Modern Applied Science
www.ccsenet.org/journal.html
Vol. 2, No. 4
July 2008

Discussion on Computation of Prestressed Concrete Beam's Bearing Capacity

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Abstract

Based on the producing mechanism of prestressed equivalent load, the prestressed effect of the frame structure is analyzed. This paper focuses on the functional characteristics of presstressed steel bars and classifies the contribution of prestressed steel bars at two aspects. Accordingly, the ultimate normal section load-bending capacity of prstressed beam is derived; the restriction effect which will counteract prestressed frame beam caused by secondary beam is presented, and the deficiency of prevail plane rigid frame method is pointed out.

Keywords: Prestressed concrete beam, Bearing capacity, Lateral confinement

1. Analysis of pre-stressed steel bars' operational mechanism

The use of pre-stressed concrete is now common in structures design. And with the increasing use of new construction, there is a need for a closer look at its design and analysis. Many experimental and analytical studies have been carried out in the past five decades.

Introducing concepts of primary moment, secondary moment & comprehensive moment into computation of statically indeterminate prestressed concrete structures brings difficulty to those structures' design. In order to profoundly reveal the essence of prestressed force, this paper analyses the role of prestressed steel bars from two aspects.

The first function of prestressed steel bars arises from the beginning of stretching prestressed bars to the establishment of effective stress σ_{pe}; at that time, the preformed holes weren't filled with grout, the prestressed reinforcements weren't bonded with concrete, either. After the stretching finished, the system comprised of prestressed reinforcement & anchor device acts on the segregate body including concrete & non-prestressed reinforcement, which can be identified as equivalent load. Prestressed reinforcement plays an active role, so prestressed force and internodes equivalent loads evoked by the action are recognized as exterior loads. By now, the effect of prestressed bar-anchor system on segregate body has fully developed in the form of equivalent resultant force. Despite no cementation between prestressed reinforcement & concrete, prestressed force still applies on the segregate body consisting of concrete & non-prestressed reinforcements after prestressed bars being stretched.

Grouting materials are injected into prepared holes after prestressed reinforcements being stretched. When the grouts condensate and get rigidity, prestressed reinforcements adhere to surrounding concrete, shaping the whole, resisting force and deforming together. As prestressed reinforcement reaches ultimate strength, the redundant tensile strength$\left(f_{py}-\sigma_{pe}\right)$, the surplus part of construct tensile strength value f_{py} exceeding effective prestressed force σ_{pe}, passively provides resisting force like ordinary reinforcement. This is the second function of prestressed reinforcement. The trend lines of prestressed reinforcement's stress are shown in Figure1.a.

In Figure1.a, point o' is related to σ_{pe}. During the process of the surrounding concrete's being strained, the stress of prestressed reinforcement moves along the stress-path 1 until reaching yield stress f_{py}. However, if the stress-strain relationship is idealized as double fold lines, the stress of prestressed reinforcement can be considered yielding at f_{py}, and still keeping f_{py} despite increment of the strain. When the surrounding concrete bears more compression force, the stress in prestressed reinforcement will change along path 2. Increase & decrease of stress-strain in bonded bar is shown in Figure1.b, where origin of coordinate is located on point o'. Unlike stage 1, the change of stress on certain section will not expand to the whole bar, and the increment of stress on this section is only determined by its section internal force. Therefore, the effect of prestressed reinforcement equals to the homalographic common bar whose tensile yield stress is $\left(f_{py}-\sigma_{pe}\right)$.

Generally, σ_{pe} is a demarcation point: prestressed steel bar is an agent which acts on the structure in the form of equivalent load on one hand; on the other hand, it is a receiver supplying redundant load bigger than σ_{pe}.

What more we need to declare here is how to choose the prestressed bars' ultimate strength. Since prestressed bar has no visible yield step, which presents difficulty to accurate calculation of prestressed beam. In China Code for Design of Concrete Structures (GB 50010-2002), constrained yield stress $f_{0.2}$ is used as nominal yield stress on the base of material stress-strain curve. Prestressed reinforcement usually can reach nominal yield stress to the damage of beam with appropriately reinforcement, so it is simple and safe to replace the yield stress of high-strength material f_{py} by $f_{0.2}$, ignoring the part of stress exceeding f_{py}.

2. The equation of bearing capacity on cross-section

In the formula of PRC (prestressed reinforcement concrete) structure's cross-section bearing capacity calculation, the problem why and how to take the secondary moment into account puzzled people in a long term, which also hindered the development of PRC structure. This paper analyses PRC structure cross-section bearing capacity on the base of exploring the prestressed reinforcement's mechanism of action.

M_D represents design bending moment; M_p represents the control section moment under the load of end prestressed force and its secondary internode load; N_p expresses axial load(compression force is positive). Both design load and equivalent load are applied on structure; accordingly the bending moment on this section is $M = M_D + M_p$. By now, the first function has been considered. And then, the bearing capacity of section can be calculated by substituting common steel bar for prestressed reinforcement with same area and at the same location. We can select a differential segment as the free body, shown in Fig.2.

Those are the equations of bearing capacity on cross-section acquired from the balanced equations for the select free body.

$$\begin{cases} M_D + M_p = A_s f_y\left(h_s - \dfrac{x}{2}\right) + A_p\left(f_{py} - \sigma_{pe}\right)\left(h_p - \dfrac{x}{2}\right) + N_p\left(h_p - e_p - \dfrac{x}{2}\right) \\ f_{cm} bx = N_p + A_s f_y + A_p\left(f_{py} - \sigma_{pe}\right) \end{cases} \tag{1}$$

3. The influence of lateral confinement action on the bearing capacity of PRC beam's cross-section

The structure that we choose in deducing process has not any lateral confinement. That is to say, neither connected member nor pedestal influences compression deformation of beam and constrains axial deformation. Therefore, the axial force in beam section always equals to the arithmetic product of effective prestressed force and area of pre-stressed reinforcement, $N_p = A_p\sigma_{pe}$. It is acceptable to continuous beam and frame beam connecting columns whose rigidity is smaller. The bearing capacity equation of those structures fit with the China Code for Design of Concrete Structures (GB 50010-2002). However, in practical structure, vertical construction member functions as lateral confinement more or less, which hampers the transmission of axial force in horizontal bending member. The bigger & harder the vertical member is, the more visible the effect is. There is $N_p < A_p\sigma_{pe}$, so it is incorrect to adopt this method into calculation of beam with lateral confinement.

As to frame construction, its top & bottom layers' prestressed beam's final axle force effect is smaller than polar pre-applied force, i.e. there is axial loss of prestress. Main reason of this loss is that pillar has lateral rigidity, and pillar's lateral limitation restricts horizontal prestress's transmission to middle part of beam. In prestressed frame construction, the equivalent load of inter-segment and polar concentrated moment affect little to beam's axial force. Moreover, the influence to beam's axial prestress is also small when in frame construction pillar's rigidity is low and stride few. But as to pillars of frame constructions with numerous layers (for example, high-level frame construction), because prestressed beam span is large, large distance between pillars causes pillars' larger load area, and the large number of layers, the result is the pillars' axial force is quite great. Size of pillars' sections are decided by axial pressing ratio, such sections are often big. The component section's rigidity is proportional with its height's cubic powers; the larger the section's height, the greater the components' rigidity. Size of frame beam's section is generally determined by the span, the section' height is usually 1/15~1/20 of the span. The pillars' rigidity here is possibly much larger than the beam's. Because rigidity of the pillars' section is great, the axial loss of pre-stressed force is great. The more rigid the pillars are and the more the continual spans are, the more the loss is.

Under action of axial pre-applied force, beam does not counter-arch, thus there won't be spatial effect caused by secondary beam's connection. Therefore, analysis of axial pre-applied force could adopt the plane frame. When analyzing pillars' influence, the pre-applied force's vertical influencing scope should first be made clear, i.e. how many layers of pillars the beam's polar pre-applied force transmits through. Scope and degree of axial pre-applied force's influence are related with pillars' rigidity. As to prestressed beam, there's a lid on its top when it's pulled by pre-stressed force. Under polar funneling effect, the beam makes axial translation, thus produces pre-stressed force in middle of the beam, simultaneously drives the connected top and bottom pillars' displacement; pillars hinder beam's axial deformation by its anti-side rigidity and participate axial pre-stressed force's distribution through form of anti-cutting. The pre-applied forces assigned by top pillars completely transmit to upper pillars through upper nodes, so the upper prestressed force's range of influence is definite. As to pillars, the pre-applied force they assigned transmits to lower pillars. Shearing force of farther pillars could be neglected. When beam of certain layer is pulled, its nearest upper & lower pillars' shearing forces are greater; shearing forces of pillars one layer farther are small; axial pressure in the neighboring layer's beams are large while axial forces in beams one layer farther are very small (and they are pulling force). Therefore, it could be demonstrated that the pre-stressed effect transmitted through pillars only need to take the neighboring upper & lower pillars into account, i.e. pillars' rigidity's influencing area is mainly at upper & lower layers while transmits through shearing force of beams' & pillars' nodes when influencing.

Due to the effect of column lateral confinement, the axial force in beam is less than the effective prestressed force, that is $N_p < A_p\sigma_{pe}$. The main axial force in section of beam can be expressed as $N_M = A_p\sigma_{pe}$. Then the secondary axial force is obtained $N_S = N_p - N_M < 0$. Obviously it is tension. Taking $N_p = N_S + N_M$ into the equation 1 and assuming $M_p = -A_p\sigma_{pe}e_p$, $N_M = A_p\sigma_{pe}$, we can get the equation 2.

$$\begin{cases} M_D + M_p = A_s f_y\left(h_s - \frac{x}{2}\right) + A_p\left(f_{py} - \sigma_{pe}\right)\left(h_p - \frac{x}{2}\right) + N_S\left(h_p - e_p - \frac{x}{2}\right) \\ f_{cm}bx = N_p + A_s f_y + A_p\left(f_{py} - \sigma_{pe}\right) \end{cases} \quad (2)$$

Because the secondary axial force is tension, what the section of prestressed frame beam bears is stretch bending force. Equation 2 accords with the rule in China Code for Design of Concrete Structures (GB 50010-2002). As a result, the section bearing capacity of stretch bending member is lower than that of pure bending member. The secondary axial force in beam originates from the secondary shear force in column, and its value increases with the resistance rigidity to lateral bending of column and the prestressed force absorbed by it. It is unsafe to bearing capacity and unpractical to crack resistance to directly apply the equation induced from the structure of continuous beam to the frame beam.

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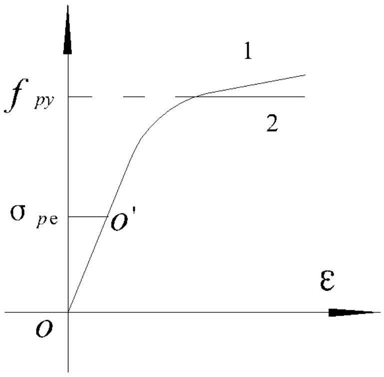

Figure 1. a Stress-strain curve of prestressed reinforcement

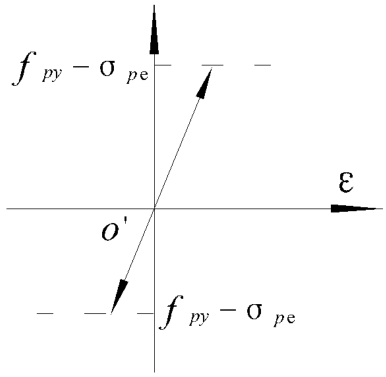

Figure 1. b Ideal stress-strain curve of prestressed reinforcement

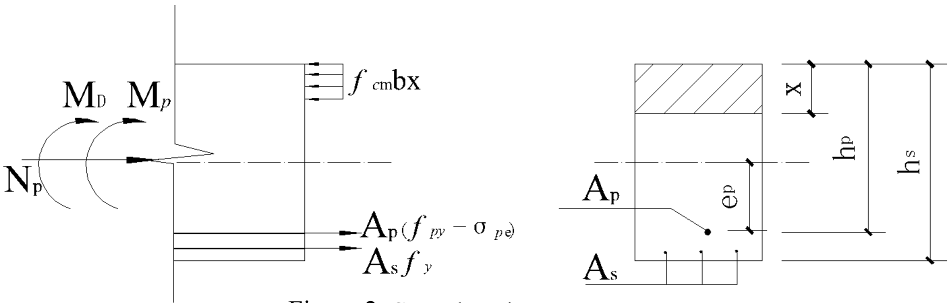

Figure 2. Control section