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Scattering Properties of Spin-polarized ^{3}He-HeII Mixtures at Low Temperature

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Abstract

The cross sections and the man free path of ^{3}He$^\uparrow$ atoms in HeII background are calculated. The Ramsauer-Townsend effect in the cross sections is found in these mixtures at low temperatures. In the low energy limit the cross sections are dominated by the S-wave scattering. The influence of S-scattering decreases with increasing magnetic field. With increasing k, S-wave scattering tends to decrease; whereas the contribution of the higher angular-momentum waves (especially P-wave) to the scattering increases. Our results of the mean free path exhibit the ^{3}He$^\uparrow$ atoms propagate through the HeII background with an exceedingly long mean free path and without friction.

Keywords: spin-polarized ^{3}He-HeII mixture, total, diffusion and viscosity cross sections, mean free path, Ramsauer-Townsend effect

1. Introduction

Spin-polarized ^{3}He-HeII mixture is an interesting quantum many-body. Theoretically, this system provides us with testing ground for the underlying quantum physics. Experimentally, it possesses completely new specific properties uncharacteristic of pure isotopes of helium, such as giant viscosity (Akimoto et al., 2007).

In this paper, we shall study the effect of the magnetic field on the scattering properties of this system. We shall apply the Lippmann-Schwinger (LS) formalism to our system so as to calculate the cross section using the highly-acclaimed interatomic helium potential HFDHE2 (Aziz et al., 1979; Jazan & Aziz, 1995) as an input.

Spin-polarized ^{3}He-HeII mixture has been studied theoretically from various perspectives. A variational method has been used to determine the viscosity and thermal conductivity of the dilute spin-polarized ^{3}He-HeII mixture (Hampson et al., 1988). The viscosity has been found to depend on the polarization. The magnetokinetic effects have been investigated at arbitrary temperatures, impurity concentrations and magnetic fields (Meyerovich, 1978). In strong magnetic fields, the kinetic coefficients have been found to increase exponentially with the field.

Experimentally, a polarization greater than 99% has been obtained in ^{3}He-HeII mixture at $B \leq 14.8$ T and $T \geq 1.5$ mK (Akimoto et al., 2007). A giant viscosity enhancement is observed using a composite vibrating wire viscometer for a fully spin-polarized ^{3}He-HeII mixture. This large growth of the viscosity with spin polarization is in agreement with the theoretical predictions (Hampson et al., 1988; Bashkin & Meyerovich, 1977; 1978). As a consequence, a large mean free path of ^{3}He$^\uparrow$ atoms in HeII background is found by Akimoto et al. (2007). Another objective of this work is to study the dependence of the mean free path of ^{3}He$^\uparrow$ on the magnetic field.

The rest of the paper is organized as follows. The underlying theoretical framework is presented in Section 2. The results are summarized and discussed in Section 3. Finally, in Section 4, the paper closes with some concluding remarks.

2. Cross Sections of ^{3}He$^\uparrow$-^{3}He$^\uparrow$ Scattering in HeII

The probability for a particle to cross, or to pass through, a unit area surrounding a stationary particle is called the differential cross section. If the force causing the scattering is central, the differential cross section for

spin-polarized fermions with spin S is defined by (Hirschfelder et al., 1954; Passel & Schermer, 1966; Al-Maaitah et al., 2011):

$$\frac{d\sigma}{d\Omega}(\theta)=\frac{1}{2}\left(\begin{array}{l}\frac{1}{2}\left(\frac{S+1}{2S+1}\right)\left(1+\frac{S}{S+1}f_n f_N\right)\left|f(\theta)-f(\pi-\theta)\right|^2\\+\frac{S}{2S+1}\left(1-f_n f_N\right)\left|f(\theta)+f(\pi-\theta)\right|^2\end{array}\right), \tag{1}$$

$f(\theta)$ being the scattering amplitude. Where f_n and f_N being the incident and target polarizations. In our system $f_n = f_N = f$. Here $f(\theta)$ is defined by (Landau, 1996):

$$f(\theta)=\frac{1}{2ik}\sum_{\substack{\ell=0\\ \lambda,\lambda'}}^{\infty}(2\ell+1)\left[exp\left(2i\delta_{\ell,\lambda\lambda'}(k;P,\beta)\right)-1\right]P_\ell(cos\,\theta), \tag{2}$$

$P_\ell(cos\,\theta)$ being the first-kind Lengendre polynomial of order ℓ, and $\delta_{\ell,\lambda\lambda'}$ is the relative phase shift. And λ denotes the spin of the particle ($\uparrow$ or $\downarrow$).

The polarization f can be calculated as

$$f=\frac{\rho_\uparrow-\rho_\downarrow}{\rho_\uparrow+\rho_\downarrow}=\frac{k_{F\uparrow}^3-k_{F\downarrow}^3}{k_{F\uparrow}^3+k_{F\downarrow}^3}. \tag{3}$$

where $k_{F\uparrow}(k_{F\downarrow})$ is the Fermi momentum for the spin-up (spin-down) subsystem and are given by

$$\boldsymbol{k_{F\uparrow}}=\sqrt{\left(\boldsymbol{k_F^2}+\frac{\boldsymbol{2m^*}}{\hbar^2}\boldsymbol{m_B B}\right)}, \tag{4}$$

$$k_{F\downarrow}=\sqrt{\left(k_F^2-\frac{2m^*}{\hbar^2}m_B B\right)}. \tag{5}$$

The Fermi momenta k_F, $k_{F\uparrow}$ and $k_{F\downarrow}$ are

$$k_F=\left(3\pi^2\rho_3\right)^{1/3};\;\; k_{F\uparrow}=\left(6\pi^2\rho_{3\uparrow}\right)^{1/3};\;\; k_{F\downarrow}=\left(6\pi^2\rho_{3\downarrow}\right)^{1/3}. \tag{6}$$

The number density of ^{3}He particles in ^{3}He-HeII mixtures ρ_3 is given by Bradley (1997)

$$\rho_3=\frac{0.6022x}{27.58(1+\alpha x)}\text{Å}^{-3}. \tag{7}$$

From Eqs. (6) and (7) k_F can be calculated at different values of x.

A general expression for integral cross sections is

$$\sigma_n=2\pi\int_0^\pi\left(1-cos^n\theta\right)\frac{d\sigma}{d\Omega}(\theta)\;\;sin\,\theta\;d\theta, \tag{8}$$

where n = 1 corresponds to the diffusion cross section σ_D; θ is the center-of-mass scattering angle. Substituting n = 1 in Eq. (8), we have

$$\sigma_D=2\pi\int_0^\pi\left(1-cos\,\theta\right)\frac{d\sigma}{d\Omega}(\theta)\;\;sin\,\theta\;d\theta \tag{9}$$

In arriving at Eq. (9), we have used the fact that the first integral is even, whereas the second is odd and therefore finishes

$$\sigma_D=2\pi\int_0^\pi\frac{d\sigma}{d\Omega}(\theta)\;\;sin\,\theta\;d\theta=\sigma_T$$

$$\sigma_D = \frac{8\pi}{k^2}\left[\begin{array}{l} \frac{1}{2}\left(\frac{S+1}{2S+1}\right)\left(1+\frac{S}{S+1}f_n f_N\right)\sum\limits_{\substack{\ell(odd)\\ \lambda=\lambda'}}^{\infty}(2\ell+1)\sin^2\left(\delta_{\ell,\lambda\lambda'}(k)\right) \\ +\left(\frac{S}{2S+1}\right)(1-f_n f_N)\sum\limits_{\substack{\ell(even)\\ \lambda\neq\lambda'}}^{\infty}(2\ell+1)\sin^2\left(\delta_{\ell,\lambda\lambda'}(k)\right) \end{array}\right] \tag{10}$$

Finally, the viscosity cross section σ_η is obtained by substituting $n = 2$ in Eq. (8)

$$\sigma_\eta = \frac{4\pi}{k^2}\left[\begin{array}{l} \frac{1}{2}\left(\frac{S+1}{2S+1}\right)\left(1+\frac{S}{S+1}f_n f_N\right)\sum\limits_{\substack{\ell(odd)\\ \lambda=\lambda'}}^{\infty}\frac{(\ell+1)(\ell+2)}{\left(\ell+\frac{3}{2}\right)}\sin^2\left(\delta_{\ell+2,\lambda\lambda'}(k)-\delta_{\ell,\lambda\lambda'}(k)\right) \\ +\left(\frac{S}{2S+1}\right)(1-f_n f_N)\sum\limits_{\substack{\ell(even)\\ \lambda\neq\lambda'}}^{\infty}\frac{(\ell+1)(\ell+2)}{\left(\ell+\frac{3}{2}\right)}\sin^2\left(\delta_{\ell+2,\lambda\lambda'}(k)-\delta^E_{\ell,\lambda\lambda'}(k)\right) \end{array}\right] \tag{11}$$

For ^{3}He where $S=\frac{1}{2}$ and $f_n = f_N = f$, Eqs. (10) and (11) become

$$\begin{aligned} \sigma_T = {} & \frac{3\pi}{k^2}\left(1+\frac{1}{3}f^2\right)\sum_{\ell(odd)}^{\infty}(2\ell+1)\left[\sin^2\left(\delta_{\ell,\uparrow\uparrow}(k)\right)+\sin^2\left(\delta_{\ell,\downarrow\downarrow}(k)\right)\right] \\ & +\frac{2\pi}{k^2}\left(1-f^2\right)\sum_{\ell(even)}^{\infty}(2\ell+1)\sin^2\left(\delta_{\ell,\uparrow\downarrow}(k)\right) \end{aligned} \tag{12}$$

$$\begin{aligned} \sigma_\eta = {} & \frac{3\pi}{2k^2}\left(1+\frac{1}{3}f^2\right)\sum_{\ell(odd)}^{\infty}\frac{(\ell+1)(\ell+2)}{\left(\ell+\frac{3}{2}\right)}\left[\begin{array}{l}\sin^2\left(\delta_{\ell+2,\uparrow\uparrow}(k)-\delta_{\ell,\uparrow\uparrow}(k)\right) \\ +\sin^2\left(\delta_{\ell+2,\downarrow\downarrow}(k)-\delta_{\ell,\downarrow\downarrow}(k)\right)\end{array}\right] \\ & +\frac{\pi}{k^2}\left(1-f^2\right)\sum_{\ell(even)}^{\infty}\frac{(\ell+1)(\ell+2)}{\left(\ell+\frac{3}{2}\right)}\sin^2\left(\delta_{\ell+2,\uparrow\downarrow}(k)-\delta_{\ell,\uparrow\downarrow}(k)\right) \end{aligned} \tag{13}$$

The starting point in computing the ^{3}He-^{3}He cross sections in spin-polarized ^{3}He-HeII mixtures is the determination of the relative phase shifts $\delta_{\ell,\lambda\lambda'}$. This can be done by solving the Lippmann-Schwinger (LS) integral equation using a matrix-inversion technique (Bishop et al., 1977). We shall treat (LS) formalism briefly—just for reference purposes and for defining the quantities involved, since this theory is well described elsewhere (Sandouqa et al., 2006; Sandouqa et al., 2010; Joudeh et al., 2010; Joudeh, 2011).

The basic input is the Campbell effective interaction potential (Campbell, 1967; Sandouqa et al., 2006). For calculating the Campbell potential, we have used the highly-acclaimed interatomic helium potential, HFDHE2 (Aziz et al., 1979; Jazan & Aziz, 1995) which is generally regarded as the most reliable He-He potential.

In case of spin-polarized systems, the t-matrix can be generalized to incorporate the interaction between the magnetic moments of the fermions with an external magnetic field $\vec{B}$ as follows:

$$t^{\lambda\lambda'}(\mathbf{p},\mathbf{p}';s_{\lambda\lambda'},\mathbf{P}) = u(|\mathbf{p}-\mathbf{p}'|)+(2\pi)^{-3}\int d\mathbf{k}\,\frac{u(|\mathbf{p}-\mathbf{k}|)t^{\lambda\lambda'}(\mathbf{k},\mathbf{p}';s_{\lambda\lambda'},\mathbf{P})}{k^2-s_{\lambda\lambda'}-i\eta}\quad . \tag{14}$$

The operator $u \equiv \frac{2\mu^*}{\hbar^2} V \equiv \frac{b}{2} V$, where V is the Fourier transform of the effective bare He-He interaction and μ^* is the effective reduced mass of the interacting pair: $\mu^* = \frac{1}{2} m_3^* = \frac{b}{2} m_3$. Throughout this work we shall use units such that $\hbar = 2m_3 = k_B = 1$, k_B being Boltzmann's constant. The conversion factor was $\frac{\hbar^2}{2m_3} = 8.0425$ K.Å^2. The other parameters shown explicitly in the t-matrix equation – namely, λ, $\boldsymbol{p}$, $\boldsymbol{p}'$, $\boldsymbol{P}$, $s_{\lambda\lambda'}$, and - denote, respectively, the relative incoming momentum, the relative outgoing momentum, the center of mass momentum, the total energy of the interacting pair in the center-of-mass frame. The quantity η is a positive infinitesimal in the scattering region and zero otherwise. $s_{\lambda\lambda'}$ being given by

$$s_{\lambda\lambda'} = b\left(P_0 + \frac{(\mu_\lambda + \mu_{\lambda'})}{2} \right) - \boldsymbol{P}^2 . \tag{15}$$

The chemical potential μ_λ is given by $\mu_\lambda = m_B.\mathbf{B}$, where m_B is the magnetic moment of ^{3}He. This represents the total energy of the interacting pair in the center-of-mass frame, including the magnetic energy, where P_0 is the kinetic energy of the pair (Sandouqa et al., 2006).

The phase shifts can be determined by parametrizing the on-energy-shell t-matrix, $t_l^{\lambda\lambda'}(p, p; s_{\lambda\lambda'}, P) \equiv t_l^{\lambda\lambda'}(p; P)$, as follows (Sandouq et al., 2006; Al-Maaitah et al., 2011):

$$t_l^{\lambda\lambda'}(p; P) = -\frac{4\pi}{p} exp\left(i\delta_{l\lambda\lambda'}^E(p; P)\right) sin\left(\delta_{l\lambda\lambda'}^E(p; P)\right); \tag{16}$$

so that

$$tan\left(\delta_{l\lambda\lambda'}^E(p; P, \beta)\right) \equiv \frac{Im\, t_l^{\lambda\lambda'}(p; P; \beta)}{Re\, t_l^{\lambda\lambda'}(p; P; \beta)}, \tag{17}$$

$Im\, t_l^{\lambda\lambda'}(p; P; \beta)$ and $Re\, t_l^{\lambda\lambda'}(p; P; \beta)$ denoting, respectively, the imaginary and real parts of $t_l^{\lambda\lambda'}(p; P; \beta)$.

3. Results and Discussion

Our results are summarized in Figures 1-7 and Tables 1-7. The principal physical quantities here are the total (= diffusion) and viscosity cross sections for ^{3}He$^\uparrow$-^{3}He$^\uparrow$ scattering in HeII. It was found necessary to include partial waves up to $\ell = 14$ so as to obtain results accurate to better than ~ 0.5%. In our figures, the velocity (upper scale) represents the corresponding velocity v_1 [m/s] of a projectile atom $v_1 = \frac{\hbar k}{m_3^*} = \frac{420.86\, k[\,Å^{-1}\,]}{m_3^*}$ on a stationary target atom (v_2=0) as a function of k [Å^{-1}].

Figures 1 and 2 display σ_T as functions of k at different values B for x = 150 ppm and x = 626 ppm, respectively. σ_T depends on the concentration and magnetic field. It is noted that σ_T tends to decrease with increasing magnetic field in the zero-energy limit; this because of Pauli's principle which forbids atoms to come close enough to have significant interactions. Tables 1 and 2 show $\sigma_T(0)$ at different values of B for x = 150 ppm and x = 626 ppm, respectively. From these figures, we have observed a minimum and a peak structure in the total cross section. The minimum appears as a result of a delicate balance between attractive short-range and repulsive zero-range interactions. The physical observation is that, at a particular value of the ^{3}He$^\uparrow$ collision energy, the total scattering cross section is anomalously small. This is indicative of the Ramsauer-Townsend (RT) effect. Our results for RT are summarized in Tables 3 and 4. These tables show the relative momentum k [Å^{-1}] and the total cross section σ_T [Å^2] at which RT occurs. Further, σ_T has a peak at a particular value of k [Å^{-1}], as shown in Tables 5 and 6. Judging from previous experience, this peak may be interpreted as an indicator

of superfluidity or a quasi-bound state (Bohm, 1979; Alm et al., 1994). Undulations due to the quantum statistics have been resolved for $k > 1$ Å^{-1}. These Undulations in the momentum (energy) dependence of σ_T originate from the indistinguishability of ^{3}He atoms (Cantini et al., 1972; Feltgen et al., 1982). The amplitude of the undulations decreases in the first approximation as the inverse of the relative velocity of the colliding atoms. This undulatory behavior was first noticed by Bernstein (1962; 1963) who also pointed out that the number of undulations was related (semiclassically) to the number of bound states of the potential.

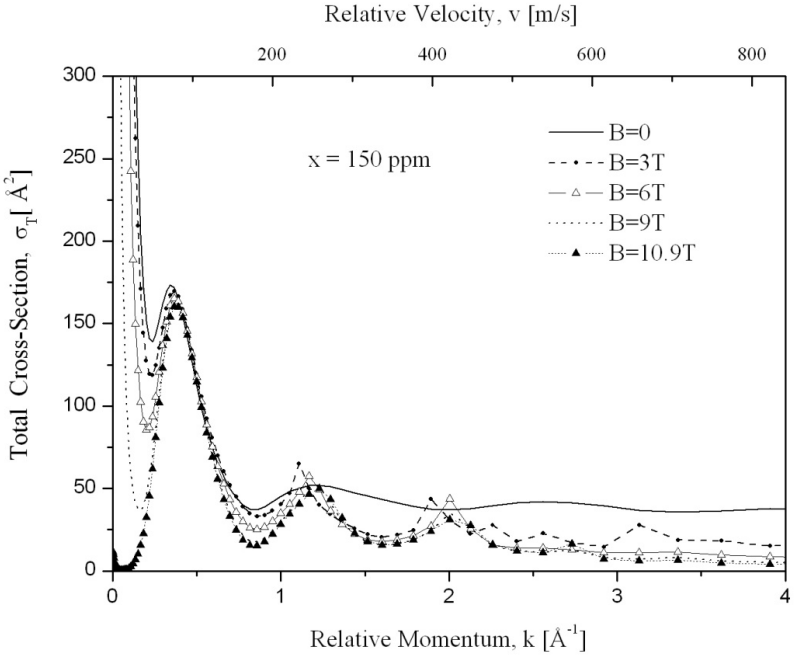

Figure 1. The total cross section σ_T [Å^2] for ^{3}He$^{\uparrow}$-^{3}He$^{\uparrow}$ scattering in HeII as a function of k [Å^{-1}] at x = 150 ppm and different values of B. The upper scale [m/s] represents the corresponding velocity v of a projectile atom on a stationary target atom

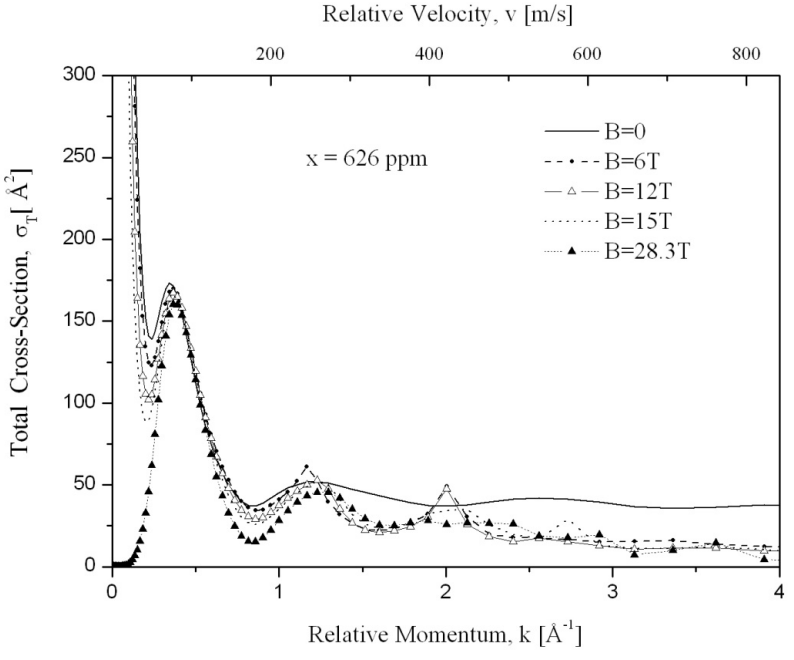

Figure 2. The same as Figure 1; but for x = 626 ppm

Table 1. $\sigma_T(0)$ [Å^2] for ^{3}He$^\uparrow$-^{3}He$^\uparrow$ scattering in HeII for x = 150 ppm at different values of B

B [T]	$\sigma_T(0)$ [Å^2]
0	58140.6
3	48893.9
6	27327
9	6636.3
10.9	11.6

Table 2. $\sigma_T(0)$ [Å^2] for ^{3}He$^\uparrow$-^{3}He$^\uparrow$ scattering in HeII for x = 626 ppm at different values of B

B [T]	$\sigma_T(0)$ [Å^2]
0	58205.5
6	52535.9
12	37889
15	26946.7
28.3	0.091

Table 3. The Ramsauer-Townsend total cross section σ_T [Å^2] for ^{3}He$^\uparrow$-^{3}He$^\uparrow$ in HeII at x = 150 ppm for different values of B

B [T]	k [Å^{-1}]	σ_T [Å^2]
0	0.234	139
3	0.217	118.8
6	0.2	85.6
9	0.166	36.6
10.9	0.062	0.61

Table 4. The Ramsauer-Townsend total cross section σ_T [Å^2] for ^{3}He$^\uparrow$-^{3}He$^\uparrow$ in HeII for x = 626 ppm at different values of B

B [T]	k [Å]	σ_T [Å^2]
0	0.234	138.9
6	0.237	122.9
12	0.218	102.1
15	0.2	88.7

Table 5. The peak in the total cross section σ_T [Å^2] for ^{3}He$^\uparrow$-^{3}He$^\uparrow$ in HeII for x = 150 ppm at different values of B

B [T]	k [Å]	σ_T [Å^2]
0	0.343	173.4
3	0.366	169.6
6	0.366	165.4
9	0.366	161.5
10.9	0.366	160.3

Table 6. The peak in the total cross section σ_T [Å^2] for ^{3}He$^\uparrow$-^{3}He$^\uparrow$ in HeII for x = 626 ppm at different values of B

B [T]	k [Å]	σ_T [Å^2]
0	0.342	173.4
6	0.366	170.05
12	0.366	167.2
15	0.366	165.6
28.3	0.366	160.2

Figure 3 shows σ_T as a function of k at B = 6T for x = 150 ppm and x = 626 ppm. It is noted that the high-concentration total cross sections are less than the corresponding low-concentration cross sections as $k \to 0$ because of the overall less attraction of the V_{eff} due to the overall repulsion of medium effects. For high k, these cross sections are independent of concentration. This is because the kinetic energy part is much larger than the interaction part; therefore the medium effects become negligible, i.e., one can define a *free-atom* cross section appropriate for the energy range where the cross section is a constant.

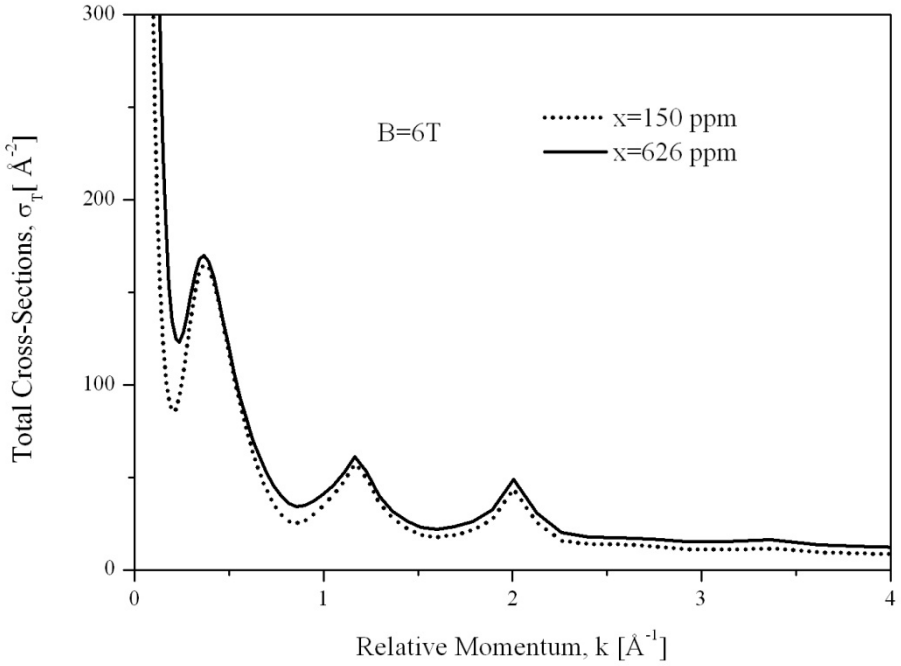

Figure 3. The total cross section σ_T [Å^2] for ^{3}He$^\uparrow$-^{3}He$^\uparrow$ scattering in HeII as a function of k [Å^{-1}] at B = 6T for x = 150 ppm and x = 626 ppm, respectively

Figure 4 shows the behavior of the total cross section σ_T and the ℓ - wave cross section σ_ℓ ($\ell = 0$ - **3**) for ^{3}He$^\uparrow$-^{3}He$^\uparrow$ scattering in HeII as functions of k at x = 150 ppm and B = 6 T. For k < 0.5 Å^{-1}, the S-wave cross section is dominant. With increasing k, S-wave scattering tends to decrease; whereas the contribution of the higher angular-momentum waves, especially P-wave, to the scattering increases. Although many relative partial waves contribute to σ_T, the undulations arise from ***odd-***ℓ scattering. For k >3 **Å^{-1}**, the total cross section is nearly constant. The presence of a potential centrifugal barrier that arises from the orbital angular momentum $\ell = 1$ of the collision leads to the existence of quasi-bound states which manifest themselves as a resonance-like behavior.

Figures 5 and 6 exhibit σ_η as functions of k at different values B for x = 150 ppm and x = 626 ppm, respectively. σ_η has the same behavior as the total cross section. It is found the high-polarization viscosity

cross sections, for $k > 3.5$ Å^{-1}, approach the corresponding low-polarization cross sections. Therefore, for $k > 3.5$Å^{-1}, there are no strong quantum effects manifesting themselves because of the relatively small scattering length for the $^3He^{\uparrow}$-$^3He^{\uparrow}$scattering in ^{3}He-HeII mixtures (Bishop et al., 1977).

Figure 7 shows σ_η as a function of k at B = 6T for x = 150 ppm and x = 626 ppm. It is noted that the effect of concentration is similar to that of a magnetic field. The RT minimum decreases by increasing the concentration due to increasing the repulsive many body effect.

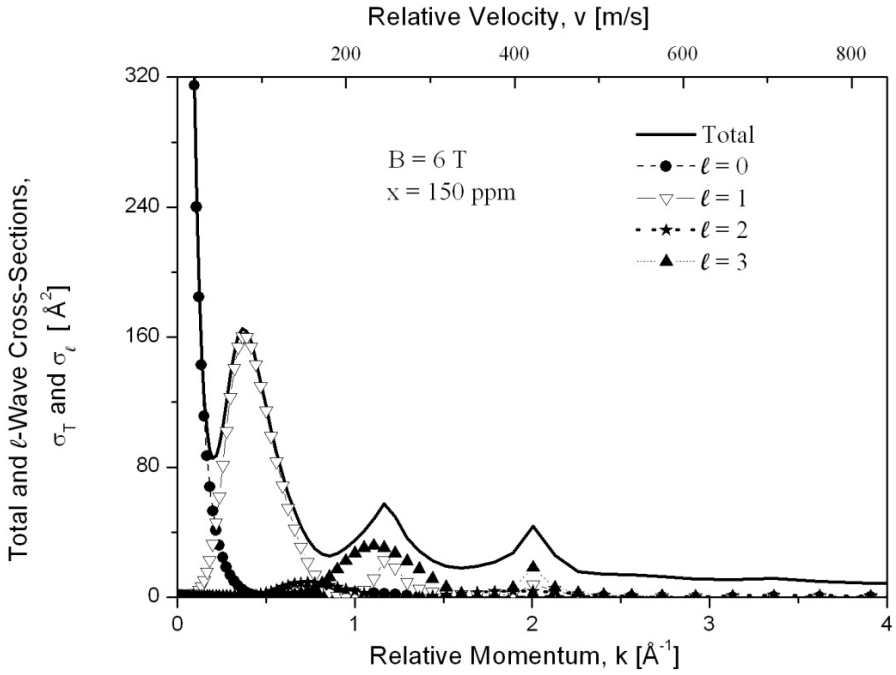

Figure 4. The ℓ -wave effective cross sections σ_ℓ [Å^2], $\ell = 0$ **-** 3 and the total cross section σ_T [Å^2] for $^3He^{\uparrow}$-$^3He^{\uparrow}$ scattering in HeII as functions of k [Å^{-1}] at B = 6T for x = 150 ppm. The upper scale [m/s] represents the corresponding velocity v of a projectile atom on a stationary target atom

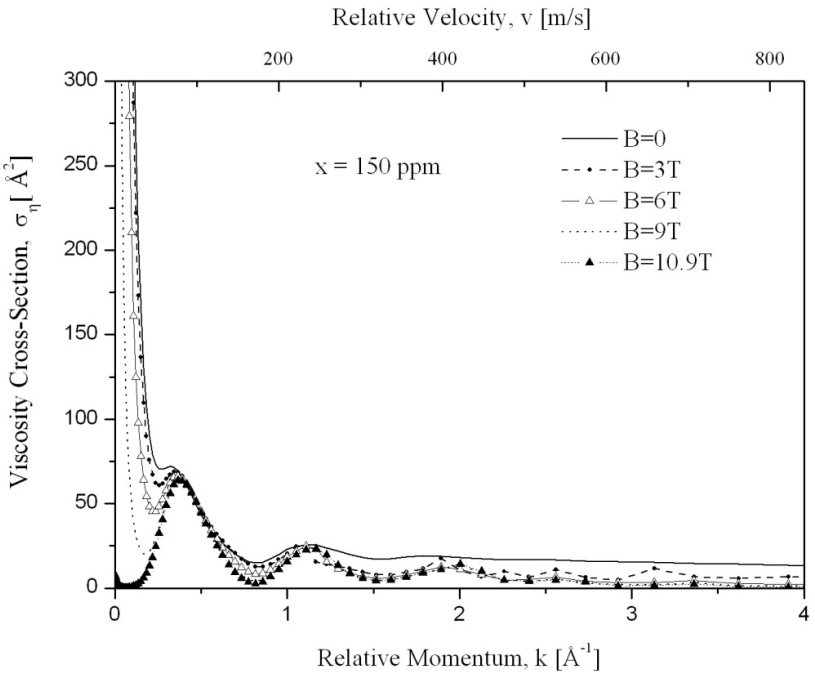

Figure 5. The viscosity cross section σ_η [Å^2] for ^{3}He$^\uparrow$-^{3}He$^\uparrow$ scattering in HeII as a function of k [Å^{-1}] at x = 150 ppm and different values of B. The upper scale [m/s] represents the corresponding velocity v of a projectile atom on a stationary target atom

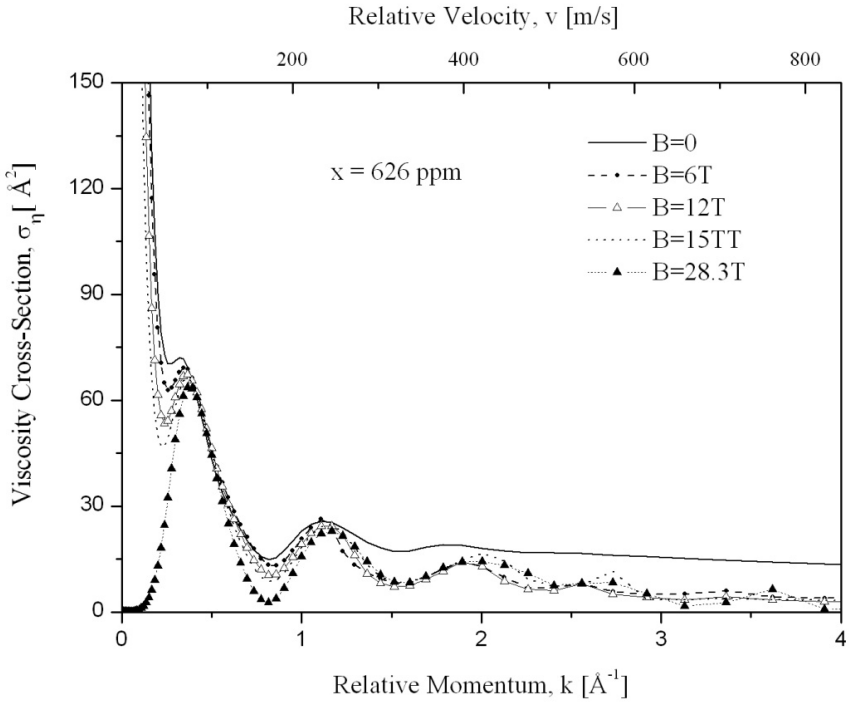

Figure 6. The same as Figure 4; but for x = 626 ppm

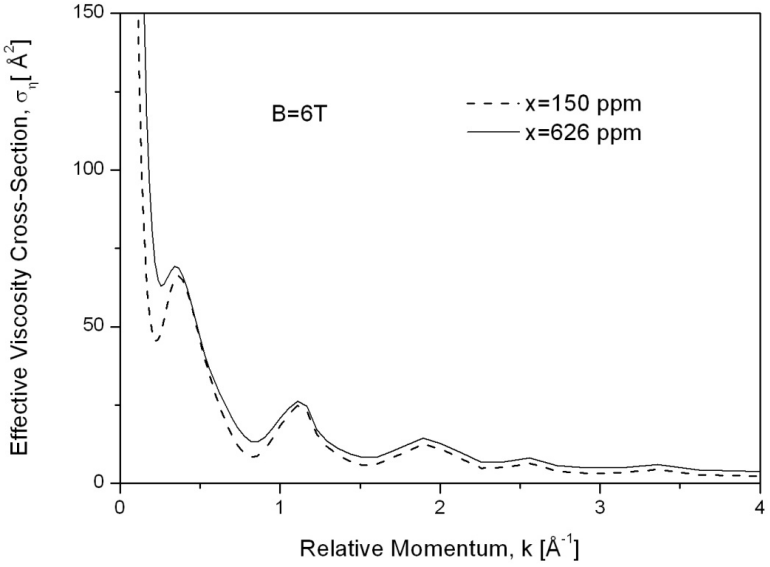

Figure 7. The viscosity cross section σ_η [Å^2] for ^{3}He$^\uparrow$-^{3}He$^\uparrow$ scattering in HeII as a function of k [Å^{-1}] at B = 6T for x = 150 ppm and x = 626 ppm, respectively

In polarized system, there are two relaxation times, one for spin-up ($\tau_\uparrow$) and the other for spin-down ($\tau_\downarrow$). Meyerovich (1982) found the following expressions for relaxation times for polarized system. The relaxation time for the spin-up subsystem $\tau_\uparrow$ is given by Meyerovich (1982):

$$\tau_\uparrow = \frac{5\hbar^3}{\sigma_T(0) m_3^* k_B^3 T^2} \frac{1}{5 - 3\left(\frac{\rho_\downarrow}{\rho_\uparrow}\right)^{\frac{2}{3}}} \frac{\rho_\uparrow}{\rho_\downarrow}; \tag{18}$$

The relaxation time for the spin-down subsystem $\tau_\downarrow$ is given by Meyerovich (1982):

$$\tau_\downarrow = \frac{5\hbar^3}{2\sigma_T(0) m_3^* k_B^3 T^2} \frac{\rho_\downarrow}{\rho_\uparrow}. \tag{19}$$

In the low-temperature limit $\sigma_T(k) \underset{k\to 0}{\to} 2\pi a_o^2$.

Multiplying these relaxation times by the Fermi velocities gives the mean free paths for spin-up subsystem $\ell_\uparrow$ and spin-down subsystem $\ell_\downarrow$. For the spin-up subsystem $\ell_\uparrow$ is given by

$$\ell_\uparrow = v_{F\uparrow}\tau_{F\uparrow};$$

$$\ell_\uparrow = \frac{5\hbar^4}{\sigma_T(0) m_3^{*2} k_B^3 T^2} \frac{k_{F\uparrow}}{5 - 3\left(\frac{\rho_\downarrow}{\rho_\uparrow}\right)^{\frac{2}{3}}} \frac{\rho_\uparrow}{\rho_\downarrow}. \tag{20}$$

And the spin-down subsystem $\ell_\downarrow$ is given by

$$\ell_\downarrow = v_{F\downarrow}\tau_{F\downarrow};$$

$$\ell_{\downarrow} = \frac{5\hbar^4}{2\sigma_T(0) m_3^{*2} k_B^3 T^2} \frac{\rho_{\downarrow}}{\rho_{\uparrow}} k_{F\uparrow}. \tag{21}$$

where $v_{F\uparrow}$ and $v_{F\downarrow}$ being the Fermi velocities for spin-up subsystem and spin-down subsystems, respectively and given by

$$v_{F\lambda} = \frac{\hbar k_{F\lambda}}{m_3^*}. \tag{22}$$

Our results for $\ell_{\uparrow}$ and $\ell_{\downarrow}$ at x = 150 ppm for different values of B are summarized in Table 7. It is found that the mean free path increases by increasing B. It is generally anticipated that the mean free path would increase because the polarization is expected to decrease the number of collisions. The spin-up mean free path $\ell_{\uparrow} \sim 2$ cm at B = 10.3 T in agreement with (Bashkin & Meyerovich, 1981; Akimoto et al., 2007). Bashkin and Meyerovich (1981) predicted that the mean free path reaches tens of centimeters at sufficiently low temperatures and high magnetic field. Akimoto et al. (2007) calculated the mean free path of $^3He^{\uparrow}$ atoms in HeII background from experimental measurements of the viscosity. They found a large ℓ under extremely high magnetic field ($\ell \approx 3$ mm). Thus, $^3He^{\uparrow}$ atoms propagate through the medium as essentially free, unscattered particles. This result indicates the existence of a state known as a *supermobility state* - characterized by the $^3He^{\uparrow}$ atoms moving in the ^{4}He-background with an exceedingly long mean free path and without friction. This state is predicted in dilute ^{3}He-HeII mixtures at very low temperature (Cohen, 1961; Bardeen et al., 1967; Landau et al., 1970; Hoffberg, 1972).

Table 7. $\ell_{\uparrow}$ and $\ell_{\downarrow}$ at x = 150 ppm for different values of B

B [T]	$\ell_{\uparrow}$ [Å]	$\ell_{\downarrow}$ [Å]
0	5.12	5.12
3	9.7	5.19
6	41.7	7.31
9	845.6	18.9
10.9	2.1×10^8	1505.5

4. Conclusion

In this paper, the cross sections and the man free path of $^3He^{\uparrow}$ atoms in HeII background are calculated. In the low energy limit, the cross sections are dominated by the S-wave scattering. The influence of S-scattering decreases with increasing magnetic field; whereas the contribution of the P-wave scattering increases. The achievements are: (1) the prediction of the Ramsauer-Townsend effect in this mixture. (2) the prediction of a phase transition due to resonance-like behavior in the total cross section and (3) studying the dependence of the mean free path of $^3He^{\uparrow}$ on the magnetic field.

In conclusion, our results present a theoretical evidence for the pervious prediction that at high magnetic field and low temperature $^3He^{\uparrow}$ atoms propagate through the HeII background with an exceedingly long mean free path and without friction.

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