Bribe-Giving and Endogenous Partnership in Oligoplies: Some Theoretical Conjectures

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Abstract
As stories of bribe-giving, like corporate contributions to campaign funds and Enron scandals, continue to rock the corporate world, it is imperative that implication of such activities on long-term market outcomes is evaluated. Can bribe-giving have long-term and far-reaching consequences for an oligopolistic market? In this paper we attempt to provide an answer to this question by developing a simple duopoly model that characterizes a perfect Nash equilibrium of bribe-giving. This allows us to establish some perplexing comparative static properties of the market equilibrium. The primary intuition behind our results is that bribe-giving can have serious impacts on the long-run equilibrium of an oligopoly through their effects on the incentives of and constraints on individual firms to form alliances, or partnerships. Bribe-giving, through its effects on individual costs, can trigger a series of endogenously-driven changes in an industry. One of the key changes is the endogenous formation of business alliances (also known as endogenous mergers), which is driven by changes in costs as a direct consequence of bribes. We construct a model of endogenous mergers, to our understanding for the first time, to shed some light on the incentives of firms to endogenously merge in the context of bribe-giving. We show that this can seriously influence the welfare of market participants.

Keywords: Bribe-giving, Perfect Nash Equilibrium

1. Introduction
Since its inception, the main thrust of the literature on forms of corporate corruption like bribery and campaign giving has centred on policy determination in representative democracies (Grossman & Helpman, 1994, 1996). Few studies have attempted to examine the feedback between bribe and campaign giving and overall market outcomes. This paper explores the pros and cons of bribe-giving in the context of a duopoly. We posit that bribe-giving by firms/duopolists typically impact on their cost and demand functions. As bribe-giving can affect the cost of production and hence demand functions, firms can strategically choose bribes in order to influence the market equilibrium. We consider a perfect Nash equilibrium volume of bribe-giving and establish important comparative-static properties of the equilibrium. While this paper considers the case of a duopoly, the framework used here can be easily extended to the case of oligopoly.

It is of utmost importance to realize that bribe-giving have a far-reaching impact on the industrial structure through its effect on cost functions of competitors. Changed cost conditions alter market shares of firms and industrial concentration ratios, which thereby induce firms to take various actions to retain/enhance their market
positions. These actions can take the form of product and technology developments, innovation and strategic acquisition and endogenous mergers. The theory of endogenous merger argues that the decision to merge relies critically on the underlying industrial structure (Note 1). As the industrial structure changes, it alters the incentives and constraints of firms to merge wherefrom one should observe new waves of merger activities. Static models of mergers typically consider mergers between exogenously given firms on the basis of pre-merger and post-merger profits and anticipated anti-trust rulings. We argue that bribe-giving can alter cost functions and thereby open up new possibilities of mergers not captured by the static models of exogenous mergers. The model also highlights entry-inducing mergers associated with bribe-giving.

A bulk of literature on mergers focuses on the cost functions of firms involved in mergers and acquisitions to be the same, which is an inhibiting factor towards a realistic assessment of the affect of a merger on the firm and in the industry in which the merger is considered. To resolve this issue we consider endogenous mergers to which the decision to merge is determined within the constraints of the firms’ operating and production choice. In an early work Gowrisankaran (1999) offers an endogenous and dynamic model of mergers. Gowrisankaran suggests that this type of analysis may be beneficial in assessing antitrust considerations. Previous work on this issue of modeling mergers applied game theory to find the implications on welfare of mergers, e.g., Farrell and Shapiro (1990) examined mergers in a static setting. Gowrisankaran (1999) argues that though a static analysis of the merger process, the conclusion, which may be intuitive to some, may suggest that a merger may lower social welfare through increased concentration and price effects. A dynamic analysis however may show that the increase in concentration in the market will be offset by the increased entry into the industry that may follow mergers. Furthermore, there is a suggestion by Gowrisankaran that static analysis may miss the benefit that a merger may have on an industry where languishing firms are merged into others thus ‘preventing the dead-weight loss of discarding capital without a significant anti-competitive effect’ (Note 2).

Non-endogenous merger analysis will therefore be unable to predict the future welfare and industry effects of current policies. Furthermore Gowrisankaran also argues that the relative profitability of merger versus no merger is not a valid guide to whether a merger may occur. This is the case due to the strategic nature of the market where a firm may hold off merging to see if another firm merges with a third firm since this will have the effect of decreasing competition in the market. Similar issues are discussed in Polasky and Mason (1998) in terms of the ignorance of short-run and long-run effects of mergers in much of the literature where only one shot static games are employed. Ignoring the distinction is a shortcoming in the literature, as firms may not have sufficient time to react to mergers in the short-run. Allowing the merging group to form endogenously in a single merger case has been featured in Kamien and Zang (1990, 1991) and Gaudet and Salant (1992). In a sequential merger scenario, Kamien and Zang (1993) analyze the possible effect of monopolisation in an industry if endogenously formed sequential mergers take place.

The much-celebrated work of Barros (1998) utilizes a model of endogenous mergers that can predict merger partners; i.e., endogenous mergers, from the cost asymmetry of merger participants given the initial degree of market concentration (Note 3). To put it differently, Barros proposes that mergers are endogenously determined by the cost asymmetry. Barros established that cost asymmetry gives rise to mergers. This paper contributes to the existing literature by arguing that bribe-giving affects the level of cost asymmetry and which contributes to merger waves.

The plan of the rest of this paper is as follows. We present a prototype model in Section 2. Section 3 contains comparative-static properties of the equilibrium. A model of endogenous mergers is developed in Section 4 and Section 5 contains some concluding remarks.

2. The Model

We consider a market that has duopolists (hereafter, agents) selling a product to a large number of buyers. The market is captured by a simple sequential game: in Stage I agents engage in choosing bribe-giving that yield financial returns to them. We posit that the effect of bribe-giving materializes in Stage II of the proposed game. The effect is assumed to be threefold. First, bribe-giving by agent $i$ enhances the market position of firm $i$. Secondly, bribe-giving by agent $i$ yields private rents to agent $i$. Finally, the cost function of agent $i$ is influenced by the bribe-giving of agent $j$ and vice versa, which implies some form of competitive bribing. Bribe-giving thus impinges on cost and demand functions in Stage II. In Stage II these agents engage in quantity competition to capture the largest possible market shares. The solution to the game is proposed in a recursive fashion. We first determine the market outcome in Stage II and then trace back to Stage I. Rationality and complete information about the structure of the game dictate that both agents will form their expectations by looking ahead and foreseeing the market outcome of Stage II. If agents behave in this fashion, they are said to have rational expectations. The resultant
equilibrium is the **perfect Nash equilibrium** (PNE) of the proposed sequential game. The PNE is an overall equilibrium in the two-stage model. It is a Nash equilibrium in bribe-giving since each agent's overall profits can be given as a function of their bribes chosen in Stage I. Thus, in essence, the agent's choice of bribe-giving is *ipsa facto* a choice of reaction functions in the postulated duopoly.

**Stage II: Quantity Competition in the Duopoly**

The postulated market is characterized by the following assumptions:

**Assumption 1:** The inverse demand function in the duopoly, without bribe-giving, is linear:

\[ p = a - bX \]  

(1a)

\( X \) is the market output and \( p \) is the price of good per unit and \( a, b > 0 \).

**Assumption 2:** We assume that bribe-giving lends a local monopoly power to these agents paying bribes. Hence, bribe-giving by the \( i_{th} \) agent increases the price of the product that he sells. We define \( p_i \) as the price that agent \( i \) receives if agent \( i \) gives bribe \( T_i \) to an agent/leader of the winning political party in the election. We define \( p_i \) as:

\[ p_i = p_i + \alpha T_i = a - bX + \alpha T_i \]  

(1b)

for \( i = 1, 2 \) and \( j = 1, 2 \). In the absence of bribe-giving, the market turns out to be a bland duopoly with the inverse demand function given by equation (1a). Alternatively, one could have assumed the individual (inverse) demand function to be as (1b) in order to model bribe-giving.

**Assumption 3:** We assume bribe-giving by agent \( j, T_j \), to increase the marginal and average cost of agent \( i \). The cost function of the \( i_{th} \) agent is \( C_i \) is postulated to be:

\[ C_i = (c + \beta T_j)X_i \]  

(1c)

c is the constant marginal cost of production in the absence of bribe-giving. \( T_j \) is the bribe-giving donated by the \( j_{th} \) agent.

Let us try to explore the imports of these two assumptions. First and foremost, in the absence of bribe-giving this model reduces to the archetypal model of duopoly. With positive bribe-giving, more importantly, this model represents a scenario like this: each firm contributes \( T_i \) to the leader of the winning political party. Each firm produces a homogenous product, say oil, or coal, domestically and sells the product to a foreign market. Bribes are a quid-pro-quo as these firms expect direct future returns/favours from the government in exchange for these bribes given at an earlier stage. There are various scenarios that can justify our basic assumptions: as an example, the local government can negotiate with the foreign government for a slice of the foreign market for those firms who paid bribes. This is reflected in equation (1b): ceteris paribus as \( T_i \) increases so does the market spoil for firm \( i \) from the market. Alternatively, the government can secure a better price from the foreign firm, ceteris paribus. In order to sell this product in the foreign market, each firm incurs a transaction cost - maybe a transport cost, and/or import duties. The government can reduce this transaction cost either by direct subsidy, or through negotiations with the foreign government. We also assume that both firms may engage in pressure politics to raise the cost of the rival in order to have a more competitive position in the foreign market. This cost component is reflected in the second assumption as captured by equation (1c). The assumption is that this cost of firm \( i \) is larger the larger is the contribution from its rival, \( T_j \) ceteris paribus. The necessity of these functional forms is to keep calculations tractable. All the results will be through with a general functional form for the profit function of firm \( i \) as in equation (2a):

\[ \Pi_i = f(T_i, T_j) \]  

such that \( \delta \Pi_i / \delta T_i < 0 \) and \( \delta \Pi_i / \delta T_j > 0 \).

Assumption 2 and Assumption 3 together capture the effect of bribe-giving on the competitive positioning of these duopolists. Profits of firm \( i, \Pi_i \) in Stage II are given by:

\[ \Pi_i = aX_i^2 - bX_i^2X_j + cX_i + \alpha X_iT_i - \beta X_iT_j - T_i \]  

(2a)

**Assumption 4:** We posit that each agent at the helm of a firm also derives some pecuniary and non-pecuniary rents from bribe-giving in Stage I. It is assumed that the rent to each agent in Stage I is a constant, ‘r’, per unit of bribe-giving. Hence the rent of agent \( i \) from bribe-giving, \( T_i \), is \( M_i \):

\[ M_i = rT_i \]  

(2b)

We now look at the market outcome at Stage II: maximizing the profit functions in (2a) with respect to \( X_i \) yields the Cournot equilibrium in the duopoly:

\[ X_1^* = [(a-c)+T_2(\alpha+2\beta)-T_1(2\alpha+\beta)]/(3b) \]  

(3a)

\[ X_2^* = [(a-c)+T_1(\alpha+2\beta)-T_3(\beta+2\alpha)]/(3b) \]  

(3b)

\[ X^* = X_1^* + X_2^* = [2(a-c)-(\alpha-\beta)(T_1+T_2)]/(3b) \]  

(3c)
\[
p^* = (a+2c)+(\alpha-\beta)(T_1+T_2)
\]
(3d)
\[
\Pi_i^* = p^* X_i^* + (\alpha T_1 - \beta T_2 - c) X_i^* - T_1
\]
(3e)

It is evident from (3a)-(3f) that the duopoly equilibrium in Stage II critically hinges on the values of \(T_1\) and \(T_2\) chosen in Stage I. This reveals the 'strategic' role of bribe-giving, i.e. the incentive for agents to use bribe-giving to change the rivals' output. This strategic role of bribe-giving is now addressed.

**Stage I: Strategic Role of Bribe-giving**

Bribe-giving has direct pecuniary and non-pecuniary rents to agents in Stage I, bribe-giving also influences the market outcome of Stage II by affecting cost and demand conditions of these agents. Rational decision-makers must consider these effects of their bribe-giving of Stage I on the duopoly outcome of Stage II. The two-stage financial return of agent \(i\), \(R_i\), are:

\[
R_i = M_i + \Pi_i = rT_i + \Pi_i(X_1^*, X_2^*)
\]
(4a)

The first component on the right-hand side of (4a) is the private rent to agent \(i\) in Stage I that is given by equation (2b) of Assumption 4. The second term is the profit of the \(i\)th agent from the duopoly outcome in Stage II and given by equation (3f). For keeping the analysis simple, we assume that the time rate of discount is zero for each agent.

Maximizing profit functions in (4a) with respect to \(T_i\) yields the overall Cournot equilibrium in bribe-giving \((T_1^*, T_2^*)\) in Stage I, whilst \(T_1^*, T_2^*\) are derived from the following first order conditions:

\[
\frac{dT_i}{dR_i} = r + \frac{d \Pi_i}{dT_i} = 0
\]
(4b)

\[
\frac{dT_j}{dR_j} = r + \frac{d \Pi_j}{dT_j} = 0
\]
(4c)

**Proposition 1:** The proposed sequential game has a unique Perfect Nash equilibrium (PNE): in Stage I agents choose optimal bribe-giving, \(T_1^*\) and \(T_2^*\), such that:

\[
T^* = T_1^* = T_2^* = \left(\frac{a-c}{2\alpha - \beta}\right)\left(\frac{5\beta - 2\alpha}{4\beta - \alpha}\right) + \frac{9br}{2(\alpha - \beta)(4\beta - \alpha)}
\]
(5a)

In Stage II these agents sell optimal outputs \(X_1^*\) and \(X_2^*\) in the duopoly:

\[
X_1^* = X_2^* = \frac{a-c}{3b} \left(\frac{\alpha - \beta}{\beta}\right) T^*
\]
(5b)

The equilibrium (total) output \(X^*\) and the equilibrium price \(p^*\) in the duopoly are respectively:

\[
X^* = \frac{2(a-c)}{3b} - \frac{2(\alpha - \beta)}{3b} T^*
\]
(5c)

\[
p^* = \frac{a+2c}{3} + \frac{2(\alpha - \beta)}{3} T^* + \beta T^*
\]
(5d)

**Proof:** \(T_1^*\) and \(T_2^*\) are derived from the simultaneous equations (4b) and (4c). Since the proposed game is a symmetric game, \(T_1^* = T_2^* = T^*\). Substituting the value of \(T^*\) from (5a) into (3a) and (3b) yields (5b). Substituting (5a) into (3c) and (3d) respectively yields (5c) and (5d)(Note 4). Q.E.D.

**3. Comparative Static Properties**

In order to examine the comparative-static properties of the PNE given by equation (5a)- equation (5d), we differentiate these equations with respect to relevant parameters to arrive at the following propositions.

**Proposition 2:** An increase (decrease) in the exogenous demand for the good, labeled as ‘\(a\)’, increases (decreases) the equilibrium bribe-giving \(T^*\) if \(\beta > (2\alpha/5)\). Thus the market buoyancy and the intensity of corporate corruption are directly related. The equilibrium output, \(X_e\), of the duopoly goes up (goes down) as the demand parameter ‘\(a\)’ increases (decreases). The equilibrium price, \(p_e\), of the duopoly increases (decreases) as the parameter ‘\(a\)’ goes up (goes down) if \((\beta+13\alpha\beta-5a2) > 0\).

**Proof:** Differentiating equation (5a) with respect to ‘\(a\)” yields:

\[
\delta T^*/\delta a = (5\beta-2\alpha)[2(\alpha-\beta)(4\beta-\alpha)] > 0
\]
(6a)

Since \((\alpha-\beta)>0\) and from the second order condition we know \((4\beta-\alpha)>0\), and hence \((5\beta-2\alpha)>0\) for \(\beta>2\alpha/5\). We know from the second order condition \((\alpha-\beta)(4\beta-\alpha)>0\). Hence, \(\delta T^*/\delta a>0\) if \(\beta>2\alpha/5\), otherwise, \(\delta T^*/\delta a<0\).
Similarly, we can find out the effect on the equilibrium price as
\[
\frac{\delta p^e}{\delta a} = \frac{[\beta^2 + 13\alpha\beta - 5\alpha^2]/[3(\alpha - \beta)(4\beta - \alpha)]}{(6c) \delta}
\]
Q.E.D.

The impact of changes in ‘a’ on the equilibrium output is unambiguous and expected. The impact of changes in 'a' on the equilibrium price is ambiguous and can be perverse since in any market an increase in demand is expected to increase both the equilibrium price and the equilibrium quantity. This perverse result arises in our model if \((\beta^2 + 13\alpha\beta - 5\alpha^2) < 0\) and, hence, \(\delta p^e/\delta a < 0\). This is due to the effect of a change in ‘a’ on the equilibrium bribe-giving, \(T^*\), that in turn impinges on the equilibrium price.

Proposition 3: An increase (decrease) in cost, \(c\), reduces (increases) the (Nash) equilibrium level of bribe-giving if \(\beta > 2\alpha/5\). These changes have ambiguous and possible perverse effects on the equilibrium output and the equilibrium price of the proposed duopoly.

**Proof:** It is instructive to note that
\[
\delta T^*/\delta c = -(5\beta - 2\alpha)/[2(\alpha - \beta)(4\beta - \alpha)]
\]
Hence, \(\delta T^*/\delta c > 0\) for \(\beta < 2\alpha/5\) and \(\delta T^*/\delta c < 0\) for \(\beta > 2\alpha/5\).

\[
\delta X^e_i/\delta c = 1/(3b)(1 + [5\beta - 2\alpha]/[b(\alpha + 4\beta)])
\]
Hence, \(\delta X^e_i/\delta c < 0\) for \(\beta < 2\alpha/5\) and \(\delta X^e_i/\delta c > 0\) for \(\beta > 2\alpha/5\). Similarly,
\[
\delta p^e/\delta c = 2\beta(\alpha - 1.75\beta)
\]
Hence, \(\delta p^e/\delta c > 0\) for \(\alpha > 1.75\beta\) and \(\delta p^e/\delta c < 0\) for \(\alpha < 1.75\beta\).

Q.E.D.

The conventional wisdom is that an increase (decrease) in cost increases (decreases) the equilibrium price and reduces (increases) the equilibrium output in a duopoly. However, due to the effects of bribes on the output and prices, we note the possibility of perverse effects of changes in cost of production on the equilibrium output and the equilibrium price: \(\delta X^e_i/\delta c > 0\) for \(b < [(5\beta - 2\alpha)/(-\alpha + 4\beta)]\) and \(\delta p^e/\delta c < 0\) for \(\alpha < 1.75\beta\).

Proposition 4: An increase (decrease) in the pecuniary rent, \(r\), from bribe-giving increases (decreases) the equilibrium bribe-giving, increases (decreases) the equilibrium price and reduces (increases) the equilibrium output of each duopolist.

**Proof:** We differentiate equation (5a), (5b) and (5d) w.r.t ‘\(r\)’ to yield the following:
\[
\delta T^*/\delta r = 9b(\alpha - \beta)/4 > 0
\]
\[
\delta X^e_i/\delta r = -(\alpha - \beta)(\delta T^*/\delta r)/(3b) < 0
\]
\[
\delta p^e/\delta r = 2(\alpha - \beta)(\delta T^*/\delta r)/(3b) > 0
\]
Q.E.D.

Proposition 4 highlights the following: as the private rent from bribe-giving increases, each agent increases his bribe-giving that, in turn, raises the total cost of production of each agent. This increase in total cost, induced by an increase in \(T^*\), results in a rise in the equilibrium price and a decline in the equilibrium output of each agent.

### 4. Bribe-Giving and Endogenous Mergers in a Myopic Model

We start off with a simple duopoly in which firms engage in quantity competition. For the sake of tractability the demand function is postulated as \(q = [1 - (p/9)]\) by assuming \(a = 1, b = 1/9\) in the demand function (1a). Both firms have constant and identical average cost of production \(C^*\) in the absence of bribe-giving. It is now assumed that bribes have no impact on prices. Bribes however impact on transaction costs. Different transaction costs cause a difference in cost conditions. It is further assumed that these firms have constant marginal costs \(C_i\) for firm \(i, i = 1, 2\). Thus \(C_1 = C^* + \lambda_1\) and \(C_2 = C^* + \lambda_2\) where \(\lambda_i\) is the transaction cost that is determined by \(T_j\). The model is myopic since firms don’t use \(T_j\)s in a strategic fashion to embark on the most desirable endogenous merger. All firms in the model have made political contributions in a myopic fashion, which means these firms did not attempt to influence the industrial structure by making these bribe-giving. These firms expect some political favours by giving bribes from the political party if it gets elected. Thus firms only consider the short-term effects of bribe-giving on their profits. We assume that \(C_1 < C_2\) since \(T_1 > T_2\). We label the cost difference as \(\Delta\), hence \(\Delta = C_2 - C_1 = \lambda_2 - \lambda_1\). Note that
the transaction costs asymmetry as given by \((\lambda_2 - \lambda_1)\) is determined by an asymmetry in bribes \((T1-T2)\). The profits of these incumbents from the Cournot-Nash equilibrium are:

\[
\Pi_1^* = (1-C_1+\Delta_1)^2
\]
\[
\Pi_2^* = (1-C_2-\Delta_2)^2
\]

Since Firm 1 has a lower transaction cost, \(\Pi_1^* > \Pi_2^*\).

We now consider a possible entry by Firm 3 who has a constant marginal cost \(C_3\). We assume this firm has committed to a prior bribe to the government and \(C_3=C^*+\lambda_3\). We examine exogenous merger possibilities between Firm 1 and Firm 2 as Firm 3 makes an entry- assuming that the merger does not attract flak from the regulatory authority. As Firm 1 and Firm 2 merge into a single entity, the merged entity with the marginal cost \(C_1\) engages in Cournot competition with Firm 3. We represent the cost difference \(C_3-C_1 = \lambda_3 - \lambda_1 = \Delta_1\). The profits from the Cournot-Nash equilibrium of the newly emerged duopoly are the following:

\[
\Pi_{1,2}^* = (1-C_1+\Delta_1)^2
\]
\[
\Pi_3^* = (1-C_3-\Delta_3)^2
\]

Equation (8c) gives the profits of the merged entity.

We now examine the incentives of Firm 1 and Firm 2 to merge. We first consider the possibility that \(C_3 > C_2\). Suppose there is no merger, then the duopoly is turned into a triopoly as Firm 3 makes an entry. We label the cost difference between Firm 2 and Firm 3 as \(\Delta_2\), \(\Delta_2 = C_3-C_2\). The profits of these three firms from the Cournot-Nash equilibrium involving three firms are:

\[
\Pi_1 = 9(1-C_1+\Delta_1+\Delta_2)^2/16
\]
\[
\Pi_2 = 9(1-C_2-\Delta_2)^2/16
\]
\[
\Pi_3 = 9(1-C_3-\Delta_2-\Delta_3)^2/16
\]

A simple comparison will give us the incentives for exogenous merger between Firm 1 and Firm 2.

**Observation 1:** The incumbents - following the entry of Firm 3- have an incentive to merge if \(C_3 > (2C_1+3\Delta_1-1)\).

Proof: Firm 1 and Firm 2 have incentives to merge if \((1c)>(2a)\). That is,

\[
(1-C_1+\Delta_1)^2 > 9(1-C_1+\Delta_1+\Delta_2)^2/16
\]

Simplification of (9d) yields the result. QED.

### 4.1 Endogenous Mergers

The entry of Firm 3 leads to the possibility of three distinct mergers - Firm 1 and Firm 2, Firm 1 and Firm 3, Firm 2 and Firm 3. Assuming \(C_1 < C_2 < C_3\) and retaining the previous symbols we label the post-merger profits of Firm i and j by \(\Pi_{i,j}^*\),

\[
\Pi_{1,2}^* = (1-C_1+\Delta_1)^2
\]
\[
\Pi_{1,3}^* = (1-C_1+\Delta_2)^2
\]
\[
\Pi_{2,3}^* = (1-C_2-\Delta_2)^2
\]

From these values we know \(\Pi_{1,2}^* > \Pi_{1,3}^* > \Pi_{2,3}^*\). Thus, both Firm 2 and Firm 3 have an incentive to merge with Firm1 whilst Firm 1 has an incentive to merge with Firm 2.

**Observation 2:** Firm 2 and Firm 3 have an incentive to merge if \(C_3 < (1+2C_2-\Delta_3)/3\).

Proof: Firm 1 and Firm 2 have an incentive to merge if \(\Pi_{1,3}^* > \Pi_{2}^*\):

Substituting (10b) and (9b) yields the result. QED.

From Observation 1 and Observation 2 we offer the first result on endogenous mergers.

**Proposition 5:** Assuming \(C_1 < C_2 < C_3\), Firm 1 and Firm 2 will merge if

\[
C_3>(1+2C_2-\Delta)/3>2C_1+3\Delta-1
\]

Firm 2 and Firm 3 will merge if

\[
C_3<2C_1+3\Delta-1<(1+2C_2-\Delta)/3
\]

There will be no merger if

\[
(1+2C_2-\Delta)/3<C_3<2C_1+3\Delta -1
\]
Firm 1 and Firm 2 will merge if
\[ 2C_1 + 3\Delta - 1 < C_3 < \frac{(1 + 2C_2 - \Delta)}{3} \]  
(11d)
since \( \Pi_1, 2^* > \Pi_1, 3^* > \Pi_2, 3^* \).

Proof: Comparisons of equations (8a), (8b) and (8c) with (9a), (9b) and (9c) respectively will yield the result. QED.

We now consider the case in which \( C_1 < C_3 < C_2 \). That is Firm 3 is more efficient than Firm 2. Let us label the cost difference as: \( C_3 - C_1 = d, C_2 - C_1 = d_1, C_2 - C_3 = d_3 \). The profits of these three firms from the Cournot-Nash equilibrium are:

\[ \Pi_1 = \frac{9(1 - C_1 + d + d_1)^2}{16} \]  
(12a)
\[ \Pi_2 = \frac{9(1 - C_2 - d + d_2)^2}{16} \]  
(12b)
\[ \Pi_3 = \frac{9(1 - C_3 - d_1 - d_2)^2}{16} \]  
(12c)

The post-merger outcomes are

\[ \Pi_{1,3}^* = (1 - C_1 + d_1)^2 \]  
(13a)
\[ \Pi_{1,2}^* = (1 - C_1 + d)^2 \]  
(13b)
\[ \Pi_{2,3}^* = (1 - C_3 - d)^2 \]  
(13c)

while \( \Pi_1, 3^* > \Pi_1, 2^* > \Pi_2, 3^* \). We now offer the result for endogenous mergers.

**Proposition 6:** Assuming \( C_1 < C_3 < C_2 \), Firm 1 and Firm 3 will merge if
\[ d < C_1 + \frac{(1 - C_1 + d_1)}{3} \]  
(14a)
That is
\[ C_3 < \frac{(1 + C_1 + C_2)}{3} \]  
(14a’)
Firm 2 and Firm 3 will merge if
\[ C_3 > 3(C_2 - C_1 - 1) \]  
(14b)
Firm 1 and Firm 2 will merge if
\[ C_3 > 3(C_2 - C_1) \]  
(14c)
There will be no merger if
\[ \frac{(1 + C_1 + C_2)}{3} < C_3 \]  
(14d)
and
\[ 3(C_2 - C_1) < C_3 \]  
(14e)
Proof: These results are derived from the above profit functions. QED.

4.2 Mimicking as a Low Cost Entrant and Endogenous Mergers

In order to introduce signaling and possible mimicking we consider two periods and three firms. At date 1 Firm 1 and Firm 2 form a duopoly and Firm 3 is an entrant. At date 1 as Firm 3 enters there are three firms, who engage in quantity competition, in this market. At date 2 this entry can trigger an endogenous merger that depends on the cost of production of Firm 3 as articulated in subsection 4.1. The entrant (Firm 3) knows its cost from the start while the incumbents do not know Firm 3’s cost. Because Firm 3 prefers to merge with Firm 1, Firm 3 intends to convey the information that it has low cost such that \( C_1 < C_3 < C_2 \), and \( C_3 > \frac{(1 + 2C_2 - \Delta)}{3} > (2C_1 + 3\Delta - 1) \). The problem is that it does not have a direct means to do that, even if the above inequalities are true. The indirect way is to signal by increasing its output. The knotty problem is that Firm 3 may want to increase its output at date 1 even if it has a high cost and the above inequalities do not hold. The loss in the first period profit may be offset by the gain in the second period gain from a merger with Firm 1. Does this mean that Firm 1 will merge with Firm 3 when observing a high output of Firm 3? It is not straightforward. A rational incumbent, Firm 1, knowing that it is in the entrant’s self-interest to “lie” in this manner, will not necessarily believe that the entrant has a low cost bounded within the above inequalities. In turn, the entrant knows about this incentive. We look at the much-famed separating equilibrium in which the entrant does not pick the same first-period price when his cost is low as opposed to the case when it is high. The first-period equilibrium then fully reveals the cost of Firm 3 to Firm 1. It is well know that there are two necessary conditions for the existence of an equilibrium. First, the low-cost type does not want to pick the high-cost type’s equilibrium output, and vice versa. In the separating equilibrium, the low-cost entrant’s output will induce merger of Firm 1 with the entrant, Firm 3. In the separating equilibrium, the high-cost entrant’s output will not induce any merger.
In order to derive the result we assume the possibility of subsequent de-merger if Firm 3 has ‘lied’ about its cost in the earlier stage. We cost of de-merger to the entrant is labeled as D. On the basis of this de-merger cost and the previous results we offer the following findings.

**Proposition 7:** An entrant, Firm 3, with cost $C_3$ such that $C_1 < C_2 < C_3$ does not mimic as a low cost firm and the entry does not induce endogenous mergers if:

\[
\frac{1+2C_2-\Delta}{3} < C_3 < 2C_1 + 3\Delta - 1
\]

(15a)

\[
2C_2 - C_1 > 1/2
\]

(15b)

\[
C_3 > 2C_2 - C_1 + 2
\]

(15c)

Proof: Algebraic manipulations yield the above.

**Proposition 8:** An entrant, Firm 3, with cost $C_3$ such that $C_3 > C_2 > C_1$ does not mimic as a low cost firm and the entry triggers an endogenous merger between incumbents if:

\[
2C_2 - C_1 > 1/2
\]

(15b)

\[
C_3 > 2C_2 - C_1 + 2
\]

(15c)

\[
C_3 > 2C_1 + 3\Delta - 1
\]

(15d)

Proof: Simplification yields the above.

**Proposition 9:** An entrant with cost $C_3$ such that $C_3 > C_2 > C_1$ does not mimic and the entry induces an endogenous merger between the entrant and the less efficient incumbent, Firm 2, if:

\[
C_3 < \frac{1+2C_2-\Delta}{3}
\]

(16a)

Proof: Details are available from the author.

**Proposition 10:** An entrant with cost $C_3$ such that $C_3 > C_2 > C_1$ mimics as a low cost firm with cost $C^{**}$ and the entry induces endogenous merger and subsequent de-merger between the entrant and the more efficient incumbent, Firm 1, if:

\[
C_3 < 2C_2 - C_1 + 2
\]

(16b)

\[
C^{**} > 2C_1 + 3\Delta - 1
\]

(16c)

\[
2C_2 - C_1 > 1/2
\]

(15b)

\[
D > 0
\]

(16d)

Proof: Details are available from the author.

**Proposition 11:** An entrant with cost $C_3$ such that $C_3 > C_2 > C_1$ mimics as a low cost firm with cost $C^{**}$ and the entry induces endogenous merger and subsequent de-merger between the entrant and the less efficient incumbent, Firm 2, if:

\[
C_3 < 2C_2 - C_1 + 2
\]

(16b)

\[
C^{**} < 2C_1 + 3\Delta - 1
\]

(16c)

Proof: Details are available from the author.

**Proposition 12:** An entrant with cost $C_3$ such that $C_3 < C_2 < C_1$ mimics as a low cost firm with cost $C^{**}$ and the entry does not trigger an endogenous merger if:

\[
C_3 < 2C_2 - C_1 + 2
\]

(16b)

\[
\frac{1+2C_2-\Delta}{3} < C^{**} < 2C_1 + 3\Delta - 1
\]

(17a)

Proof: Details are available from the author.

The above are all pooling-strategy equilibria. Let us now consider the possibility of signaling equilibrium.

**Proposition 13:** A truly low-cost entrant signals a cost $C^{***} = (C_2 - C_1 - 2)$ and the entry triggers an endogenous merger between the entrant and the more efficient incumbent if:

\[
C^{***} < \frac{1+C_1+C_2}{3}
\]

(17b)

This is possible if

\[
C_2 < 2C_1 - 3
\]

(17c)

Proof: Details are available from the author.
Proposition 14: A truly low-cost entrant signals a cost \( C^{**} = (C_2 - C_1 - 2) \) and the entry triggers an endogenous merger between the entrant and the less efficient incumbent if:

\[
C_2 > \frac{3}{2}
\]  
(17d)

Proof: Details are available from the author.

Proposition 15: A truly low-cost entrant signals a cost \( C^{**} = (C_2 - C_1 - 2) \) and the entry does not trigger an endogenous merger if:

\[
C_2 > 2C_1 - 3
\]  
(18a)

Proof: Details are available from the author.

Proposition 16: A truly low-cost entrant signals a cost \( C^{**} = (C_2 - C_1 - 2) \) and the entry triggers an endogenous merger between incumbents if:

\[
C_2 < 1
\]  
(18b)

Proof: Details are available from the author.

5. Concluding Comments

We suppose bribe-giving impinge on cost and demand functions of duopolists and, thereby, influence their competitive positions. Duopolists engage in bribe-giving to capture the largest possible market shares. From this extended model of duopoly we derive the perfect Nash equilibrium in terms of bribe-giving. We note that increased market buoyness and decrease in marginal cost of production and increase in private rents from bribe-giving funds will increase the equilibrium bribe that each duopolist offers - given parametric restrictions. We similarly expect the equilibrium bribe-giving to decline with decreased market buoyness and increase in marginal cost of production and decrease in private rents from bribe-giving - given parametric restrictions. We also noted perplexing comparative-static properties of the duopoly equilibrium in the presence of bribe-giving: parametric changes in demand and cost functions can have perverse effects on the equilibrium price and the equilibrium quantity of the proposed duopoly with bribe-giving.

Our second proposal is that bribe-giving can have asymmetric effects on costs and will, therefore, cause cost asymmetry only (Note 5). This cost asymmetry can be shown to trigger a gamut of endogenous changes. In this work we show the influence of this cost asymmetry on the incentives of firms to merge. A whole set of new results are offered: under a set of restrictions we establish that the most efficient firm to merge with the least efficient one. With a new restriction we establish the less efficient firms will merge. Under another set of restrictions we establish most efficient firms will merge. What is important is that these restrictions are solely driven by differential impacts of bribe-giving on individual costs. Differences in bribe-giving can therefore drive a wave of endogenous mergers that cannot be explained by any other economic factors. The other important finding is that differences in campaign funds can create situations that induce entry of firms in an industry that will in turn trigger endogenous mergers. Once again, such mergers and merger-inducing entry cannot be explained by any other models.

References


**Notes**

1. Recent studies that consider the general issue of incentive for mergers include Amir, Diamantoudi and Xue (2009), Lewis (2011) and Borla (2012).
3. Recent studies that have considered the issue of endogenous mergers include Dargaurd (2012). Furthermore, within the context of cost asymmetry, Engel, et al. (212) highlight the trade-off between deterrence and law enforcement.
4. The second order condition is satisfied if \((\alpha-\beta)(4^-)=0\).
5. Within the context of cost asymmetry, Engel, et al. (212) highlight the trade-off between deterrence and law enforcement.