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Estimation for Product Life Expectancy Paramaters under Interval Censored Samples

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Abstract

From basic consept for reliability theory, we computed the moment and maximum likehood estimation for product life expectancy paramaters by means of interval censor data. This is a feasible and efficient estimator for life parameters

Keywords: Exponential distribution, Reliability function, Interval censored data, Moment estimation, Maximum likehood estimation

1. Introduction

In survival analysis and reliable research, often because of the restriction of objective conditions,

the time lapse could not be accurately observed values, which they can only be observed by their interval. Generally this kind of data is called interval censored data . In 1972 Hole and Walburg had the research and the application in the medicine clinical test domain to the interval censored data . In 1991 Keiding and walburg gave the definition of the interval censored data theoretically. When the survival variable turns to the product life, the products to maintain their performance time are an imporant quality indicator, such reliability and product life are linked each other. When the censored variable turns to a time variable, we assume that this variable be a continuous random variable, the probability density function have un-known parameters which need to be estimated. In this paper, by means of interval censored data, we gave the monment estimation and maximum likehood estimation.

2. Suppose that survival variable and the interruption variable all obey the single parameter exponential distribution

Let X be a survival variable, which is a continuous random variable, the probability density function of X is $f(x)$, the distribution function of X is $F(x)$. Suppose that survival variable obeys the single parameter exponential distribution, $f(x)=e^{-x/\theta}/\theta \quad (x \ge 0)$, $f(x)=0 \quad (x<0)$. Let Y be a survival variable, which is a continuous random variable, the probability density function of Y is $g(y)$, the distribution function of Y is $G(y)$. Suppose that survival variable obeys the single parameter exponential distribution, $g(y)=e^{-y/\theta_0}/\theta_0 \quad (y \ge 0), g(y)=0 \quad (y<0)$. Assume X and Y be mutually independent, $\theta > \theta_0$ means that the average life span of censored variables not less than that of test objects . Let $(x_1, x_2, ..., x_n)$ be a simple sample which comes from the overall X, $(y_1, y_2, ..., y_n)$ be a simple sample which comes from the overall Y. When we observe them actually, we can obtain the sample (Y_i, δ_i), where $\delta_i = 1$ $(X_i \le Y_i)$ or $\delta_i = 0 \quad (X_i > Y_i) \quad (i=1,2,...,n)$. Now we consider the monment estimation of interal $X_i \le Y_i$, the mathematical expectation of them

$$EZ = \iint xyf(x)g(y)dxdy = \int_0^\infty\int_x^\infty xy\, e^{-x/\theta}/\theta \; e^{-y/\theta_0}/\theta_0 \, dxdy$$

$$= \int_0^\infty e^{-(\theta+\theta_0)x/\theta\theta_0} \, x/\theta \, dx = \theta_0/(\theta+\theta_0)$$

Let $EZ = \bar{\delta}$, where $\bar{\delta} = \sum_i \delta_i / n$, we have $\bar{\delta} = \theta_0/(\theta+\theta_0)$, then $\bar{\theta} = (1-\bar{\delta})\theta_0/\bar{\delta}$

3. Suppose that survival variable obeys the single parameter exponential distribution and the interruption variable obeys the even distribution

Let X be a survival variable, which is a continuous random variable, the probability density function of X is $f(x)$, the distribution function of X is $F(x)$. Suppose that survival variable obeys the single parameter exponential distribution, $f(x) = e^{-x/\theta} / \theta \quad (x \geq 0)$, $f(x) = 0 \quad (x < 0)$. Let Y be a survival variable, which is a continuous random variable, the probability density function of Y is $g(y)$, the distribution function of Y is $G(y)$. Suppose that survival variable obeys the even distribution, $g(y) = 1/\theta_0 \quad (y \in (0,\theta_0))$, $g(y) = 0 \quad (y \notin (0,\theta_0))$. Assume X and Y be mutually independent, $\theta > \theta_0$ means that the average life span of censored variables not less than that of test objects. Let $(x_1, x_2, ..., x_n)$ be a simple sample which comes from the overall X, $(y_1, y_2, ..., y_n)$ be a simple sample which comes from the overall Y. When we observe them actually, we can obtain the sample (Y_i, δ_i), where $\delta_i = 1 \quad (X_i \leq Y_i)$ or $\delta_i = 0 \quad (X_i > Y_i) \quad (i = 1,2,...,n)$. Now we consider the monment estimation of interal $X_i \leq Y_i$, the mathematical expectation of them

$$EZ = \iint xyf(x)g(y)dxdy = \int_0^{\infty}\int_x^{\theta_0} xye^{-x/\theta} / (\theta\theta_0)dxdy$$

$$= \int_0^{\infty} e^{-x/\theta}(\theta_0 - x)xd\,x / (\theta\theta_0) = \theta - \frac{2\theta^2}{\theta_0}$$

Let $EZ = \overline{\delta}$, where $\overline{\delta} = \sum_i \delta_i / n$, we have $2\theta^2 - \theta_0\theta + \theta_0\overline{\delta} = 0$

$\overline{\theta} = (\theta_0 - \sqrt{\theta_0^2 - 8\theta_0\overline{\delta}}) / 2$ (Charities) then $\overline{\theta} = (\theta_0 + \sqrt{\theta_0^2 - 8\theta_0\overline{\delta}}) / 2$

4. Suppose that survival variable obeys the single parameter exponential distribution and the censored variable is a time variable

Let X be a survival variable, which is a continuous random variable, the probability density function of X is $f(x)$, the distribution function of X is $F(x)$. Suppose that survival variable obeys the single parameter exponential distribution, $f(x) = e^{-x/\theta} / \theta \quad (x \geq 0)$, $f(x) = 0 \quad (x < 0)$. In the moment $t_0 = 0$ we start to admit experimental n-products. In the moment t_1, t_2, …, t_i, we remove for examination, these n-products in the product life have ended to remove, the remaining time puts the latter to continue testing. In the period of $(0,t_1]$, $(t_1,t_2]$, $(t_2,t_3]$, $...,(t_{i-1},t_i]$, we assume that the number of products which products lives are lost be respectively $m_1, m_2, ..., m_i$, the number of their remaining products $n - m_1 - m_2 - ... - m_i = c$. These c-experimental products will be lost their life during (t_i, ∞). Let the interval of $(0,t_1]$, $(t_1,t_2]$, $(t_2,t_3]$, $...,(t_{i-1},t_i]$ be the same, which means $t_j = jt_1 \quad (j = 1,2,...,i)$, during the period of $(0,t_1]$, $(t_1,t_2]$, $(t_2,t_3]$, $...,(t_{i-1},t_i]$, we assume that the probability of $m_1 + m_2 + ... + m_i$ products lives be lost, but c-products lives haven't lost , then we give a reliability function

$$L(\theta) = (\int_{t_i}^{\infty} f(x)dx)^c (\int_0^{t_1} f(x)dx)^{m_1} ...(\int_{t_{i-1}}^{t_i} f(x)dx)^{m_i}$$

Then $$\ln L(\theta) = c\ln\int_{t_i}^{\infty} f(x)dx + m_1\ln\int_0^{t_1} f(x)dx + ... + m_i\ln\int_{t_{i-1}}^{t_i} f(x)dx$$

Let $(\ln L(\theta))'_{\theta} = 0$, we have their maximum likehood estimation

$$\overline{\theta} = t_1 \Big/ \ln[1 + (n-c) \Big/ (ci + \sum_{j=2}^{i} (j-1)m_j)]$$

Let $EX = \overline{X}$, we have the monment estimation $\overline{\theta} = \int_0^{\infty} f(x)dx = \overline{X}$

5. Example

Let the product life obey the single parameter exponential distribution, we extract 12- products to carry on the experiment. When 8-products lives have already finished we stop experiment, the products lives closure time presses the arranged in order is 2, 10, 18, 36, 60, 180, 720, 2880, we discuss the maximum likehood estimate and the monment estimate solution. From the time order 2, 10, 18, 36, 60, 180, 720 and 2880, we know that 8 time compartments separately belong to $(0,2],(4\times 2,5\times 2]$, $(8\times 2,9\times 2]$, $(17\times 2,18\times 2],...,(1439\times 2,1440\times 2]$. Let the time-gap be the same, then $t_i = jt_1$, $t_1 = 2$, $j = 1$, 4, 8, 17, 29, 89, 179 and 1439. Because 8 time compartments products life finished, then $m_1 = m_2 = ... = m_8 = 1$. Sutituting the maximum likehood estimate formula, we have $\overline{\theta} = 460.769$. Sutituting the monment formula we have $\overline{\theta} = 460.769$

6. conclusion

The monment estimate and the maximum likehood estimate to obtain the product life only to be able to small partially to carry on the experiment, the sample which we can take are quite small, but the monment estimate and the maximum likehood estimate to the unknown parameter is a kind of feasible and efficient estimate method.

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