{2}$$

We can see from (2) that the pitching channel exists a forward effect from the rudder's deflexion to the vertical overload.

The design of pitching channel autopilot is to make sure the vertical overload n_y of missile body can trace homing order n_{yc} effectively:

$$\lim_{t \to \infty} (n_{yc} - n_y) = 0 \tag{3}$$

When ignoring the forward effect, the vertical overload of missile body is

$$n_{y1} = V a_4 \alpha / g \tag{4}$$

For this situation, according to (1) and (4), literature(Lin W, Qian C J. 2000) concluded a pitching channel control law of BTT missile as follows:

$$\delta_z = f(\alpha, \beta, \omega_x, \omega_y, \omega_z, V, n_{y1}) \tag{5}$$

The control law (5) can ensure n_{y1} to trace n_{yc} , and $n_{y1} = n_{yc}$ in steady state; however, due to the existence of the forward effect, the actual vertical overload of missile body n_y doesn't trace n_{yc} in steady state, and n_y is

$$n_{y} = n_{y1} + \frac{Va_{5}\delta_{z}}{g} = n_{yc} + \frac{Va_{5}\delta_{z}}{g} \neq n_{yc}$$
(6)

So we can see that the control law (5) fails to achieve the design aim of pitching channel autopilot (3). To solve this problem, we need to design a new forward controller.

According to (1) and (2), the overload control system model of autopilot is

$$\begin{cases} \dot{n}_{y} = \frac{Va_{4}}{g}w_{z} - a_{4}n_{y} - \frac{Va_{4}}{g}w_{x}\beta + \frac{Va_{5}}{g}\dot{\delta}_{z} \\ \dot{w}_{z} = (-a_{1} - a_{1}')w_{z} + \frac{g}{V}(a_{1}' - \frac{a_{2}}{a_{4}})n_{y} + \left(\frac{a_{2}a_{5}}{a_{4}} - a_{3}\right)\delta_{z} + \frac{J_{x} - J_{y}}{J_{z}}w_{x}w_{y} \end{cases}$$
(7)

Taking $X = [n_y \ w_z]^T$ as the system states, and δ_z as the control variable, we can obtain from (7) the pitching channel mathematical model of missile body as follows:

$$\begin{cases} \prod_{x_1}^{\Box} = \frac{Va_4}{g} x_2 - a_4 x_1 - \frac{Va_4}{g} w_x \beta + \frac{Va_5}{g} \dot{\delta}_z \\ \prod_{x_2}^{\Box} = \left(\frac{a_2 a_5}{a_4} - a_3\right) u_1 - (a_1 + a_1') x_2 + \frac{g}{V} (a_1' - \frac{a_2}{a_4}) x_1 + \frac{J_x - J_y}{J_z} w_x w_y \end{cases}$$
(8)

where $x_1 = n_y$, $x_2 = w_z$ and $u_1 = \delta_z$. The disturbing items in (8) include all the coupling effects on the pitching channel.

3. Controller design with the impact of feedforward

3.1 Controller design of the nonlinear system

We firstly study the output tracking problem of SISO nonlinear system, which can be expressed as(Wang Q D, Jing Y W, Zhang S Y, 2004)(Wang Qiang-de, Jing Yuan-wei, Zhang Si-ying, 2006):

$$\begin{cases} x_{1} = cx_{2} + \phi_{1}(x, u, d(t)) \\ x_{2} = cx_{3} + \phi_{2}(x, u, d(t)) \\ \vdots \\ x_{n} = g(t)u + \phi_{n}(x, u, d(t)) \\ y = x_{1} \end{cases}$$
(9)

where $x = (x_1, ..., x_n)^T \in \mathbb{R}^n$, $u \in \mathbb{R}$ and $y \in \mathbb{R}$ are the system states, input and output respectively; $g(t) \neq 0$ and d(t) are two unknown limit piecewise continuous time-variation functions (their borders are unknown), ϕ_i is a continuous function of its variables. In the following, combining Nussbaum-type gain technology with back-stepping design method, we study the output problem of nonlinear system (9).

For the output tracing problem of nonlinear system (9), the controlling aim is to construct a robust adaptive state feedback nonlinear controller, so as to make the system's output tracing error $y(t) - y_r(t)$ globally uniformly bounded, which is to say that:

$$\lim_{t \to \infty} |y - y_r(t)| < \varepsilon \tag{10}$$

Meanwhile, the controller also needs to ensure all the other signals of the closed-loop system globally uniformly bounded.

For the convenience of controller design, we give some Lemmas and definitions as follows:

Lemma 1(Wang Qiang-de, Jing Yuan-wei, Zhang Si-ying, 2006): For any positive integer m, n and any real-value function r(x, y) > 0, the following inequality exists:

$$|x|^{m} |y|^{n} \le \frac{m}{m+n} r(x,y) |x|^{m+n} + \frac{n}{m+n} r^{-m/n}(x,y) |y|^{m+n}$$
(11)

Lemma2(Wang Qiang-de, Jing Yuan-wei, Zhang Si-ying, 2006): For $\phi_i(x, u, d(t))$ in system (1), there exists a known smooth function $\overline{f}_i(x_1, ..., x_i) \ge 1$ and an unknown constant $\Theta \ge 1$ to make $|\phi_i(x, u, d(t))| \le \overline{f}_i(x_1, ..., x_i)\Theta$.

Lemma 3(Wang Qiang-de, Jing Yuan-wei, Zhang Si-ying, 2006): For any real numbers $a \ge 0$, b > 0 and $m \ge 1$, a inequality $a \le b + |a/m|^m |(m-1)/b|^{m-1}$ holds.

We use the Nussbaum-type gain technology to overcome the difficulty of the control direction unknown, and submit some correlative definition and lemma as following:

Definition 1(Nussbaum R D, 1983): Function $N(\chi)$ is named Nussbaum-type function, if it has the following properties:

$$\begin{cases} \limsup_{s \to \infty} \sup \frac{1}{s} \int_0^s N(\chi) d\chi = +\infty \\ \liminf_{s \to \infty} \inf \frac{1}{s} \int_0^s N(\chi) d\chi = -\infty \end{cases}$$
(12)

where $N(\chi) = \exp(\chi^2) \cos |\pi\chi/2|$.

Lemma 4(Nussbaum R D, 1983): Suppose V(t) and $\chi(t)$ are two smooth functions which are defined in the interval $\begin{bmatrix} 0 & t_f \end{bmatrix}$, for $\forall t \in \begin{bmatrix} 0 & t_f \end{bmatrix}$ and $V(t) \ge 0$, $N(\chi)$ is a suitable and smooth Nussbaum-type function, if the following inequality holds:

$$V(t) \le c_0 + e^{-c_1 t} \int_0^t (g(\tau)N(\chi) + 1)\dot{\chi} e^{c_1 \tau} d\tau$$
(13)

where c_0 is a suitable constant, $c_1 > 0$, g(t) is the variable time-varying parameter which is in a unknown closed interval $I = [l^-, l^+](0 \notin I)$, thus V(t), $\chi(t)$ and $\int_0^t (g(\tau)N(\chi) + 1)\dot{\chi}d\tau$ must have borders in $[0, t_f)$.

The controller design of nonlinear system can be described as follows:

Step 1: Give a bounded smooth reference signal y_r , let $\xi_1 = x_1 - y_r$ as the error signal, then construct a positive-define and proper Lyapunov function $V_1(\xi_1) = \xi_1^2 / 2$, and a direct calculation is given as

$$V_1 = \xi_1[cx_2 + \phi_1(x, u, d(t)) - \dot{y}_r(t)]$$
(14)

Since y_r is bounded, there exists a smooth function $\gamma_1(\xi_1)$ satisfying the following inequality.

$$\left|\phi_{1}(x,u,d(t)) - \dot{y}_{r}(t)\right| \le \gamma_{1}(\xi_{1})$$
(15)

According to Lemma 3, for any constant $\delta_1 > 0$, there exists a smooth function $\rho_1(\xi_1) \ge 0$; let $a = |\xi_1| \gamma_1(\xi_1)$, $b = \delta_1$ and m = 2, we can obtain the following result:

$$\left|\xi_{1}\right|\gamma_{1}(\xi_{1}) \leq \xi_{1}^{2}\rho_{1}(\xi_{1}) + \delta_{1}$$
(16)

where $\rho_1(\xi_1) = \delta_1 + \xi_1^2 \gamma_1^2(\xi_1) / 4\delta_1$

Combining (14) with (16), we have the following result:

$$\bigvee_{1}^{\cup} = \xi_{1} \left[c x_{2} + \xi_{1} \rho_{1}(\xi_{1}) \right] + \delta_{1}$$
(17)

After we choose $x_2^* = -\xi_1[2 + \rho_1(\xi_1)] = -\xi_1\beta_1(\xi_1)$ as the virtual smooth controller, (17) can be rewritten as:

$$V_{1}^{\sqcup} = -2\xi_{1}^{2} + \xi_{1} \Big[cx_{2} - x_{2}^{*} \Big] + \delta_{1}$$
(18)

Step2: Suppose at step k-1, there are a set of smooth virtual controllers and coordinate changes, which are defined by

where $\beta_1(\xi_1) > 0, \dots, \beta_{k-1}(\xi_{k-1}) > 0$ are smooth. There exists a smooth Lyapunov function $V_{k-1} = \sum_{i=1}^{k-1} \frac{1}{2} \xi_i^2$ satisfying the following inequality.

$$\dot{V}_{k-1} \le -\sum_{i=1}^{k-2} \xi_i^2 - 2\xi_{k-1}^2 + \xi_{k-1} [cx_k - x_k^*] + \delta_{k-1}$$
(19)

According to Lemma 1 and Lemma 2, and based on a Lyapunov function $V_k = V_{k-1} + \xi_k^2 / 2$, we can acquire the following inequality:

$$V_{k} \leq -\sum_{i=1}^{k-2} \xi_{i}^{2} - 2\xi_{k-1}^{2} + \xi_{k-1} [cx_{k} - x_{k}^{*}] + \delta_{k-1} + \xi_{k} \cdot [x_{k+1} + \phi_{k}(x, u, d(t)) - \sum_{i=1}^{k-1} \frac{\partial x_{k}^{*}}{\partial x_{i}} (x_{i+1} + \phi_{i}(\cdot)) - \frac{\partial x_{k}^{*}}{\partial y_{r}} y_{r}^{\Box}]$$

$$(20)$$

According to $|x+y|^p \le 2^{p-1}(|x|^p + |y|^p)$, there exists a smooth function $\overline{\rho}_k(\xi_1,...,\xi_k) \ge 0$, and we have $|\xi_{k-1}[cx_k - x_k^*]| \le \xi_{k-1}^2 + \xi_k^2 \overline{\rho}_k(\xi_1,...,\xi_k)$ (21)

According to Lemma 1, it is not difficult to prove the following relations:

$$\left|\xi_{k}\left[\phi_{k}\left(x,u,d\left(t\right)\right)-\sum_{i=1}^{k-1}\frac{\partial x_{k}^{*}}{\partial x_{i}}\left(x_{i+1}+\phi_{i}\left(\cdot\right)\right)-\frac{\partial x_{k}^{*}}{\partial y_{r}}y_{r}^{\Box}\right]\right|\leq\xi_{k}^{2}\rho_{k}\left(\xi_{1},...,\xi_{k}\right)+\delta_{k}$$

$$(22)$$

where $\delta_k = k \delta_1$, $\rho_k(\square)$ is a nonnegative smooth function.

Putting the above relations and (20) together, we have

$$V_{k} \leq -\sum_{i=1}^{k-1} \xi_{i}^{2} + \xi_{k} x_{k+1} + \xi_{k}^{2} (\rho_{k}(\xi_{1}, ..., \xi_{k}) + \overline{\rho_{k}}(\xi_{1}, ..., \xi_{k})) + \delta_{k}$$
(23)

Clearly, the virtual smooth controller $x_k^* = -\xi_k (2 + (\rho_k + \overline{\rho_k})) = -\xi_k \beta_k (\xi_1, ..., \xi_k)$ can yields the following inequality.

$$\dot{V}_{k} \leq -\sum_{i=1}^{k-1} \xi_{i}^{2} - 2\xi_{k}^{2} + \xi_{k} (cx_{k+1} - x_{k+1}^{*}) + \delta_{k}$$

$$\tag{24}$$

Step 3: Using the derivation above, we conclude that at the nth step, there are a group of transformations of the form in the step 2, a smooth Lyapunov function $V_n = V_{n-1} + \xi_n^2 / 2$ and a smooth controller u can yield the following inequality.

$$\dot{V}_{n} = -\sum_{i=1}^{n-1} \xi_{i}^{2} - \xi_{n-1}^{2} + \xi_{n-1} [cx_{n} - x_{n}^{*}] + \xi_{n} [g(t)u + \phi_{n} - x_{n}^{*'}]$$
(25)

Using the method above, it is very easy to get the equation as follows:

$$\dot{V}_{n} = -\sum_{i=1}^{n} \xi_{i}^{2} + \xi_{n} g(t) u + \xi_{n}^{2} (1 + \rho_{n}(\Box) + \overline{\rho}_{n}(\Box)) + \delta_{n}$$
(26)

where $\rho_n(\Box)$ and $\overline{\rho}_n(\Box)$ are two nonnegative smooth functions, δ_n is an unknown positive constant. We choose the smooth adaptive control law as follows:

$$\begin{cases} u = \Gamma \xi_n N(\chi) (1 + \rho_n(\Box) + \overline{\rho}_n(\Box)) \\ \vdots \\ \chi = \lambda \xi_n^2 (1 + \rho_n(\Box) + \overline{\rho}_n(\Box)) \end{cases}$$
(27)

where $\Gamma > 0$ and $\lambda > 0$ are the designed constants. Based on the control law, we obtain the following relations:

$$\dot{V}_{n} \leq -\sum_{i=1}^{n} \xi_{i}^{2} + \frac{\Gamma}{\lambda} g(t) N(\chi) \dot{\chi} + \frac{\dot{\chi}}{\lambda} + \delta_{n}$$
(28)

3.2 BTT missile controller design with the impact of forward

We make the rudder model of BTT missile equal to a first-order actuator model:

$$\delta_z^{\Box} = -\frac{\delta_z}{\tau_1} + \frac{\delta_{zc}}{\tau_2}$$
(29)

where τ_1 and τ_2 are two coefficients of rudder model, δ_{zc} is the rudder angle order. Putting (29)and (8) together, we have

$$\begin{cases} \Box_{1} = \frac{Va_{4}}{g} x_{2} - a_{4}x_{1} - \frac{Va_{4}}{g} w_{x}\beta - \frac{Va_{5}}{g\tau_{1}} x_{3} + \frac{Va_{5}}{g\tau_{2}} u \\ \Box_{2} = \left(\frac{a_{2}a_{5}}{a_{4}} - a_{3}\right) x_{3} - (a_{1} + a_{1}')x_{2} + \frac{g}{V}(a_{1}' - \frac{a_{2}}{a_{4}})x_{1} + \frac{J_{x} - J_{y}}{J_{z}} w_{x}w_{y} \end{cases}$$

$$\Box_{3} = -\frac{1}{\tau_{1}} x_{3} + \frac{1}{\tau_{2}} u$$

$$(30)$$

and

$$\begin{cases} \phi(x, u, d(t)) = -a_4 x_1 - \frac{Va_4}{g} w_x \beta - \frac{Va_5}{g\tau_1} x_3 + \frac{Va_5}{g\tau_2} u \\ \phi_2(x, u, d(t)) = -(a_1 + a_1') x_2 + \frac{g}{V} (a_1' - \frac{a_2}{a_4}) x_1 + \frac{J_x - J_y}{J_z} w_x w_y \\ g(t) = 1/\tau_2, \phi(x, u, d(t)) = -x_3/\tau_2 \end{cases}$$
(31)

There exists an unknown constant $M_1 \ge 0$, render $|Va_4w_x\beta/g| \le M_1$, such that

$$\begin{cases} V_{1}(\xi_{1}) = \frac{1}{2}\xi_{1}^{2} \\ V_{2}(\xi_{1},\xi_{2}) = \frac{1}{2}\xi_{1}^{2} + \frac{1}{2}\xi_{2}^{2} \\ V_{3}(\xi_{1},\xi_{2},\xi_{3}) = \frac{1}{2}\xi_{1}^{2} + \frac{1}{2}\xi_{2}^{2} + \frac{1}{2}\xi_{3}^{2} \\ V_{1} \leq -2\xi_{1}^{2} + \xi_{1}\left[cx_{2} - x_{2}^{*}\right] + \delta_{1} \\ V_{3} \leq -\xi_{1}^{2} - \xi_{2}^{2} - \xi_{3}^{2} + \frac{\Gamma}{\lambda}g(t)N(\chi)\chi + \frac{\chi}{\lambda} + \delta_{3} \end{cases}$$
(32)

where $\xi_1 = x_1 - y_r$, $\xi_2 = x_2 - x_2^*$, $x_2^* = -\xi_1 \beta_1(\xi_1)$, $\xi_3 = x_3 - x_3^*$, $x_3^* = -\xi_2 \beta_2(\xi_2)$

The control aim is to design a smooth adaptive control law to make the practical output trace the reference signal $y_r(t)(|y_r(t)| \le M$, $|\dot{y}_r(t)| \le M$, and $M \ge 0$ is an unknown constant). According to (28), we have

$$\begin{cases} u = \Gamma \xi_3 N(\chi) (1 + \rho_3(\Box) + \overline{\rho}_3(\Box)) \\ \vdots \\ \chi = \lambda \xi_3^2 (1 + \rho_3(\Box) + \overline{\rho}_3(\Box)) \end{cases}$$
(33)

Applying the designed controller into system (8) and (9), we can make the following conclusions:

For any initial conditions, all the closed-loop system's signal is bounded in the interval $[0, +\infty)$. In addition, if adjusting the design parameter properly, we can make the output tracking error become appropriately small in a finite time.

Proof: The design controller above is smooth, so the closed-loop system's solution has a definition in the interval $[0, t_f)$. We convert (28) into:

$$V_{n}^{\Box} \leq -2V_{n} + \frac{\Gamma}{\lambda}g(t)N(\chi)\chi + \frac{\chi}{\lambda} + \delta_{n}$$
(34)

From (28), it follows that

$$0 \le V_n(t) \le V_n(0) + \frac{1}{\lambda} e^{-2t} \int_0^t (\Gamma g(\tau) N(x) + 1) \chi e^{2\tau} d\tau + e^{-2t} \int_0^t \left| \delta_n \right| e^{2\tau} d\tau$$
(35)

Since δ_n is a constant, $e^{-2t} \int_0^t |\delta_n| e^{2\tau} d\tau$ is bounded; According to Lemma 4, V(t), $\chi(t)$ and $\int_0^t (g(\tau)N(\chi)+1)\dot{\chi}d\tau$ are all bounded in the interval $[0,t_f)$, thus the closed-loop system's state variables are bounded in $[0,t_f)$, further it is easy to infer that the virtual control law x_i^* and control law u are all bounded. By the controller design and the conclusions above, we can see that, properly adjusting the parameter Γ, λ and δ can make output tracking error appropriately small in a finite time.

4. Simulation

We perform a simulation taking into account of three-channel coupling. In the simulation, the initial conditions are set to $x_1(0) = 0.1$, $x_2(0) = 0.1$ and $\chi(0) = 0$; the other parameters are designed as $\Gamma_1 = 0.1$, $\lambda_1 = 1$ and $\delta_1 = 10$; and the lateral overload n_y keeps track the square wave overload instruction n_{yc} . The simulation results are shown as follows:Fig1-fig5.

5. Conclusion

In this paper, we studied a design method of BTT missile autopilot with the impact of feedforward; and we obtained the conclusions as following:

(1) From the comparison between Fig. 1 and Fig. 2, we can see that the designed controller with the impact of feedforward can not only reduce the stable errors but also make tracking precision is obviously better than the controller without any consideration of feedforward effect.

(2) By employing the Nussbaum-type gain technique and the adding a power integrator design, a global nonlinear control strategy was proposed to obtain the continuous autopilot. The stability of the control system in finite time was strictly proved applying the method of Lyapunov stability theory.

(3) A simulation example is given to demonstrate that the designed control system not only overcome the influence of uncertain factors and the problem of the stable error, but also improved the tracking precision by adjusting the design parameter properly.

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Figure 1. Output response of vertical overload n_y to instruction n_{yc} with the impact of feedforward



Figure 2. Output response of vertical overload n_y to instruction n_{yc} without the impact of feedforward



Figure 3. Output response of rotational angular velocity w_z to instruction n_{vc} with the impact of feedforward



Figure 4. Output response of adaptive parameter χ to instruction n_{yc} with the impact of feedforward



Figure 5. Output response of sideslip angle β to instruction n_{yc} with the impact of feedforward