Another Look at Rectangularization: Variability of Age at Death within a Given Population

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Abstract

The study investigated the factors responsible for compression of mortality usually observed in human survival curves. The concept of rectangularisation was adopted to study the changes that occur in human survival curves at the older ages under the assumption that the basic mortality table is occurring in time zero. Simulations were carried out to examine the behavior of survival and cumulative death curves under varying conditions. The result revealed that the survival curve shifts to the right of the base mortality survival curve l_x^0 as mortality situation improves. Furthermore, the study indicated that expansion of mortality spectrum will occur if mortality curve is allowed to flow smoothly throughout the human life.

Keywords: Rectangularisation, Age at death, Inter-quartile range

1. Introduction

With the fall of man from grace to grass, he has been exposed to hardship, diseases, hunger, pains, sorrows which will eventually culminate into death if not properly handled.

However, death remains the biggest challenge to man not by coincidence but by divine design. Man has over the years been fighting various causes of death especially those that result in premature death. Although, death remains inevitable, man appears to be wining the battle up to some extent. Some of these causes that shortened the life span of man are gradually being eliminated by scientific discoveries and advancement in medical science. Some diseases that require special attention are now being treated in the normal hospital because of the discoveries of acceptable and reliable cures.

Man on the average lives longer nowadays than his predecessors. However, as man tries to fight one disease others do surface. For instance, smallpox, malaria, tuberculosis, leprosy to mention a few have been fought and won. Surprisingly, before man could have any relief, cardiopulmonary diseases, cancer, HIV and AIDS have started dominating the mortality spectrum with their attendant high death rates in some parts of the worlds. If man succeeds in fighting those diseases, who knows what will happen next?

Two important demographic consequences of improved human mortality are increased life expectancy and aging of the population.

Expectation of life at birth as one of the parameters of measuring the improvement in human mortality is the average age at death in a stationary population. Experience has shown that the higher it is, the better is mortality situation of that population. The aging of a population is a direct indication that people are living to old ages which might be due to the reduction in the death rates at various ages hence, aging population is the focus of this study as it intends to examine the changes in the shape of the survival curve associated with the high concentration of death around the mode (at old ages) of the death curve. More specifically, the study attempts to examine the changes in the variability of the distribution of the ages at death due to mortality improvement.

Mortality improvement in any human population would always manifest as reduction in the number of deaths in some age brackets. The study therefore, examines rectangularization and variability in the distribution of ages at death using the changes in the number of deaths at various ages as the basic parameter.

Unlike some investigators that carried out similar empirical studies on those issues, the study adopts a theoretical approach by carrying out simulations on a basic mortality table (1958 C.S.O. mortality table for male) to find out what happens if certain conditions are imposed on a given population. Simulations were carried out to examine the behaviour of the survival and cumulative death curves under varying conditions

2. Literature

Fries (1980) started the concept of rectangularization and made variability of the distribution of the ages at death of human population very popular. The author predicted that human life expectancy is approaching its maximum potential value and pointed out that rectangularization would make possible a compression of morbidity such that the period of debilitating diseases in late life is minimized. Cenfort (1979), Upton (1977) and Pearl (1940) joined in the application of rectangulisation and this generated reactions from many other investigations which led to a lot of criticisms especially in Fries's claims and predictions. Myers and Menton (1984) suggested that rectangularization is a "myth" while Menton and Tolley (1991) held the view that it is an "ill posed question".

Renthenberg, et al (1991) was of the opinion that mortality is now undergoing an expansion rather than a compression at the oldest ages. Eakin and Witten (1995) supporting Fries (1980) stated that rectangularization has been an important characteristic of historical mortality change in humans, although sometimes it would appear as if rectangularization has become slower in recent decades. Nusselder and Mackenback (1996) challenged Fries's claim about the compression of morbidity.

According to Menton and Singer (1994) the compression of morbidity refers to an increasing concentration of illness and disability in later years of life, which is the context applied in this study.

According to Myers and Menton (1984), a historical view of the compression of mortality reveals that it may depend critically on the age range and time–frame of the analysis although, variability of age at death may decline in some periods if the full age is considered. The authors added that this compression may remain stable if the analysis is limited to older ages and there is no reason to expect that any process of compression (of mortality or morbidity) should continue indefinitely.

However, Fries's (1980) argument about an upper limit for human life expectancy which he thought was around 85 years was based on a specific premise linking the two concepts together. With the belief that the maximum human life span is fixed the author however, claimed that despite a great change in average life expectancy there has been no detectable change in the number of people living longer than 100 years or in the maximum age of persons dying in a given year. Hence, if we assume that Fries premise were true, then the distribution of age at death would be bounded on the right. Under such situation, as mortality improves and death is delayed, the distribution of age at death might be compressed and the survival curve in turn might become more rectangular. Myers and Menton (1984) observed that the standard deviation of deaths above age 60 in the United States increased rather than decreased between 1962 and 1979. Rothenberg, *et al* (1991) confirmed the above result when they carried out the analysis of ages of death in the United States between 1962 and 1984.

Although, rectangularization is not a proof of the existence of a fixed upper limit to the human life span, the opposite is a logically valid proposition. If an upper limit exists for human life span, then rectangularization is a necessary consequence of prolonged mortality improvement.

The above argument notwithstanding, two empirical findings have attested to the fact that rectangularization or the compression of mortality has not continued in recent decades for elderly population. Wilmoth and Horiuchi (1999) have disproved the claim that the proportion of centenarians in a population has been constant historically or that the maximum age at death is unchanging over time.

The authors were of the opinion that a fixed maximum human life spam may be of necessity result in a continued compression of mortality as death rates decline. Consequently, the failure to observe such a compression suggests that either no limit exists or that it is not currently in sight.

A combination of these two sets of findings undermine the empirical basis for arguing that rectangularization might be a necessary concomitant of mortality improvement which questions the Fries's conclusion about limits that affect the maximum or average human life span. Eakin and Witten (1995), Myers and Menton (1984), Nusselder and Mackenbach (1996), looked at the possibility of establishing biological limits for human life expectancy to the detriment of other relevant issues and failed to show little agreement about how rectangularization and variability should be measured. None of these authors directed effort towards considering differences across contemporary populations in variability of age at death. One can however expect that variability should be low in countries with high life expectancies, population with comparable mortality levels still may differ notably in their variability of age at death perhaps due to differences in social structure and disease environments. Also, to the best of our knowledge, not much has been done on the issue of how long term changes in the variability of age at death may affects the perception, attitudes and behaviours of individuals and the society at large. As a result, these broad consequences of historical compression deserve full investigation.

Most recently, Cheung,et al (2005) couched the issue of the rectangularization of a populations survival curve in terms of the epidemiological transition. They proposed three key components to describe rectangularization as "horizontalization," "verticalization," and "longevity extension". Horizontalization corresponds to how long a cohort can live and how many cohort members survive before aging-related deaths significantly decrease the proportion of survivors verticalization corresponds to how concentrated aging-related deaths are around the modal age at death and the longevity extension corresponds to how far the right hand tail of the survival curve, representing the highest normal life durations, can exceed the modal age at death. Drawing upon the work of Kannisto (2001) ,the authors developed a method to summarize these three dimensions of the survival curve and applied the measures they develop to Hong Kong life table data from 1976 to 2001. Their results showed that the survival curve in Hong Kong became increasingly horizontal and vertical leading the mortality to become increasingly compressed around the modal age of death over the period and concluded that rectangularization occurred in Hong Kong between 1976 and 2001.

Authors such as Myers and Manton (1984), Wilmoth (1998; 2000), Wilmoth and Horiuchi (1999), stated that interpreting evidence of rectangularization as an indication that a biologically based upper limit to human longevity exists is in itself problematic. The presence of rectangularization alone cannot conclusively establish the existence of biological limits to human longevity. However, if biological limits to the human lifespan exist, rectangularization will occur, but rectangularization although it may also occur in the absence of biological limits. Myers and Manton mentioned that causally attributing rectangularization to the existence of biological limits that govern the human lifespan is impossible without detailed information concerning both the socio-environmental and biological pathways leading up to death. On the other hand Wilmoth and Horiuchi (1999) said that there is little current evidence to suggest that we are approaching a biologically determined limit to life expectancy.

Wilmoth (1998) claimed that although gains in life expectancy have slowed in recent decades, the populations of many developed nations have nonetheless continued to experience a decline in mortality rates.

Vaupel and Yashin (2001) stated that research examining the complex causal associations between the biological and socio-environmental determinants of longevity suggests that socio-environmental factors play an important role. Taking together, these facts fundamentally challenge the assertion that average life expectancy at birth is biologically limited to 85 years of age.

While the existence of an upper limit to life expectancy is contested, many would agree that the reasons underlying the dramatic increases in life expectancy that occurred in the developed world over the course of the 20th century are largely attributable to socio-environmental factors.

Cheung, et al (2005) examined three components of rectangularization, which are horizontalization, verticalization and longevity extension and concluded that the transformation occurring in the survival function for Hong-Kong may be interpreted as resistance to human longevity.

However, the core assumption underlying the existence of rectangularization is that socio-environmental and medical advances tied to socioeconomic development are responsible for the increasingly rectangular shape of the human survival curve. This is why researchers typically examine survival curves over time. In those analyses, time essentially becomes a proxy used to gauge socioeconomic improvements. Omran (1971) stated that, as the epidemiologic transition took hold, improvements in the standard of living, public health, sanitation, and medical interventions ushered in a new era of historically unprecedented reductions in mortality at all ages. Wilmoth and Horiuchi (1999) said that these reductions led to substantial increases in life expectancy in many parts of the world and, over time, fundamentally transformed the shape of the survival curve (The more socioeconomic advancement within a nation, the greater the likelihood that the population as a whole will be healthier, live longer, and exhibit less variability around the average age of death (e.g., rectangularization will occur).

Hayward, et al (2006) said that the diffusion of mass education throughout a population often occurs alongside improvements in the infrastructure of the healthcare system. Hidajat, et al. (2007), said that the individual and institutional factors associated with the spread of education converge to increase the social capacity for population health.

Dustin, et al (2009) examined sex-education specific variations in the rectangularization of the survival curve among the 50+ population in the United States unlike other previous researchers who examined rectangularization between population and/or over time under the assumption that differences in the degree of rectangularization between nations and over time reflect levels of socioeconomic and technological advancement. Adopting this line of thought, they hypothesized that the degree of rectangularization would be more pronounced among groups with higher educational attainment because, relative to persons with less education, people with more education are better able to access a wide variety of resources that allow than to optimize their health and ultimately, increase the length of their lives. The authors examined two dimensions of the survival curve ("Verticalization" and "longevity extension") and applied the method developed by Cheung et al. (2005) to test their hypothesis. The authors observed that there were evidence supporting the notion that survival curves among males and females with higher levels of education exhibit a greater amount of rectangularization than the survival curves of less educated. Studies have shown that mortality is currently concentrated at older ages in most countries. Life expentancy has slowed down its rapid increase and approached the modal age at death. De Benetictis (2001) claimed that the study of the modal age at death provides an opportunity to understand changes in the distribution of death, and to explain the changes in mortality at older ages.

The opinion has been held that shifting mortality scenario, where the compression of mortality has stopped may be realistic.Robine (2005) had stimulated a new debate about how to interpret period life expectancy when rates of death vary over time. Bongaarts (2005) studied separately the two components of the logistic model, senescent and background mortality with this second term not varying over age.

3. Scope of Study

As Wilmoth and Horiuchi (1999) stressed, there is a great need to distinguish between the compression of mortality which leads to the increasing concentration of the ages of death thus making the survival curve to become more rectangular and the compression of morbidity which refers to an increasing concentration of illness and disability in the later years of life. Menton and Singer (1994) suggested that both types of compressions would occur simultaneously and at same time have two separate phenomena. The study focused more attention on the former which is the compression of mortality or rectangularization of the human survival curve issues.

4. Mathematical Framework

Let l_0 be the radix of a stationary population and d_x the number of people dying between ages x and $x+1,$ we therefore have the number of people alive at age$x$ denoted by l_x as

$$l_x = l_0 - \sum_{t=0}^{x-1} d_t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

4.1 Calculating the Number of People Alive as a Result of Mortality Improvement

As stated in the introduction, we shall regard the basic mortality table being adopted in this study as occurring in time $0.$ while other mortality improvement shall be considered as occurring at times $t = 1, 2, 3$e.t.c

We shall recall also that mortality improvement at any time $t.$ will be measured vide the changes in the number of deaths at various ages. If the change, in the number of death at arbitrary age x at time $t = 1$ is $\theta_x\%,$ we have the number of people alive at age x time 1 denoted by l_x^1 as

$$l_x^1 = l_0 - \sum_{t=0}^{x-1} (1 - 0.0\theta_t)\, d_t^0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

$$= l_0 - \sum_{t=0}^{x-1} d_t^0 + \sum_{t=0}^{x-1} 0.0\theta_t d_t^0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.\ldots\ldots\ldots\ldots\ldots \quad (3)$$

From (1) , (3) can be written as

$$= l_x^0 + \sum_{t=0}^{x-1} 0.0\, \theta_t d_t^0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. \ldots \quad (4)$$

Equation (4) enables us to express the number of people alive at age x time $t = 1$ denoted by l_x^1 in terms of l_x^0 and the changes in the number of deaths from age zero to $x - 1$

A critical examination of (4) shows that if all the $\theta_{x's}$ are positive then l_x^1 will be more than l_x^1

$$\text{by} \sum_{x=0}^{x-1} 0.0\theta_x\, d_x^0.$$

$$\text{i.e. } l_x^1 - l_x^0 = \sum_{x=0}^{x-1} 0.0\theta_t d_t^0 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.(5)$$

On the other hand, if the $\theta_{x's}$ are all negative, then there is a reduction in the number of people

$$\text{alive at age } x \text{ time } t = 1 \text{ by} \sum_{x=0}^{x-1} 0.0\theta_t d_t^0$$

$$\text{That is } l_x^0 - l_x^1 = \sum_{x=0}^{x-1} 0.0\theta_t d_t^0 \dots\dots\dots\dots\dots.\dots\dots\dots\dots\dots\dots..(6)$$

This equation also reveals that the effect of mortality improvement is felt more at advanced ages.

As x increases the value of the summation $\sum_{t=0}^{x-1} \theta_t\, d_t^0$ increases because of the people who survive death at the various age brackets more sequentially to higher age brackets it is the concentration of many people within the old age brackets that is the subject of discussion of the next section of this paper.

4.2 Rectangularisation

From equation (3), it is shown that

$$l_x^1 = l_0 - \sum_{t=0}^{x-1} d_t + \sum_{=0}^{x-1} 0.0\theta_t d_t^0\,,$$

It is easy to see that as the percentage reduction in the number of deaths at the various ages approach 100% the l_x^1 approaches l_0. The shape of the survival curve changes and approaches what is regarded as rectangularization because of the concentration of many people within the old age, brackets. The decay process being experienced within the universe leads to the failure of the immune system of these old people and death sets in at very advanced ages (between 80 and 90years) (See annals of life insurance medicine) The experience in many parts of the world shows that only few people survive to 100 years and beyond.

The reason for this is not far fetched. In this age bracket man has entered into his second childhood and life is no longer enjoyable. Even the essence of living has become meaningless and the economic worth of the individual is nothing to write home about.

The younger generations feel relieved when such an individual passes on especially if they have to sustain him or her from their earnings. It is this apparent limitation of the human life span that brings about rectangularization if mortality improvement is experienced for a considerable length of time.

The initial impression created by the concept of rectangularization is that the human genome contains some sort of time bomb, a built in self destructive device that is activated after some 80 to 90 years. So far, the search for death causing genes has yielded no positive result. Rather, there is evidence to show that age related deaths are caused by the progressive improper functioning of the human organs at old age thus making it difficult for the aged to cope with environmental changes that confront man on a daily basis.

Rectangularization can be regarded as child of circumstances. By this we mean that if mortality improvement occurs within a given period of time and nature within the same time frame restricts the life span of man to certain age, then rectangularization becomes a necessary consequence of this restriction. On the other hand, if nature permits the extension of the mortality spectrum a phenomenon called expansion rather than compression of mortality occurs.

Before proceeding, it is necessary to define “nature” Nature shall mean the consequences of man’s activities to his environment and being. It is a known fact that man’s activities on earth have led to the global warming which gave birth to the recent Copenhagen’s Conference in Demark the conference became necessary because man’s existence is threatened by this phenomenon called global warming. Some other negative contributions of men to his environment are desertification and pollution. As this paper is not intended for this type of discussion, interested readers can for example look at the effect of particular air pollution on infant deaths.

As stated in the introduction, man has also contributed positive to his well being by fighting various types of diseases and other things that threatened his existence on earth. This to a large extent is responsible for the improved life expectancy of man on earth.

To drive home our discussion, let us consider two scenarios, one positive the other negative. For simplicity, we shall consider a uniform change of 5% in the number of deaths at all ages. Invoking (4) for the positive scenario, we have

$$l_x^1 = l_x^0 + 0.05 \sum_{t=0}^{x-1} d_t^0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

and

$$l_x^2 = l_x^0 - 0.05 \sum_{t=0}^{x-1} d_x^0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

for the negative scenario

The graphs of l_x^0, l_x^1 andl_x^2 are shown in figure I

A critical examination of figure1 shows that l_x^1 is to the right of l_x^0, while l_x^2 is to left of l_x^0 . This indicates that as mortality improves, the survival curve moves to the right of the base mortality survival curve l_x^0

The tail ends of l_x^1shows a sudden drop from around age 99 to 100. On the contrary, l_x^2 is a smooth curve to age 100 which is the terminal age of the adopted mortality table? The phenomenon exhibited by l_x^1 above, is the gradual building up of the process of rectangularization as a result of the restriction of the terminal age of survival to 100 years.

The process of rectangularization becomes more pronounced when we further decrease the number of deaths at each age and at the same time maintain the restriction of 100 years as the terminal age of the mortality table. See figure II below. In which the number of deaths at each age were decreased by 10%, 15% and 20% respectively

On the contrary, if this restriction is removed and the resulting mortality curve is allowed to flow smoothly to intersect the x axis, a new phenomenon called the expansion of the mortality spectrum will occurs. As stated earlier in the study, the occurrence of this new phenomenon is subject to the permission of nature which in turn depends to some extent on the activities of man on earth.

4.3 Measurement of Rectangularization

Literature on rectangularization shows little agreement about the standard to be adopted in measuring rectangularity and variability of the age at death. Earlier researcher such as Fries depended on visual inspection of survival curves for their findings. Wilmoth and Horuchi (1999) made no systematic effort has been made to list and compare the various possible measures in this regard. However, the study adopted the inter-quartile range (IQR) of the cumulative death curve of the life table as a measurement of rectangularity of the survival curve. Using this measure, the (IQR) for the base mortality curve 19.2.

This shall serve as the measure of the pressure on the survival curve resulting from the compression of mortality which refers to an increasing concentration of ages at death thus leading to a more rectangular survival curve. As could be seen from table 1 below, the pressure increased from 19.2, the base mortality pressure, to 33.4 when there was a 25% reduction in the number of deaths at all ages. Beyond this level, the inter-quartile range began to drop with further improvement in the mortality rate and continued down wards as long as the restriction on the terminal age is maintained. A logical explanation for this is that rather than have the usual compression, a decompression process has set in. This implies that the process of compression can not continue indefinitely. A new phenomenon known as expansion of the mortality spectrum will invariably occur thus making the terminal age to shift to a higher age.

5. Conclusion

A close look at table 1 above shows that the interquatile ranges increased gradually from 19.80 for the base table to 33.4 for 25% reduction in the number of death at all ages. Beyond this level the interquartile range fail progressively. 25% decrease in the number of deaths at all ages appears to be the turning point where decompression begins. This phenomenal change attest to the fact that compression can not continue indefinitely, and that there must of necessity be an expansion of the mortality spectrum if progressive mortality improvements are experienced over a long period of time worthy of note is the behaviour of the third.

The third quartile range increased from 80.44 for the base table to 88.64 after 20% reduction in the number of deaths at all ages 25% gave a figure of 99.002 which is more than 10 years difference. If there has been no restriction on the terminal age of 100, the third quartile for 25% reduction should have bear 100 years when 7 500 000 people would have died leaving a balance of 2 500 000 to be staggered over some years with 30% reduction in the number of deaths at all ages, 7 000 000 would have died at age 100 if the restriction was removed.

These observations show that if the age restriction is removed, the expansion process of the mortality spectrum will begin within this zone? That is from 25% reduction upwards.

A better confident zone would be from 20% reduction downwards whereas sharp increase in the third quartile value was recorded between 20% to 20% reduction in the number of deaths at all ages.

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Table 1. Results of Inter-Quartile Range for Various Reductions in Death Rate

Degree of reduction of no. of deaths	Q_1	Q_3	$Q_3 - Q_1 = \text{IQR}$
Base table	61.25	80.44	80.44 – 61.25 = 19.19 $\cong$ 19.20
5% reduction in no. of death	62.03	81.83	81.83 – 62.03 = 19.80
10% reduction in no. of death	62.86	83.50	83.50 – 62.86 = 20.64
15% reduction in no. of death	63.65	85.64	85.64 – 63.65 = 21.99 $\cong$ 22
20% reduction in no. of death	64.63	88.64	88.64 – 64.63 = 24.01
25% reduction in no. of death	65.62	99.002	99.002 – 65.62 = 33.4
30% reduction in no. of death	66.7	99.02	99.02 – 66.7 = 32.32
35% reduction in no. of death	67.84	99.3	99.3 – 67.84 = 31.46
40% reduction in no. of death	69.1	99.4	99.4 – 69.1 = 30.3
45% reduction in no. of death	70.40	99.45	99.45 – 70.40 = 29.05
50% reduction in no. of death	72.1	99.5	99.5 – 72.1 = 27.4

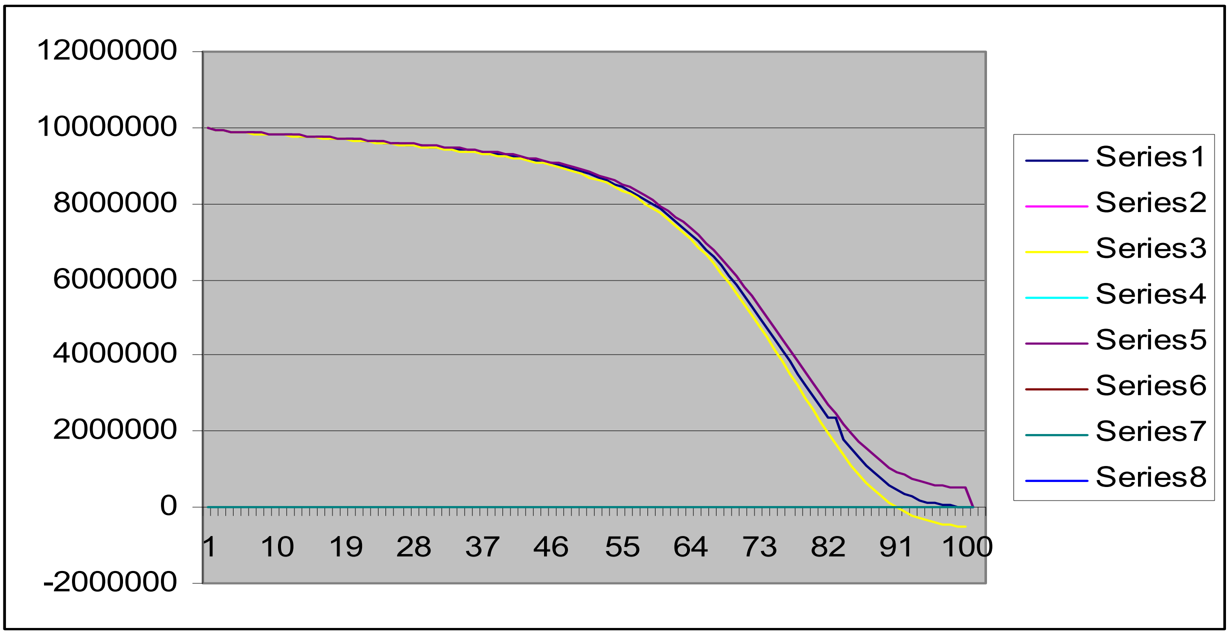

Figure 1. Graph of Various Degrees of Reductions Plotted Against Age

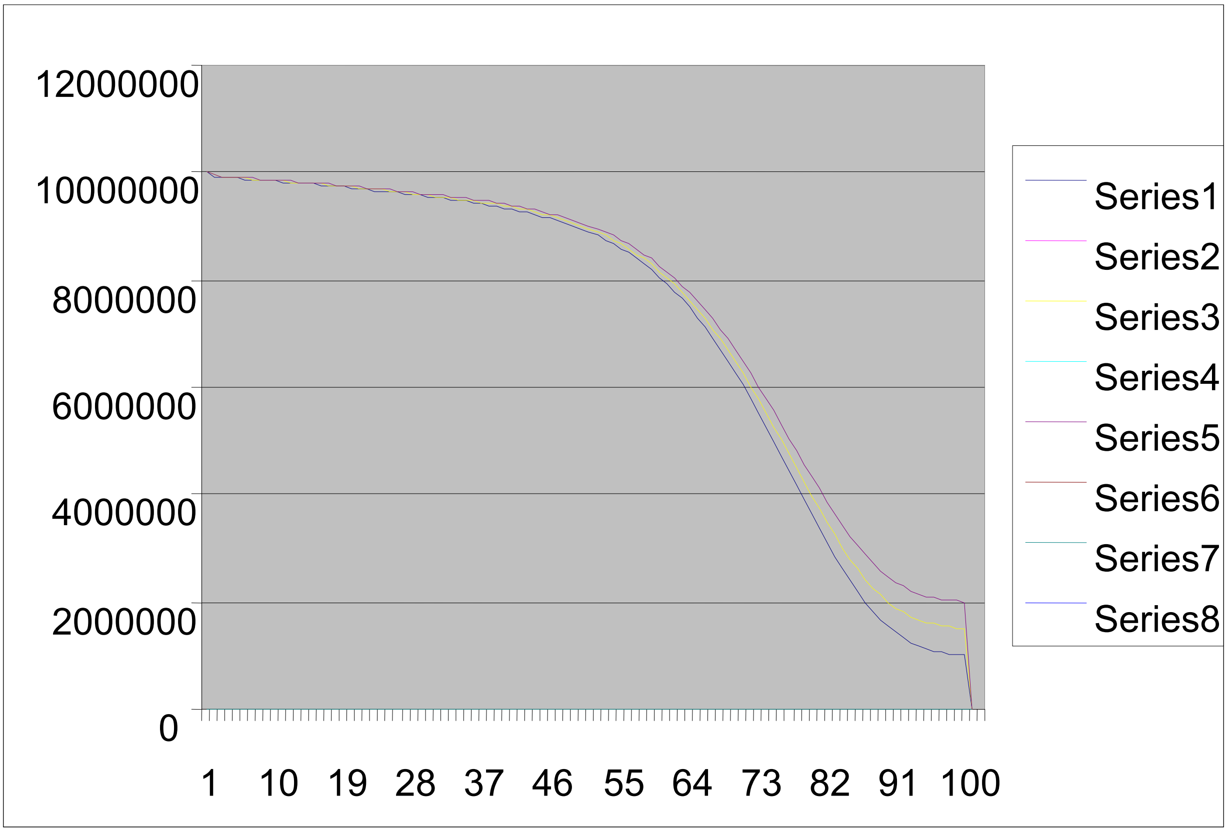

Figure 2. Various Degrees of Reductions Plotted Against Age