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Some Sufficient Conditions for Oscillation of Symmetric Cellular Neural Networks with Delay

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Abstract

In this paper, some sufficient conditions for oscillation of symmetric cellular neural networks with delay (DCNNs) are introduced. These conditions impose constrains on the system of dynamic state equations when the cells are working in linear region $\dot{x}(t) - Hx(t) - A_1 x(t-\tau) = 0$, where the coefficients H and A_1 are real 2×2 matrix $\mathbf{R}^{2\times 2}$, $\tau \in \mathbf{R}^{+}$, and $x(t), \dot{x}(t) \in \mathbf{R}^{2\times 1}$ are the state vector and its derivative at time t.

Keywords: Cellular neural networks with delay (DCNNs), Oscillation

1. Introduction

Cellular neural networks (CNNs) were introduced by (Chua, L O et al, 1988, p. 1257-1272; Chua, L O et al, 1988, 1273-1290); they found important applications in pattern recognition and signal processing, especially in statistic image treatment. The stability of CCNs has been investigated in (Chua, L O et al, 1990, p. 1520-1527; Zou, F et al, 1991, p. 38:675-677; Savaci, F A et al, 1992, p. 240 – 245). Processing of moving images requires the introduction of a delay in the signal transmitted among the cells, which led to introducing delayed cellular neural networks (DCNNs) by (Chua, L O et al, 1990, p. 12-25; Chua, L O, 1992, p. 449-459) and the consideration of their dynamic behavior in (Roska, T et al, 1992, p. 487-490). The most important uses of DCNNs (signal and image processing, non-linear/transcendental equation solving and so on) rely on stability. The stability of DCCNs has been investigated in many papers (Liao, T L et al, 1999, p. 1347-1349; Arik, S et al, 2000, p. 571-574; Liao, T L et al; 2000, p. 1481-1484; Takahshi, N N, 2000, p. 793-799; Arik, S, 2002, p. 1211-1214; Arik, S, 2003, p. 156-160). However, this work presents some sufficient conditions for oscillation of symmetric cellular neural networks with delay (DCNNs) for the dynamic state equations.

2. Cellular neural networks with delay

A CNN is a massively parallel computing architecture made of simple processing elements (cells) which are locally connected. A cell is the basic circuit unit containing linear and nonlinear circuit elements, which are linear resistors, linear capacitors, linear and nonlinear controlled sources and independent sources. Any cell in a CNN is connected only to its neighbor cells. Due to the propagations effects on continuous time dynamics of CCNs, the cells that not directly connected to each other can affect each other indirectly (Chua, L O et al, 1988, p. 985- 988). These properties of the CCNs are similar to the cellular automata.

In this paper we refer to Civalleri and Gilli study on some stability properties of DCNNs with consideration to reciprocal and non-reciprocal of the networks. They showed that a symmetric DCNN can become unstable if the delay is suitably chosen and they present a sufficient condition to assure the complete stability (Civalleri, P P et al, 1992, p. 94 -99; Civalleri, P P, 1993, p. 157-165). This paper gives new sufficient conditions for oscillation of symmetric cellular neural networks with a delay τ. These conditions impose constrain on the system of dynamic state equations, $x(t)$, when the cells are working in linear region.

Now, suppose the cells of the DCNNs are working in linear region, i.e.: (in matrix notation)

$$\dot{x}(t) - Hx(t) - A_1 x(t-\tau) = 0, \tag{1}$$

for a symmetric DCNN with a delay τ , which can oscillate, where

$$x(t)=\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} \quad , \quad H=\begin{pmatrix} h & a_{12}^0 \\ a_{12}^0 & h \end{pmatrix} \quad , \quad A_1=\begin{pmatrix} a_{11}^1 & a_{12}^1 \\ a_{12}^1 & a_{11}^1 \end{pmatrix}, \text{ and } \; h=a_{11}^0-1>0 \,,$$

where H and A_1 are coefficients of real 2×2 matrix $\mathbf{R}^{2\times 2}$, τ is non-negative real number.

The dynamic equations of a (2 $\times$1)-DCNN (cells are arranged in 2rows and 1 column) for equation (1) are the following systems of equations:

$$\frac{dx_1(t)}{dt}=-x_1(t)+a_{11}^0x_1(t)+a_{12}^0x_2(t)+a_{11}^1x_1(t-\tau)+a_{12}^1x_2(t-\tau)\,,$$

$$\frac{dx_2(t)}{dt}=-x_2(t)+a_{12}^0x_1(t)+a_{11}^0x_2(t)+a_{12}^1x_1(t-\tau)+a_{11}^1x_2(t-\tau)\,.$$

In this paper we will use two definitions for the systems of the differential equations $x_1(t)$ and $x_2(t)$ to define the theory of oscillation of solutions.

Definition 1: A solution $x(t)=\left[x_1(t),x_2(t),\ldots\ldots,x_m(t)\right]^T$ is said to oscillate if every component $x_j(t)$ of then solution has arbitrary large zeros. Otherwise the solution is called non-oscillatory.

Definition 2: A solution $x(t)=\left[x_1(t),x_2(t),\ldots\ldots,x_m(t)\right]^T$ is said to oscillate if it is eventually trivial or if at least one component does not have eventually constant signum. Otherwise the solution is called non-oscillatory.

We will use definition (2) to obtain the sufficient conditions for the oscillation of all solutions. However, as we will prove in theorem (2) for the system of differential equations, if all solutions of the system oscillate with respect to one of the two definitions (1) and (2), then they also oscillate with respect to the other.

We have found two explicit sufficient conditions for the oscillations of the linear autonomous delay symmetric DCNN. The results are based on the work of Ferreira and Györi in (Ferreira, José M et al, 1987, p. 332-346). We utilize the logarithmic norm of $\mu(A_1)=\max_{\|u\|=1}(Hu,u)$ and $\mu(H)=\max_{\|u\|=1}(A_1u,u)$ of the matrices H and A_1 to obtain explicit conditions for the oscillation and for the non-oscillation of all solutions of equation (1).

Theorem 1: Assume that $H, A_1\in\mathbf{R}^{2\times 2}$ and $\tau\in\mathbf{R}^+$, then, the following statements are equivalent.

A). every solution of equation (1) oscillates componentwise.

B). the characteristic equation $\det(\lambda I-H-A_1e^{-\lambda\tau})=0$ has no real roots, where I is 2×2 identity matrix.

Theorem 2: Assume that $H, A_1\in\mathbf{R}^{2\times 2}$, $\tau\in\mathbf{R}^+$ and $\mu(H)\le 0$, $\mu(A_1)\le 0$.

Then each of the following two conditions is sufficient for the oscillations of all solutions of equation (1):

A). $$-\mu(A_1)\tau>\frac{1}{e}\,, \tag{2}$$

B). $$\sqrt{\mu(H)\cdot\mu(A_1)}>\frac{1}{2e\tau}\,. \tag{3}$$

Lemma 1: Assume that $H, A_1\in\mathbf{R}^{2\times 2}$ and $\tau>0$ are such that

$$\mu(H)+\mu(A_1)e^{-\gamma\tau}<0\,, \text{ for } \gamma\in\mathbf{R}^+ \,, \tag{4}$$

and

$$\inf_{\gamma<0}\left[\frac{1}{\gamma}\mu(H)+\frac{1}{\gamma}\mu(A_1)e^{-\gamma\tau}\right]>1\,. \tag{5}$$

Then every solution of equation (1) oscillates.

Proof: Assume, for the sake of contradiction, that equation (1) has a non-oscillatory solution. Then, by theorem (1), the characteristic equation $\det(\lambda I-H-A_1e^{-\lambda\tau})=0$ has a real root λ_0 . But, then there exists a vector $u\in\mathbf{R}^2$ with $\|u\|=1$ such that $(\lambda_0I-H-A_1e^{-\lambda_0\tau})u=0$, hence,

$\lambda_0 = \left(Hu + A_1 e^{-\lambda_0 \tau} u, u\right) = \left(Hu + A_1 u, u\right) e^{-\lambda_0 \tau} \le \mu(H) + \mu(A_1) e^{-\lambda_0 \tau}$, and so, by equation (4), $\lambda_0 < 0$, and $1 \ge \frac{1}{\lambda_0}\left(\mu(H) + \mu(A_1) e^{-\lambda_0 \tau}\right)$. This contradicts equation (5) and completes the proof.

Proof of Theorem 1: We apply lemma (1). As $\mu(H) \le 0$ and $\mu(A_1) \le 0$, equation (4) is satisfied and so it suffices to establish equation (5). First, assume that equation (2) holds. Then, by using the inequality $e^x \ge ex$, we see that for all $\gamma < 0$,

$$\frac{1}{\gamma}\left[\mu(H) + \mu(A_1) e^{-\gamma \tau}\right] \ge \frac{1}{\gamma}\left[\mu(H) + \mu(A_1) e(-\gamma \tau)\right] = e\left[-\mu(H) + \mu(A_1)\tau\right] \cdot$$

From this and equation (2), it follows that equation (5) holds. Next, assume that equation (3) hold. Then, by using the arithmetic mean-geometric mean inequality, we see that for all $\gamma < 0$,

$$\begin{aligned} \frac{1}{\gamma}\left[\mu(H) + \mu(A_1) e^{-\gamma \tau}\right] &= -\frac{1}{\gamma}\left[-\mu(H) - \mu(A_1) e^{-\gamma \tau}\right] \\ &\ge -\frac{2}{\gamma}\sqrt{-\mu(H)\mu(A_1) e^{-\gamma \tau}} \\ &= -\frac{2}{\gamma}\sqrt{-\mu(H)\mu(A_1)} \cdot e^{-\frac{\gamma \tau}{2}} \\ &\ge -\frac{2}{\gamma}\sqrt{\left[-\mu(H)\mu(A_1)\right]} \cdot e\left(-\frac{\gamma \tau}{2}\right) \\ &= e\sqrt{-\mu(H)\mu(A_1)} \cdot \tau , \end{aligned}$$

from this and equation (3), it follows that equation (5) holds. The proof is complete.

3. Conclusion

This paper successfully established some new sufficient conditions for oscillation of symmetric cellular neural networks with delay. These conditions impose constrains on the system of dynamic state equations when the cells are working in linear region $\dot{x}(t) - Hx(t) - A_1 x(t-\tau) = 0$.

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