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Vibration Suppression Techniques in Feedback Control Loop of a Flexible Robot Manipulator

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Abstract

This paper presents the use of angular position control approaches for a flexible robot manipulator with disturbances effect in the dynamic system. Delayed Feedback Signal (DFS), Linear Quadratic Regulator (LQR) and Proportional-Derivative (PD) controller are the techniques used in this investigation to actively control the vibrations of flexible structure. A constrained planar single-link flexible manipulator is considered and the dynamic model of the system is derived using the assumed mode method. A complete analysis of simulation results for each technique is presented in time domain and frequency domain respectively. Performances of the controller are examined in terms of vibration suppression and disturbances cancellation. Finally, a comparative assessment of the impact of each controller on the system performance is presented and discussed.

Keywords: Flexible manipulator, Vibration control, Delayed feedback signal, Linear quadratic regulator, PD controller.

1. Introduction

Flexible robot manipulators exhibit several advantages over the rigid link manipulators as they require less material, are lighter in weight, have higher manipulation speed, lower power consumption, require smaller actuators, are more manoeuvrable and transportable, have less overall cost and higher payload to robot weight ratio (Martins et al., 2003). However, the control of flexible manipulators to maintain accurate positioning is a challenging problem. A flexible manipulator is a distributed parameter system and has infinitely many degrees of freedom. Moreover, the dynamics are highly non-linear and complex. Problems arise due to precise positioning requirements, system flexibility leading to vibration, the difficulty in obtaining accurate model of the system and non-minimum phase characteristics of the system (Yurkovich, 1992). To attain end-point positional accuracy, a control mechanism that accounts for both the rigid body and flexural motions of the system is required. If the advantages associated with lightness are not to be sacrificed, precise models and efficient control strategies for flexible manipulators have to be developed.

The requirement of precise position control of flexible manipulators implies that residual vibration of the system should be zero or near zero. Over the years, investigations have been carried out to devise efficient approaches to reduce the vibration of flexible manipulators. The considered vibration control schemes can be divided into two main categories: feed-forward control and feedback control techniques. Feed-forward techniques for vibration suppression involve developing the control input through consideration of the physical and vibrational properties of the system, so that system vibrations at dominant response modes are reduced. This method does not require additional sensors or actuators and does not account for changes in the system once the input is developed. On the other hand, feedback-control techniques use measurement and estimations of the system states to reduce vibration. Feedback controllers can be designed to be robust to parameter uncertainty.

In general, control of flexible manipulators can be made easier by locating every sensor exactly at the location of the actuator, as collocation of sensors and actuators guarantees stable servo control (Gevarter, 1970). In the case of flexible manipulator systems, the end-point position is controlled by obtaining the parameters at the hub and end-point of the manipulator and using the measurements as a basis for applying control torque at the hub. Thus, the feedback control can be divided into collocated and non-collocated control. By applying control torque based on non-collocated sensors, the problem of non-minimum phase and of achieving stability is of concern. Several approaches utilising closed-loop control strategies have been reported for control of flexible manipulators. These include linear state feedback control (Cannon and Schmitz, 1984; Hasting and Book, 1987), adaptive control (Feliu et al., 1990; Yang et al., 1992), robust control techniques based on H-infinity (Moser, 1993) and variable structure control (Moallem et al., 1998) and intelligent control based on neural networks (Gutierrez et al., 1998) and fuzzy logic control schemes (Moudgal et al., 1994).

Another method of controlling flexible structures is based on time delay control (TDC). In the TDC method, time delay is used to estimates the effects of unknown dynamics and unpredictable disturbances (Youcef and Ito, 1990a; Youcef and Bobbett, 1992a). The TDC introduces delay terms in the closed-loop of the system in order to cancel the unwanted dynamics. In (Youcef and Wu, 1992b), time delay has been used to achieve an input/output linearization of a class of nonlinear systems with a special application to the position control of a single-link flexible arm. In general, time delays occur in real systems in several forms. Transport delays and acoustic feedback are considered the main sources. The stability of systems with delay has been dealt with extensively in the literature (Youcef and Reddy, 1990b; Malek and Jamshidi, 1987; Kharitonov, 1979). Recently, a generalized approach to investigate the stability of time delay systems has been presented in (Olgac and Sipahi, 2001). This approach resembles the Routh-Hurwitz technique for linear systems and can be used to select the time delay parameters that lead to a stable closed-loop system. More recently, TDC has been used in the control of aerodynamic systems. In (Ramesh and Narayanan, 2001), a time-delayed feedback to control the chaotic motions in a two-dimensional airfoil was used and, a similar technique to stabilize the motion of helicopter rotor blades was used in (Krodkiewski and Faragher, 2000) except that the time delay in this case was selected to be the period of the motion to be stabilized. A method for determining the stability switches for time delayed dynamic systems with unknown parameters has been presented in (Wang and Hu, 2000; Jnifene, 2007). In the present paper, the time delay has been introduced to generate the control signal and the delay time is considered as the design parameter.

This paper presents investigations of angular position control approach in order to eliminate the effect of disturbances applied to the single-link flexible robot manipulator. A simulation environment is developed within Simulink and Matlab for evaluation of the control strategies. In this work, the dynamic model of the flexible manipulator is derived using the assumed mode method (AMM). To demonstrate the effectiveness of the proposed control strategy, the disturbances effect is applied at the tip of the flexible link. This is then extended to develop a feedback control strategy for vibration reduction and disturbances rejection. Three feedback control strategies which are DFS, LQR and PD controller are developed in this simulation work. Performances of each controller are examined in terms of vibration suppression and disturbances rejection. Finally, a comparative assessment of the impact of each controller on the system performance is presented and discussed.

2. The Flexible Manipulator System

The single-link flexible manipulator system considered in this work is shown in Figure 1, where X_oOY_o and XOY represent the stationary and moving coordinates frames respectively, τ represents the applied torque at the hub. E, I, ρ, A, I_h and m_p represent the Young modulus, area moment of inertia, mass density per unit volume, cross-sectional area, hub inertia and payload mass of the manipulator respectively. In this work, the motion of the manipulator is confined to X_oOY_o plane. Transverse shear and rotary inertia effects are neglected, since the manipulator is long and slender. Thus, the Bernoulli-Euler beam theory is allowed to be used to model the elastic behaviour of the manipulator. The manipulator is assumed to be stiff in vertical bending and torsion, allowing it to vibrate dominantly in the horizontal direction and thus, the gravity effects are neglected. Moreover, the manipulator is considered to have a constant cross-section and uniform material properties throughout. In this study, an aluminium type flexible manipulator of dimensions 900 × 19.008 × 3.2004 mm³, $E = 71 \times 10^9$ N/m², $I = 5.1924 \times 10^{11}$ m^4 , $\rho = 2710$ kg/m^3 and $I_h = 5.8598 \times 10^{-4}$ kgm^2 is considered.

3. Modelling of the Flexible Manipulator

This section provides a brief description on the modelling of the flexible robot manipulator system, as a basis of a simulation environment for development and assessment of the control techniques. The AMM with two modal displacements is considered in characterising the dynamic behaviour of the manipulator incorporating structural damping and hub inertia. Further detailed of the description and derivation of the dynamic model of the system can be found in (Subudhi et al., 2002). The dynamic model is validated with an actual experimental rig to study the performance of the model in (Martin et al., 2003).

Considering revolute joints and motion of the manipulator on a two-dimensional plane, the kinetic energy of the system can thus be formulated as

$$T = \frac{1}{2}(I_H + I_b)\dot{\theta}^2 + \frac{1}{2}\rho\int_0^L (\dot{v}^2 + 2\dot{v}\dot{\theta}x)dx \tag{1}$$

where I_b is the beam rotation inertia about the origin O_0 as if it were rigid. The potential energy of the beam can be formulated as

$$U = \frac{1}{2}\int_0^L EI\left(\frac{\partial^2 v}{\partial x^2}\right)^2 dx \tag{2}$$

This expression states the internal energy due to the elastic deformation of the link as it bends. The potential energy due to gravity is not accounted for since only motion in the plane perpendicular to the gravitational field is considered.

To obtain a closed-form dynamic model of the manipulator, the energy expressions in (1) and (2) are used to formulate the Lagrangian $L = T - U$. Assembling the mass and stiffness matrices and utilizing the Euler-Lagrange equation of motion, the dynamic equation of motion of the flexible manipulator system can be obtained as

$$M\ddot{Q}(t) + D\dot{Q}(t) + KQ(t) = F(t) \tag{3}$$

where *M, D* and *K* are global mass, damping and stiffness matrices of the manipulator respectively. The damping matrix is obtained by assuming the manipulator exhibit the characteristic of Rayleigh damping. *F(t)* is a vector of external forces and *Q(t)* is a modal displacement vector given as

$$Q(t) = [\theta \quad q_1 \quad q_2 \quad \ldots \quad q_n]^T = [\theta \quad q^T]^T \tag{4}$$

$$F(t) = [\tau \quad 0 \quad 0 \quad \ldots \quad 0]^T \tag{5}$$

Here, q_n is the modal amplitude of the i th clamped-free mode considered in the AMM procedure and n represents the total number of assumed modes. The model of the uncontrolled system can be represented in a state-space form as

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{6}$$

with the vector $x = [\theta \quad \dot{\theta} \quad q_1 \quad q_2 \quad \dot{q}_1 \quad \dot{q}_2]^T$ and the matrices A and B are given by

$$\mathbf{A} = \left[\begin{array}{c|c} 0_{3\times3} & I_{3\times3} \\ \hline -M^{-1}K & -M^{-1}D \end{array}\right], \quad \mathbf{B} = \left[\begin{array}{c} 0_{3\times1} \\ \hline M^{-1} \end{array}\right]$$
$$\mathbf{C} = [I_{1\times3} \mid 0_{1\times3}], \qquad \mathbf{D} = [0] \tag{7}$$

4. Controller Design

In this section, three feedback control strategies (DFS, LQR and PD controller) are proposed and described in detail. The main objective of the feedback controller in this study is to maintain the angular position of flexible manipulator while suppressing the vibration due to disturbances effect. All the feedback control strategies are incorporated in the closed-loop system in order to eliminate the effect of disturbances.

4.1 DFS Controller

In this section, the control signal is calculated based on the delayed position feedback approach described in Equation (8) and illustrated by the block diagram shown in Figure 2.

$$u(t) = k(y(t) - y(t-\tau)) \tag{8}$$

Substituting Equation (8) into Equation (6) and taking the Laplace transform gives

$$sIx(s) = Ax(s) - kBC(1 - e^{-s\tau})x(s) \tag{9}$$

The stability of the system given in Equation (9) depends on the roots of the characteristic equation

$$\Delta(s,\tau) = |\, sI - A + kBC(1 - e^{-s\tau})\,| = 0 \tag{10}$$

Equation (10) is transcendental and results in an infinite number of characteristic roots (Olgac and Sipahi, 2001). Several approaches dealing with solving retarded differential equations have been widely explored. In this study, the approach described in (Ramesh and Narayanan, 2001) will be used on determining the critical values of the time delay τ that result in characteristic roots of crossing the imaginary axes. This approach suggests that Equation (10) can be written in the form

$$\Delta(s,\tau) = P(s) + Q(s)e^{-s\tau} \tag{11}$$

P(s) and *Q(s)* are polynomials in *s* with real coefficients and deg(*P(s)*=n>deg(*Q(s)*) where *n* is the order of the system. In order to find the critical time delay τ that leads to marginal stability, the characteristic equation is evaluated at $s=j\omega$. Separating the polynomials *P(s)* and *Q(s)* into real and imaginary parts and replacing $e^{-j\omega\tau}$ by $\cos(\omega\tau)-j\sin(\omega\tau)$, Equation (11) can be written as

$$\Delta(j\omega,\tau) = P_R(\omega) + jP_I(\omega) + (Q_R(\omega) + jQ_I(\omega))(\cos(\omega\tau) - j\sin(\omega\tau)) \tag{12}$$

The characteristic equation $\Delta(s,\tau)=0$ has roots on the imaginary axis for some values of $\tau \geq 0$ if Equation (12) has positive real roots. A solution of $\Delta(j\omega,\tau)=0$ exists if the magnitude $|\Delta(j\omega,\tau)|=0$. Taking the square of the magnitude of $\Delta(j\omega,\tau)$ and setting it to zero lead to the following equation

$$P_R^{\ 2} + P_I^{\ 2} - (Q_R^{\ 2} + Q_I^{\ 2}) = 0 \tag{13}$$

By setting the real and imaginary parts of Equation (13) to zero, the equation is rearranged as below

$$\begin{bmatrix} Q_R & Q_I \\ Q_I & -Q_R \end{bmatrix}\begin{bmatrix} \cos\beta \\ \sin\beta \end{bmatrix} = \begin{bmatrix} -P_R \\ -P_I \end{bmatrix}, \tag{14}$$

where $\beta=\omega\tau$.

Solving for $\sin\beta$ and $\cos\beta$ gives

$$\sin(\beta) = \frac{(-P_RQ_I + P_IQ_R)}{(Q_R^{\ 2} + Q_I^{\ 2})} \text{ and } \cos(\beta) = \frac{(-P_RQ_R - P_IQ_I)}{(Q_R^{\ 2} + Q_I^{\ 2})}$$

The critical values of time delay can be determined as follows: if a positive root of Equation (13) exists, the corresponding time delay τ can be found by

$$\tau_k = \frac{\beta}{\omega} + \frac{2k\pi}{\omega} \tag{15}$$

where $\beta \in [0\ 2\pi]$. At these time delays, the root loci of the closed-loop system are crossing the imaginary axis of the s-plane. This crossing can be from stable to unstable or from unstable to stable. In order to investigate the above method further, the time-delayed feedback controller is applied to the single-link flexible manipulator. Practically, the control signal for the DFS controller requires only one position sensor and uses only the current output of this sensor and the output τ second in past. There is only two control parameter: *k* and τ that needs to be set. Using the stability analysis described in (Wang and Hu, 2000), the gain and time-delayed of the system is set at $k=55$ and $\tau=0.005$. The control signal of DFS controller can be written as below

$$u_{DFS}(t) = -55(\theta(t) - \theta(t-0.005))$$

4.2 LQR Controller

A more common approach in the control of manipulator systems involves the utilization LQR design [31]. Such an approach is adopted at this stage of the investigation here. In order to design the LQR controller a linear state-space model of the flexible manipulator was obtained by linearising the equations of motion of the system. For a LTI system in Equation (6), the technique involves choosing a control law $u=\psi(x)$ which stabilizes the origin (i.e., regulates x to zero) while minimizing the quadratic cost function

$$J = \int_0^\infty x(t)^T Qx(t) + u(t)^T Ru(t)dt \tag{16}$$

where $Q=Q^T \geq 0$ and $R=R^T > 0$. The term “linear-quadratic” refers to the linear system dynamics and the quadratic cost function.

The matrices Q and R are called the state and control penalty matrices, respectively. If the components of Q are chosen large relative to those of R, then deviations of x from zero will be penalized heavily relative to deviations of u from zero. On the other hand, if the components of R are large relative to those of Q, then control effort will be more costly and the state will not converge to zero as quickly.

A famous and somewhat surprising result due to Kalman is that the control law which minimizes J always takes the form $u = \psi(x) = -Kx$. The optimal regulator for a Linear Time Invariant (LTI) system with respect to the quadratic cost function above is always a linear control law. With this observation in mind, the closed-loop system takes the form

$$\dot{x} = (A - BK)x \tag{17}$$

and the cost function J takes the form

$$J = \int_0^\infty x(t)^T (Q + K^T RK)x(t)dt \tag{18}$$

Assuming that the closed-loop system is internally stable, which is a fundamental requirement for any feedback controller, the following theorem allows the computation value of the cost function for a given control gain matrix *K*.

In order to implement the LQR controller, all the state variables need to be available either through direct measurement or through estimation. The block diagram of LQR controller is illustrated in Figure 3. For the single-link flexible manipulator described by the state-space model given by Equation (6) and with *M*, *K*, and *D* matrices calculated earlier, the LQR gain matrix for

$$Q = 10\left[\begin{array}{c:c} I_{3\times3} & 0_{3\times3} \\ \hdashline 0_{3\times3} & 0_{3\times3} \end{array}\right] \qquad \text{and} \qquad R = 1$$

was calculated using Matlab and was found to be

$$K_{LQR} = [3.1623 \quad 2.1991 \quad 8.1477 \quad 0.4476 \quad 0.5333 \quad 5.1509]$$

The control signal for the LQR is calculated as

$$u_{LQR} = -3.1623\theta - 2.1991\dot{\theta} - 8.1477q_1 - 0.4476q_2 - 0.5333\dot{q}_1 - 5.1509\dot{q}_2$$

4.3 PD Controller

To demonstrate the performance of the PD controller in dealing with vibration and disturbances, a PD feedback of collocated sensor signals is adopted for control the angular position of the flexible manipulator. A block diagram of the PD controller is shown in Figure 4, where K_P and K_D are the proportional and derivative gains respectively, θ and $\dot{\theta}$ represent hub angle and hub velocity, respectively. Essentially the task of this controller is to position the flexible arm to the specified angle of demand. The hub angle and hub velocity signals are fed back and used to control the hub angle of the manipulator.

To design the PD controller, a linear state-space model of the flexible manipulator was obtained by linearising the equations of motion of the system. The control signal $u(s)$ in Figure 4 can be written as

$$u_{PD}(s) = -[K_P\theta(s) + K_D\dot{\theta}(s)]$$

where s is the Laplace variable. In this study, the Ziegler-Nichols approach is utilized to design the PD controller. Analyses the tuning process of the proportional and derivative gains using Ziegler-Nichols technique shows that the optimum response of PD controller is achieved by setting $K_P = 1.2$ and $K_D = 0.8$.

5. Simulation Results

In this section, the proposed control schemes are implemented and tested within the simulation environment of the flexible manipulator and the corresponding results are presented. The control strategies were designed by undertaking a computer simulation using the fourth-order Runge-Kutta integration method at a sampling frequency of 1 kHz. Three system responses namely the angular position, hub-angular velocity, and modal displacement are obtained. Moreover, the power spectral density (PSD) of the modal displacement is evaluated to investigate the dynamic behaviour of the system in frequency domain. Two criteria are used to evaluate the performances of the control strategies:

(1) Level of vibration reduction at the natural frequencies. This is accomplished by comparing the responses of the controller with the response to the open loop system.

(2) Disturbance cancellation. The capability of the controller to achieved steady state conditions at zero angular position.

In all simulations, the initial condition $x_o = [0\ \ 0\ \ 1\times10^{-3}\ \ 0\ \ 1\times10^{-5}\ \ 0]^T$ was used. This initial condition is considered as the disturbances applied to the flexible manipulator system. The first two modes of vibration of the system are considered, as these dominate the dynamic of the system.

Figure 5 shows the open loop response of the free end of the flexible arm which consist of angular position, modal displacement, angular velocity and PSD results. These results were considered as the system response with disturbances effect and will be used to evaluate the performance of feedback control strategies. It is noted that, in open loop configuration, the steady-state angular position for the flexible manipulator system was achieved at 0.014 radian within the settling times of 1 s. The angular velocity response shows the maximum oscillation between -4 and 6 rad/sec, whereas the modal displacement oscillate between ±0.03 m. Resonance frequencies of the system were obtained by transforming the time-domain representation of the system responses into frequency domain using power spectral analysis. The vibration frequencies of the flexible manipulator system under disturbances effect were obtained as 16 and 55 Hz for the first two modes as demonstrated in Figure 5.

The system responses of the flexible manipulator with the DFS controller are shown in Figure 6. It shows that, with the gain and time delay of 55 and 0.005s respectively, the effect of the disturbances has been successfully eliminated. This is evidenced in angular position response whereas the flexible manipulator system maintained its steady-state conditions at zero radian in a very fast response. It is noted that the vibration in the angular position, angular velocity and modal displacement responses were reduced as compared to the open loop response. This can be clearly demonstrated in frequency domain results as the magnitudes of the PSD at the natural frequencies were significantly reduced.

Figure 7 shows the response of the closed loop system using the LQR controller. The angular position result demonstrates that, the LQR controller can handle the effect of disturbances in the system by optimizing the feedback gain in order to achieve zero radian steady state conditions. It is noted that the overall system vibrations were significantly reduced with the LQR controller even though the level of vibration reduction was less than the case with the DFS controller. It is noted that, the magnitude reduction of the PSD only effect the first mode of the natural frequencies as demonstrated in Figure 7.

The closed loop system responses of the flexible manipulator under PD controller are shown in Figure 8. The angular position, angular velocity and modal displacement response shows a similar pattern as the case of LQR and DFS controller. The results also demonstrated that the conventional PD controller can eliminate the effect of disturbances in the system. It is noted that, the overall time response results exhibits small magnitude of oscillation as compared to the LQR and DFS controller. Besides, the vibration in the angular position, angular velocity and modal displacement responses in overall were reduced as compared to the open loop response. The PSD result shows that the magnitudes of vibration were significantly reduced especially for the first mode of vibration.

By comparing the results of DFS, LQR and PD controller, it is noted that higher performance in the reduction of vibration of the system is achieved with the DFS control strategies. This is observed and compared to the LQR and PD controller at the first two modes of vibration. For comparative assessment, the levels of vibration reduction with the modal displacement using DFS, LQR and PD controller are shown with the bar graphs in Figure 9. The result shows that highest level of vibration reduction is achieved with the DFS controller, followed by PD and LQR controller. Therefore, it can be concluded that overall the delayed feedback signal (DFS) provide better performance in vibration reduction as compared to the both PD and LQR controller. Moreover, the vibration of the system settles within 0.25 s for DFS, LQR and PD controller, which is fourfold improvement as compared to the open loop response. This is evidenced in the angular position, hub-angular velocity, and modal displacement responses for each controller respectively.

6. Conclusion

Investigations into vibration suppression of a flexible robot manipulator with disturbances effect using the DFS, LQR and PD controller have been presented. Performances of the controller are examined in terms of vibration suppression and disturbances cancellation. The results demonstrated that the effect of the disturbances in the system can successfully be handled by DFS, LQR and PD controller. A significant reduction in the system vibration has been achieved with the DFS controller as compared to the LQR and PD controller. The result reveals that the proposed controllers provide a high speed system response in cater the disturbances effect to the flexible manipulator system.

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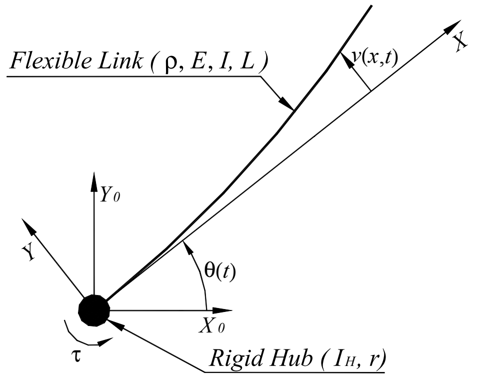


Figure 1. Description of the flexible manipulator system.

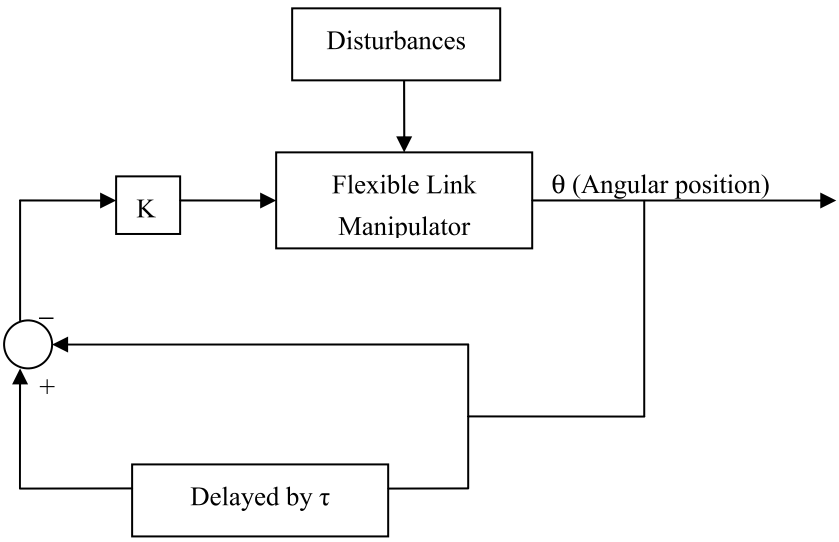


Figure 2. The delayed feedback signal (DFS) controller structure.

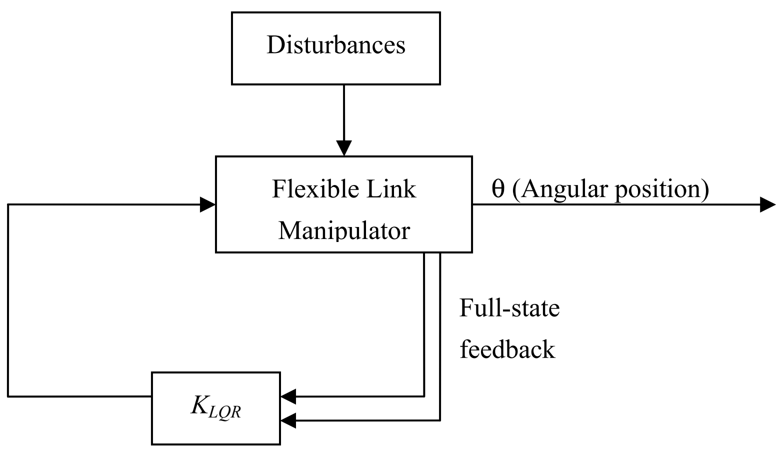


Figure 3. The linear quadratic regulator (LQR) controller structure.

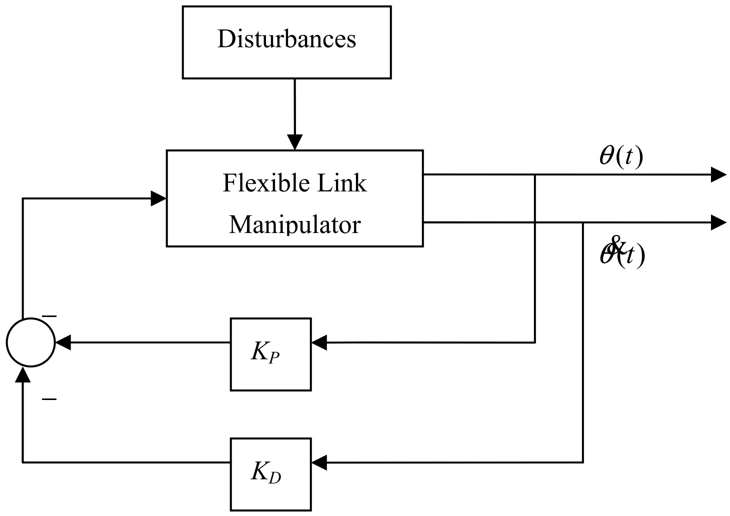
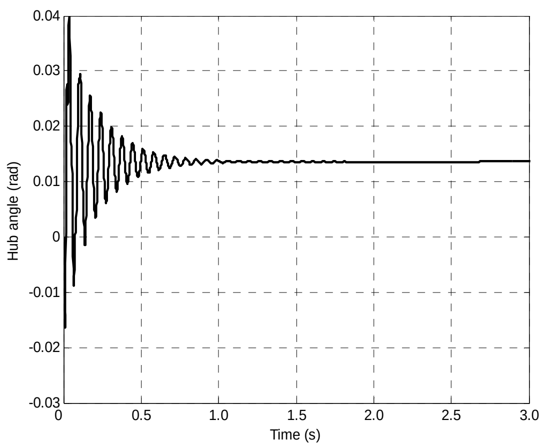
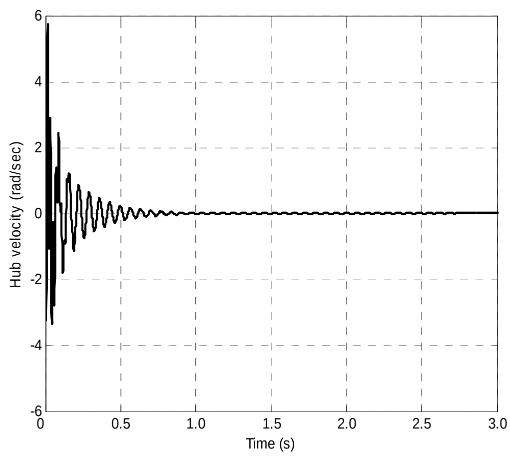


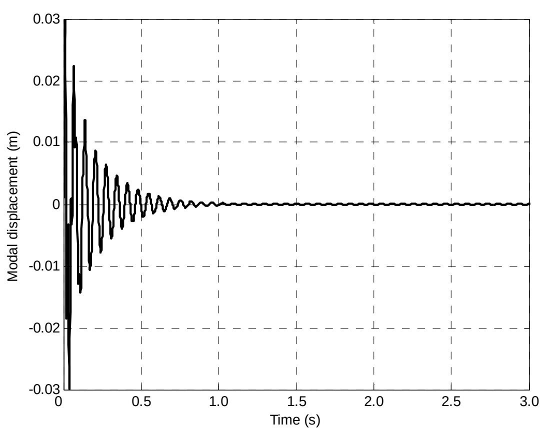
Figure 4. The proportional-derivative (PD) controller structure.



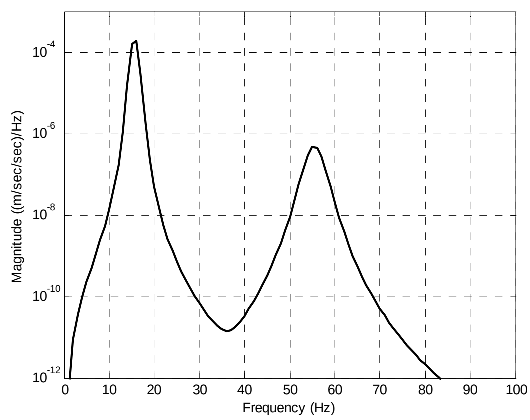
(a) Angular position.



(b) Angular velocity.

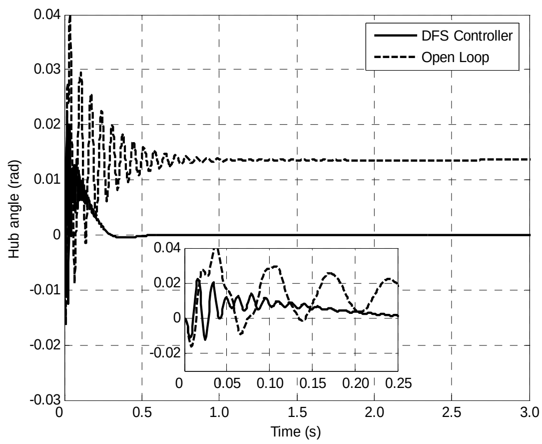


(c) Modal displacement.

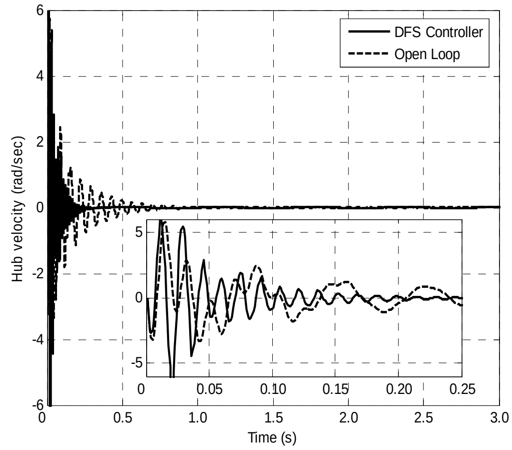


(d) PSD of modal displacement.

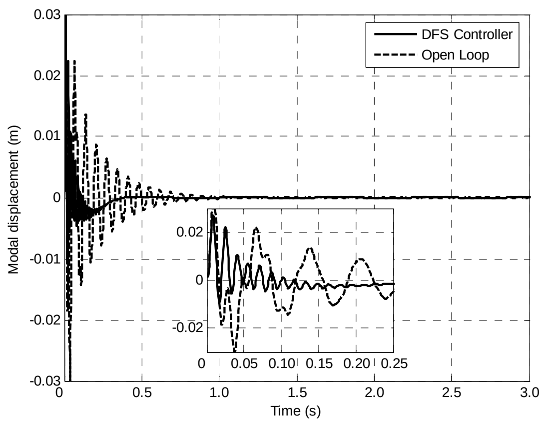
Figure 5. Open loop response of the flexible manipulator.



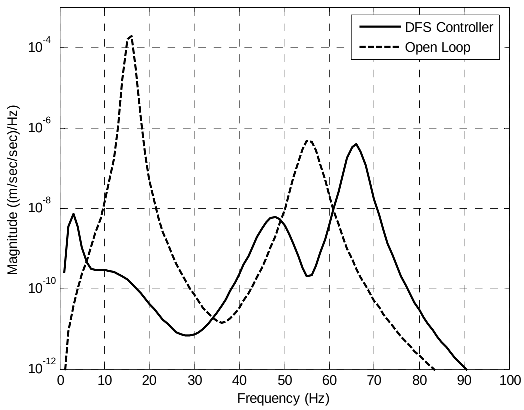
(a) Angular position.



(b) Angular velocity.

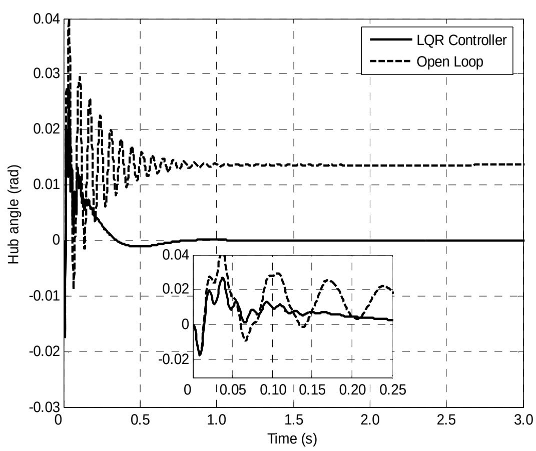


(c) Modal displacement.

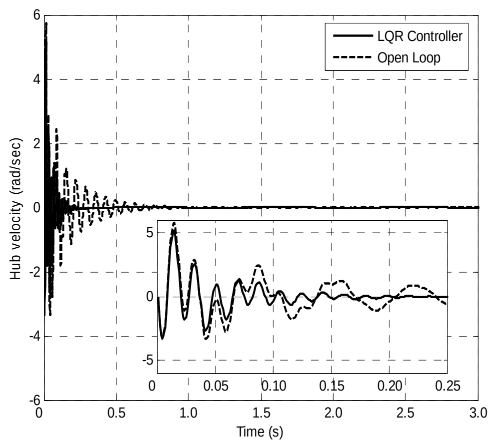


(d) PSD of modal displacement.

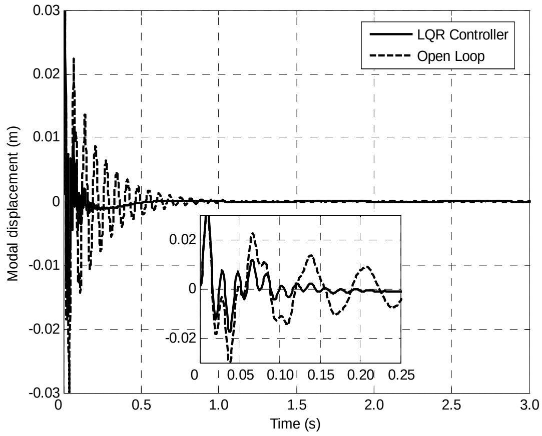
Figure 6. Response of the flexible manipulator with DFS Controller.



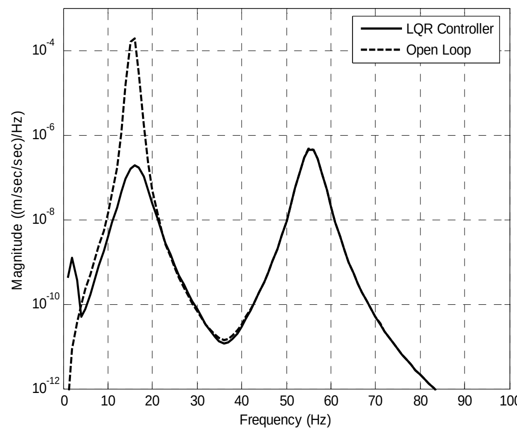
(a) Angular position.



(b) Angular velocity.

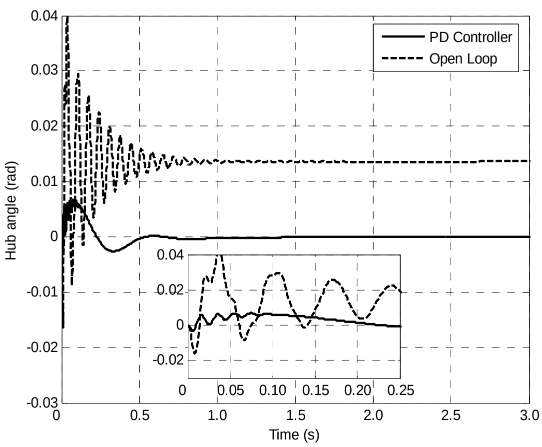


(c) Modal displacement.

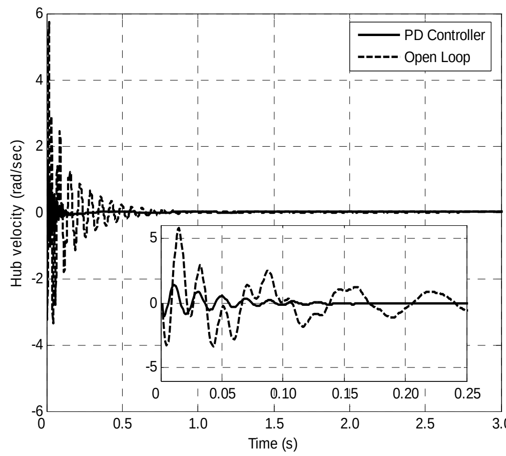


(d) PSD of modal displacement.

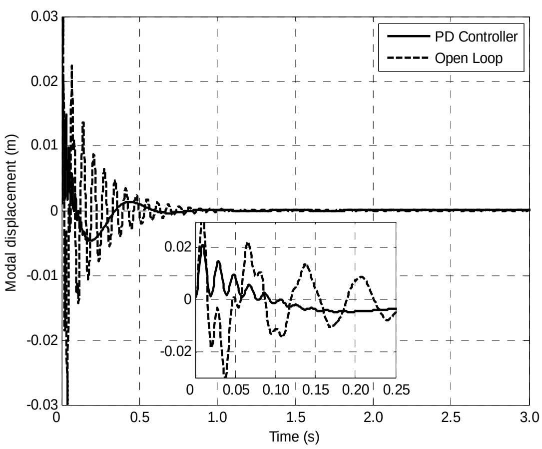
Figure 7. Response of the flexible manipulator with LQR Controller.



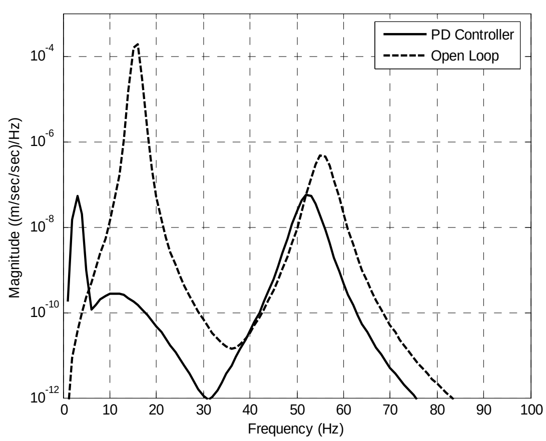
(a) Angular position.



(b) Angular velocity.



(c) Modal displacement.



(d) PSD of modal displacement.

Figure 8. Response of the flexible manipulator with PD Controller.

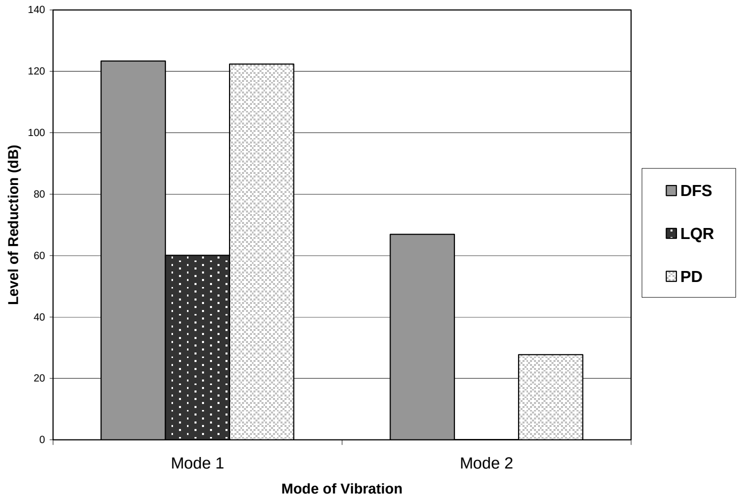


Figure 9. Level of vibration reduction using DFS, LQR and PD controller.