



A New Variant of ARFIMA¹ Process and Its Predictive Ability

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Abstract

ARFIMA models generated an enormous amount of interest in the literature about three decades ago. However, this interest waned after Granger (1999) showed that an ARFIMA process might have stochastic properties that do not mimic the properties of the data at all. The empirical results of our research in which we used exchange rate data for the analysis, show that a variant of an ARFIMA process indeed can beat the ARFIMA, the Random Walk and the ARMA process of the order one in out of sample forecasting. This indirectly indicates that our variant of the ARFIMA process can be considered as the data generating process for the long memory time series.

Keywords: Forecast evaluation, A new variant of ARFIMA process

1. Introduction

The search for a model which can outperform random walk in out of sample forecasting was started about two decades ago in two important areas of study: volatility modelling and purchasing power parity (PPP) hypothesis. In volatility modelling, it has been recognized that the simple random walk can outperform many sophisticated volatility models in out of sample forecasting, while in purchasing power parity research, the existence of mean reverting behaviour in exchange rates has not been established convincingly yet. In research on mean reversion behaviour, the most significant negative results were obtained by Meese and Rogoff (1983a, b). They evaluated the predictive ability of a series of linear structural exchange rate models and found that none was able to consistently outperform a simple random walk for all the known exchange rates and horizons. This seemingly robust result was then put into dispute when Mark and Sul (2002), Rapach and Wohar (2002), and Faust, Rogers and Wright (2003) obtained some evidence of linear structural models outperforming random walk models. Recent work done by Taylor, Sarno, Clarida and Valente (2003) using nonlinear models show the promising positive result that there are structural models which can outperform random walk models in out of sample forecasting, or to put it differently, there is mean reverting behaviour in exchange rate.

In this paper, we offer another structural model based on a long memory process, which can outperform the random walk soundly. Our approach is by adding an explanatory variable into the ARFIMA long memory model. This explanatory variable is chosen based on the research finding of Taylor, Sarno, Clarida and Valente (2003) that there is nonlinearity in exchange rates. Our dependent variable is made up of two components: the intrinsic y component and its logarithmic counterpart. By using this transformation, we are able to use this compound dependent variable to capture the nonlinear behaviour of the exchange rate. This is because $\ln y$ is a nonlinear function. Equivalently, we can view the $\ln y$ component as an explanatory variable just as the first lag of y is acting as an explanatory variable in a first order autoregressive model.

The rest of the paper is organized as follows. In section 2, we briefly review ARFIMA processes and outline the empirical estimation methodology used in the rest of the paper. Section 3 describes how we construct our YQ-ARFIMA model. Section 4 discusses three predictive accuracy techniques, while in section 5, we outline the predictive model selection procedures used in the research. In section 6, we present the empirical analysis. Section 7 presents one application of our model and we conclude this paper in section 8.

2. The ARFIMA Long Memory Process

There are basically two experimental evidence, which show that a long memory process is very useful in long horizon forecasting. However, there is also one experimental finding that a long memory process is not suitable for forecasting. The two experimental evidences are as follows:

Going back to the 1960s, experience of nonparametric spectral estimation for many economic time series has suggested very marked peakedness around the zero frequency. This essentially suggests that the maximum likelihood of events happening will be at a low frequency. Moreover, a low frequency component is closely related to the long run dynamics of the process. However, at frequency $\lambda = 0$, we can have two conditions: one, the spectral density function is bounded and the other one is unbounded. In a long memory process, an ARFIMA model would have its spectral density function unbounded at frequency $\lambda = 0$, making it more suitable for long horizon forecasting. In terms of autocorrelation, ARFIMA models show characteristics of hyperbolic autocorrelation decay patterns when modelling economic and financial time series. Despite these positive evidences, one piece of negative evidence has emerged around 1999, when Granger acknowledges that an ARFIMA model might have stochastic properties that essentially do not mimic the properties of the data at all. With this experimental evidence in mind, we shall now present the prototypical ARFIMA model examined in the literature.

The standard ARFIMA model is shown below,

$$B(L)(1-L)^d y_t = A(L)u_t \quad (1)$$

where $B(L) = 1 - B_1 L - \dots - B_p L^p$ and $A(L) = 1 + A_1 L + \dots + A_q L^q$, d is the fractional differencing parameter or rather the memory parameter, and u_t is the white noise. This process is covariance stationary if $-0.5 < d < 0.5$, with mean reversion when $d < 1$. The lag polynomials shown in equation (1) can be easily expanded to reveal the importance of the hyperbolic decay property of ARFIMA. This is shown in equation (2)

$$(1-L)^d = 1 - dL + \frac{d(d-1)}{2!} L^2 - \frac{d(d-1)(d-2)}{3!} L^3 + \dots = \sum_{j=0}^{\infty} g_j(d) \quad (2)$$

2.1 Long Memory Model Estimation

Long memory model estimation essentially boils down to the estimation of the memory parameter d , which describes the characteristics of the autocorrelation of the series. There are a few estimation techniques for the value of d , notably the semi-parametric estimation procedure of Geweke and Porter-Hudak(1983), and Robinson(1995), modified rescaled range estimator of Lo(1991), and the exact local Whittle estimator of Shimotsu and Phillips (2004). We focus only on the GPH estimator (Geweke and Porter-Hudak) since its computation is easier and its range of errors is acceptable when it is used for comparison purposes.

The GPH estimation is basically a two step procedure. It begins with the estimation of d , which is based on the log-periodogram regression. It is given by equation (3)

$$\ln[I(\lambda_j)] = \beta_0 + \beta_1 \ln \left[4 \sin^2 \left(\frac{\lambda_j}{2} \right) \right] + u_j \quad (3)$$

where $\lambda_j = \frac{2\pi j}{T}$, $j = 1, 2, \dots, m$ and λ_j represents the $m = \sqrt{T}$ Fourier frequencies.

$I(\lambda_j)$ which denotes the sample periodogram is defined as follows:

$$I(\lambda_j) = \frac{1}{2\pi T} \left| \sum_{t=1}^T y_t e^{i\lambda_j t} \right|^2 \quad (4)$$

The critical assumption for this GPH estimator is that the spectrum of the ARFIMA(p, d, q) process is the same as that of an ARFIMA(0, d , 0) process. Robinson (1995a) shows the following asymptotic result,

$$\left(\frac{\pi^2}{24m} \right)^{-1/2} (\hat{d}_{GPH} - d) \rightarrow N(0, 1) \quad (5)$$

for $-0.5 < d < 0.5$ and $j = 1, \dots, m$ in the equation for λ_j above. Equation (5) is due to Robinson, and essentially

implies that for d between -0.5 and 0.5, the long memory process is stationary and invertible. If the value of d is outside this range, Robinson suggested that we difference the series until d is within the specified range for stationarity and invertibility.

3. YQ (Note 2)-ARFIMA Model

Recently, intense interest has been growing in nonlinear modelling especially in modelling exchange rates. This is largely due to the positive result obtained through nonlinear modelling; for example, a research finding by Taylor, Sarno, Clarida and Valente (2003), shows that exchange rate has nonlinear characteristics. By constructing a three regime switching intercept heteroskedastic VECM model, they are able to show that their model can beat the random walk. We follow the direction used by this piece of research, that is, we try to capture the nonlinearity of the exchange rate by incorporating another component into the dependent variable of the standard ARFIMA model which is given by equation (1). We assume that exchange rate is constructed by an additive method with a linear component added to a nonlinear component. The nonlinear component is generated by a nonlinear transformation of the linear component. Thus, we have the YQ-ARFIMA model given by

$$B(L)(1-L)^d z_t = A(L)u_t, \quad u_t \sim iid(0, \sigma_t^2) \quad (6)$$

$$z_t = y_t - \Phi(L) \ln y_t \quad (7)$$

$$B(L)(1-L)^d \Phi(L) = c \quad (8)$$

where $B(z) = 0$ has roots inside the lag polynomials and c is a constant.

3.1 Theoretical Analysis of the YQ-ARFIMA Model

We are interested in two special cases of the general model set up as in equations (6), (7) and (8). They are:

Combining equation (6), (7) and (8), and after recognizing that $B(L)$ has an inverse, we obtain the following equation

$$B(L)(1-L)^d y_t = A(L)u_t + c \ln y_t \quad (9)$$

When $\Phi(L) = 1$, equation (6) would become

$$B(L)(1-L)^d (y_t - \ln y_t) = A(L)u_t, \quad (10)$$

$$y_t = \ln y_t + [B(L)(1-L)^d]^{-1} A(L)u_t \quad (11)$$

For the first case, we use mainly equation (9) for empirical analysis. Notice that in equation (9), we can treat $\ln y_t$ as an explanatory variable for the standard ARFIMA. We can use Hermite polynomials to verify equation (10).

A special mention must be made of equation (11), which reveals very clearly that the forecast values are accurate because this equation (11) essentially means regressing y_t on $\ln y_t$. We know that $y_t \approx \ln y_t$ when y_t is small after expanding it using the Taylor expansion. This simply means that the R squared value is close to unity, which in turn implies that it is a very good forecasting model.

4. Predictive Ability Evaluation Techniques

We shall use three evaluation techniques for evaluating the predictive ability of YQ-ARFIMA, ARFIMA, RW and ARMA. The first one is by using root mean square error (RMSE). It is essentially similar to mean square prediction error (MSPE). The difference is that we do not assume the loss function to be quadratic in nature.

4.1 Ratio G of RMSE Measure

We compare the predictive ability of two models by defining the ratio of their respective RMSE measure, where the RMSE measure of the benchmark model 0 is the denominator and the RMSE of the model 1 is the numerator. Thus, we define the following:

$$G = \frac{RMSE \text{ Model 1}}{RMSE \text{ Model 0}} \quad (12)$$

If the ratio $G > 1$, this means that model 0 is a better forecasting model than model 1 by G times. On the other hand, if $G < 1$, then model 1 is a better forecasting model than model 0. One word of caution, this ratio G can be taken as a rough guide only because we have not established its distributional properties yet. To fulfil this deficiency, we shall present a simple but accepted to be good enough model evaluation technique for non-nested models in the next subsection.

4.2 The Diebold & Mariano (DM) Statistic (1995)

We use the simple version of the DM statistical test, that is, the Sign Test. This simple test is used because we have obtained an affirmative result from the ratio of RMSE test, and by this test, we have extended the result from the ratio of RMSE test for the sample to that of the population.

Define the loss differential d as follows:

$$d_t = [g(e_{it}) - g(e_{jt})] \quad (13)$$

We have made the assumption that the loss function is a direct function of the forecast error and that the absolute value of the forecast error is a direct function of the forecast error itself. Thus, we have the simple case where

$$\begin{aligned} g(e_{it}) &= |e_{it}| \\ g(e_{jt}) &= |e_{jt}| \end{aligned} \quad (14)$$

We further assume that the loss differentials are iid, which simply means that the models must be non-nested, and that the number of positive loss differential observations in a sample of size T has the binomial distribution with parameters T and 0.5 under the null hypothesis that the two models have similar forecasting abilities against the alternative hypothesis that the two models have different forecasting abilities. With that, we have the following results:

For small samples, we have the sign-test statistic as given by

$$S_a = \sum_{t=1}^T I_+(d_t) \quad (15)$$

where

$$\begin{aligned} I_+(d_t) &= 1 \quad \text{if } d_t > 0 \\ &= 0 \quad \text{otherwise} \end{aligned} \quad (16)$$

Using a table of the cumulative binomial distribution, we may obtain and assess the significance of the test. However, if the sample size is large, we use the studentized t test to approximate the distribution. The studentized statistic is given by

$$S_b = \frac{S_a - 0.5T}{\sqrt{0.25T}} \sim N(0,1) \quad (17)$$

One word of caution: because we have assumed that the loss differential series is iid, the above test can only be applied to non-nested models. Otherwise, the limiting distribution will be non standard. However, if we use a Newey-West (1987) type estimator for the DM test, then the tabulated critical values are quite close to those for the $N(0,1)$. Moreover, the non-standard limit distribution is reasonably approximated by a standard normal in many contexts (see McCracken (1999) for tabulated critical values). Furthermore, this DM test is suitable for evaluating the predictive ability of non-nested models only.

4.3 The Clark and McCracken Encompassing Test (CM statistic)(2001)

This CM statistic is designed for comparing nested models. This CM test is conducted for the purpose of further confirming the empirical results obtained by using the ratio of RMSE measure and the DM test, as the latter two tests have not eliminated the chances that the two models concerned may be nested. This test has the same null hypotheses as the DM test, except that the alternative is model 1 can outperform model 0. Thus, CM statistics is given by:

$$CMt = \sqrt{(P-1)} \frac{\bar{u}}{[P^{-1} \sum_{t=R}^{T-1} (u_{t+h} - \bar{u})^2]^{1/2}} \quad (18)$$

where $u_{t+h} = e_{0,t+h}(e_{0,t+h} - e_{1,t+h})$ and $\bar{u} = P^{-1} \sum_{t=R}^{T-1} u_{t+h}$, and P is the prediction period. The limiting distribution for $h > 1$ is non-standard, but if we use the Newey-West type estimator the tabulated critical values are quite close to those for the $N(0,1)$.

5. Predictive Model Selection

We use forecast horizons of 5 steps, 20 steps, 60 steps and 240 steps to correspond to 1 week, 1 month, 3 months and 1 year, respectively. We first split the sample of size T into two equal halves. The first half is used to produce 0.25T recursive (and rolling) predictions. The other 0.25T observations are used as the initial sample for estimating the parameters for the next step of the predictions. To put it differently, parameters are updated before each new prediction is constructed. These predictions are then used to obtain the best YQ-ARFIMA, ARFIMA, RW and ARMA by comparing the out of sample root mean square forecast errors.

After selecting the best YQ-ARFIMA, ARFIMA, RW and ARMA models, we fix the respective specifications for the ratio of RMSE test, DM test and CM test for the evaluation of predictive ability. Table 1 shows the respective specifications selected for the various best models. (See Table 1)

6. The Empirical Analysis

In this empirical analysis, we have conducted two experiments: one, we determined the values of the RMSE measure for 15 exchange rates around the globe and two, we determine the RMSE measure, the DM statistics and the CM statistics for 6 exchange rates specially selected to represent different parts of the globe. For the first experiment, we conducted only point forecasts. However, for the second experiment, we performed point forecasts as well as interval forecasts. The results of the first experiment are recorded in Table 2 and the results from the second experiment are recorded in Table 3 to Table 13.

6.1 Analysis of the first experimental results

This first experiment was aimed at testing the robustness of the YQ-ARFIMA with regard to different exchange rates around the globe. In this experiment, we performed only the recursive forecasts and the forecast horizon is 8 periods only. Moreover, the sample size is fixed at 1000. Column 6 of Table 2 shows that there are 6 exchange rates and 9 exchange rates where the YQ-ARFIMA model is better than RW in out of sample forecasting on the average about 49 times and 5 times, respectively. As for the YQ-ARFIMA model versus ARFIMA model, the former is about 45 times and 6 times better than the latter for same 6 exchange rates, and for the other 9 exchange rates, the YQ-ARFIMA model is better about 6 times. With regard to RW model versus ARFIMA model, the former is better than the later for 9 out of the 15 exchange rates.

Thus, on the whole, the YQ-ARFIMA model can outperform the RW model and ARFIMA model in out of sample forecasting soundly. In addition RW model can outperform ARFIMA model in 9 out of the 15 exchange rates. However, we have used the ratio of the RMSE measure for comparison which is accurate for the samples concerned. These spectacular results may not be valid for population in general as we have not accounted for the distributional properties of the ratio of the RMSE measure. (See Table 2)

6.2 Analysis of the second experimental results

In this experiment, we have conducted three investigations: one, we determined the robustness of the first experimental results with regard to variation in sample sizes and forecast horizons, two, we tested these results by using the Diebold and Mariano statistical test and three, we tested the results again by using the Clark and McCracken test. This last test is necessary in order to ensure that the nestedness of the two models will not affect our final results.

6.2.1 Comparison of the ratio of RMSE measures

With respect to sample size, in column 3 of Tables 3, 4, 5 and 6, the YQ-ARFIMA model beat random walk soundly for sample sizes 500, 1000, 2000 and 5000, except that in the case of the recursive scheme for sample size 1000, it lost to the RW model for the case of the British pound at a forecast horizon of 20 periods only, and for sample size 2000 for case of the Euro dollar also at a forecast horizon 20 periods. However, for the case of rolling scheme, the YQ-ARFIMA model performs much better. It lost to RW only for one case, i.e., the British pound at forecast horizon 240 periods. In general, the YQ-ARFIMA performs better at the short horizons of 5 periods and 20 periods, both for the recursive and the rolling schemes of forecast.

For the case of the YQ-ARFIMA model versus the ARFIMA model as in column 4 of Tables 3, 4, 5 and 6, a similar situation arises, where the ARFIMA model beat the YQ-ARFIMA model only for case of the British pound, sample size 1000 at a forecast horizon of 60 periods, and for case of the Singapore dollar, sample size 2000 at a forecast horizon of 240 periods.

For the case of the YQ-ARFIMA model versus the ARMA model as shown in column 5 of Tables 3, 4, 5, and 6, the former is better than the latter on the average by more than 10 times. Only at three instances for the case of recursive forecast, was the ARMA model is better than the YQ-ARFIMA model that is, for the British pound with sample size 500 at a forecast horizon of 20 periods; with sample size 2000 at a forecast horizon of 240 periods, and for the Euro dollar with sample size 2000 at a forecast horizon of 240 periods. For the case of rolling forecast, there are two instances where the ARMA model is better than the YQ-ARFIMA model, that is, for the British pound with sample size 2000 at a forecast horizon of 240 periods, and for the Malaysian ringgit with sample size 2000 at a forecast horizon of 5 periods.

As for the RW model versus the ARFIMA model, the former can still beat the latter marginally. However, for sample size 2000, the two models are equal on the average for recursive forecast but for rolling forecast, the RW model lose to the ARFIMA model. (See Table 3, 4, 5 and 6)

6.2.2 Evaluation of predictive ability- DM statistic

For the YQ-ARFIMA model versus the RW model, the DM statistics are all negative with absolute values more than 2 for sample sizes 500, 1000, 2000 and 5000, and for horizons 5 periods, 20 periods, 60 periods and 240 periods. This simply means that the null hypothesis of equal models is rejected and that the loss differential for the YQ-ARFIMA

model is smaller than the RW model, which implies that the YQ-ARFIMA is a better forecasting model than the RW model in out of sample forecasting.

As for the YQ-ARFIMA model versus the ARFIMA model, we obtain a similar result except that with sample size 5000 at a forecast horizon 20 periods, the null hypothesis is accepted. As for the ARFIMA model versus the ARMA model, we find that the ARMA model is better than the ARFIMA model for about 26 instances, whereas the ARFIMA model is better the ARMA model for about 32 instances. (See Table 7, 8, 9, and 10)

6.2.3 Evaluation of predictive ability- CM statistic

It must be noted that the DM statistic is intended to be applied to non-nested models. What if the two models are nested? In that case, we have to use the Clark and McCracken statistic (CM statistic). As the main objective of this paper is to prove that the YQ-ARFIMA model can beat the random walk model, we shall conduct this CM test only for the YQ-ARFIMA model versus the RW model for the selected 6 exchange rates for sample sizes 500, 1000, 2000, 5000 and at forecast horizons 5 periods, 20 periods, 60 periods and 240 periods. The results are recorded in Table 11, which shows that the YQ-ARFIMA model beat the random walk (RW) model rather convincingly. (See Table 11)

7. One application of the YQ-ARFIMA model

Since our model can outperform many other existing models in out of sample forecasting, there would be many uses for the YQ-ARFIMA model. In this section, we shall present one application of the YQ-ARFIMA model. For the past two decades, many research papers have been investigating the mean reversion behaviour of exchange rates. Until now, this controversy still remains to be settled convincingly, especially with regard to the exchange rate after the breakdown of the Bretton Woods System in 1973. Many tools for testing the mean reverting behaviour have used the notably more powerful unit root test, panel data analysis and fractional cointegration. However, no consensus has been reached on the mean reversion property of exchange rates. Since our model can beat the random walk model soundly, it can be used to show this mean reverting behaviour. The idea is as follows: If the YQ-ARFIMA model can beat the random walk model for one particular exchange rate, then this exchange rate cannot show 100 % persistency since the random walk is not the data generating process of the exchange rate. If not fully persistent, this implies that the exchange rate has some degree of stationarity, or to put it differently, there is mean reverting behaviour for this set of data. We test this idea by using the exchange rate British pound per unit US dollar (UKEX)

UKEX has been shown to exhibit mean reverting behaviour by using two centuries of exchange rate data (see Lothian, James and M. Taylor, 1996). We shall show the same result by using data after the breakdown of the Bretton Woods System. We test whether the YQ- ARFIMA model can beat the random walk model in out of sample forecasting by using the said data set. If a positive result is obtained, we shall conclude that indeed, the British pound per unit US dollar exhibits mean reverting behaviour.

Table 13(a) and Table 13(b) show the experimental results. We select randomly five samples from the UKEX exchange rate series for testing. These five samples are of sizes 259, 530, 780, 1038 and 1230. It is very clear from the experimental figures shown in Table 13(a) and Tale 13(b), that the YQ-ARFIMA model outperforms the random walk model soundly. Thus, indeed, UKEX exhibits mean reverting behaviour. (SeeTables 12(a) and (b))

8. Conclusion

We have shown convincingly that the YQ-ARFIMA model can beat the random walk model in out of sample forecasting. We have used the ratio of RMSE measure, DM test and CM test to verify the above result. However, we have not used a Newey and West type of estimator for the variance used in these two tests. We think that it is not necessary because the ratio of RMSE measure is very large indeed for it to be invalid in statistical testing.

As for the uses for our model, we have shown how to use it to test the mean reverting behaviour of exchange rates. With this new tool, we hope to put to rest the controversy of mean reversion and its existing testing tools. With this accurate model, we can devise more accurate volatility models with a long memory or a short memory data generating process. We also can use the YQ-ARFIMA model to devise an early warning system for currency attack.

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Notes

Note 1. ARFIMA stands for autoregressive fractional integrated moving average process

Note 2. YQ-ARFIMA stands for the new variant of ARFIMA. Y denotes Yip and Q denotes Quah

Note 3. The 15 sets of exchange rate data are obtained from the Federal Reserve Bank of US.

Table 1. Best specifications for YQ-ARFIMA, ARFIMA, RW and ARMA with respect to the AUDS, UKPD, CAND, SIND, MALR and EURO

Models	AUSD (Australian Dollar)	UKPD (British Pound)	CAN (Canadian Dollar)	SIND (Singapore Dollar)	MALR (Malaysian Ringgit)	EURO (Euro Dollar)
YQ ARFIMA	2,[-0.070],1	2,[0.208],1	2,[-0.053],1	1,[0.094],1	1,[0.493],0	2,[-0.112],1
ARFIMA	1,[0.065],1	1,[0.035],1	1,[0.473],1	2,[0.209],1	1,[0.111],1	1,[0.039],0
RW	0,1,0	0,1,0	0,1,0	0,1,0	0,1,0	0,1,0
ARMA	1,0,1	1,0,1	1,0,1	1,0,1	1,0,1	1,0,1

Note: The first, second and third values in each cell denote autoregressive parameter, difference parameter and moving average parameter. RW denotes random walk while ARMA is the autoregressive moving average process.

Table 2. Comparing the 8 periods forecasting ability of ARFIMA, YQ-ARFIMA and Random Walk (RW) in terms of RMSE values for all the 15 exchange rate series.

Name of the exchange rate (Note 1)	ARFIMA (P)	YQ-ARFIMA (Q)	R.W (R)	R/P	R/Q	P/Q
Aus dollar	0.0031	0.00006	0.0032	1.05	48.53	46.17
Mal ringgit	0.0016	0.00005	0.0027	1.67	51.61	30.91
Thai bath	0.2946	0.00293	0.2923	0.99	99.75	100.52
Sin dollar	0.0085	0.00034	0.0082	0.98	24.07	24.58
Jap yen	0.3792	0.02358	0.4016	1.06	17.03	16.08
UK pound	0.0048	0.00089	0.0037	0.78	4.14	5.32
Euro Eur	0.0062	0.00113	0.0071	1.13	6.24	5.50
China yuan	0.0112	0.00133	0.0107	0.95	7.96	8.38
HK dollar	0.0231	0.00307	0.0186	0.81	6.07	7.51
S.Afri Rand	0.1628	0.03376	0.1638	1.01	4.85	4.82
Den Kronor	0.0689	0.02062	0.0487	0.71	2.36	3.35
Swiss Franc	0.0125	0.00193	0.0085	0.68	4.43	6.46
Can dollar	0.0092	0.00017	0.0094	1.02	56.37	55.38
Mexi Pesos	0.1620	0.01831	0.1473	0.91	8.05	8.85
Brazil Real	0.1209	0.01302	0.0586	0.48	4.49	9.28

Note: RMSE denotes root mean square error, Aus-Australia, Mal- Malaysia, Thai-Thailand, Sin-Singapore, Jap-Japan, UK-United Kingdom, Eur-Europe, HK-Hong Kong, S.Afri-South Africa, Den-Denmark, Can-Canadian, And Mexi-Mexico

Note 1. These 15 exchange rate data are obtained from Federal Reserve Bank of US.

Table 3. Comparison ratio of RMSE measure for sample size 500 with recursive forecast, and rolling forecast in bracket

Exrate	h	R/Q	F/Q	M/Q	F/R	M/R
AUD\$ (Australian Dollar)	5	16.9(13.3)	21.5(16.4)	22.1(16.8)	1.28(1.23)	1.31(1.27)
	20	22.7(11.5)	34.4(16.6)	34.8(17.3)	1.51(1.44)	1.53(1.51)
	60	108(9.49)	53.4(16.3)	52.7(17.1)	0.49(1.72)	0.48(1.79)
	240	33.6(90.3)	30.7(107)	30.6(108)	0.91(1.18)	0.91(1.19)
CAD\$ (Canadian Dollar)	5	28.1(32.2)	31.4(27.1)	23.7(26.5)	1.11(0.84)	0.84(0.82)
	20	22.1(19.2)	31.9(25.5)	32.7(26.8)	1.44(1.32)	1.48(1.39)
	60	30.0(21.7)	55.9(34.3)	74.2(49.2)	1.47(1.58)	1.95(2.26)
	240	31.0(18.1)	61.0(26.6)	2.08(31.4)	1.96(1.46)	2.08(1.73)
GBP£ (British Pound)	5	4.42(5.16)	4.59(7.19)	4.25(5.59)	1.04(1.38)	0.96(1.08)
	20	6.32(6.51)	1.03(2.23)	0.93(2.37)	0.16(0.34)	0.15(0.36)
	60	7.11(5.91)	7.80(5.53)	1.19(7.12)	1.09(0.94)	1.19(1.20)
	240	1.65(0.70)	22.9(10.4)	22.6(10.4)	13.9(15.1)	13.7(14.8)
S\$ (Singapore Dollar)	5	9.06(13.4)	8.22(9.51)	11.3(23.5)	0.91(0.71)	1.25(1.76)
	20	6.27(19.5)	7.51(10.6)	1.86(28.4)	1.19(0.54)	0.30(1.45)
	60	7.37(16.8)	5.13(16.6)	11.3(16.5)	0.69(0.98)	1.54(0.98)
	240	9.83(8.81)	1.13(6.85)	7.69(6.89)	0.11(0.78)	0.78(0.79)
MYR RM (Malaysian Ringgit)	5	51.1(60.6)	45.0(98.0)	152(87.0)	0.88(1.61)	2.97(1.43)
	20	185(66.2)	407(254)	276(48.1)	2.20(3.84)	1.49(0.73)
	60	61.3(60.4)	127(232)	123(240)	2.08(3.84)	2.00(3.98)
	240	224(173)	176(124)	125(130)	0.79(0.71)	0.56(0.75)
EUR€ (Euro Dollar)	5	17.2(12.0)	19.8(14.1)	18.6(10.6)	1.15(1.17)	1.08(0.88)
	20	14.4(26.7)	11.7(18.0)	11.4(18.0)	0.81(0.67)	0.79(0.67)
	60	67.5(19.3)	83.4(22.4)	79.1(21.8)	1.23(1.16)	1.17(1.13)
	240	8.13(211)	15.6(47.5)	14.5(239)	1.91(0.23)	1.78(1.13)

R = RMSE for random walk; Q = RMSE for YQ-ARFIMA; F = RMSE for ARFIMA; M = RMSE for ARMA, Exrate = Exchange rate, h = Forecast horizon

Table 4. Comparison ratio of RMSE measure for sample size 1000 with recursive forecast and rolling forecast in bracket

Exrate	h	R/Q	F/Q	M/Q	F/R	M/R
AUD (Australian Dollar)	5	50.2(11.6)	58.5(10.5)	58.7(15.6)	1.16(0.91)	1.17(1.35)
	20	13.5(12.3)	18.5(10.4)	16.3(9.5)	1.37(0.85)	1.21(0.77)
	60	20.1(16.3)	16.0(13.4)	23.2(13.4)	0.79(0.82)	1.15(0.80)
	240	20.9(158)	11.2(61.2)	11.3(140)	0.54(0.39)	0.54(0.89)
CAD (Canadian Dollar)	5	7.53(10.8)	13.8(1.15)	12.3(12.1)	1.83(1/83)	1.64(1.11)
	20	20.0(39.0)	25.2(1.15)	1.63(1.15)	11.8(1.26)	1.64(1.11)
	60	43.9(37.9)	50.1(20.0)	52.2(21.3)	1.14(0.53)	1.18(55.4)
	240	19.5(39.3)	39.3(60.0)	39.7(62.2)	2.01(0.80)	2.03(0.83)
UKPD (British Pound)	5	3.17(2.48)	10.7(0.75)	3.20(2.47)	3.29(0.30)	1.00(0.97)
	20	0.59(2.76)	1.59(1.18)	2.81(2.38)	2.49(0.43)	4.71(0.86)
	60	1.65(2.85)	0.98(1.87)	2.97(1.87)	0.35(0.66)	1.06(1.07)
	240	3.07(4.15)	3.55(2.10)	3.65(3.82)	1.15(0.51)	1.18(0.92)
SIND (Singapore Dollar)	5	10.4(12.6)	9.67(11.5)	10.3(12.4)	0.93(0.91)	0.99(0.98)
	20	8.81(13.7)	8.58(10.1)	8.22(12.8)	0.93(0.73)	0.93(0.93)
	60	10.2(14.4)	11.5(45.5)	11.6(36.0)	1.12(3.16)	1.14(2.50)
	240	17.2(5.15)	22.9(6.45)	20.1(5.59)	1.32(1.25)	1.16(1.08)
MALR (Malaysian Ringgit)	5	11.9(7.92)	173(3.99)	177(3.99)	14.5(0.50)	14.8(0.50)
	20	22.4(5.33)	25.18(0.4)	25.6(8.04)	1.12(1.51)	1.14(1.51)
	60	39.5(9.07)	45.5(1.20)	45.0(10.6)	1.15(1.20)	1.13(1.16)
	240	61.4(13.7)	62.1(0.15)	63.7(19.5)	1.01(0.15)	1.04(1.41)
EURO (Euro Dollar)	5	8.08(7.87)	10.1(15.0)	10.1(6.02)	1.25(1.90)	1.25(0.76)
	20	8.71(15.6)	8.16(15.0)	8.11(6.02)	0.94(0.96)	0.93(0.38)
	60	6.25(5.03)	7.54(5.65)	7.56(5.63)	1.21(1.12)	1.21(1.12)
	240	26.8(4.29)	21.8(6.14)	22.5(5.83)	0.81(1.43)	0.84(1.36)

R = RMSE for random walk; Q = RMSE for YQ-ARFIMA; F = RMSE for ARFIMA; M = RMSE for ARMA,
Exrate = Exchange rate, h = Forecast horizon

Table 5. Comparison ratio of RMSE measure for sample size 2000 with recursive forecast and rolling forecast in bracket

Exrate	h	R/Q	F/Q	M/Q	F/R	M/R
AUSD (Australian Dollar)	5	7.09(5.74)	6.25(5.99)	6.58(5.84)	0.88(1.04)	0.93(1.02)
	20	7.97(6.73)	8.60(6.04)	8.60(6.18)	1.08(0.89)	1.07(0.92)
	60	6.65(7.04)	6.42(4.86)	6.12(5.03)	0.96(0.69)	0.92(0.71)
	240	5.27(15.9)	4.80(13.7)	4.85(13.7)	0.91(0.86)	0.92(0.84)
CAND (Canadian Dollar)	5	17.9(24.6)	17.4(1.04)	17.6(1.02)	0.97(1.04)	0.98(1.02)
	20	16.5(21.9)	16.3(22.8)	16.2(22.4)	0.99(1.04)	0.98(1.02)
	60	12.7(18.5)	12.6(19.2)	12.9(18.8)	0.99(1.04)	1.01(1.02)
	240	52.3(13.9)	46.6(15.1)	47.6(14.7)	0.89(1.08)	0.91(1.05)
UKPD (British Pound)	5	12.4(12.2)	12.2(9.38)	12.2(9.38)	0.98(0.76)	0.98(0.98)
	20	12.5(14.5)	13.1(12.8)	13.1(16.4)	1.04(0.88)	1.04(1.13)
	60	8.98(9.48)	9.15(9.12)	9.06(9.12)	1.02(0.96)	1.01(0.96)
	240	10.5(7.18)	9.87(0.99)	0.94(0.99)	0.93(0.99)	0.94(0.99)
SIND (Singapore Dollar)	5	15.4(19.4)	16.9(19.6)	15.7(19.6)	1.09(1.01)	1.02(1.01)
	20	20.2(19.8)	17.4(12.5)	28.9(12.5)	0.86(0.63)	1.43(1.77)
	60	15.9(8.58)	33.9(25.7)	16.5(35.1)	2.13(2.99)	1.04(4.09)
	240	18.9(10.9)	0.73(11.8)	16.4(11.3)	0.73(1.07)	0.86(1.03)
SIND (Singapore Dollar)	5	15.4(19.4)	16.9(19.6)	15.7(19.6)	1.09(1.01)	1.02(1.01)
	20	20.2(19.8)	17.4(12.5)	28.9(12.5)	0.86(0.63)	1.43(1.77)
	60	15.9(8.58)	33.9(25.7)	16.5(35.1)	2.13(2.99)	1.04(4.09)
	240	18.9(10.9)	0.73(11.8)	16.4(11.3)	0.73(1.07)	0.86(1.03)
MALR (Malaysian Ringgit)	5	72.1(36.4)	83.1(26.6)	85.4(0.73)	1.15(0.73)	1.18(0.72)
	20	121(25.0)	149(17.0)	147(16.7)	1.23(0.68)	1.21(0.67)
	60	48.6(13.4)	61.6(25.7)	53.7(25.7)	1.27(1.92)	1.10(1.35)
	240	15.6(20.7)	23.9(38.8)	23.6(37.3)	1.53(1.87)	1.51(1.80)
EURO (Euro Dollar) [1200]	5	4.67(4.97)	4.64(4.66)	4.83(5.06)	0.99(0.94)	1.03(1.02)
	20	0.45(7.59)	4.56(3.71)	4.89(3.36)	10.2(0.49)	10.9(0.44)
	60	3.79(1.42)	4.42(1.32)	4.43(1.19)	1.16(0.93)	1.16(0.84)
	240	4.43(4.37)	6.03(4.03)	0.58(3.80)	1.36(0.92)	0.13(0.87)

R = RMSE for random walk; Q = RMSE for YQ-ARFIMA; F = RMSE for ARFIMA; M = RMSE for ARMA,
Exrate = Exchange rate, h = Forecast horizon

Table 6. Comparison ratio of RMSE measure for sample size 5000 with recursive forecast and rolling forecast in bracket

Exrate	h	R/Q	F/Q	M/Q	F/R	M/R
AUD\$ (Australian Dollar)	5	3.93(7.00)	4.62(7.22)	4.00(6.00)	1.17(1.03)	1.01(0.86)
	20	3.79(3.45)	3.96(3.98)	3.85(3.96)	1.04(1.15)	1.01(1.15)
	60	4.11(3.63)	4.80(3.86)	4.55(3.79)	1.16(1.06)	1.11(1.04)
	240	4.86(3.78)	6.22(3.21)	5.58(3.42)	1.28(3.21)	1.15(0.91)
CAD\$ (Canadian Dollar)	5	11.8(12.3)	13.3(12.3)	11.6(12.3)	1.12(1.00)	0.98(1.03)
	20	12.5(11.6)	12.9(11.9)	1.08(1.04)	1.03(1.02)	1.08(1.05)
	60	11.9(10.8)	10.8(11.3)	10.7(11.7)	0.91(1.05)	0.89(1.08)
	240	12.5(3.33)	10.1(11.5)	9.53(21.7)	0.80(3.46)	0.76(3.65)
GBP\$ (British Pound)	5	95.0(62.6)	100(58.8)	98.3(1.01)	1.06(0.94)	1.03(1.01)
	20	72.3(36.4)	73.5(35.9)	57.1(38.8)	1.02(0.99)	0.79(1.06)
	60	27.4(34.5)	30.5(36.4)	59.3(37.1)	1.11(1.05)	2.16(1.07)
	240	6.54(50.7)	6.98(54.6)	7.04(55.5)	1.06(1.08)	1.07(1.09)
S\$ (Singapore Dollar) [3000]	5	62.2(3.67)	20.9(7.62)	76.6(7.62)	1.24(2.07)	76.6(2.06)
	20	78.2(6.74)	56.1(8.38)	50.6(8.93)	0.72(1.24)	0.65(1.32)
	60	85.0(5.79)	127(6.95)	122(7.38)	1.49(1.19)	1.43(1.27)
	240	16.7(13.7)	24.0(7.51)	13.5(7.12)	1.44(0.55)	0.81(0.52)
MYR\$ (Malaysian Ringgit) [3500]	5	124(80.6)	114(96.6)	124(65.6)	0.92(1.19)	1.00(0.81)
	20	104(55.9)	99.7(59.6)	101(50.0)	0.96(1.07)	0.97(0.89)
	60	44.5(25.6)	41.9(26.8)	42.3(24.2)	0.94(1.05)	0.95(0.94)
	240	62.1(37.7)	66.4(32.5)	65.2(31.3)	1.07(0.86)	1.05(0.83)

R = RMSE for random walk; Q = RMSE for YQ-ARFIMA; F = RMSE for ARFIMA; M = RMSE for ARMA,
Exrate = Exchange rate, h = Forecast horizon

Table 7. DM test statistics (YQ-ARFIMA versus RANDOM WALK)

Exrate	h	Sample size			
		500	1000	2000	5000
AUSD (Australian Dollar)	5	-2.24[0.00]	-2.24[0.00]	-2.24[0.00]	-2.24[0.00]
	20	-4.47[0.00]	-4.47[0.00]	-4.47[0.00]	0.00[0.58]
	60	-7.74	-7.74	-7.23	-7.74
	240	-48.9	-15.4	-15.1	-15.4
UKPD (British Pound)	5	-2.24[0.00]	-2.24[0.00]	-2.24[0.00]	-2.24[0.00]
	20	-4.47[0.00]	-4.47[0.00]	-4.47[0.00]	-4.47[0.00]
	60	-7.74	-7.48	-7.48	-7.48
	240	-15.4	-15.4	-15.4	-12.0
CAND (Canadian Dollar)	5	-2.24[0.00]	-2.24[0.00]	-2.24[0.00]	-2.24[0.00]
	20	-4.47[0.00]	-4.47[0.00]	-4.02[0.00]	-4.47[0.00]
	60	-7.74	-7.74	-7.48	-7.74
	240	-15.4	-15.4	-15.4	-15.2
SIND (Singapore Dollar)	5	-2.24[0.00]	-2.24[0.00]	-1.34[0.18]	-2.24[0.00]
	20	-4.47[0.00]	-4.47[0.00]	-4.02[0.00]	-4.47[0.00]
	60	-7.74	-7.74	-7.74	-7.48
	240	-15.4	-15.4	-13.1	-13.9
MALR (Malaysian Ringgit)	5	-2.24[0.00]	-2.24[0.00]	-2.24[0.00]	-2.24[0.00]
	20	-4.47[0.00]	-4.47[0.00]	-4.47[0.00]	-4.47[0.00]
	60	-7.74	-7.74	-7.48	-7.74
	240	-15.4	-15.4	-15.2	-15.2
EURO (Euro Dollar)	5	-2.24[0.00]	-2.24[0.00]	-2.24[0.00]	-2.24[0.00]
	20	-4.47[0.00]	-4.47[0.00]	-4.47[0.00]	-4.47[0.00]
	60	-7.74	-7.74	-7.23	-7.74
	240	-14.5	-15.4	-14.3	-15.2

 H_0 : Model 0 = Model 1 H_1 : Model 0 $\neq$ Model 1Reject H_0 when DM statistic > 2

When DM statistics < 0 , Model 0 performs better than Model 1. When DM statistics > 0 , Model 1 performs better than Model 0. Exrate = Exchange rate

DM statistics = Diebold and Mariano Statistics

[] = Binomial probability of accepting null hypothesis () = sample size

Model 0 = YQ-ARFIMA, and Model 1 = RANDOM WALK are equal.

Table 8. DM test statistics (YQ-ARFIMA versus ARFIMA)

Exrate = Exchange rate

Exrate	h	Sample size			
		500	1000	2000	5000
AUD\$ (Australian Dollar)	5	-2.24[0.00]	-2.24[0.00]	-2.24[0.00]	-2.24[0.00]
	20	-4.47[0.00]	-4.47[0.00]	-4.47[0.00]	0.00[0.58]
	60	-7.48	-7.48	-7.74	-7.74
	240	-15.1	-15.3	-14.1	-15.4
GBP£ (British Pound)	5	-2.24[0.00]	-1.34[0.18]	-2.24[0.00]	-2.24[0.00]
	20	-4.47[0.00]	-4.47[0.00]	-4.47[0.00]	-4.47[0.00]
	60	-7.74	-7.48	-7.48	-7.74
	240	-15.4	-15.3	-15.4	-13.5
CAD\$ (Canadian Dollar)	5	-2.24[0.00]	-2.24[0.00]	-2.24[0.00]	-2.24[0.00]
	20	-4.47[0.00]	-4.47[0.00]	-4.47[0.00]	-4.47[0.00]
	60	-7.74	-7.74	-7.74	-7.74
	240	-15.4	-15.4	-15.1	-15.1
SGD\$ (Singapore Dollar)	5	-2.24[0.00]	-2.24[0.00]	-2.24[0.00]	-2.24[0.00]
	20	-4.47[0.00]	-4.47[0.00]	-4.47[0.00]	-4.02[0.00]
	60	-7.48	-7.48	-7.74	-7.74
	240	-15.4	-14.8	-13.4	-15.4
MYR RM (Malaysian Ringgit)	5	-2.24[0.00]	-2.24[0.00]	-2.24[0.00]	(3500) -2.24[0.00]
	20	-4.47[0.00]	-4.47[0.00]	-4.47[0.00]	-4.47[0.00]
	60	-7.74	-7.74	-7.74	-7.74
	240	-15.4	-15.1	-15.4	-15.2
EUR€ (Euro Dollar)	5	-2.24[0.00]	-2.24[0.00]	-2.24[0.00]	(1200) -
	20	-4.47[0.00]	-4.47[0.00]	-4.47[0.00]	-
	60	-7.74	-7.48	-7.74	-
	240	-15.4	-15.4	-15.1	-
H_0 : Model 0 = Model 1		H_1 : Model 0 $\neq$ Model 1		Reject H_0 when DM statistic > 2	

When DM statistics < 0, Model 0 performs better than Model 1. When DM statistics > 0, Model 1 performs better than Model 0.

DM statistics = Diebold and Mariano Statistics

[] = Binomial probability of accepting null hypothesis () = sample size

Model 0 = YQ-ARFIMA and Model 1 = ARFIMA are equal.

Table 9. DM test statistics (RW versus ARFIMA)

Exrate = Exchange rate

Exrate	h	Sample size			
		500	1000	2000	5000
AUSD (Australian Dollar)	5	-2.24[0.00]	-2.24[0.00]	2.24[0.00]	-2.24[0.00]
	20	-3.57[0.0002]	-3.13[0.0013]	-2.68[0.50]	-3.13[0.0013]
	60	-5.93	5.93	2.84	-0.26
	240	9.29	15.2	-4.64	-14.9
UKPD (British Pound)	5	-1.34[0.31]	-2.24[0.00]	2.24[0.00]	-2.24[0.00]
	20	1.78[0.98]	2.24[0.00]	-3.13[0.0013]	-0.89[0.25]
	60	-7.23	4.46	-7.74	-2.84
	240	-14.9	-4.91	15.1	-4.00
CAND (Canadian Dollar)	5	-2.24[0.00]	0.44[0.81]	2.24[0.00]	-2.24[0.00]
	20	-3.13[0.0013]	-4.47[0.00]	4.02[0.00]	-4.02[0.00]
	60	-6.71	-6.19	1.29	6.45
	240	-9.03	-7.10	10.8	14.5
SIND (Singapore Dollar)	5	1.34[0.97]	0.44[0.81]	-0.44[0.5]	-0.44[0.5]
	20	-3.57[0.0002]	3.13[0.0002]	1.78[0.97]	0.44[0.5]
	60	7.74	-2.32	-1.29	-1.03
	240	-15.3	-8.00	-2.32	-1.29
MALR (Malaysian Ringgit)	5	0.44[0.81]	-1.34[0.97]	-1.34[0.97]	(3500) -2.24[0.00]
	20	-8.49[0.000]	-4.47[0.00]	-4.46[0.00]	-4.47[0.00]
	60	-7.74	-3.61	-6.97	6.19
	240	14.7	1.67	-8.13	-5.81
EURO (Euro Dollar)	5	-0.45[0.81]	-1.34[0.97]	2.24[0.00]	(1200) -
	20	3.13[0.98]	0.89[1.00]	-0.44[0.5]	-
	60	-7.74	-7.23	-2.58	-
	240	-12.0	14.9	-13.4	-

 H_0 : Model 0 = Model 1 H_1 : Model 0 $\neq$ Model 1Reject H_0 when DM statistic > 2

When DM statistics < 0 , Model 0 performs better than Model 1. When DM statistics > 0 , Model 1 performs better than Model 0.

DM statistics = Diebold and Mariano Statistics

[] = Binomial probability of accepting null hypothesis

() = sample size

Model 0 = RW and Model 1 = ARFIMA are equal.

Table 10. DM test statistics (ARFIMA versus ARMA)

Exrate = Exchange rate

Exrate	h	Sample size			
		500	1000	2000	5000
AUSD (Australian Dollar)	5	0.44[0.81]	0.44[0.81]	-2.24[0.00]	2.24[0.00]
	20	-4.47[0.02]	4.02[0.0013]	-3.13[0.0013]	4.47[0.02]
	60	0.00	-5.93	5.42	0.52
	240	-14.1	-4.51	-8.13	12.9
UKPD (British Pound)	5	1.34[0.31]	2.24[0.00]	2.24[0.00]	-2.24[0.00]
	20	-3.57[0.002]	-2.23[0.02]	-3.57[0.002]	0.89[0.25]
	60	-7.48	-4.91	7.74	0.00
	240	4.38	17.2	-12.1	-5.55
CAND (Canadian Dollar)	5	2.24[0.00]	0.44[0.81]	-2.24[0.00]	2.24[0.00]
	20	-3.13[0.0013]	-3.57[0.0002]	4.47[0.00]	-4.02[0.00]
	60	-7.48	0.26	-4.13	3.09
	240	-9.94	-7.74	-9.81	14.2
SIND (Singapore Dollar)	5	-1.34[0.97]	1.34[0.97]	1.34[0.97]	-0.44[0.00]
	20	2.68[0.005]	0.89[0.99]	1.34[0.97]	3.57[0.002]
	60	-7.22	-3.09	5.42	7.22
	240	5.93	8.77	-9.94	8.00
MALR (Malaysian Ringgit)	5	-1.34[0.31]	2.24[0.00]	-0.44[0.81]	(3500) 2.24[0.00]
	20	3.57[0.002]	-4.47[0.00]	2.23[0.02]	-4.47[0.00]
	60	1.29	4.46	6.45	-5.42
	240	8.13	-7.23	7.87	-0.75
EURO (Euro Dollar)	5	-0.44[0.81]	-1.34[0.97]	(1200) -2.24[0.00]	-
	20	2.68[0.005]	0.89[0.99]	-3.57[0.00]	-
	60	7.74	-2.06	-0.77	-
	240	15.4	-14.9	13.7	-
H_0 : Model 0 = Model 1		H_1 : Model 0 $\neq$ Model 1		Reject H_0 when DM statistic > 2	

When DM statistics < 0, Model 0 performs better than Model 1. When DM statistics > 0, Model 1 performs better than Model 0.

DM statistics = Diebold and Mariano Statistics

[] = Binomial probability of accepting null hypothesis () = sample size

Model 0 = ARFIMA and Model 1 = ARMA are equal.

Table 11. CM test statistics (YQ-ARFIMA versus RW)

Exrate = Exchange rate

Exrate	h	Sample size			
		500	1000	2000	5000
AUSD (Australian Dollar)	5	-2.82	-3.17	-2.37	-2.72
	20	4.97	-6.26	-5.37	0.04
	60	-9.42	-12.8	-11.2	-8.92
	240	-16.1	-19.3	-17.5	-25.5
UKPD (British Pound)	5	-2.51	-2.09	-2.97	-2.37
	20	-6.22	8.04	-6.52	-5.63
	60	-6.57	-9.39	-19.4	-9.23
	240	-18.3	-14.6	-20.3	-15.6
CAND (Canadian Dollar)	5	-3.18	-1.85	-4.23	-3.27
	20	-4.06	-6.38	-8.67	-4.99
	60	-12.0	-7.38	-11.1	-14.5
	240	-13.9	-15.0	-14.7	-24.6
SIND (Singapore Dollar)	5	-2.64	-1.48	-2.53	-3.15
	20	-4.48	-3.43	-4.26	-4.31
	60	-11.1	-14.4	-10.4	-0.06
	240	-16.8	-13.8	-19.9	-15.4
MALR (Malaysian Ringgit)	5	-2.25	-2.55	-2.79	-2.57
	20	-4.89	-9.16	-6.34	-23.8
	60	-8.21	-6.26	-12.5	-7.76
	240	-27.2	-12.8	-19.5	-14.7
EURO (Euro Dollar)	5	-2.04	3.69	-3.23	-
	20	-5.85	5.56	-5.20	-
	60	-10.3	8.37	-7.05	-
	240	-14.7	14.4	18.1	-
H_0 : Model 0 = Model 1		H_1 : Model 0 > Model 1	Reject H_0 when CM statistic > absolute 2		

When CM statistics < absolute 2, Model 0 performs better than Model 1.

CM statistics = Clark and McCracken Statistics

Null hypothesis: The two competing models are equal

Alternate hypothesis: Model 1 is a better forecasting model than model 0

Model 0 = YQ-ARFIMA and Model 1 = RW are equal.

Table 12. (a)– Comparison of the RMSE values for the case of British pound per unit US dollar by using the YQ-ARFIMA model and the Random Walk.

Sample size	Model Specifications		
	ARFIMA(2, d , 1) (M)	Random Walk (W)	W/M
Euro			
259	0.0013225	0.026926	20.4
530	0.0025324	0.013686	5.40
780	0.0057584	0.021163	3.68
1038	0.0054069	0.015672	2.89
1240	0.0094798	0.078815	8.31

Table 12. (b) – Comparison of the MAPE values for the case of British pound per unit US dollar by using the YQ-ARFIMA model and the Random Walk.

Sample size	Model Specifications		
	ARFIMA(2, d , 1) (M)	Random Walk (W)	W/M
Euro			
259	0.065827	1.3474	20.5
530	0.104560	0.5556	5.31
780	0.374140	1.3611	3.64
1038	0.398240	1.1654	2.93
1241	0.609850	5.3464	8.77