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R/S Analysis with Computer Algebra

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Abstract

In this paper, Hurst exponent is applied to study Shenzhen Stock component (SSC), which shows that the market is a chaotic system and Hurst exponent is valuable in stock market. The program package presented can be used in calculating R/S .

Keywords: Chaos, Hurst exponent, R/S

1. Introduction

Generally, one of the most commonly used indicators to study the fractal and chaos theory is the Hurst exponent based on the analysis of rescaled range. Hurst discovered that instead of following Brown motion and Gauss distribution, the most of nature phenomena, including river levels, temperatures, rainfalls, and sunspots do follow biased random walk. Hurst extended Einstein's work on Brownian motion that the distance that a random particle covers increases with the square root of time used to measure it:

$$R = T^{0.5} \tag{1}$$

Where R is the distance covered, T a time index. He found that the following was a more general form of equation (1):

$$(R/S)_N = (bN)^H \tag{2}$$

Where R/S is rescaled range, b is a constant, N is time index of a time series $\{x_t\}$ $(t = 1,2,\cdots,N)$, H is called as Hurst exponent. Equivalently, in logarithm, equation (2) becomes

$$\log(R/S) = H(\ln N + \ln b) \tag{3}$$

R/S analysis is a nonparametric method raised by Hurst studying ample empirical analysis. The procedure for calculating R/S is as follow:

Step1: For a given time series $\{x_t\}$ of length M , divide this time period into A contiguous subperiods of length N , such that $AN = M$. Label each subperiod I_a , with $a = 1,2,\cdots,A$. Each element in I_a is labeled $x_{k,a}$ such that $k = 1,2,\cdots,N$. For each I_a of length N , the average value is defined as:

$$e_a = \frac{1}{N}\sum_{k=1}^{N} x_{k,a}$$

Where e_a is average value of the M contained in subperiod I_a of length N .

Step2: The time series of accumulated departures $y_{k,a}$ from the mean value for each subperiod I_a is defined as:

$$y_{k,a} = \sum_{i=1}^{k}(x_{i,a} - e_a)\ (k = 1,2,\cdots,N)$$

Step3: The range is defined as the maximum minus the minimum value of $y_{k,a}$ within each subperiod I_a :

$$R_a = \max_{1\le k\le N}\{y_{k,a}\} - \min_{1\le k\le N}\{y_{k,a}\}$$

Step4: The sample standard deviation calculated for each subperiod I_a :

$$S_a = [\frac{1}{N-1}\sum_{k=1}^{N}(x_{k,a} - e_a)^2]^{\frac{1}{2}}$$

Step5: Each range, R_a, is now normalized by dividing by the S_a corresponding to it. Therefore, the rescaled range for each I_a subperiod is equal to R_a/S_a. From step1 above, we get A contiguous subperiods of length N. Therefore, the average R/S value for length N is defined as:

$$(R/S)_N = \frac{1}{A}\sum_{a=1}^{A}(R_a/S_a)$$

We can now apply equation (3) by performing an ordinary least squares regression on $\ln(N)$ as the independent variable and $\ln(R/S)_N$ as the dependent variable. The slope of the equation is the estimate of the Hurst exponent H.

Simple discussions are given as follows: If $0 \le H < 0.5$, the time series is anti-persistent. If $H = 0.5$, the time series is standard Brownian motion. If $0.5 < H \le 1$, the time series is persistent. Let the measure of the length of the cycle V be $V_N = \frac{(R/S)_N}{\sqrt{N}}$.

2. Design ideas

With applied computer algebra system Mathematica, we get some program packages as follows:

Hrra [ndata_,num_]:the average R/S value for each length N.

Hurstzh [ndata_]:the estimate of the Hurst exponent.

Where the variable ndata is the original data, the variable num is the length N for each I_a.

And the module hurstzh [ndata_] contains the module hrra[ndata_,num_].

3. Results

In this study, total 2392 Shenzhen stock components from 1992 to 2000 are used. From figure 2, we can easily observe the sequence data has a strong linear correlation. In order to eliminate the linearly dependence, applying $AR(1)$ regression to the component, we have

$$y_t = x_t - (a + bx_{t-1})$$

Where a and b is the coefficient of the $AR(1)$ regression model.

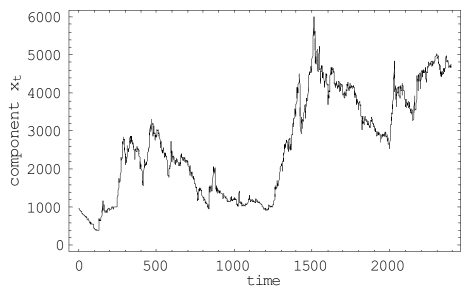

Figure1. Stock component original time series diagram

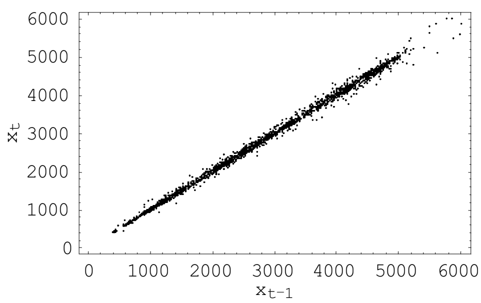

Figure 2. $x_{t-1} - x_t$ Phase diagram

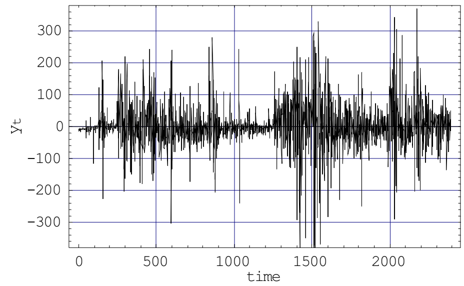

Figure 3. The sequence data after adjusting component to eliminate trend

According to the above theory, we get $y_t = x_t - (4.42191 + 0.998925 x_{t-1})$. And $F = 9.41441 > F_{0.05}(1,2377) = 3.84$ shows that there exists the dependency significant between x_t and x_{t-1} (figure 2). However, $DW = 1.93$ shows that the series $\{y_t\}$ is not clear dependency, that is to say, the linearly dependency of stock component series has been removed. Thus, we get ideal results by R/S analysis method.

We obtain the values $(R/S)_N$ and N (table 1) to be carried out fitting. Hence we get the fitted curve: $f(x) = -0.520362 + 0.659262x$, thus, Hurst exponent $H = 0.659262 > 0.5$ indicates that Shenzhen stock market has distinct fractal characteristics.

Table 1. The value $(R/S)_N$ and V_N of Shenzhen stock component series

N	$(R/S)_N$	V_N	N	$(R/S)_N$	V_N	N	$(R/S)_N$	V_N
2	0.707107	0.50000	24	5.12472	1.04608	108	13.1557	1.26591
3	1.10077	0.635527	27	5.5742	1.07285	132	15.7062	1.36705
4	1.42804	0.714022	33	6.09506	1.06101	198	19.5645	1.39039
6	1.95637	0.798834	36	6.55034	1.09172	216	20.7775	1.41373
8	2..43735	0.861734	44	7.40883	1.11692	264	24.9546	1.53585
9	2.65252	0.884173	54	8.43224	1.14748	297	23.3234	1.35336
11	3.0429	0.917468	66	9.78581	1.20455	396	29.0291	1.45877
12	3.22352	0.930551	72	10.4268	1.22881	594	33.6387	1.38021
18	4.28143	1.00914	88	11.9419	1.27301	792	46.8194	1.66365
22	4.81813	1.02723	99	12.3931	1.2355	1188	59.1688	1.71666

As shown in table 1, it is easily observed that the average orbital period of the system is 264 trade days (It can be judged by the Hurst exponent declining for the first time). To a certain extent, the objective of the state continued existence and the long-term memory cycle of Shenzhen stock market (SSMS) can be verified through this result.

4. Conclusion

Hurst exponent $H = 0.659262 > 0.5$ shows the fractal characteristics of SSMS. The calculations of Hurst exponent shows that the average orbital period of SSC is about 264 days(approximately 53 weeks), which shows SSMS has state persistence and long-term memory, and the memory cycle is 53 weeks, which means Hurst exponent is valuable in the investment in stock market. This is broadly consistent with the conclusion of some literature on SSMS. Furthermore, this conclusion confirmed the objective existence of the fractal characteristics of SSMS.

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