Based on Linear Matrix Inequality Stability Analysis of Neutral Delay Systems

Long Yu
College of Science, Yanshan University, Qinhuangdao 066004, China
E-mail: yulongyulong2010@163.com
Hongxia Sun
College of Science, Yanshan University, Qinhuangdao 066004, China

Supported by the Foundation for the natural science of He Bei province of China (200700038)

Abstract

The asymptotic stability of the neutral systems with norm-bounded uncertainties and time-varying delays were discussed by using the method of LMI. The results were expressed in terms of linear matrix inequalities. Compared with some existing results, the criteria obtained in our paper are less conservative.

Keywords: Time-delay system, Uncertainty, Stability, Linear matrix inequality, Neutral systems

1. Introduction

In recent years, in the neutral delay system stability studies, the main concern of neutral discrete time-delay system, for many time-delay system stability problems, some related time-delay dependent or delay-independent stability criteria was given .At the same time, many people pay attention to variable delay neutral systems, by making the identity transformation, structure reasonable Lyapunov function, we can obtain delay-depend stability criteria.

Y.He studied the discrete time-delay system of neutral with the constant delays, by using the free weight matrix to indicate Newton-Leibniz formula, they got delay-depend stability criteria. This method reduced conservative. This paper will extend the result of Y.He to the time-varying delay neutral system. Using Lyapunov stability theory and free weight matrix approach, global stability of the system is transformed into a linear matrix inequality optimization problem, we can get the delay-depend stability criteria.

2. Problem Formulations

Considering the following neutral uncertain time-delay system

$$\dot{x}(t)-(C+\Delta C(t))\,\dot{x}(t-h)=(A+\Delta A(t))x(t)+(B+\Delta B(t))x(t-d(t)) \tag{1}$$

$$x(t)=\varphi(t) \quad t\in[-d,0]$$

Where $x(t)\in R^n$ is the neuron state vector; $A\in R^{n\times n}$, $B\in R^{n\times n}$, $C\in R^{n\times n}$ are the constant matrices with appropriate dimensions; $d(t)$ denote the time-varying delay satisfies

$$0<d(t)\le d \quad , \quad \dot{d}(t)\le \mu \tag{2}$$

in which d and μ are the constants ; scalar $h>0$ is the state derivative of the delay ; $\Delta A(t)$, $\Delta B(t)$ and $\Delta C(t)$ reflect the system model in the time-varying parameter uncertainty in real matrix ; $\varphi(t)$ is a continuous vector initial .

Assumption 1 In system (1) $\Delta A(t)$, $\Delta B(t)$ and $\Delta C(t)$ satisfies

$$\Delta A(t)=DF(t)E_1 \quad , \quad \Delta B(t)=DF(t)E_2 \quad , \quad \Delta C(t)=DF(t)E_3 \tag{3}$$

In which $D\,,E_i\ (i=1,2,3)$ is a constant matrix of appropriate dimension ; $F(t)$ satisfies

$$F^T(t)\ F(t)\le I \tag{4}$$

3. Main Results

Considering the following neutral uncertain time-delay system

$$\dot{x}(t) = Ax(t) + Bx(t - d(t)) + C\dot{x}(t - h) \tag{5}$$

Theorem1 On the assumption (3) , for a given constant d , if there exist symmetric positive definite matrices $P, R, Q_1, Q_2, X_{ii}, Y_{ii} (i = 1,\cdots,5)$ and arbitrary matrices $X_{ij}, Y_{ij} (i = 1,\cdots,5, i \le j)$, makes the establishment of the following matrix inequality, then system (5) In the equilibrium is globally asymptotically stable

$$\Omega = \begin{pmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & A^T S \\ * & \Phi_{22} & \Phi_{23} & \Phi_{24} & B^T S \\ * & * & \Phi_{33} & \Phi_{34} & C^T S \\ * & * & * & \Phi_{44} & 0 \\ * & * & * & * & -S \end{pmatrix} < 0 \tag{6}$$

Where

$$\Phi_{11} = A^T P + PA + Q_1 + Q_2 + \frac{1}{d}(X_{15} + X_{15}^T) + \frac{1}{h}(Y_{15} + Y_{15}^T) + \frac{1}{d}X_{11} + \frac{1}{h}Y_{11}$$

$$\Phi_{12} = PB - \frac{1}{d}(X_{15} + X_{25}^T - X_{12}) + \frac{1}{h}(Y_{25} + Y_{12})$$

$$\Phi_{13} = A^T PC + \frac{1}{d}(X_{13} + X_{35}^T) + \frac{1}{h}(Y_{15}^T + Y_{35}^T)$$

$$\Phi_{14} = \frac{1}{d}(X_{14} + X_{45}^T) + \frac{1}{h}(Y_{14} + Y_{45}^T)$$

$$\Phi_{22} = -(1-\mu)Q_1 - \frac{1}{d}(X_{25} + X_{25}^T) + \frac{1}{d}X_{22} + \frac{1}{h}Y_{22}$$

$$\Phi_{23} = -B^T PC + \frac{1}{d}(X_{23} - X_{35}^T) + \frac{1}{h}(Y_{23} - Y_{25})$$

$$\Phi_{24} = \frac{1}{d}(X_{24} - X_{45}^T) + \frac{1}{h}Y_{24}$$

$$\Phi_{33} = -Q_2 - \frac{1}{h}(Y_{35} + X_{35}^T) + \frac{1}{d}X_{33} + \frac{1}{h}Y_{33}$$

$$\Phi_{34} = \frac{1}{d}X_{34} + \frac{1}{h}(Y_{34} - Y_{45}^T)$$

$$\Phi_{55} = R + \frac{d}{1-\mu}X_{55} + \frac{1}{h}Y_{55}$$

*stand for Symmetry elements of matrix transpose

Proof Construct the Lyapunov-Krasovksii functional:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t) + V_6(t)$$

Now the derivative of $V(t)$ along the trajectories of system (5) yields

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \dot{V}_4(t) + \dot{V}_5(t) + \dot{V}_6(t)$$

Where

$$\dot{V}_1(t) = 2x^T(t)P\dot{x}(t) = 2x^T(t)P(Ax(t) + Bx(t-d(t)) + Cx(t-h))$$

$$\dot{V}_2(t) = \le x^T(t)Q_1x(t) - (1-\mu)x^T(t-d(t))Q_1x(t-d(t))$$

$$\dot{V}_3(t) = x^T(t)Q_2x(t) - x^T(t-h)Q_2x(t-h)$$

$$\dot{V}_4(t) = x^T(t)Rx(t) - x^T(t-h)Rx(t-h)$$

$$\dot{V}_5(t) \le \frac{d}{1-\mu}\dot{x}^T(t)X_{55}\dot{x}(t) - \frac{1}{d}(\int_{t-d(t)}^{t}\dot{x}(s)ds)^T X_{55}(\int_{t-d(t)}^{t}\dot{x}(s)ds)$$

$$\dot{V}_6(t) \le h\,\dot{x}^T(t)Y_{55}\dot{x}(t) - \frac{1}{h}(\int_{t-h}^{t}\dot{x}(s)ds)^T Y_{55}(\int_{t-h}^{t}\dot{x}(s)ds)$$

$$\dot{x}^T(t)S\dot{x}(t) = (Ax(t) + Bx(t-d(t)) - C\dot{x}(t-h))^T \times S(Ax(t) + Bx(t-d(t)) - C\dot{x}(t-h))$$

In which $S = R + \frac{d}{1-\mu}X_{55} + hY_{55}$

Base on Leibniz-Newton formula and Ref. (Y.He, M.Wu, J.H.She, G.P.Liu. 2004), we can get

$$\dot{V}(t) = \xi_1^T(t)\Psi\xi_1(t) - \frac{1}{d}\xi_2^T(t,s)\mathrm{X}\xi_2(t,s) - \frac{1}{d}\xi_3^T(t,s)\mathrm{Y}\xi_3(t,s) \tag{9}$$

Where

$$\xi_1(t) = \begin{pmatrix} x(t) & \dot{x}^T(t) & x(t-d(t)) & x(t-h) \end{pmatrix}^T,\ \xi_2(t,s) = (\int_{t-d(t)}^{t}\dot{x}(s)ds)\ \xi_1^T(t)$$

$$\xi_3(t,s) = (\int_{t-h}^{t}\dot{x}(s)ds)\ \xi_1^T(t)$$

$$\Psi = \begin{pmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} \\ * & \Phi_{22} & \Phi_{23} & \Phi_{24} \\ * & * & \Phi_{33} & \Phi_{34} \\ * & * & * & \Phi_{44} \end{pmatrix} < 0$$

Theorem is proven.

Theorem2 On the assumption (3), for a given constant d , if there exist symmetric positive definite matrices P , R , Q_1 , Q_2 , X_{ii} , $Y_{ii}(i = 1,\cdots,5)$ and arbitrary matrices $X_{ij}, Y_{ij}(i = 1,\cdots,5, i \le j)$, makes the establishment of the following matrix inequality, then system (1) In the equilibrium is globally asymptotically stable

$$\begin{pmatrix} \Phi_{11} + E_1^TE_1 & \Phi_{12} + E_1^TE_2 & \Phi_{13} + E_1^TE_3 & \Phi_{14} & A^TS & PD \\ * & \Phi_{22} + E_2^TE_2 & \Phi_{23} + E_2^TE_3 & \Phi_{24} & B^TS & 0 \\ * & * & \Phi_{33} + E_3^TE_3 & \Phi_{34} & C^TS & 0 \\ * & * & * & \Phi_{44} & 0 & 0 \\ * & * & * & * & -S & SD \\ * & * & * & * & * & -I \end{pmatrix} < 0 \tag{10}$$

Proof In the proof of theorem 1 , using $A + DF(t)E_1, B + DF(t)E_2, C + DF(t)E_3$ replaced A , B

and C , then get

$$\Omega+\lambda\begin{pmatrix} PD \\ 0 \\ -D^T PD \\ 0 \\ SD \end{pmatrix}\begin{pmatrix} E_1 & E_2 & E_3 & 0 \end{pmatrix}+\lambda^{-1}\begin{pmatrix} PD \\ 0 \\ -D^T PD \\ 0 \\ SD \end{pmatrix}\begin{pmatrix} PD & 0 & -D^T PD & 0 & SD \end{pmatrix}<0$$

Theorem is proven.

4. Numerical Examples

Examples 1 Consider the following system

$$\dot{x}(t)-(C+DF(t)E_3)\dot{x}(t-h)=(A+DF(t)E_1)x(t)+(B+DF(t)E_2)x(t-d(t))$$

Where

$$A=\begin{pmatrix} -2 & 0 \\ 0 & -1 \end{pmatrix},\ B=\begin{pmatrix} -1 & 0 \\ -1 & -1 \end{pmatrix},\ C=\begin{pmatrix} c & 0 \\ 0 & c \end{pmatrix},\ 0\le |c|<1$$

$$E_1=\begin{pmatrix} 1.6 & 0 \\ 0 & 0.05 \end{pmatrix},\ E_2=\begin{pmatrix} 0.1 & 0 \\ 0 & 0.3 \end{pmatrix},\ E_3=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},\ D=I$$

When $c=0.1$, the result can be seen from table 1 , with the incremental of d , $\bar{h}$ regressive .

When $d=0.1$, the result can be seen from table 2 , with the incremental of $|c|$, $\bar{h}$ regressive .

Compared with the results of Q.L.Han, the criteria obtained in our paper are less conservative.

5. Conclusion

In this paper, with the method of Lyapunov and LMI, One sufficient condition for the neutral systems with norm-bounded uncertainties and time-varying delays is derived. Finally compared with some existing results, the criteria obtained in our paper are less conservative.

References

D.Andrea. (2001). Linear Matrix Inequality Conditions for Robustness and Control Design. *International Journal of Robust and Nonlinear Control,* 2001,11(6):541-554.

Q.L.Han. (2005). On Stability of Linear Neutral Systems with Mixed Time Delays: A Discretized Lyapunov Functional Approach. *Automatica*, 2005, 41: 1209-1218.

X.J.Jing, D.L.Tan, Y.C.Wang. (2004). An LMI Approach to Stability of Systems with Severe Time-Delay. *IEEE Trans Automat. Contr.*, 2004, 49(7):1192-1195.

Y.He, M.Wu, J.H.She, G.P.Liu. (2004). Delay Dependent Robust Stability Criteria for Uncertainty Neutral Systems with Mixed Delays. *Systems and control letters,* 2004, 51:57-65.

Table 1. The results when $c=0.1$

d	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Han's	0.80	0.73	0.65	0.57	0.49	0.41	0.33	0.24	0.16	0.07
Theorem1	0.97	0.91	0.87	0.86	0.82	0.79	0.75	0.71	0.65	0.62

Table 2. The results when $d = 0.1$

$\|c\|$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7
Han's	0.92	0.73	0.55	0.41	0.29	0.19	0.11	0.04
Theorem1	1.09	0.91	0.73	0.59	0.47	0.34	0.23	0.11