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Robust Tracking Control of Robot Manipulator Using Dissipativity Theory

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Abstract

The robust H_∞ controller is designed for the problem of rigid robot tracking, which based on the dissipativity theory. The quadratic form dissipative feedback control law was given for interference suppression under the condition of existing model error and external disturbance. The scheme improved the robustness of the system. The simulation results show that the algorithm can achieve rapid tracking of the robot system.

Keywords: Robot, Passivity, Dissipativity, Robustness

1. Introduction

Robot system is a very complex multi-input multi-output nonlinear system with time-varying, and the strong coupling of nonlinear dynamics. dissipativity theory has been put forward by the people's concern and reached a wide range of research results (Van der Schaft A.J., 1999; Feng, 1998) in the 1970s. Its substance is that the internal energy system loss is always less than the external energy supply rate. H_∞ control (Mei,2003)and passive control are a special cases of dissipative control. The H_∞ control is a kind of interference suppression control, which can not only guarantee the stability of the system, but also achieve smallest degree requested by the interference to system output. If the supply rate is the product of input and output, the state will be passivity problem. Passivity theory has been widely used in many engineering problems, such as electrical systems and thermal power systems. A strict passive dynamic system generally has excellent dynamic characteristics and satisfactory robustness. (Feng, 1999,pp. 577-582)

Stabilization controller is designed by using passivity theory on the definite part of the robot system, and then uses the dissipativity theory for interference suppression under the condition of existing model error and external disturbance. The scheme enhances the robustness of the robot tracking, at the same time, and improves the tracking accuracy and speed.

2. Dissipative Control for Robot

2.1 Model Control Law Design

Considering the following n-degree of robot, the dynamic equation is as follows

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \tag{1}$$

Where the vector $q(t)$ is the $n\times 1$ joint angle; $M(q)$ is the $n\times n$ symmetric positive definite inertia matrix; $C(q,\dot{q})\dot{q}$ is the $n\times 1$ vector of coriolis and centrifugal torques; $G(q)$ is the $n\times 1$ vector of gravitational torques; τ is the $n\times 1$ vector of actuator joint torques.

The description of robot systems in Eq (1), has the following characteristics:

Property 1: $M(q)$ is a bounded, symmetric positive definite matrix, its inverse matrix is a bounded. There are positive number λ_1 and λ_2, $0<\lambda_1 I \le M \le \lambda_2 I$.

Property 2: $\dot{M}(q)-2C(q,\dot{q})$ is skew symmetric matrix, both M an C in Eq (1) satisfy the following equations

$$x^T\left[\dot{M}(q)-2C(q,\dot{q})\right]x=0$$
$$\dot{M}=C(q,\dot{q})+C(q,\dot{q})^T$$

Suppose that the desired trajectory of system is described by q_d, $\dot{q}_d$ and $\ddot{q}_d$, then the corresponding error is defined as $e=q-q_d$, $\dot{e}=\dot{q}-\dot{q}_d$, $\ddot{e}=\ddot{q}-\ddot{q}_d$

The τ can be given as

$$\tau=u+M(q)\ddot{q}_d+C(q,\dot{q})\dot{q}_d+G(q) \tag{2}$$

u is a control input signal.

For system in Eq(1), we can give non-linear compensation in Eq(2), receive error dynamic equation:

$$M\ddot{e}+C\dot{e}=u \tag{3}$$

The state vector is defined:

$$x_1=e, x_2=\dot{e}+e \tag{4}$$

Obviously, when $\lim_{t\to\infty} e=0$, $\lim_{t\to\infty}\dot{e}=0$, if and only if

$$\lim_{t\to\infty} x(t)=0 \tag{5}$$

In the new coordinates, and Eq(3) can be translated into the following equation of state:

$$\begin{cases}\dot{x}_1=-x_1+x_2\\ M\dot{x}_2=-(M-C)x_1+(M-C)x_2+u\end{cases} \tag{6}$$

The output signal is defined:

$$y=x_2 \tag{7}$$

Choose the form of Lyapunov function as follows:

$$V(t,x)=\frac{1}{2}x_1{}^T x_1+\frac{1}{2}x_2{}^T M(q)x_2 \tag{8}$$

Along the state trajectory, and its time derivative is as follows:

$$\begin{aligned}\dot{V}(t,x)&=x_1{}^T\dot{x}_1+\dot{x}_2{}^T M(q)x_2+\frac{1}{2}x_2{}^T\dot{M}(q)x_2\\ &=x_1{}^T\dot{x}_1+x_2{}^T\left\{M(x_2-x_1)+Cx_1+u\right\}+\frac{1}{2}x_2{}^T\left\{\dot{M}(q)-2C\right\}x_2\\ &=-x_1{}^T x_1+x_2{}^T\left\{M(x_2-x_1)+Cx_1+x_1+u\right\}\end{aligned}$$

The feedback control law is as follows

$$u=\beta(x)+v \tag{9}$$

With

$$\beta(x)=-M(x_2-x_1)-Cx_1-x_1$$

So $\dot{V}(t,x) = -x_1^T x_1 + y^T v \le y^T v \quad \forall v$

This shows that the closed-loop system is passive from the input, viz. v, to the output, viz. y. According to relations of passivity and asymptotic stability, let $v = -y = -x_2$, the closed-loop system is gradual and stable.

The equation of state (6) is replaced by:

$$\begin{cases} \dot{x}_1 = -x_1 + x_2 \\ \dot{x}_2 = -M^{-1}(x_1 + Cx_2) + M^{-1}v \\ y = x_2 \end{cases} \tag{10}$$

If formula (10) is passive, which can be written for the general form:

$$\begin{cases} \dot{x} = f(x) + g(x)v \\ y = h(x) \end{cases} \tag{11}$$

According to a KYP lemma, then:

$$\begin{cases} \dfrac{\partial V}{\partial x} f(x) \le 0 \\ \dfrac{\partial V}{\partial x} g(x) = h^T(x) \end{cases} \tag{12}$$

2.2 Robust Controller Design

Consider the model error and external disturbances, the robot model:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau + \omega \tag{13}$$

The equation of state:

$$\begin{cases} \dot{x}_1 = -x_1 + x_2 \\ M\dot{x}_2 = -(M-C)x_1 + (M-C)x_2 + u + \omega \end{cases} \tag{14}$$

Combining (9) and (14), we have

$$\begin{cases} \dot{x}_1 = -x_1 + x_2 \\ \dot{x}_2 = -M^{-1}(x_1 + Cx_2) + M^{-1}(v + \omega) \\ y = x_2 \end{cases} \tag{15}$$

Theorem: If the definite part (10) of system (15) is the passive, for any positive number γ, system (15) is the dissipative for the supply rate $s(\omega, y) = \gamma^2 \|\omega\|^2 - \|y\|^2$, under the condition of the negative feedback:

$$v = -y - \frac{1}{4\gamma^2} y \tag{16}$$

where $V(x)$ is that the energy storage function.

Proof: give a general form for system (15)

$$\begin{cases} \dot{x} = f(x) + g(x)(v + \omega) \\ y = h(x) \end{cases} \tag{17}$$

For any positive number γ, combining formula (16), hence

$$\dot{V} = L_f V + L_g V(v + \omega) = L_f V - y^T y - \frac{1}{4\gamma^2} y^T y + y^T \omega$$

$$\le L_f V - \|y\|^2 - \frac{1}{4\gamma^2} y^T y + \gamma^2 \|\omega\|^2 + \frac{1}{4\gamma^2} y^T y$$

$$= L_f V - \|y\|^2 + \gamma^2 \|\omega\|^2 \le -\|y\|^2 + \gamma^2 \|\omega\|^2$$

So the closed-loop system is dissipative on supply rate $s(w, y) = \gamma^2 \|\omega\|^2 - \|y\|^2$. $V(x)$ is the storage function.

The closed-loop system is dissipative on $s(w,y)=\gamma^2\|\omega\|^2-\|y\|^2$ supply rate. That is to say, the γ is rejection ratio of closed-loop system from interference ω to output y, that is H_∞ control.

3. Simulation Research

In order to verify the control strategy, the objects are simulated, based on the MIMO dynamics model of two-DOF robot manipulator (Liu, 2005).

The dynamics model is given by

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q)=u(t)+\omega$$

where

$$M(q)=\begin{bmatrix} m_1l_1^2+m_2(l_1^2+l_2^2)+2m_2l_1l_2\cos(q_2) & m_2l_2^2+m_2l_1l_2\cos(q_2) \\ m_2l_2^2+m_2l_1l_2\cos(q_2) & m_2l_2^2 \end{bmatrix},\ C(q,\dot{q})=\begin{bmatrix} -m_2l_1l_2\dot{q}_2\sin(q_2) & -m_2l_1l_2(\dot{q}_1+\dot{q}_2)\sin(q_2) \\ m_2l_1l_2\dot{q}_1\sin(q_2) & 0 \end{bmatrix},$$

$$g(q)=\begin{bmatrix} (m_1+m_2)l_1g\cos(q_1)+m_2l_2g\cos(q_1+q_2) \\ m_2l_2g\cos(q_1+q_2) \end{bmatrix},$$

$m_1=4kg$, $m_2=2kg$, $l_1=1m$, $l_2=1m$, g=9.8 is acceleration of gravity

The model error and external disturbance: $\omega(t)=\begin{bmatrix} q_1\dot{q}_1\sin(t) & q_2\dot{q}_2\cos(t)\end{bmatrix}^T$

The position orders of joint 1 and joint 2: $q_{1d}=\sin(t)$; $q_{2d}=\cos(t)$

The initial joint angle position: $\begin{bmatrix} q_1(0), & q_2(0)\end{bmatrix}^T=\begin{bmatrix}0.1 & 0.9\end{bmatrix}^T$

The initial angular speed: $\begin{bmatrix} \dot{q}_1(0), & \dot{q}_2(0)\end{bmatrix}^T=\begin{bmatrix}0 & 0\end{bmatrix}^T$

The rejection ratio from interference ω to output y: $\gamma=0.03$

Simulation results show that the application of dissipative control algorithm can ensure the smooth controller and gradual convergence of the tracking error.

4. Conclusion

In this paper, considering this robot system in model error and external disturbances conditions, robust H_∞ tracking controller is designed by using disposition theory. The H_∞ tracking controller can effectively inhibit interference, and the robot system can achieve quickly and accurately tracking.

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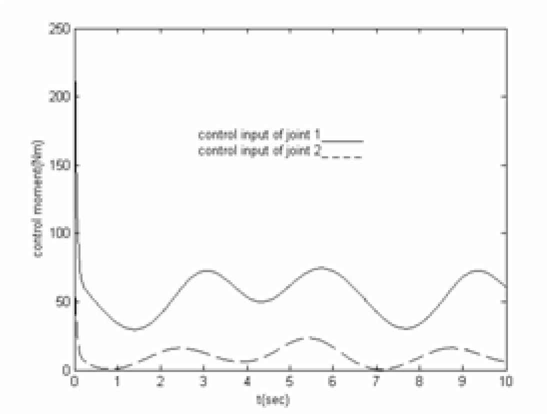

Figure 1. The control input of joint 1 and joint 2

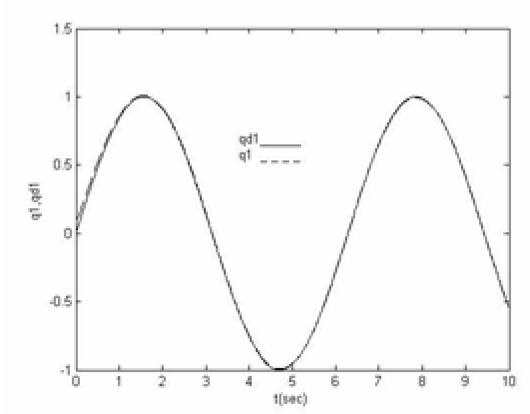

Figure 3. The tracking of joint 1

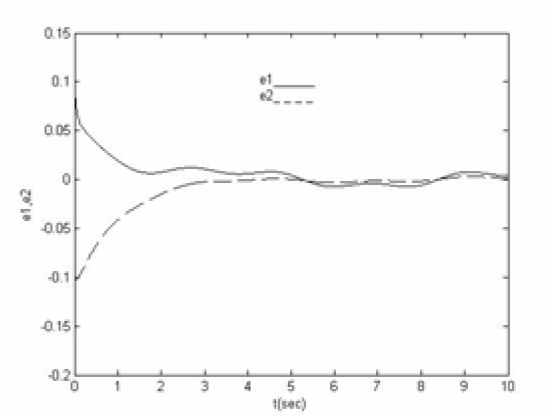

Figure 2. The tracking error of joint 1 and joint 2

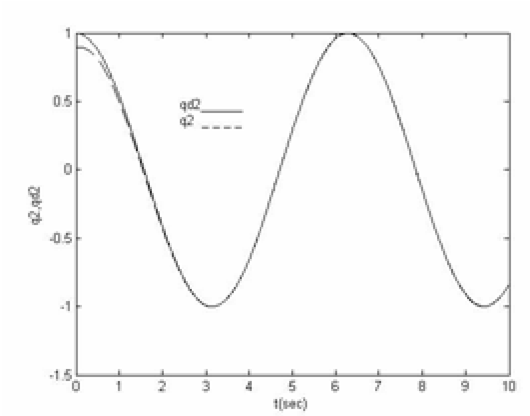

Figure 4. The tracking of joint 2