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Nonoscillation of First-order Neutral Difference Equation

Jianqiang Jia (Corresponding author)
Department of Science, Yanshan University
438 West of He Bei Avenue, Qinhuangdao 066004, China
E-mail: wyy_1246@sina.com.cn

Xiaozhu Zhong, Xiaohui Gong, Rui Ouyang & Hongqiang Han
Department of Science, Yanshan University
438 West of He Bei Avenue, Qinhuangdao 066004, China

Abstract

The oscillation of the first order neutral difference equation $\Delta[x(n)-px(n-\tau)]+qx(n-\sigma)=0$ is studied in this paper, where $p>0$ or $p<0$, q is a positive constant, σ is a non-negative integer, τ is a positive integer. The sufficient conditions for nonoscillation of the equation is obtained by suitable inequality and characteristic equation.

Keywords: Difference equation, Neutral, Nonoscillation

1. Introduction

Qualitative behavior of solutions of difference equations has received considerable interest recently. In [1] the oscillation of the first order neutral difference equation

$$\Delta[x(n)-px(n-\tau)]+qx(n-\sigma)=0\,, \tag{1}$$

was considered and some oscillation criterias were given, q is a positive constant, σ is a non-negative integer, τ is a positive integer. In this paper nonoscillation of the solutions of the equation (1) are studied, where $p>0$ or $p<0$ and Δ is the forward difference, i.e., $\Delta x_n = x_{n+1}-x_n$.

A solution of equation (1) is called oscillatory, if it is neither finally positive nor negative. Otherwise it is called nonoscillatory.

2. main result

Lemma 1 [2] A necessary and sufficient condition for all solutions of equation (1) to oscillate is that the characteristic equation

$$F(\lambda)=(\lambda-1)\lambda^{\sigma-\tau}(\lambda^{\tau}-p)+q=0 \tag{2}$$

has no positive real root.

Theorem 1 Assume that $p<0.q$ is a positive constant, σ is a non-negative integer and τ is a positive integer, all solutions of equation (1) are nonoscillatory.

Proof: Since $F(\lambda)=(\lambda-1)\lambda^{\sigma-\tau}(\lambda^{\tau}-p)+q=0$ and $q<0$, the equation (2) possibly has roots on interval $(1,\infty)$. Obviously $F(1)=q<0$ .From $p<0$ we can get

$$F(\lambda)=(\lambda-1)\lambda^{\sigma-\tau}\left(\lambda^{\tau}-p\right)+q=(\lambda-1)\left(\lambda^{\sigma}-\frac{p}{\lambda^{\tau-\sigma}}\right)+q>(\lambda-1)\lambda^{\sigma}+q$$

Assume that $G(\lambda)=(\lambda-1)\lambda^{\sigma}+q$,the values of $G(\lambda)$ increase on interval $(1,\infty)$ .Obviously when $\lambda\to\infty$, $G(\lambda)\to\infty$ .So there is a positive constant N ,such that $G(N)>0$ .From $F(\lambda)>G(\lambda)$ we can get $F(N)>0$ .So $F(N)F(1)<0$,and $F(\lambda)$ is continuous on interval $[1,N]$,we can get there is at least a point ζ on interval $(1,N)$ such that $F(\zeta)=0$. Therefore the equation (2) has roots on $(1,\infty)$, so all solutions of equation (1) are nonoscillatory.

Theorem 2 Assume that $p>0.q$ is a positive constant, σ is a non-negative integer and τ is a positive integer, all solutions of equation (1) are nonoscillatory.

Proof: When $1>p>0$, Obviously the equation (2) has positive roots only on $\lambda \in (1,\infty)\cup\left(0, p^{\frac{1}{\tau}}\right)$. $F(1)=q<0$.

When $\sigma-\tau \ge 0$, the values of $F(\lambda)$ increase on interval $(1,\infty)$. Obviously when $\lambda\to\infty$, we can get $F(\lambda)\to\infty$.So there is a positive constant M_1, such that $F(M_1)>0$.So $F(M_1)F(1)<0$,and $F(\lambda)$ is continuous on interval $[1, M_1]$, we can get there is at least a point ξ_1 on $(1, M_1)$ such that $F(\xi_1)=0$. Therefore the equation (2) has a root at least on interval $(1,\infty)$.So the equation (2) has a root at least on interval $(1,\infty)\cup\left(0, p^{\frac{1}{\tau}}\right)$ and all solutions of equation (1) are nonoscillatory.

When $\sigma-\tau<0$, we can get

$$F(\lambda)=(\lambda-1)\lambda^{\sigma-\tau}\left(\lambda^{\tau}-p\right)+q=(\lambda-1)\left(\lambda^{\sigma}-\frac{p}{\lambda^{\tau-\sigma}}\right)+q$$

Obviously when $\lambda\to\infty$, we have $\frac{p}{\lambda^{\tau-\sigma}}\to 0$ and $F(\lambda)\to\infty$. So there is a positive constant M_2 such that $F(M_2)>0$.So $F(M_2)F(1)<0$, and $F(\lambda)$ is continuous on interval $[1, M_2]$, we can get there is at least a point ξ_2 on interval $(1, M_2)$ such that $F(\xi_2)=0$. Therefore the equation (2) has a root at least on interval $(1,\infty)$.So the equation (2) has a root at least on interval $(1,\infty)\cup\left(0, p^{\frac{1}{\tau}}\right)$ and all solutions of equation (1) are nonoscillatory.

When $p=1$, obviously the equation (2) has positive roots only on interval $(1,\infty)\cup(0,1)$.The process of the proof is similar with above-mentioned. We can get the equation (2) has a root at least on interval $(1,\infty)\cup(0,1)$.So all solutions of equation (1) are nonoscillatory.

When $p>1$, obviously the equation (2) has positive roots only on interval $\left(p^{\frac{1}{\tau}},\infty\right)\cup(0,1)$.The process of the proof is similar with above-mentioned. We can get the equation (2) has a root at least on interval $\left(p^{\frac{1}{\tau}},\infty\right)\cup(0,1)$.So all solutions of equation (1) are nonoscillatory.

3. Examples

Example 1. Consider difference equation $\Delta(x_n+2x_{n-3})-4x_{n-2}=0$ where $p=-2$, $\tau=3$, $\sigma=2$, $q=-4$.

So the conditions in theorem 1 are satisfied and the characteristic equation is

$$F(\lambda)=(\lambda-1)\lambda^{-1}\left(\lambda^3+2\right)-4=0 \tag{3}$$

From the figure 1 we can see the equation (2) has a real root. Therefore all solutions of equation (1) are nonoscillatory.

Example 2. Consider difference equation $\Delta\left(x_n-\frac{1}{2}x_{n-1}\right)-2x_{n-3}=0$ where $p=\frac{1}{2}$, $\tau=1$, $\sigma=3$, $q=-2$.

So the conditions in theorem 2 are satisfied and the characteristic equation is

$$F(\lambda)=(\lambda-1)\lambda^2\left(\lambda-\frac{1}{2}\right)-2=0 \tag{4}$$

From the figure 2 we can see the equation (2) has a real root. Therefore all solutions of equation (1) are nonoscillatory.

Example 3. Consider difference equation $\Delta\left(x_n - \frac{1}{2}x_{n-3}\right) - 2x_{n-1} = 0$, where $p = \frac{1}{2}, \tau = 3$, $\sigma = 1, q = -2$.

So the conditions in theorem 2 are satisfied and the characteristic equation is

$$F(\lambda) = (\lambda - 1)\lambda^{-2}\left(\lambda^3 - \frac{1}{2}\right) - 2 = 0 \tag{5}$$

From the figure 3 we can see the equation (2) has no real root. Therefore all solutions of equation (1) are oscillatory.

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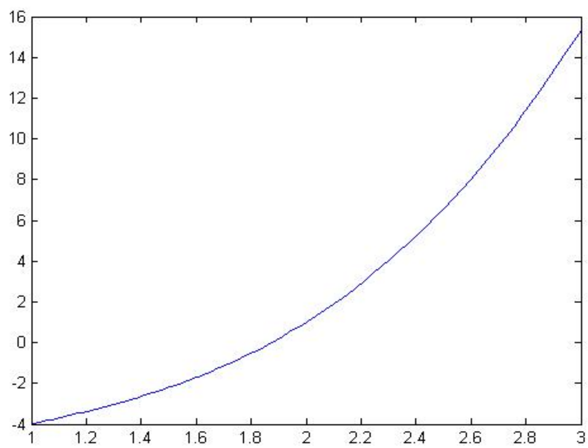

Figure 1. The figure of (3) on interval（-4,16）

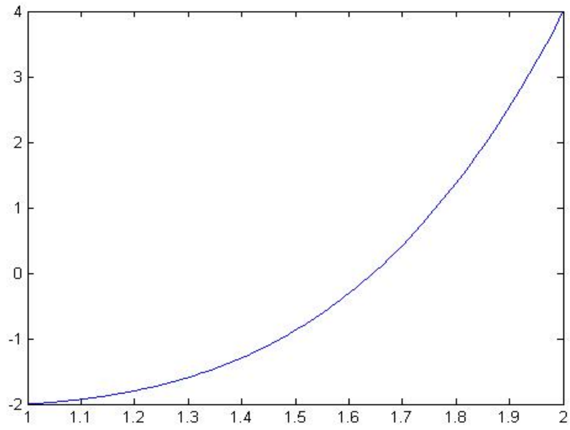

Figure 2. The figure of (4) on interval $(-2,4)$

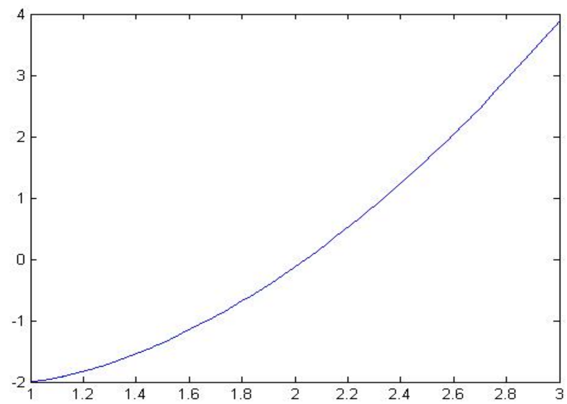

Figure 3. The figure of (5) on interval $(-2,4)$