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The Wehrl's Entorpy of Schrödinger-cat States

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Abstract

The Wehrl's entropy of the Schrödinger-cat states has been calculated numerically in this paper. In particular, we show the dependence of the Wehrl's entropy on the two parameters of the Schrödinger-cat states, which act as, respectively, the photon number parameter α and the phase difference ϕ. The Wehrl's entropy has been found to be in the range $1 \le W < 1.7$, and its changes with respect to α, ϕ are not homogenized. Furthermore, we obtain both the dependence of the Wehrl's entropy on the average photon number and the relation between the Wehrl's entropy and the statistical entropy.

Keywords: Wehrl's entropy, Schrödinger-cat states, Statistical entropy

1. Introduction

Entropy is a crucial concept in the thermodynamics, statistical mechanics and information theory. Its sovereign role regarding the behavior of macroscopic systems was recognized about one century ago by Clausius, Kelvin, Maxwell, Boltzmann and many others. Traditionally, entropy is derived from thermodynamically phenomenological consideration based upon the second law of thermodynamics. Entropy relates macroscopic and microscopic aspects of nature and determines the behavior of macroscopic systems.

In quantum statistical mechanics entropy is defined by von Neumann in terms of the density operator as (E.T.Jaynes R.D.Levine and M.Tribus, 1978).

$$S = -\mathrm{Tr}\rho\ln\rho \tag{1.1}$$

where ρ is density operator.

However, the quantum entropy has the problem that it gives the zero value for all pure states, whether or not they are classical or non-classical. Wehrl introduced a new definition of entropy for radiation filed in terms of coherent states by (A Wehrl, 1979)

$$W = -\frac{1}{\pi}\int Q(\beta)\ln Q(\beta)d^2\beta \tag{1.2 a}$$

where

$$Q(\beta) = \langle\beta|\rho|\beta\rangle \tag{1.2 b}$$

are the diagonal elements of the density operator in coherent states $|\beta\rangle$, subject to the constraint

$$\int Q(\beta)d^2\beta = \pi \tag{1.2 c}$$

We recognized that $Q(\beta)$ is exactly the distribution function corresponding to the antinormally ordering of operators, and it is called the Q representation. The Wehrl entropy has bellow characteristics:

(1) The Q representation is always non-negative.

(2) $Q(\beta)$ is a bounded function lying within unit. Thus $Q(\beta)$ serves as a standard distribution function since $0 \le Q(\beta) \le 1$.

(3) As a c-number form, it is calculated much more easily than the quantum entropy**.**

The different pure states may be distinguished in terms of the Wehrl entropy. It takes its minimum value $W = 1$ for coherent states. This may be used as a reference to measure the degree of squeezed states (C.T.Lee, 1988).

Qrlowski (Arkadiusz Orlowski, 1993) discussed the Wehrl's entropy of the two-photon coherent states (squeezed states), photon-number states, ideal laser light, chaotic (thermal) radiation, and displaced photon-number states. They are fond to displace the same shape respect to the average photon number $\langle N\rangle$**.** Then Qrlowski pointed out that the Wehrl's entropy can be considered as a good measure of the strength of the coherent component and there is a possibility of using the Wehrl's entropy as a reasonable classification of quantum states.

The difficulty with the quantum entropy and the Wehrl's entropy lies in the fact that they remain immeasurable at present. Actually, neither eigenvalues of a density operator or its Q representation have yet been found to be measurable by present quantum optical technique, although some possible methods have proposed (L.Knoll and A.Orlowski, 1995). However, the statistical entropy (Qiao, Gu, 2003) can be measured by means of the photon counting data.

A question may be put as follows: whether Wehrl's entropy of Schrödinger-cat states displace the same shape respect to the average photon number $\langle N\rangle$ as (Arkadiusz Orlowski, 1993)?whether statistical entropy of the Schrödinger-cat states can replace the Wehrl's entropy to describe the coherence of such states (Qiao, Gu, 2003)? To answer these questions, there are three main works in the present paper.

Firstly, we calculate the Wehrl's entropy of the Schrödinger-cat states.

Secondly, we discuss the dependence of the Wehrl's entropy of the Schrödinger-cat states on the average photon number.

Thirdly, we discuss the relation of the Wehrl's entropy and the statistical entropy.

2. The Wehrl's Entropy of the Schrödinger-cat states

2.1 Calculation of the Wehrl's Entropy of the Schrödinger-cat states

The Schrödinger-cat states have the explicit definition: it is a quantum superposition of two macroscopically distinguishable states. The Schrödinger cat states can be formulated in different ways [6]. One of them is expressed by

$$|\alpha_c\rangle = \frac{1}{\sqrt{A}}[|\alpha_1\rangle + |\alpha_2\rangle] \tag{2.1 a}$$

where $|\alpha_1\rangle$ and $|\alpha_2\rangle$ are coherent states, which have the same amplitude $|\alpha|$ but different phases, that is

$$\alpha_1 = |\alpha|e^{i\phi_1}, \quad \alpha_2 = |\alpha|e^{i\phi_2} \tag{2.1 b}$$

The normalization constant A is determined by $\langle\alpha_c|\alpha_c\rangle = 1$

For simplicity, we take $\phi_1 = 0$ and $\phi_2 = \phi$, so ϕ acts as a phase difference

We can get that

$$A = 2 + 2\exp(-|\alpha|^2 + |\alpha|^2\cos\phi)\cos(|\alpha|^2\sin\phi) \tag{2.2}$$

Now we calculate the Wehrl's entropy for density operator

$$\rho = |\alpha_c\rangle\langle\alpha_c| \tag{2.3}$$

For this purpose, we first write the distribution function in terms of $\alpha = r\exp(i\theta)$.

From (1.2), a straightforward and strenuous calculation gives

$$W = -\frac{1}{\pi}\int Q(\alpha)\ln Q(\alpha)d^2\alpha \tag{2.4}$$

$Q(\alpha) = \frac{1}{A}(B + C + D);$

where

$B = \exp(-r^2 - \alpha^2 + 2r\alpha\cos\theta)$

$C = 2\exp(E)\cos(r\alpha F)$

$D = \exp(-r^2 - \alpha^2 + 2r\alpha\cos(\theta - \phi))$

$E = -r^2 - \alpha^2 + r\alpha\cos\theta + r\alpha\cos(\theta - \phi)$

$F = \sin(\theta - \phi) - \sin\theta$

From (2.4), we know that the Wehrl' entropy depends on the parameters α and ϕ .In the present case, the Wehrl' entropy cannot be calculate analytically, but it can be evaluated numerically.

2.2 Discussing the results

From Fig.1to Fig.5, we can get many information concerning the Wehrl's entropy of the Schrödinger-cat states.

(1) The values of the Wehrl's entropy of the Schrödinger-cat states have a limit: $1 \le W < 1.7$.

(2) The wehrl's entropy with respect to α and ϕ have different characters.

3. Dependence of the Wehrl's Entropy on the Average Photon Number

The average photon number is given generally by

$$N = \langle a^+ a \rangle = \mathrm{tr}(\rho a^+ a) \tag{3.1}$$

Substituting the (2.3) into (3.1) and calculating the (3.1) we get :

$$N = \frac{1}{A}2|\alpha|^2[1 + \exp(-|\alpha|^2 + |\alpha|^2\cos\phi)\cos\Delta)] \tag{3.2}$$

where

$\Delta = \cos(\phi + |\alpha|^2\sin\phi)$

On the basis of (3.2), we can calculate numerically the Wehrl's entropy as a function of the average photon number, and the result is shown in Fig.6.

From Fig.5, one can see that the shape of wehrl'entropy with respect to *N* is similar to that in Ref. (Arkadiusz Orlowski, 1993).

4. The Statistical Entropy and The Wehrl's Entropy

We know that Wehrl entropy remains immeasurable at present. However, we can discuss the relation between the statistical entropy and Wehrl's entropy.

Some researchers (V.Perinova,J.Krepelka and J.Perina, 1986) have proposed to use the entropy

$$G = -\sum_{n=0}^{\infty} p(n)\ln p(n) \tag{4.1}$$

as an alternative measurement of entropy for the radiation field on account of the measurability of photon number distribution

$$p(n) = \langle n|\rho|n\rangle . \tag{4.2}$$

Calculating $p(n)$ of the cat states defined by (2.1), we get that:

$$p(n) = \frac{1}{A}\frac{\exp(-|\alpha|^2)|\alpha|^{2n}}{n!}(2 + 2\cos n\phi) \tag{4.3}$$

Substituting (4.3) into (4.1), we can get the statistical entropy of the cat states. The numerical result is shown in Fig7.

We can distinctly conclude that wehrl's entropy and the statistical entropy is the one-to-one corresponding relation. So cat states can be can described by statistical entropy as wehrl' entropy do.

5. Brief summary

We get the dependence of Wehrl's entropy of cat states on two parameters α,ϕ. Wehrl's entropy has a limit ($1 \le W < 1.7$) and the density of Wehrl's entropy respect to α,ϕ are not homogenized.

The Wehrl's entropy can classify cat states. Both the average photon number and the statistical entropy are one to one relation to Wehrl's entropy, so Wehrl' entropy can replace to both the average photon number and the statistical entropy to describe the cat states.

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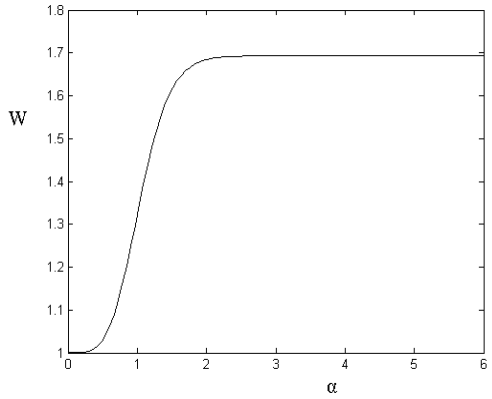

Figure 1. Wehrl's entropy with respect to α, where $\phi = \pi$.

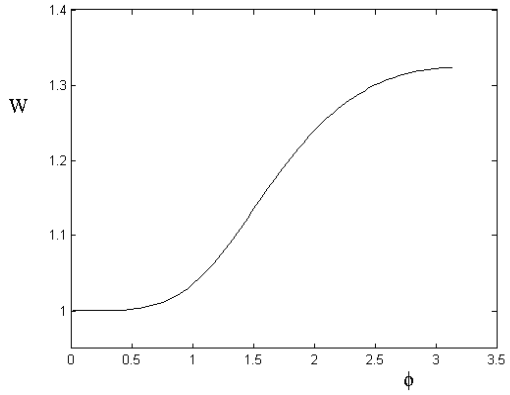

Figure 2. Wehrl's entropy with respect to ϕ, where $\alpha = 1$.

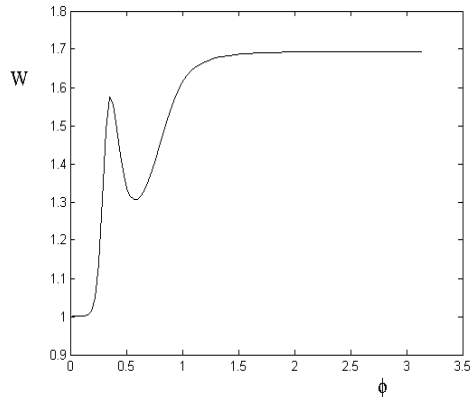

Figure 3. Wehrl's entropy with respect to ϕ ,where $\alpha = 3$.

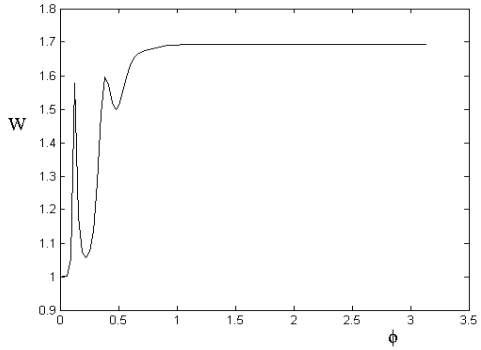

Figure 4. wehrl's entropy respect to ϕ , where $\alpha = 5$.

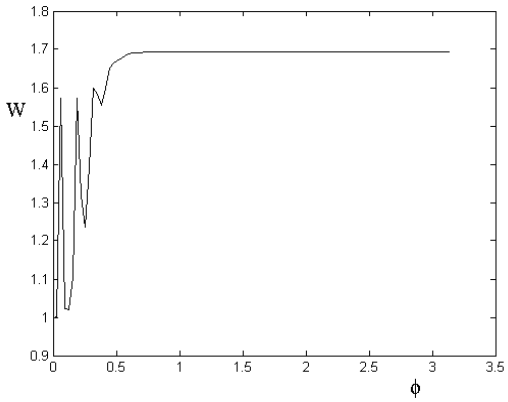

Figure 5. Wehrl's entropy respect to ϕ , where $\alpha = 7$.

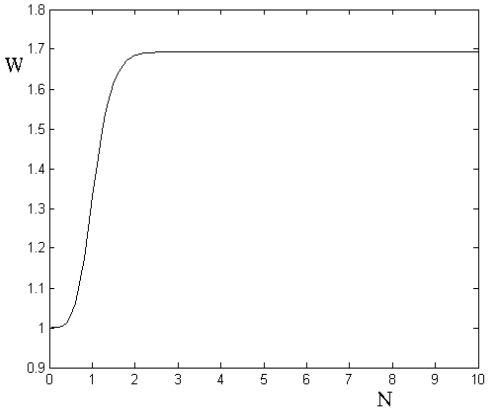

Figure 6. Wehrl's entropy with respect to *N*.

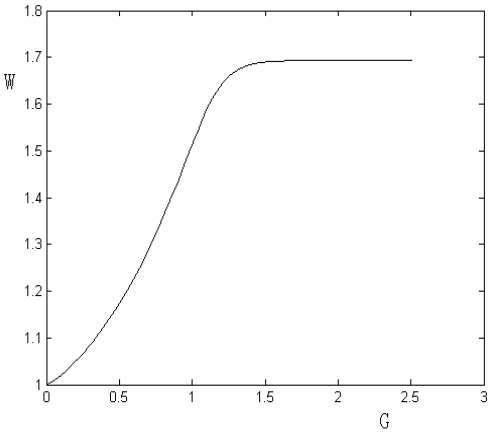

Figure 7. The relation between the Wehrl's entropy and the statistical entropy.