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A Regionalized Stochastic Rainfall Model for the Generation of High Resolution Data in Peninsular Malaysia

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Abstract

High resolution rainfall data is an important input for studies on hydrological systems. Often time synthetic data has to be generated in the absence of historical data. The stochastic Neyman Scott Rectangular Pulse (NSRP) model has been developed to produce synthetic data of high resolution. To take into account the spatial and temporal nature of rainfall, an extra domain had been added producing the Spatial Temporal NSRP (ST-NSRP) model. However these models require the repeat estimation of all the model parameters for each location. This work develops the Regionalized ST-NSRP model which produces a single model for a region. The regionalization approach is carried out by the station-year method using ten years (2001-2010) of records from sixteen stations. The statistical characteristics used in estimating the model's parameters are hourly and daily coefficient of variation, hourly and daily lag one auto correlation, hourly cross correlation between sites and hourly skewness of rainfall series. Simulation process is conducted at two independent stations with distinctly different rainfall profile. The Regionalized ST-NSRP model produced statistical characteristics of the ungauged sites which matched those of the observed series fairly well even though there is a tendency to underestimate hourly covariance and lag one autocorrelation. Considering that spatial variability of rainfall is high within the studied region, this model is sufficiently robust in producing reasonable synthetic hourly rainfall series which captured the characteristics and pattern of observed rainfall series. Therefore the model has shown a promising potential in generating high resolution synthetic data for any ungauged site within the region.

Keywords: stochastic model, regionalized, spatial temporal, neyman scott model

1. Introduction

1.1 Background

Water is one of the most essential elements for life on earth. Fresh water is mainly sourced from rivers, ponds, lakes, and underground aquifers. Malaysia, being a tropical country has an abundance of rain and is rich in water resources. However management issues and public apathy causes water crisis to be increasingly critical. Globally, growth in population and expansion in urbanization, industrialization and irrigated agriculture are among factors contributing to the increasing demand on water resources, besides contributing to rising water pollution. The managing of hydrological system is therefore crucial in minimizing the negative impacts on water resources.

Rainfall models are important tools in aiding studies of hydrological scheme. The ability to model rainfall process accurately is important for hydrological analysis towards better planning, design and operation of the hydrologic systems. Historical rainfall data are important inputs to formulate and calibrate models that would describe the characteristics of rainfall distribution. Outputs from such models could be useful in studies pertaining to risk of natural hazards and climate change related problems. However, the availability of rainfall data is limited to areas where the rain gauges are successfully and efficiently installed. Hence, hydrologists have to rely on synthetic rainfall series generated using some mathematical models. Some of these models use data from single rain gauge which only caters for the temporal aspect whence rainfall process is known to be extremely variable not only in time but also in space. Henceforth, a more practical approach is to use spatial temporal rainfall models. The magnitude and frequency of precipitation in ungauged sites can be further addressed by incorporating values from other sites which points to a regionalized model.

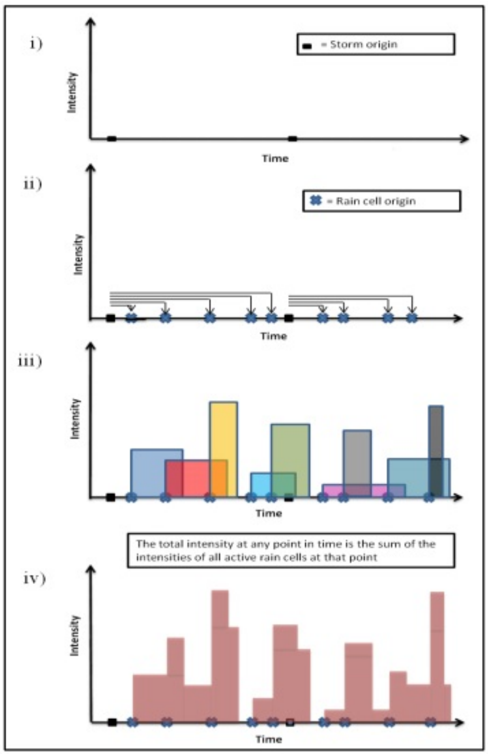

Figure 1. Schematic diagram of the Neyman-Scott model

1.2 Stochastic Rainfall Model

In general, stochastic rainfall models are defined as "empirical" statistical models (Cox and Isham, (1994). These models are statistical models fitted directly to the observed data, and does not include a lot of physics in the process, except for a few simplifying assumptions made in cluster model. At present, there are various works using stochastic rainfall models for the purpose of generating synthetic rainfall series. Two approaches are available; one assesses in continuous time (e.g. Rodriguez et al. (1987); Najem (1988); Westra et al. (2010)) whereas the other uses discrete time (e.g. Gabriel and Neumann (1962); Dawdy and Lettenmaier (1987)). Rainfall process can be described in detail by using the Neyman Scott model (Cowpertwait et.al (1995) and Anne et al. (2004)) or the Barlett-Lewis model (Wheater et. al (2000) and Campo et. al (2012)). The current classes of point process cluster models are Neyman Scott, Bartlet Lewis and hybrid. Both the Neyman Scott and Barlet Lewis cluster models use rectangular pulses for the random precipitation intensity and duration burst. In general, both cluster models have five to seven parameters that typically describe the duration of activity, number of cells, cell depth, duration of cell and cell arrival. Neyman Scott Rectangular Pulse (NSRP) model proposed by Rodriguez et al. (1987) is one of the most advanced approaches to stochastic rainfall simulation currently available. A clustered Neyman Scott model was employed to simulate hourly and daily data at single sites in the U.K. (Cowpertwait et al., 1996a) which used rainfall records of about 20 years in length. Carlo et. al (2006) highlight the usefulness of NSRP model for producing reliable synthetic data that preserved the historical quantile rainfall depth of 1-hour and 24-hour. Continuing the promising study, Carlo et al. (2010) showed the ability of the model in generating consistent synthetic extreme values relative to historical data. Kim and Olivera (2012) also commended on the ability of Neyman Scott model in producing reasonable synthetic data whenever real data is not available. Work on single site and multi-site Neyman-Scott Rectangular Pulse modelling were also carried out by several other researchers such as Bierkens et al. (1990), Fadhilah et al. (2008)), Srikanthan (2009); and Kashid et al. (2010). Muamaraldin and Bauwens (2012) however found single site model fail to capture the characteristics of monthly observed rainfall.

Cowpertwait(1995) extended the NSRP model by adding another domain to represent spatial coverage of rainfall. Cowpertwait (1998) added a third moment function to improve its fitting procedure. Numerous studies were carried out in relation to spatial and temporal distribution in rainfall modeling. Bell and More (2000) emphasized the importance of spatial variability of rainfall specifically with convective rainfall events which tend to vary considerably in the spatial dimension. For urban drainage and early flood forecasting system, a research incorporating spatial and temporal variable was performed at Dalmuir, UK by Marie et al. (2007). The result showed the simulated data at individual sites within the region produced properties that match the observed statistics. The application of spatial temporal model had also been carried out for daily rainfall in a semi-arid area in Iran by Mirshashi et al. (2008). The result indicated rainfall properties are well captured if spatial variable is included in the model construction process. Park and Young (2009) also substantiate the use of spatial temporal model in identifying the different pattern of rainfall occurence based on region.

Sites required for hydrological modeling are sometimes located in areas where rain gauges are not installed. Hence data need to be estimated from neighbouring stations. Several approaches are available in the literature such as using artificial neural network (Dawson et.al, 2006), ordinary kriging (Chokmani and Ouarda, 2004) and a regression-based approach as well as a nearest neighbour based approach (Young, 2006). Another new dimension was also explored in producing synthetic data for ungauged sites using NSRP. Cowpertwait et al. (1996b) extended the NSRP model for regionalization to ungaged sites using regression of harmonic variables on site variables. In a study using 112 sites, the percentage error of predicted values using the regionalized model were found to be less than the sampling error typically found in a 20 year historical record. Norzaida et. al. (2012) used spatial temporal NSRP model with the mixed-exponential distribution representing cell intensity to explore its ability in producing synthetic data in Peninsular Malaysia. Despite the commendable results, a shortcoming of the method is all nine parameters of the ST-NSRP have to be estimated for each different station or region. Hence this study proposes a stochastic Regionalized Spatial Temporal Neyman Scott Rectangular Pulse (RST-NSRP) model which would reduce the model to a single regionalized model. The RST-NSRP model is built upon several parameters to represent the physical characteristics of rainfall process. Following Norzaida (2012) Mixed-Exponential distribution will represent rain cell intensity as it was shown that the statistical and physical properties of observed rainfall series using Mixed-Exponential were more accurate compared to Weibull (Cowpertwait, 2002) in presenting rain cell intensity.

2. Methodology

Figure 1 illustrates the processes involved in the temporal domain of NSRP model. The first time line describes storm origins occur with mean arrival rate, λ following a Poison process while waiting time for each cell origin after the storm occurred is exponentially distributed with parameter β. Each storm origin generates a random number of cell origins. The mean number of cell origin is described by Poisson process with parameter μ_C as shown in the second time line. No cell origin is located at the storm origin. The third time line subsequently shows the physical processes of duration of rain cell and intensity of the pulse. Duration of rain cell is distributed exponentially with parameter η while the distribution of rain cell intensity is not fixed. In this study, intensity of rain cell follows a Mixed-Exponential distribution with three parameters; mean intensity of heavy cell, θ, mean intensity of light cell, ξ, and mixing probability of cell intensity, α. The total rainfall intensity is then the superposition of the effects of these random cell intensities as shown in the fourth time line.

For spatial domain in RST-NSRP model, mean number of cell origin generated by each storm, μ_C is related to mean number of cell per unit area, φ and radius of rain cell, ϕ as follows

$$\mu_C = 2\pi\varphi\phi^{-2} \tag{1}$$

Hence, there are a total nine parameters in RST-NSRP model. The following relations between the values of α, ξ and θ are as given by Fadhilah (2008).

$$\alpha = 0.65, \qquad \xi = 0.25\theta, \qquad \theta = \frac{\mu_1\eta}{0.5125\lambda\mu_C} \tag{2}$$

where μ_1 is mean amount of rainfall at 1-hour timescale. The ST-NSRP model forms the basis of the RST-NSRP model. Table 1 list the parameters involved in model building.

2.1 Study Region

This study involved historical hourly and daily rainfall data from 16 rainfall stations in Peninsular Malaysia. A list of the stations is given in Table 2 and the location is mapped onto Figure 2.

Table 1. The parameters involved in regionalizing ST-NSRP model

Physical process	Distribution	Parameter	PDF
Arrival rate of storm origin	Poisson	λ	$f(x,\lambda)=\frac{e^{-\lambda}\lambda^{x}}{x!}$
Waiting time for each cell origin after the storm occur	Exponential	Β	$f(x)=\beta e^{-\beta x}$
Mean number of cells generated by each storm	Poison	μ_c	$f(x,\mu_c)=\frac{e^{-\mu_c}{\mu_c}^{x}}{x!}$
Duration of cells	Exponential	η	$f(x)=\eta e^{-\eta x}$
Radius of cell	Exponential	ϕ	$f(x)=\phi e^{-\phi x}$
Number of cell per unit area	Poisson	φ	$f(x,\varphi)=\frac{e^{-\varphi}\varphi^{x}}{x!}$
Mean intensity of heavy cell	Mix exponential	θ	$f(x)=\frac{\alpha}{\xi}e^{-\frac{x}{\xi}}+\frac{(1-\alpha)}{\alpha}e^{-\frac{x}{\theta}}$
Mean intensity of light cell		ξ	
Mixing probability of cell intensity		α	

Table 2. List of sixteen rainfall stations used in model construction

Code	Station Number	Station Name	Code	Station Number	Station Name
S1	6019004	Rumah Kastam	S9	5504035	Lahar Ikan Mati
S2	5524001	Kg. La	S10	5210069	STN. Pemeriksaan Hutan Lawin
S3	4726001	Gunung Gagau	S11	4207048	Pejabat JPS Sitiawan
S4	5331048	JPS Kuala Terengganu	S12	3710006	Rumah Pam JPS
S5	4234109	JPS Kemaman	S13	3117070	Ampang
S6	3930012	Sg. Lembing PCCL Mill	S14	2719001	Sikamat
S7	3533102	Rumah Pahang Tua	S15	2025001	Pintu Kawalan Tg. Agas
S8	6103047	Alor Star	S16	1534002	Pusat Kemajuan Per. Pekan Nanas

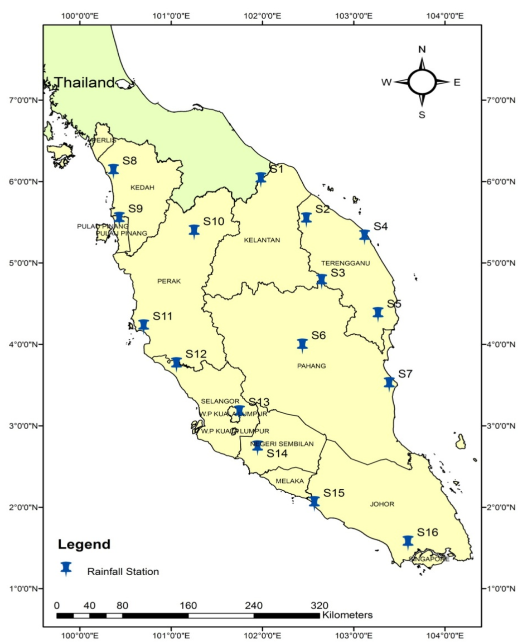

Figure 2. Location of stations used in the study

Rainfall process is non-stationary whilst the ST-NSRP model viewed rainfall process as spatially and temporally stationary. To overcome this discrepancy, dimensionless statistics were calculated based on the merging of rainfall data across the sites and across the years; and scaling the data at each site according to the site mean. The scaling of data also tend to reduce the spatial variability between sites. The regionalization approach is carried out by station-year method by which the *n* years of records from *m* stations are combined and treated as a single station (Ragunath, 2006).

2.2 Parameter Estimation

Parameters are estimated using the method of moment whereby the sum of squared differences between observed and theoretical properties is minimized. The theoretical equations are given in detail by Cowpertwait (1995). The method of moment provides a practical means of estimating parameters with simple computation whereas estimators such as maximum likelihood may result in complicated equations (Smith and Karr (1985). Statistical characteristics of sampled value hourly coefficient of variation ($\hat{v}_1$), hourly lag one auto correlation ($\hat{\rho}_1$), hourly coefficient of skewness ($\hat{\kappa}_1$), daily coefficient of variation ($\hat{v}_{24}$), daily lag one autocorrelation ($\hat{\rho}_{24}$) and hourly cross correlation between sites ($\rho_{x,y}$) were used in estimating the parameters. This method involved minimizing the objective function, F.

$$F = w_1\left(1-\frac{v_1}{\hat{v}_2}\right)^2 + w_2\left(1-\frac{v_{24}}{\hat{v}_{24}}\right)^2 + w_3\left(1-\frac{\rho_1}{\hat{\rho}_2}\right)^2 + w_4\left(1-\frac{\rho_{24}}{\hat{\rho}_{24}}\right)^2 + w_5\left(1-\frac{\kappa_1}{\hat{\kappa}_2}\right)^2 + w_6\sum_{(x,y)\epsilon A}(1-\frac{\rho_{x,y}}{\hat{\rho}_{x,y}})^2$$

Table 3. Estimated parameters of the RST-NSRP model.

Month	λ	β	μ_c	η	ϕ	φ
Jan	0.0202	0.0096	1.0565	2.0457	0.1030	0.0033
Feb	0.0191	0.0054	1.0041	1.6673	0.1671	0.0045
Mar	0.0408	0.0082	1.0686	1.9258	0.1528	0.0040
Apr	0.0475	0.0014	1.1108	1.9876	0.1186	0.0025
May	0.0730	0.0015	1.0085	3.4684	0.1352	0.0029
June	0.0281	0.0012	1.0691	1.0011	0.1443	0.0035
July	0.1032	0.0015	1.0222	3.9329	0.1261	0.0026
Aug	0.0429	0.0010	1.0600	2.9433	0.2566	0.0111
Sept	0.1087	0.0013	1.0017	5.7030	0.1833	0.0054
Oct	0.0818	0.0027	1.0017	2.1421	0.1930	0.0059
Nov	0.0717	0.0012	1.0822	2.5258	0.2117	0.0077
Dec	0.0710	0.0011	1.0058	3.3635	0.1656	0.0044

where w_i, i = 1, 2, …, 6 are flexible weights assigned to handle the domination of items in the objective function. A is the set of ${}^{n}C_2$ pairs of points related to the *n* sites. Hence, the total number of sixteen stations implies ${}^{16}C_2 = 120$ cross correlation between sites per month. The method of moment together with Shuffle Complex Evolution (SCE-UA) optimization technique is used to minimize the objective function. Table 3 shows the estimated parameters of the RST-NSRP model. These parameters are used throughout the whole study. However, parameters of Mixed-Exponential distribution representing rain cell intensity were calculated based on the site mean of selected station to be tested.

3. Results and Discussions

Simulation of hourly rainfall series using the proposed RST-NSRP model was conducted at Ampang station, Selangor and Dabong station, Kelantan. Ampang station is located in an urban area on the west coast region. It receives rain mainly from convective activities. On the other hand, monsoonal rains bring most of the precipitaton for Dabong station which is located in a rural area on the east coast region. Both stations were not included in model construction to ensure unbiasedness in the simulation result. The parameters of Mixed-Exponential distribution representing rain cell intensity for Ampang and Dabong stations are shown in Table 3 and Table 4 respectively. The nine parameters given in Table 2 were used to simulate ten years (2001-2010) of hourly rainfall series for both stations. Comparison between the observed data and the simulated series of RST-NSRP for these two stations is shown in Figures 3 and 4.

Table 4. Estimated parameters of Mixed-Exponential distribution for stations Ampang and Dabong.

	Ampang			Dabong		
Month	θ	α	ξ	θ	α	ξ
Jan	21.47	0.65	5.36	10.12	0.65	2.53
Feb	23.22	0.65	5.80	8.35	0.65	2.08
Mac	26.39	0.65	6.60	11.18	0.65	2.79
Apr	44.55	0.65	11.13	13.22	0.65	3.30
May	40.64	0.65	10.16	10.70	0.65	2.67
June	37.16	0.65	9.29	12.70	0.65	3.17
July	25.58	0.65	6.39	13.57	0.65	3.39
Aug	35.65	0.65	8.91	16.53	0.65	4.13
Sept	29.05	0.65	7.26	14.38	0.65	3.59
Oct	37.22	0.65	9.30	16.54	0.65	4.13
Nov	28.78	0.65	7.19	17.82	0.65	4.45
Dec	24.93	0.65	6.23	6.86	0.65	1.71

The simulated series produced by RST-NSRP model is able to capture the main characteristics such as hourly mean, standard deviation, maximum value and skewness satisfactorily (Figures 3 (a-d) and 4 (a-d). However, it has less ability to capture the hourly autocorrelation, covariance and probability dry period accurately. Nevertheless, it does exhibit the ability to preserve the pattern of those statistics over the different months. For the probability of dry period, the simulation for Ampang station exhibited a poorer fit. This could be due to its location being in the urban area on the west coast which has a very highly variable rainfall pattern. Since the model is regionalized for the whole of the peninsular using rainfall data from several stations for model construction, this could lead to the underestimating or overestimating of lag one autocorrelation and auto covariance as well as probability of dry period for Ampang station. The two stations chosen have very different rainfall profiles; one with mostly high intensity convective rains while the other with long duration low intensity rains. Subsequently, the lag one autocorrelation and auto covariance for these rainfall events is different. Nevertheless the results are quite satisfactory especially since the rainfall model is able to preserve the statistical profiles. The results verified the potential of the proposed model to simulate rainfall data at sites with different rainfall profiles.

A comparison is also done on the performance of the regionalized model against the spatial-temporal model. The main difference between the existing and the proposed models is in the parameter estimation procedure whereby for the ST-NSRP, all the nine parameters have to be estimated at each new site whereas for the RST-NSRP model, only three parameters of the mixed exponential model need to be estimated for each new site. Root mean squared error of the median statistics produced by each model against that of the observed actual values at each tested site was calculated and given in Table 5. To compare the performance of the proposed regionalized model against the existing spatial-temporal model, the percentage of difference of the two RMSE is given in the fourth and seventh columns respectively. These errors range between 0.03 to 3.35 percent which are relatively small. It can thus be concluded that despite using a single model for generating synthetic data at different sites, the performance of the regionalized model is still comparable to the spatial-temporal model. Hence, the proposed regionalized model is more efficient, would be able to avoid the tedious work of analyzing data and estimating parameters repeatedly, and can produce commendable synthetic data for any ungauged site within the region.

4. Conclusion

Considering the fact that spatial variability of tropical rainfall is high within the studied region, RST-NSRP model has performed reasonably well in capturing the pattern of rainfall process. The regionalized approach allows the model to generate rainfall series in a large region using a single model. This enables a single set of parameters to be used in generating data for any ungauged site within the peninsular. The model also incorporate spatial correlation between sites which enhances the ability of the model to simulate data at any site even if the sites are not involved in model construction. The RST-NSRP model has also shown comparable performance in relation to the ST-NSRP model. Hence the ease in computational procedure and simplicity of Figure 3(a-g): Comparison of statistical characteristics between simulated hourly series (box plots) and observed series (dotted line) for station Ampang, Selangor (2001-2010).

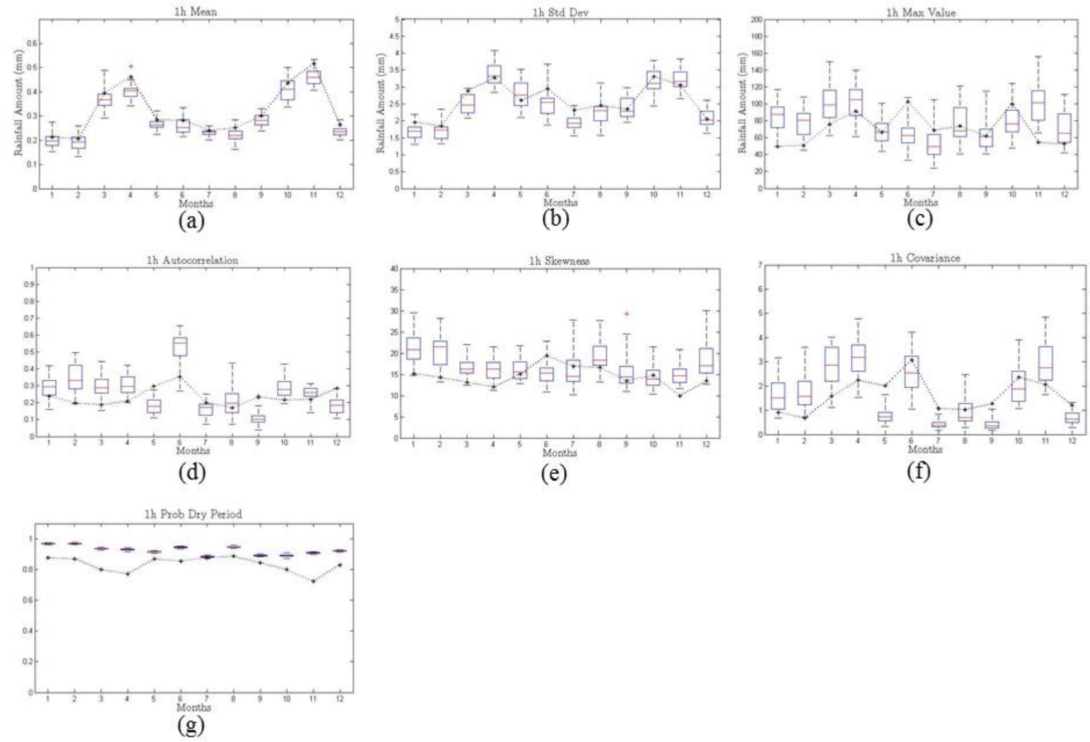

Figure 3(a-g). Comparison of statistical characteristics between simulated hourly series (box plots) and observed series (dotted line) for station Ampang, Selangor (2001-2010)

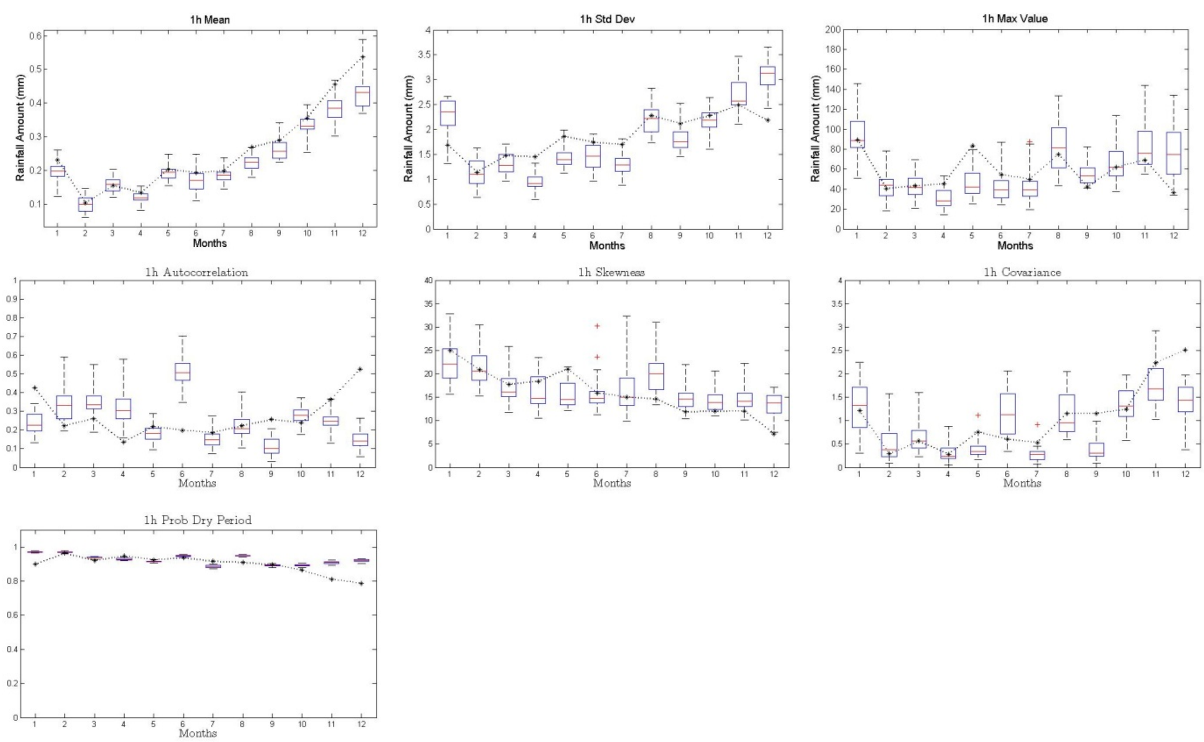

Figure 4(a-g). Comparison of statistical characteristics between simulated hourly series (boxplots) and observed series (dotted line) for station Dabong, Kelantan (2001-2010)

Table 5. The RMSE and percentage difference between RST-NSRP and ST-NSRP simulation results at tested stations Ampang and Dabong

	Ampang			Dabong		
	RST-NSRP	ST-NSRP	% difference	RST-NSRP	ST-NSRP	% difference
1h Mean	0.0320	0.0125	1.5600	0.0427	0.0098	3.3571
1h Std Dev	0.4833	0.2380	1.0307	1.2559	0.9873	0.2721
1h Skewness	3.8226	2.4027	0.5910	3.6182	4.3877	0.1754
1h Max Value	25.6409	16.4794	0.5559	18.3352	21.8971	0.1627
1h Covariance	0.8226	0.4585	0.7941	0.5928	0.1756	2.3759
1h Autocorrelation	0.1026	0.0456	1.2500	0.1737	0.0977	0.7779
1h Prob Dry	0.1042	0.1005	0.0368	0.0555	0.0242	1.2934

The RST-NSRP model justifies the implementation of the regionalized model as a synthetic data generator for any ungauged site within the peninsular region

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