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The Optimal Size of {b, t} - Blocking Set When t = 3,4 by Intersection the Tangent in PG (2, q)

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Abstract

In this research, we have been able to construct a triple blocking set of optimal size - {4q, 3} Based on the theorem (1.4.7) (Maruta, 2017, pp. 1-47).Without improving the minimum constraint of the projection level PG (2, q) We have also been able to develop the theorem (2.3.2) to construct quadratic blocking set of optimal size {5q + 1,4} - After we have engineered a quadratic blocking set of an optimal size for the projection plane PG (2,1.3) In the example (2.3.1).In general, we were able to conclude theorems (2.3.3) and (2.3.4) for construct engineered blocking sets with an optimal size when t = 3,4.

Keywords: projective plane PG(2,q), Secant- Tangent, algebraic curves, blocking set - { b, t }

1. Introduction

1.1 Introduce the Problem

The process of engineering construction of the projection level PG (2, q) It requires effort and time as studying these subjects needs long periods of time. Studies of algebraic geometry and its applications have been found in many sources, and its most important applications are encryption in embedded systems (Nilsson, Johansson, & Wagner 2019. pp. 238-258) (Kasm & Hamad 2019, pp. 130-139)., As well as in digital systems (Shukla, Khare, Rizvi, Stalin, & Kumar 2015, pp. 1387-1410). This research includes some basic definitions and theorems in optimal engineering construction for quadruple blocking set and in the plane projective PG (2, q). We have construct an optimal geometric design for blocking set by applying the projection level PG (2,17) theorem (1.4.7) (Maruta, 2017, pp. 1-47) (Yahya & Salim, 2018, pp.). To prove it and get an optimal geometrical size {4q, 3} By selecting random points from a cone and adding points resulting from junction tangles, We also called for the application of the theorem (1.4.9) (Ball, 1996, pp. 1387-1410) (Yahya, 2018,pp.24-40) Yahya, & Salim, 2019,pp.312-333).

To confirm the results. We were also able to get a quadruple set of optimal size {5q + 1,4} In the projection plane PG (2,13) In the example (2.3.1). Thus obtaining theorems (2.3.2) and (2.3.3) and (2.3.4), we also have points and lines of the fields that we used q = 13.17.

1.2 Explore Importance of the Problem

Definition (1.2.1): Suppose that V (n + 1, K) It is a vector space with distance (n + 1) identified on the field K Let it be R Is a relationship of equal cognition on points {0} - V with:(a 0, a 1, a 2,..., a n) R (b 0, b 1, b 2,..., b n) Q if and only if there is $m \in K - \{0\}$ in which bi a i =m Per I = 0,1,2,...,n Then (V- {0}, R))Is a projection space with a dimension n, And symbolizes it PG (n, k).if it was K = GF(q) Symbolizes PG (n, q),Called a projection level n = 2 (Hirschfeld,1979)

1.3 Describe Relevant Scholarship

Definition (1.3.1): Let it be c Curved-class n in the level PG (2, q) And p Is a point in c they say that p Repetitive - S (of multiplicity - s) If any line passes through P Cuts c (n-s) of the points at most

If it is S = 1 it's called p Simple point for c

And if they are S = 2 it's called p Double point And so on.

if it was S >1 it's called p Single point for c (Hirschfeld, 1979)(Yahya, & Salim, 2019,pp.).

1.4 State Hypotheses and Their Correspondence to Research Design

Definition (1.4.1): The curve that does not have single points is called a non- single curve (Hirschfeld, 1979).

Definition (1.4.2) : Quadratic is the first of the order 2 (Hirschfeld,1979).

Definition (1.4.3): defines the cone (Conic) As a separate quadrature contains (q + 1) Of the points of the projective plane PG (2, q) (Hirschfeld,1979).

An example: $C = V(x_0^2 + x_1x_2)$ It is conical at the plane PG (2, q).

Definition (1.4.4): Reported to the group S It template group (t-blocking set) If each line is in the projective plane PG (2, q) Cuts S What not Less than t Of the points where $|S| = b$ There is a dividing line S with t Of the exact points. (Cheon, Maruta, & okazaki, 2016, pp. 96–101)

Definition (1.4.5): blocking set B In the projection plane PG (2, q) is

{b, t}-blocking set If : -

t= $\min\{|l \cap B|: l \text{ is line}\}$

When t = 3 Called trigeminal aggregates triple blocking set)

When t = 4 Called quadratic quadrilateral groups (Quadrant blocking set) (Maruta, 2017, pp. 1-47).

Note (1.4.6) : [a b c] or [a, b, c] Indication of the line

$\{(x, y, z) \in pG(2, q): ax + by + cz = 0\}$ For two points Q, P

Indication of the line passing through the two points Q, P (Maruta, 2017, pp. 1-47)

Theorem (1.4.7) : for odd $q \geq 5$, Let C be a conic in the projection plane PG (2, q) For any three points P 1, P 2, P 3 in C, Let Li be the tangent of C Through the point Pi, And L_{ij} be the secant of C Through the points Pj, Pi, and let $p_{ij} = l_i \cap l_j$ For all $1 \leq i \leq j \leq 3$. We take any two points P, Q Of the three points p_{23}, p_{13}, p_{12} We assume that B = CUL 12 UL 13 UL 23 U {P, Q} then: -

B is blocking set - (4q, 3) (Maruta, 2017, pp. 1-47)

Theorem (1.4.8) : for odd q ≥ 5,Let C be a conic in the projection plane PG (2, q) For any three points p_1, p_2, p_3 in a C, Suppose Li It is tangent to C Through the point Pi, assume L_{ij}cross C Through Points p i p j, We assume that $P_{ij} = L_i \cap L_j$ for every 1 ≤i ≤j ≤3, Our Take any two points Q, P Of the three points P 23, P 13, P 12, And assume $B = C \cup L_{12} \cup L_{23} \cup L_{13} \cup \{P, Q\}$ For any line l.

$* \ |l \cap (L_{12} \cup L_{13} \cup L_{23}| \geq 2$

if $|l \cap (L_{12} \cup L_{13} \cup L_{23})| = 2$ For some fonts The Contains one of Points p_1, p_2, p_3 .

if l was Contains one of the points p_1, p_2, p_3 If l is tangent to C The

$|l \cap B| = 3$ or 4.

* If l Contains one of the points p_1, p_2, p_3 if l It is categorical

$|l \cap P| = 3$ (Maruta, 2017, pp. 1-47)

Theorem (1.4.9) : Let B be Triple blocking set in the projection plane PG (2, q) if it was :

1. q = 5, 7, 9 The B Own less than 4q From Points, if they are q = 8, then B Own less than 31 Of points.
2. if it was q = 11, 13,17, Then $|B| \geq (7q + 9)/2$
3. If it is >17 $< q = p^{2d+1}$ Then,

$|B| \geq 3q + p^d\lceil (p^{d+1} + 1)/(p^d + 1\rceil + 3$

4. if it was q > 4, q Square pairs or, q = 25,49,81,121 Then

$|B| \geq 3q - 2\sqrt{q} + 3$

5. if q>121 single square, then $|B| \geq 3q + 3\sqrt{2} + 3$ (Ball, 1996, pp. 1387-1410)(Yahya & Hamad,2019,pp.)

2.1 Some Basic Concepts

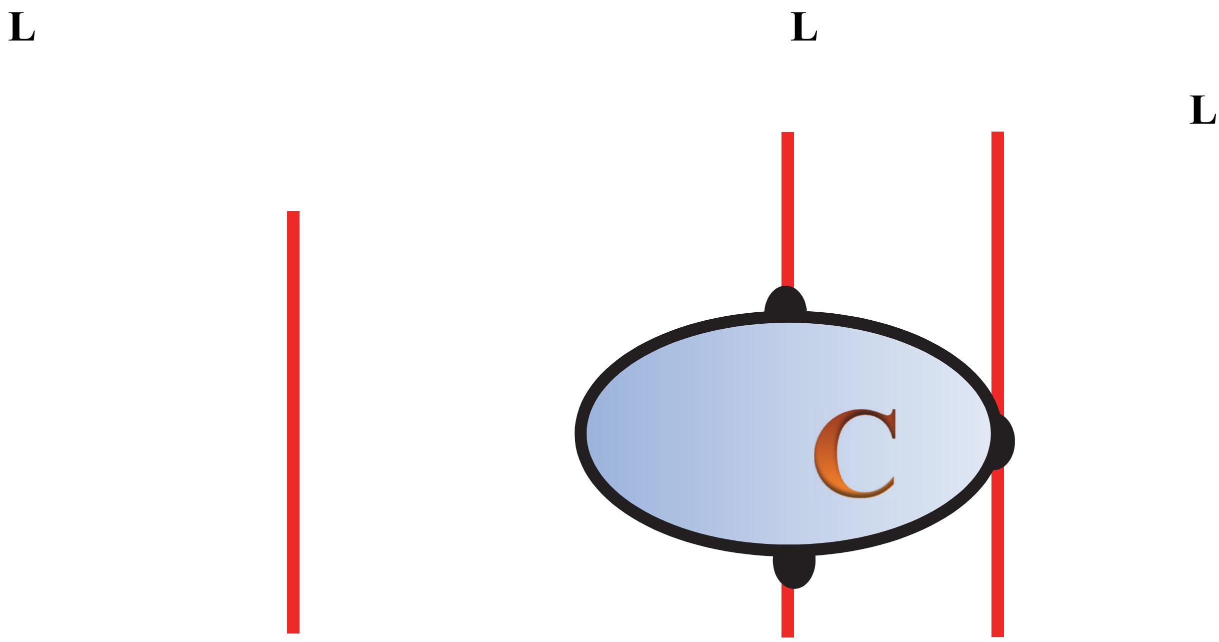
2.1.1 Tangents and secants in the projection plane PG (2, q) :

Let it be C Conical in the projection plane PG (2, q) The : -

L It called for an error externally C if it was $|C \cap L| = 0$

L is called error tangent C If it is $|C \cap L| = 1$

L is called cross error for C If it is $|C \cap L| = 2$ As it is shown in the drawing below (Figure 1) (Maruta, 2017, pp. 1-47)



(external) (secant) (tangent)

(Figure 1)

2.2 Construction of Optimal Triple Blocking Set

We will obtain the construction of optimal triple blocking set in proof of the theorem (1.4.7). We will take the PG (2,17) and find the construction of optimal triple blocking set of the field q = 17 (Maruta, 2017, pp. 1-47)

proof:

The field q = 17 consists of (307) points and (307) lines and each point is located on q + 1 of the lines and each line passes through q + 1 of the points.as shown in the Table B2. We find the points of the field q = 17 that verify the conic base

$C=x_0{}^{\wedge 2}+x_1\, x_2$

C = { (0, 1,0), (0, 0, 1), (1, 10, 5), (1, 12, 7), (1, 15,9), (1, 11,2 (1, 9, 15), (1, 11, 3) (1, 8, 2), (1, 3,11),(1, 13, 13), (1, 7, 12), (1, 6, 14), (1, 14, 6), (1, 16,1), (1, 4,4),(1,5,10),(1,2,8)}

Note that the conic contains q + 1 of the points, we choose (3) three random points of the conic for the projected plane PG (2,17) and let it be:

p_1 (1,1,16)

p_2 (0,1,0)

p_3 (0,0,1)

Each point passes q + 1 of lines that achieve the following equations

L_1:x+y+16z=0

L_2:y=0

L_3:z=0

P1 located L1

P2 located L2

P3 located L3

We begin by looking for the points that achieve the three-line equations of the field q = 17, as follows

L1 = { (1, 14,15), (1, 13, 14), (1, 2, 3), (1, 1, 2), (1, 10,11), (1, 3,4), (1, 11, 12), (1, 9, 10) (1, 5, 6), (1, 16,0),(1, 8, 9), (1, 12, 13), (1, 4, 5), (1, 0, 1), (0, 1,1), (1, 15,16),(1,6,7),(1,7,8)}

L2 = { (1, 0,0), (0, 0, 1), (1, 0, 8), (1, 0, 15), (1, 0,12), (1, 0,11), (1, 0, 10), (1, 0, 2) (1, 0, 6), (1, 0,13),(1, 0, 16), (1, 0, 14), (1, 0, 7), (1, 0, 1), (1, 0,9), (1, 0,3),(1,0,4),(1,0,5)}

L3 = { (1, 0,0), (0, 1, 0), (1, 5,0), (1, 13, 0), (1, 6,0), (1, 9,0), (1, 14, 0), (1, 8, 0) (1, 7, 0), (1, 15,0),(1, 3, 0), (1, 16, 0), (1, 2, 0), (1, 12, 0), (1, 1,0), (1, 10,0),(1,4,0),(1,11,0)}

P12, P13, and P23 are the points on which the secants are located, where point P12 is obtained by the L1 junction with L2 and P13 and P23 are points from the L1 junction with L3 and L2 with L3, respectively, as shown below

$p_{12}=L_1\cap L_2=(1,0,1)$

$p_{13}=L_1\cap L_3=(1,16,0)$

$p_{23}=L_2\cap L_3=(1,0,0)$

The points we obtained represent {P, Q} the points in which the secants pass.

Now we suppose

$B=C\cup L_{12}\cup L_{13}\cup L_{23}\cup \{P,Q\}$……*

Now we find the points of lines L_{23}, L_{13}, L_{12} These lines are secant cutoff the conic C in Pi, Pj of points for each $1\le i\le j\le 3$

L_{12}:x+z=0

L_{13}:x+16y=0

L_{23}:x=0

We start finding these points of lines (secants)

L_{12} = {(0, 1,0), (1, 6, 16), (1, 9,16), (1, 16, 16), (1, 5,16), (1, 1,16), (1, 14, 16), (1, 2, 16) (1, 10, 16), (1, 0,16),(1, 3, 16), (1, 8, 16), (1, 17, 16), (1, 12, 16), (1, 13,16), (1, 4,16),(1,15,16),(1,11,16)}

L_{13} = { (0, 0,1), (1, 1, 14), (1, 1,16), (1, 16, 7), (1, 1,4), (1, 1,11), (1, 1, 5), (1, 1, 12) (1, 1, 1), (1, 1,2),(1, 1, 6), (1, 1, 3), (1, 1, 15), (1, 1, 7), (1, 1,8), (1, 1,0),(1,1,10),(1,1,13)}

L_{23} = { (0, 1,0), (0, 0, 1), (0, 1,5), (0, 1, 13), (0, 1,6), (0, 1,9), (0, 1, 14), (0, 1, 8) (0, 1, 7), (0, 1,15),(0, 1, 3), (0, 1, 16), (0, 1, 2), (0, 1, 12), (0, 1,1), (0, 1,10),(0,1,4),(0,1,11)}

We exclude the points between the secants which are (6) points and then make up for all the points we got in (*)With the addition of some points Q, P, which we found to achieve the equation (*) and follows

B = { (0, 1,0), (0, 0, 1), (1, 10,5), (1, 12, 7), (1, 15,9), (1, 11,2), (1, 9, 15), (1, 11, 3) (1, 8, 2), (1, 3,11),(1,13, 13), (1, 7, 12), (1, 6, 14), (1, 14, 6), (1, 16,1), (1, 4,4),(1,5,10),(1,2,8), (1, 6,16), (1, 9, 16), (1, 16,16), (1, 5, 16), (1, 14,16), (1, 2,16), (1, 10, 16), (1, 0, 16) (1, 3, 16), (1, 8,16),(1, 7, 16), (1, 12, 16), (1, 13, 16), (1, 4, 16), (1, 15,16), (1, 11,16),(1,1,14),(1,16,7), (1, 1,4), (1, 1, 11), (1, 1,5), (1, 1, 12), (1, 1,1), (1, 1,2), (1, 1, 6), (1, 1, 3) (1, 1, 15), (1, 1,7),(1, 1, 8), (1, 1, 0), (1, 1, 10), (1, 1, 13), (0, 1,5), (0, 1,13),(0,1,6),(0,1,9),(0, 1,14), (0, 1, 8), (0, 1,7), (0, 1, 15), (0, 1,3), (0, 1,16), (0, 1, 2), (0, 1, 12) (0, 1, 1), (0, 1,10),(0, 1, 4), (0, 1, 11), (1, 0, 1), (1, 16, 0)}

In this way, we obtained {68,3}- blocking set of an optimal size for the

PG (2,17)

|B|=q+1+3(q+1)-6+2={68,3}-blocking set

|B|=18+3(18)-6+2=68

Table B2. Points and lines of projective plane PG (2,17)

I	Pi (Points)	i	Pi(Points)	i	Pi(Points)	I	Pi(Points)
1	1 0 0	78	1 6 3	156	1 15 2	233	1 11 0
2	0 1 0	79	1 6 10	157	1 9 7	234	0 1 11
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
76	1 6 9	154	1 0 6	231	1 13 5	306	1 2 0
77	1 2 3	155	1 3 8	232	1 7 14	307	0 1 2

lines																		
L1	1	2	10	16	87	110	120	152	176	180	192	211	233	254	259	272	279	306
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
L307	307	1	9	15	86	109	119	151	175	179	191	210	232	253	258	271	278	305

We conclude from the proof of the theorem (1.4.7) that it is possible to construct the blocking set without improving the minimum constraint for the PG (2, 17) is also optimal because it is geometrically constructed from the addition of points resulting from intersection of tangents.

The graphic below (Figure 2) shows the geometric construction method for the triple blocking set of PG (2,17)

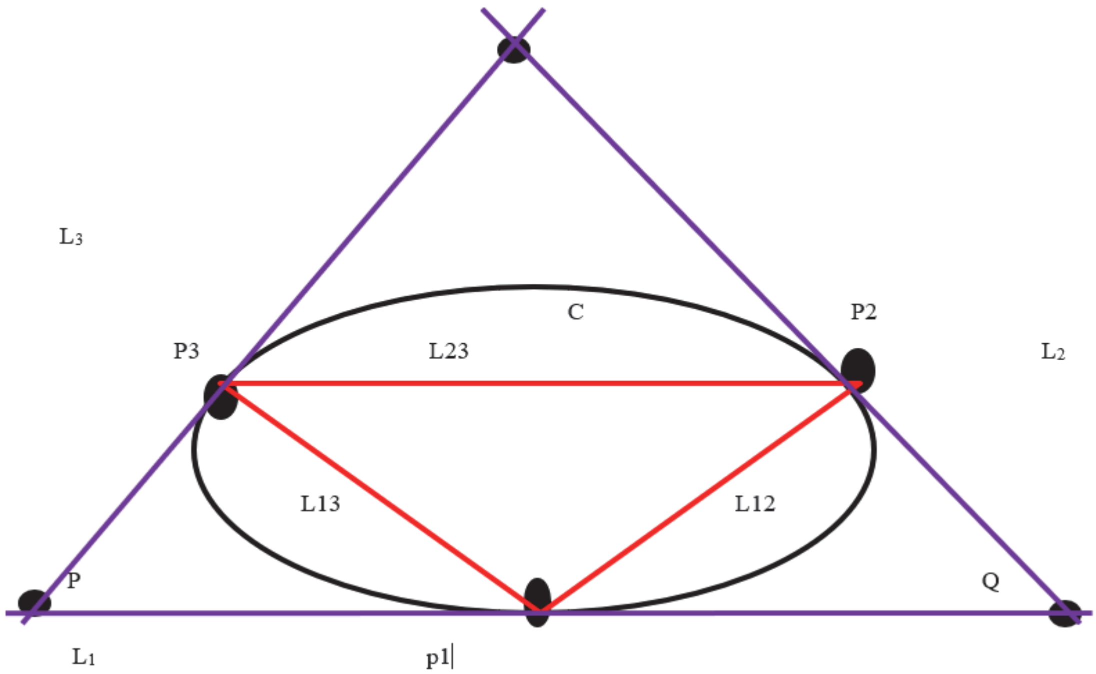


Figure 2.

To verify the results apply theorem (1.4.9) (Ball, 1996, pp. 1387-1410), as follows:

|B|≥(7q+9)/2

|68|≥(7(17)+9/2

68≥64

2.3 Construction of Optimal Quadrant Blocking Set

Plane level PG (2,13) It consists of (183) points and (183) lines and each point is located on q + 1 Of lines and every line Passes (q + 1) Of the points.as shown in the Table B1.According to the theorem (1.4.8) (Maruta, 2017, pp. 1-47)

Table B1. Points and lines of projective plane PG (2,13):

I	Pi(POINTS)	i	Pi(POINTS)	i	Pi(POINTS)	i	Pi(POINTS)
1	1 0 0	15	1 5 5	29	1 5 12	43	0 1 1
2	0 1 0	16	1 4 1	30	1 6 11	44	1 0 1
⋮							
74	1 6 9	110	1 0 3	146	1 12 8	182	1 12 0
75	1 8 3	111	1 7 11	147	1 9 11	183	0 1 12

lines														
L1	1	2	9	25	38	42	60	108	120	129	135	140	154	182
L2	2	3	10	26	39	43	61	109	121	130	136	141	155	183
L3	3	4	11	27	40	44	62	110	122	131	137	142	156	1
⋮														
L183	183	1	8	24	37	41	59	107	119	128	134	139	153	181

We take random line points from the plane lines PG (2,13) Let it be

L = { (0, 1,9), (1, 0, 2), (1, 6,4), (1, 11, 10), (1, 2,7), (1, 8,9), (1, 9, 5),

(1, 1,11), (1, 12, 6), (1, 7,0),(1, 4, 12), (1, 10, 1), (1, 3, 3), (1, 5, 8)}

If we cross line points L With points of crosses $L_{12} \cup L_{13} \cup L_{23}$ For the projection level PG (2,13) We will get points {(1,9,5), (1,7,0), (1,10,1)}

And so we got the (3) Common points that have been achieved rule

$$|l \cap (L_{12} \cup L_{13} \cup L_{33})| \geq 2$$

For any other random line of plane lines PG (2,13)

Now take another line of random lines from the plane PG (2,13) Let it be

L (1, 3,4), (1, 5, 9), (1, 0.3), (1, 8, 10), (1, 1,12), (1, 6, 5), (0, 1, 9), (1, 4, 0), (1, 7.1), (1.1 1, 1 1), (1, 10, 2), (1,2, 8), (1, 9, 6)}

$\boldsymbol{|l \cap (L_{12} \cup L_{13} \cup L_{23})| = \{(1,3,4),(1,12,7),(1,8,10)\}}$

If L Contains one of the points P 1, P 2, P 3 This is the point P 1 = (1,3,4) At the same time L Join two common points with the cross $L_{12} \cup L_{13} \cup L_{23}$ which mean that $|L \cap B| = 3$, then that L cross B with (3) Points according to the definition of (1.1.7), the t = 3 triplets blocking set Let's take another random line from the plane of the plane PG (2,13) Let it be

L = {(1, 2,6), (1, 12, 5), (1, 3,2), (1, 10, 0), (1, 11,9), (0, 1,9), (1, 6, 3), (1, 8, 8), (1, 4,11), (1.9, 4), (1, 1, 10) 1,5, 7), (1, 7, 12)}

$$\boldsymbol{|l \cap (L_{12} \cup L_{13} \cup L_{23})| = \{(1,2,6),(1,4,11)\}}$$

L Contains one of the points P 1, P 2, P 3 The point P 3 =(1,2,6) Subscribe with $(L_{12} \cup L_{13} \cup L_{23})$ Point (1,4,11) If L It is tangent, then $|L \cap B| = 3$ or 4.

This means L cross B with (3) or (4) Points and by definition (1.1.7) t = 3 that mean triple blocking set t = 4 which is quad blocking set.

From above we conclude that the size of the quadratic quadrilateral groups can be found for the projection plane PG (2, q) Drawing on the idea of the theoretician (1.4.7) and (1.4.8) (Maruta, 2017, pp. 1-47) The conclusion is as follows:

2.3.1 Example

Let's take the field q = 13 We find the perfect engineering construction of it quadrilateral assemblies

Solution / Find conical points C By achieving its equivalence

$C = x_0^2 + x_1 x_2$

C={ (1, 12,1), (1, 6, 2), (0, 1,0), (1, 10, 9), (1, 3,4), (0, 0,1), (1, 5,5) (0,11,7) (1, 8, 8), (1, 9,10), (1,4, 3), (1, 1, 12), (1,7, 11), (1, 2, 6)}

We choose (4) Random points of the conical points and let be

$p_1 = (1,3,4), p_2 = (1,5,5), p_3 = (1,2,6),\ p_4 = (0,1,0)$

for every Li, i = 1,2,3,4 we create the equations lattes located on P 1, P 2, P 3, P 4.

$$L_1: x + 3y + 4z = 0$$

$$L_2: x + 5y + 5z = 0$$

$$L_3: x + 2y + 6z = 0$$

$$L_4: y = 0$$

Points that achieve the straight equation L_1 is

L_1 = { (1, 11,11), (0, 1, 9), (1, 4, 0), (1, 10, 2), (1, 3,4), (1, 0,3), (1, 8, 10), (1, 7, 1), (1, 5, 9), (1, 2,8), (1,9, 6), (1, 6, 5), (1,1, 12), (1, 12, 7)}

Points that achieve the straight equation L 2is

L_2 = { (1, 1,4), (1, 6, 12), (1, 3,2), (1, 8, 10), (1, 2,3), (1, 10,8), (1, 11, 7), (1, 0, 5), (1, 4, 1), (1, 5,0), (1,1 2, 6), (1, 9, 9), (1,7, 11), (0, 1, 12)} Points that achieve the straight equation L 3is

L_3 = { (1, 9,12), (1, 12, 11), (1, 3,1), (1, 0, 2), (0, 1,4), (1, 10,3), (1, 5, 9), (1, 11, 7), (1, 2, 10), (1, 4,7), (1,8, 8), (1, 4, 5), (1,6, 0), (1, 1, 6)}

Points that achieve the straight equation L_4 is

L_4 = { (1, 0,0), (1, 0, 10), (1, 0,3), (1, 0, 11), (0, 0,1), (1, 0,9), (1, 0, 2), (1, 0, 7), (1, 0, 5), (1, 0,12), (1,0, 1), (1, 0, 4), (1,0, 6), (1, 0, 8)}

Now we find points that are located on secants

$$p_{12} = L_1 \cap L_2 = (1,8,10)$$

$$p_{13} = L_1 \cap L_3 = (1,5,9)$$

$$p_{14} = L_1 \cap L_4 = (1,0,3)$$

$$p_{23} = L_2 \cap L_3 = (1,11,7)$$

$$p_{24} = L_2 \cap L_4 = (1,0,5)$$

$$p_{34} = L_3 \cap L_4 = (1,0,2)$$

Suppose that

$$\boldsymbol{B = C \cup L_{12} \cup L_{23} \cup L_{14} \cup L_{34} \cup \{P, Q\}}$$

Now we find the lines (secants) on which the points P_{14}, P_{12}, P_{23}, P_{34} are located, to each

$$\boldsymbol{1 \leq i \leq j \leq 4}$$

The Point P_{12} = (1,8,10) Check the line L_{12}

Meaning that L_{12} is the secant which cutoff the conic C in the PG (2, 13) with points (p_1,p_2),since it is passes through the point P_{12} it achieves quation

$\boldsymbol{L_{12}: x + 8y + 10z = 0}$

The equation of the secant L_{14}

$\boldsymbol{L_{14}: x + 3z = 0}$

The equation of the secant L_{23}

$\boldsymbol{L_{23}: x + 11y + 7z = 0}$

The equation of the secant L_{34}

$\boldsymbol{L_{34}: x + 2z = 0}$

Now we find the points that achieve these equations and begin with the L_{12}, whose points are

L_{12} = { (1, 6,12), (1, 12, 2), (1, 11,8), (1, 4, 11), (1, 3,4), (1, 0,9), (1, 5, 5), (1, 2, 10), (1, 8, 0), (1, 9,7), (1,10, 1), (1, 7, 6), (1,1, 3), (0, 1, 7)}

L_{14} = { (1, 1,4), (1, 8, 4), (1, 5,4), (1, 12, 4), (1, 9,4), (1, 7,4), (1, 11, 4), (1, 0, 4), (1, 10, 4), (1, 6,4), (1,2, 4), (1, 4, 4), (1,3, 4), (0, 1, 0)}

L_{23} = { (1, 1,2), (1, 6, 9), (1, 0,11), (1, 3, 10), (1, 5,5), (1, 8,4), (0, 1, 4), (1, 4, 1), (1, 11, 3), (1, 10,12), (1,9, 8), (1, 7, 0), (1,12, 7), (1, 2, 6)}

L_{34} = { (1, 11,6), (0, 1, 0), (1, 3,6), (1, 10, 6), (1, 4,6), (1, 8,6), (1, 5, 6), (1, 9, 6), (1, 12, 6), (1, 7,6), (1,0, 6), (1, 1, 6), (1,2, 6), (1, 6, 6)}

Common points between the secants are (10) points

$$|B| = q + 1 + 4(q + 1) - 10 + 6$$
$$= 14 + 4(14) - 10 + 6$$

=66

={5q+1,4}-blocking set

We obtained the quadratic blocking set of the optimal size in PG(2,13) and his points are:-

B={(1,12,1),(1,6,2),(0,1,0),(1,10,9),(1,3,4),(0,0,1),(1,5,5),(0,11,7),(1,8,8),(1,9,10),(1,4,3),(1,1,12),(1,7,11),(1,2,6),(1,6,12),(1,12,2),(1,11,8),(1,4,11),(1,0,9),(1,2,10),(1,8,0),(1,9,7),(1,10,1),(1,7,6),(1,1,3),(0,1,7),(1,1,2),(1,6,9),(1,0,11),(1,3,10),(1,8,4),(0,1,4),(1,4,1),(1,11,3),(1,10,12),(1,9,8),(1,7,0),(1,12,7),(1,1,4),(1,5,4),(1,12,4),(1,9,4),(1,7,4),(1,11,4),(1,0,4),(1,10,4),(1,6,4),(1,2,4),(1,4,4),(1,11,6),(1,3,6),(1,10,6),(1,4,6),(1,8,6),(1,5,6),(1,9,6),(1,12,6),(1,0.6),(1,1,6),(1,6,6),(1,8,10),(1,5,9),(1,0,3),(1,11,7),(1,0,5),(1,0,2)}

To achieve, we apply the theorem (1.4.9)(Ball, 1996, pp. 1387-1410).

$$|B| \geq \frac{7q + 9}{2}$$

$$|66| \geq \frac{7(13) + 9}{2}$$

$|66| \geq 50$

From above we conclude

2.3.2 Theorem

For odd q ≥ 5, let C be a conic in PG(2,q). for any four points p_1,p_2,p_3,p_4 in C, let Li be the tangent of C through p_i and L_{ij} be the secant of C through p_i and p_j, and let

$$P_{ij} = L_i \cap L_j \ , \quad 1 \leq i \leq j \leq 4$$

Take any points P and Q from the points $p_{12},p_{14},p_{23},p_{34}$ and suppose that

B=C U L_{12}U L_{14}U L_{23}U L_{34}U{P,Q}

Then B is {5q+1,4}-blocking set

Thus, in general, we can conclude the following assumptions for the construction of engineering blocking set when t = 3, 4.

2.3.3 Theorem

In the projection plane PG (2, q) for every $q \geq 5$ The optimal size of the set Trilogy { b, t } When t = 3 and is

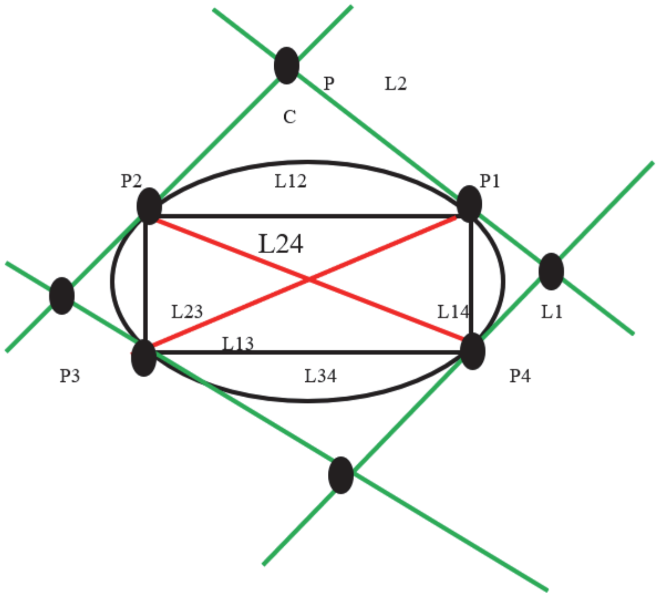
$nq \quad ,$ when $\quad n = t + 1$

2.3.4 Theorem

In the projection plane PG (2, q) for every q ≥ 5 The ideal size of the Quartet blocking set { b, t } When t = 4 is

$nq + 1 \quad , \qquad \text{when} \quad n = t + 1$

The graph (Figure 3) shows the construction of optimal quadratic $\{5q + 1,4\}$-blocking set in PG(2,13)



(Figure 3)

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