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Abstract Contraflow Models and Solution Procedures for Evacuation Planning

Ram Chandra Dhungana[1], Urmila Pyakurel[1] & Tanka Nath Dhamala[1]

[1] Central Department of Mathematics, Tribhuvan University, P.O.Box 13143, Kathmandu, Nepal

Correspondence: Tanka Nath Dhamala, Central Department of Mathematics, Tribhuvan University, P.O.Box 13143, Kathmandu, Nepal.

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Abstract

The abstract flow model deals with the flow paths (routes) that satisfy the switching property. Contraflow is a widely accepted solution approach that increases the flow and decreases the evacuation time making the traffic smooth during evacuation by reversing the required road directions from the risk areas to the safe places. In this paper, we integrate the concepts of abstract flow and contraflow, give mathematical formulations of these models and present efficient algorithms for solving the abstract contraflow problems. The efficient solution procedures are presented for maximum dynamic, lexicographically maximum and earliest arrival abstract contraflow problems. This approach maximizes the flow value in given time and seeks to eliminate the crossing conflicts.

Keywords: Abstract flow, evacuation planning, contraflow, maximum flow, earliest arrival flow

MSC(2010) Primary: 90B10, 90C27, 68Q25; Secondary: 90B06, 90B20.

1. Introduction

Due to different disasters, challenges of emergency management have been increased day by day. It is necessary to evacuate as many people as possible within the limited time horizon under dynamic hazard from the dangerous states (sources) to the safe places (sinks). For the effective evacuation planning different issues such as identifying transportation routes, notifying evacuees, common traffic delays, etc, should be managed. During the evacuation, the prominent dynamic network flow models have been widely used to find out the efficient evacuation routes that were firstly introduced and investigated by (Ford & Fulkerson, 1958). There exist different models and algorithms for buildings, stadiums, ships, districts, cities or whole sub-national region evacuation based on their scenarios. For different variants of the dynamic flow problems and their corresponding results, we refer to (Altay & Green III, 2006; Dhamala, 2015; Hamacher & Tjandra, 2001; Jarvis & Ratliff, 1982; Moriarty *et al.*, 2007; Pel *et al.*, 2012; Schadschneider *et al.*, 2009). The evacuation network is interpreted by a directed graph where the intersections of roads (i.e., rooms in a building or intersection of streets in a region) are represented by nodes, road segments between nodes (i.e., doors between rooms, or streets in region) are represented by arcs and routes are taken as paths. The places where evacuees are situated and start to move are considered as source nodes and the safe places where they are supposed to arrive are destination (sink) nodes. Each node and arc has a non-negative integer capacity, where the capacity gives the maximum number of evacuees allowed at the element.

After the development of maximal static flow and maximum dynamic flow models and algorithms by (Ford & Fulkerson, 1956; Ford & Fulkerson, 1958), the existence of lexicographically maximal (lex-max) flow was shown by (Minieka, 1973) in the time expanded network for multiterminal problems. For the single source multi-sink lex-max problem with the restriction that the source should be ranked first without pre-specified ordering of the sinks, (Megiddo, 1974) presented a solution procedure that seeks a maximum flow out of sources while maximizing the minimum flow entering any sink and any pair of sinks. Both procedures have pseudo-polynomial time complexity. A polynomial time algorithm to solve the general lex-max dynamic flow problem on the original graph by using chain decomposable flows has been presented in (Hoppe & Tardos, 1994; Hoppe & Tardos, 2000).

The abstract flows introduced in (Hoffman, 1974) generalizes the concept of paths by replacing the underlying network configuration. The maximum dynamic abstract flow problem and its solution procedure has been investigated by (Kappmeier *et al.*, 2014). The lexicographically maximum abstract flow problem has been investigated by (Kappmeier, 2015). This approach makes the use of, so-called switching property that eliminates the crossing at intersections. Some of the important lane-based routing strategy for reducing the delays that reduce (or eliminate) crossing and merging conflicts at intersections have been studied in (Zhao *et al.*, 2016). In a lane-based routing plan, selecting and turning options at intersections are restricted to improve traffic flow away from a hazardous area. Intersections with potentially significant delays can be temporarily transformed into an uninterrupted flow facility which is the most beneficial aspect of this routing.

Contraflow approach is another emerging and widely accepted model for evacuation planning. It increases the outbound road capacities by reversing the direction of roads towards the safe destinations. (Kim *et al.*, 2008) give first integer programming formulation and presented different heuristic solutions for large scale evacuations, where the evacuation time has been improved by at least 40 percent with at most 30 percent of the total arc reversals. They showed that the problem of minimizing the evacuation time is $\mathcal{NP}$-hard. The application of contraflow is not only limited to evacuation planning but also in traffic planning that reduces congestion and traffic jams during the day-to-day office hours, some accident management cases or some street exhibitions. Various mathematical models, heuristics, optimization and simulation techniques taking into account of macroscopic and microscopic behavioral characteristics deal with contraflow for this transportation network, however an acceptable contraflow solution even approximately is a lacking due to very high computational costs.

The maximum dynamic contraflow problem has been introduced in (Arulselvan, 2009; Rebennack *et al.*, 2010) and solved with polynomial algorithm for two terminal general network. They proved that the problem is $\mathcal{NP}$-hard for multi-terminal network. Authors in (Dhamala & Pyakurel, 2013; Dhungana *et al.*, 2015; Pyakurel & Dhamala, 2015; Pyakurel & D-hamala, 2017a; Pyakurel & Dhamala, 2016) introduced the earliest arrival, quickest transshipment and lex-maximum dynamic contraflow problems in both discrete and continuous time settings. The former is solved in strongly polynomial and pseudo-polynomial time on two terminal series-parallel and general networks, respectively. They presented polynomial algorithm for the quickest transshipment contraflow problem. For the given priority ordering, the lex-maximum dynamic contraflow problem is solved in polynomial time for multi-terminal network. For the fixed the supply and demands, the earliest arrival transshipment contraflow has been introduced in (Pyakurel & Dhamala, 2017a; Pyakurel & Dhamala, 2017b) in discrete and continuous times. They solved the problem in multi-source and single sink network as well as single source and multi-sink network. In both cases, their algorithms have polynomial time complexity. Moreover, they presented approximation algorithms to compute the approximate earliest arrival transshipment contraflow for multi-terminal networks, (Pyakurel & Dhamala, 2017b; Pyakurel *et al.*, 2017). The contraflow approach is generalized on lossy network in (Pyakurel *et al.*, 2014). They solved the generalized maximum dynamic and generalized earliest arrival contraflow problems on two terminal lossy network. For more details, we refer to (Pyakurel, 2016). Moreover, authors in (Pyakurel *et al.*, 2017) introduced the abstract contraflow approach with path reversal capability. They presented a polynomial time algorithm to solve the abstract maximum dynamic contraflow in continuous time setting. The quickest contraflow problems with constant and load dependent transit times are solved with computational experiments, (Pyakurel *et al.*, 2018).

Authors in (Xie & Turnquist, 2011) solved the lane based contraflow and crossing elimination strategies at intersections jointly. A network optimization model to integrate these problems has been introduced in (Zhao *et al.*, 2016). The bi-level lane-based network optimization and simulation model have been formulated in (Xie *et al.*, 2010), where the upper level optimizes the network evacuation performance subject to the contraflow and crossing-elimination constraints, and the lower level simulates dynamic evacuation flows. To deal the uncertain arrivals of evacuees with low mobility population, the multi-model integrated contraflow has been presented in (Hua *et al.*, 2014). It contains non-contraflow to shorten the strategy setup time, full-lane contraflow to minimize the evacuation network capacity and bus contraflow to realize the transit cycle operation. A more realistic contraflow problem with evacuation priorities and the setup time has been considered in (Wang *et al.*, 2013).

In this work, we introduce and investigate abstract contraflow models by combining the concepts of abstract flow and contraflow approaches. We present some efficient solution procedures to maximize the achievable number of evacuees at minimum time by reducing the possible crossing conflicts. Our contributions resemble the results in contraflow on non-abstract networks presented by the different authors (Pyakurel & Dhamala, 2015; Pyakurel & Dhamala, 2016; Rebennack *et al.*, 2010). Throughout the paper, we reverse the paths to increase the path capacities whenever it is necessary by applying the concepts of (Rebennack *et al.*, 2010) and (Pyakurel *et al.*, 2017). Then the problems are solved by using abstract flow algorithms of (Kappmeier *et al.*, 2014) and (Kappmeier, 2015).

The structure of the paper is as follows. The notations and prior works in network flows are presented in Section 2. We propose maximum dynamic and earliest arrival abstract contraflow models with efficient solution procedures for two-terminal abstract networks in Section 3. The lexicographically maximum contraflow model and algorithm for multiterminal abstract networks are presented in Section 3. The paper concludes with Section 4.

2. Some of the Basic Notations and Models

Let $N = (E, \Gamma, b, \tau, S, D, T)$ be a multi-terminal evacuation network, where E and Γ represent the sets of elements (also called ground set) and paths, respectively. The nonnegative capacity and travel time vectors of $e \in E$ are $b_e \in R^E_+$ and $\tau_e \in Z^E_+$, respectively. The given nonnegative integer time horizon T may be represented as discrete time periods

$\mathbf{T} = \{0, 1, \cdots, T\}$. A network N is called abstract if for every $\gamma \in \Gamma$ there is a linear order $<_\gamma$ of elements and the switching property is satisfied in Γ. A switching property requires that for every $\gamma_1, \gamma_2 \in \Gamma$ and $e \in \gamma_1 \cap \gamma_2$, there exist paths $\gamma_1 \times_e \gamma_2 \subseteq \{a \in \gamma_1 : a \leqslant_{\gamma_1} e\} \cup \{a \in \gamma_2 : a \geqslant_{\gamma_2} e\}$ and $\gamma_2 \times_e \gamma_1 \subseteq \{a \in \gamma_2 : a \leqslant_{\gamma_2} e\} \cup \{a \in \gamma_1 : a \geqslant_{\gamma_1} e\}$. We denote the parts of the path γ before and after an element e excluding it as $\gamma_{\to e} = (\gamma, e) = \{p \in \gamma : p <_\gamma e\}$ and $\gamma_{e\to} = (e, \gamma) = \{p \in \gamma : p >_\gamma e\}$. A network is considered as static or dynamic depending on time.

Example 1. *Let $E = \{s, a, b, c, d, e, z\}$ and $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6\}$, where $\gamma_1 = \{s, a, c, z\}, \gamma_2 = \{s, b, d, z\}, \gamma_3 = \{s, a, e, d, z\}, \gamma_4 = \{s, b, e, c, z\}, \gamma_5 = \{s, a, e, c, z\}, \gamma_6 = \{s, b, e, d, z\}$. The path system satisfies the switching property with defined linear order on each path and (E, Γ) is an abstract network (cf. Figure 1 (iii)). For example, $\gamma_3 \times_e \gamma_4 = \{s, a, e, c, z\}$ and $\gamma_4 \times_e \gamma_3 = \{s, b, e, d, z\}$ both exist in Γ.*

The maximum static abstract flow problem is to assign the nonnegative flow values $f(\gamma) \in R_+^\Gamma$ to the paths so that the total flow value is maximized and the element capacity restrictions are respected. The problem can be generalized by introducing a weight function $\omega \in R_+^\Gamma$ that specifies the reward per unit of flow sent along each path. As the weights make the general problem $\mathcal{NP}$-hard, (Hoffman, 1974), their choice is restricted to supermodular functions, i.e., $\omega(\gamma_1 \times_e \gamma_2) + \omega(\gamma_2 \times_e \gamma_1) \geq \omega(\gamma_1) + \omega(\gamma_2)$ for every $\gamma_1, \gamma_2 \in \Gamma$ and $e \in \gamma_1 \cap \gamma_2$.

For the static case, the generalized maximum weighted abstract flow and the generalized minimum weighted abstract cut problems are primal (1) and dual (2) LP problems to each other, (Hoffman, 1974).

$$\max\{\sum_{\gamma\in\Gamma} \omega(\gamma) f(\gamma) \mid \sum_{\gamma\in\Gamma:e\in\gamma} f(\gamma) \leq b_e, f(\gamma) \geq 0,\ \forall\ \gamma \in \Gamma, e \in E\} \tag{1}$$

$$\min\{\sum_{e\in E} b_e y(e) \mid \sum_{e\in\gamma} y(e) \geq \omega(\gamma), y(e) \geq 0,\ \forall\ \gamma \in \Gamma, e \in E\} \tag{2}$$

where a value $y(e)$ is assigned to every element $e \in E$ covering every path according to its weight.

The dynamic abstract flow (DAF) problems are dealt with the corresponding time expended networks. (Kappmeier *et al.*, 2014) introduced the holdover of flow at intermediate nodes to construct an abstract time expanded network (ATEN). Let $\sigma : \Gamma \to \{1, 2, \cdots, T\}$ be waiting periods for each elements of Γ. Flow enters to $e \in \gamma$ at time $\sum_{q\in\gamma_{\to e}} (\sigma_q + \tau_q) + \tau_e$ as it traveling along γ waits σ_q time units before passing through e. The set $\gamma^\sigma = \{e^\kappa \in E_T = E \times T | e \in \gamma, \sum_{q\in\gamma_{\to e}} (\sigma_q + \tau_q) + \tau_e = \kappa\}$ represents temporal paths with intermediate waiting and satisfy $q^\kappa <_{\gamma^\sigma} e^{\kappa'}$ if and only if $q <_\gamma e$. Such paths that arrive within T is $\Gamma_T^\sigma = \{\gamma^\sigma | \gamma \in \Gamma, \sigma \in \{1, 2, \cdots, T\}^\Gamma, \sum_{e\in\gamma} \{\sigma_e + \tau_e\} \leq T\}$. The set (E_T, Γ_T^σ) represent the ATEN of (E, Γ).

The maximum dynamic flow problem or the $\mathcal{NP}$-hard minimum cost dynamic flow problem have optimal solutions with local flow conservation at each intermediate elements (Fleischer & Skutella, 2003; Ford & Fulkerson, 1958). For the DAF problem, the waiting has no influence on the optimality and the temporally repeated optimal solutions can be obtained even if waiting is allowed, (Kappmeier *et al.*, 2014). The DAF is a function $f_{dyn} : \Gamma_T^\sigma \to R_+$ which is feasible if and only if the capacity of every element at every point of time is satisfied. The maximum dynamic abstract flow (MDAF) problem (3) is to maximize the total flow value respecting the given restrictions, (Kappmeier *et al.*, 2014).

$$\max\left\{\sum_{\gamma_t\in\Gamma_T^\sigma} f_{dyn}(\gamma_t) \mid \sum_{\gamma_t\in\Gamma_T^\sigma:(e,\theta)\in\gamma_t} f_{dyn}(\gamma_t) \leq b_e, f_{dyn}(\gamma_t) \geq 0,\ \forall\ \gamma_t \in \Gamma_T^\sigma, e \in E, \theta \in \mathbf{T}\right\} \tag{3}$$

The dynamic cut is $C_{dyn} = \{(e, \theta) \in E_T : \alpha(e) \leq \theta < \alpha(e) + \tilde{y}(e)\}$, where $\tilde{y}$ is the static weighted abstract dual integral optimal solution with weight $\omega(\gamma)$ and $\alpha(e) := \min_{\gamma\in\Gamma} \sum_{\gamma\in(\Gamma,e)} (\tau(\gamma) + \tilde{y}(\gamma))$.

The outflow and inflow of a source and a sink are $|f|_s^+ = \sum_{\gamma\in\delta_s^+} f_\gamma$, and $|f|_z^- = \sum_{\gamma\in\delta_z^-} f_\gamma$, respectively. Let f^1 and f^2 be maximum abstract flows, and let $s_1, s_2, \cdots, s_k$ and $z_1, z_2, \cdots, z_k$ be the orders of sources and sinks, respectively. We say that f^2 is lexicographically smaller than f^1, denoted by $f^1 \geq_L f^2$, if there exists either an $l \in \{0, 1, \cdots, k-1\}$ such that $|f^1|_{s_{l+1}}^+ > |f^2|_{s_{l+1}}^+$ and $|f^1|_{s_i}^+ = |f^2|_{s_i}^+$ for $i = 1, 2, \cdots, l$, or all $|f^1|_{s_i}^+ = |f^2|_{s_i}^+$ for $i = 1, \ldots, k$. Similarly, $f^1 \geq_L f^2$ if either $|f^1|_{z_{l+1}}^- > |f^2|_{z_{l+1}}^-$ and $|f^1|_{z_i}^- = |f^2|_{z_i}^-$ for some $l \in \{0, 1, \cdots, k-1\}$ and $i = 1, 2, \cdots, l$ or $|f^1|_{z_i}^- = |f^2|_{z_i}^-$ for all $i = 1, \cdots, k$. The lexicographically maximum abstract flow (LMAF) f^* is a maximum abstract flow (MAF) respecting the terminal orders, i.e. $f^* \geq_L f$ for all abstract flows f.

A sequence of terminals is compatible if the terminal elements respect their rank for more than one terminals of the same type appeared on a path. For sources $s_1, s_2, \cdots, s_k$, it holds that $\gamma \in \Gamma, s_i \neq s_j \in \gamma : j < i \Rightarrow s_i \leq_\gamma s_j$. But a sequence of sinks $z_1, z_2, \cdots, z_k$ has to assure $\gamma \in \Gamma, z_i \neq z_j \in \gamma : i < j \Rightarrow z_i \leq_\gamma z_j$. For given a compatible sequence of sources and sinks, we define abstract networks (E, Γ_s^i) with increasing subsets of paths $\Gamma_s^i \subset \Gamma$ for $i = 1, 2, \cdots, k$, where $\Gamma_s^i =$

$\Gamma_s^{i-1} \cup \{\gamma \in \Gamma | s_i = \text{first}(\gamma)\}$ with $\Gamma_s^0 = \emptyset$. Similarly, we define (E, Γ_z^i) with paths $\Gamma_z^i \subset \Gamma$, where $\Gamma_z^i = \Gamma_z^{i-1} \cup \{\gamma \in \Gamma | z_i = \text{last}(\gamma)\}$ with $\Gamma_z^0 = \emptyset$. Each of (E, Γ_s^i) and (E, Γ_z^i) contains the paths starting and ending in the first i sources and sinks, respectively.

The central idea behind the contraflow technique is to improve the outbound capacity by adopting the arc or path reversals toward the safer places keeping the same travel time in the evacuation network. As a result the flow value is increased, evacuation time is decreased and traffic flow is made smooth. Let $\Gamma = \{\overrightarrow{\gamma}, \overleftarrow{\gamma}\}$ be the set of all paths in contraflow abstract network N with capacities $b(\overrightarrow{\gamma}) = \min\{b_e : e \in \overrightarrow{\gamma}\}$ and $b(\overleftarrow{\gamma}) = \min\{b_e : e \in \overleftarrow{\gamma}\}$. We define the undirected auxiliary network $\tilde{N}$ by adding the capacities on the corresponding two-way paths and keeping the transit time (if any) fixed. The set of elements and paths are denoted by $\tilde{E}$ and $\tilde{\Gamma}$, where $\tilde{e} \in \tilde{E}$ and $\tilde{\gamma} \in \tilde{\Gamma}$. Then the capacity function is defined as $b(\tilde{\gamma}) = \min\{b_{\tilde{e}} : \tilde{e} \in \tilde{\gamma}\}$ while the travel times (if any) on paths remains the same.

By construction of auxiliary network with path reversal, $\tilde{\Gamma}$ satisfies the switching property and the order of elements holds for each $\tilde{\gamma} \in \tilde{\Gamma}$. This implies that the auxiliary network of the abstract evacuation network is also an abstract network.

3. Abstract Contraflow Problems

In this section, we define abstract contraflow problems and present efficient solution procedures to solve them. This approach uses path reversals in abstract network at time zero without any switching costs.

Example 2. *Let* $E = \{s, a, b, c, d, e, z\}$ *and* $\Gamma = \{\overrightarrow{\gamma_1}, \overrightarrow{\gamma_2}, \overrightarrow{\gamma_3}, \overrightarrow{\gamma_4}, \overrightarrow{\gamma_5}, \overrightarrow{\gamma_6}, \overleftarrow{\gamma_1}, \overleftarrow{\gamma_2}, \overleftarrow{\gamma_3}, \overleftarrow{\gamma_4}, \overleftarrow{\gamma_5}, \overleftarrow{\gamma_6}\}$ *with* $\overrightarrow{\gamma_1} = \{s, a, c, z\}$, $\overrightarrow{\gamma_2} = \{s, b, d, z\}$, $\overrightarrow{\gamma_3} = \{s, a, e, d, z\}$, $\overrightarrow{\gamma_4} = \{s, b, e, c, z\}$, $\overrightarrow{\gamma_5} = \{s, a, e, c, z\}$, $\overrightarrow{\gamma_6} = \{s, b, e, d, z\}$, $\overleftarrow{\gamma_1} = \{z, c, a, s\}$, $\overleftarrow{\gamma_2} = \{z, d, b, s\}$, $\overleftarrow{\gamma_3} = \{z, d, e, a, s\}$, $\overleftarrow{\gamma_4} = \{z, c, e, b, s\}$, $\overleftarrow{\gamma_5} = \{z, c, e, a, s\}$ *and* $\overleftarrow{\gamma_6} = \{z, d, e, b, s\}$. *We forget the direction of paths and reformulate it by adding the capacities of paths between the terminals. Set of paths in abstract auxiliary network is* $\tilde{\Gamma} = \{\tilde{\gamma}_1, \tilde{\gamma}_2, \tilde{\gamma}_3, \tilde{\gamma}_4, \tilde{\gamma}_5, \tilde{\gamma}_6\}$ *(cf. Figure 1).*

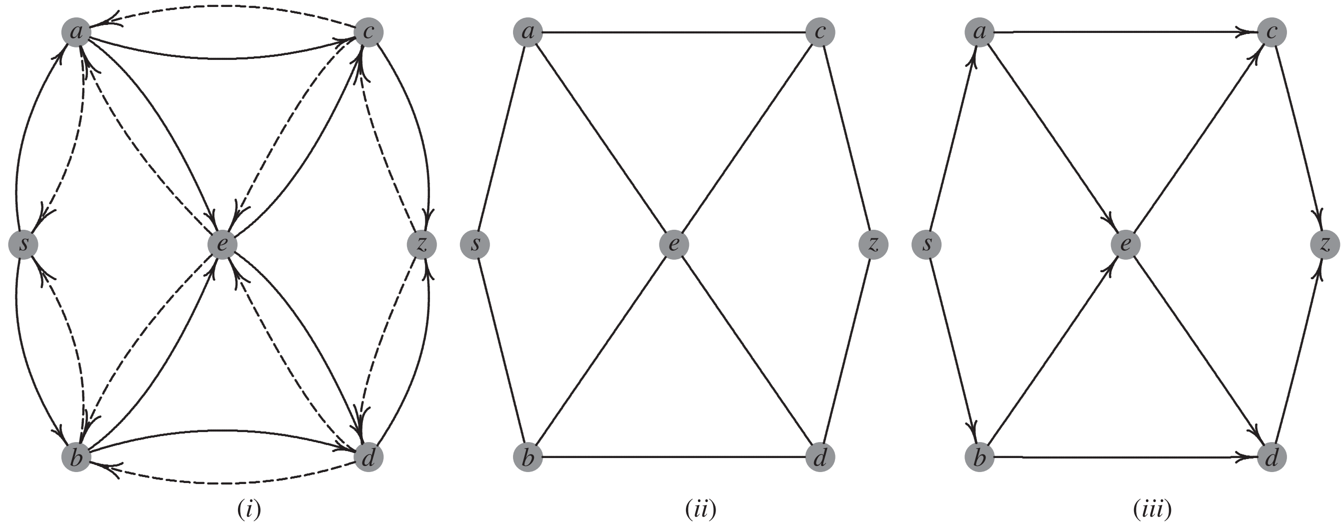

Figure 1. Abstract evacuation network, auxiliary network and network after contraflow reconfiguration, respectively

3.1 Maximum Abstract Contraflow

We define the maximum abstract contraflow (MACF) by integrating the contraflow model introduced in (Arulselvan, 2009; Rebennack *et al.*, 2010) and the MAF problem solved in (McCormick, 1996). With simple extension, we also give an efficient algorithm for solving Problem 1.

Problem 1. *Given an abstract network $N = (E, \Gamma, b, s, z)$, the MACF problem is to find the MAF with path reversals allowed initially.*

Theorem 1. *Let $\tilde{b} \in Z^+$ be the capacities and $\omega_{\tilde{\gamma}}$ be the supermodular weights on paths $\tilde{\gamma} \in \tilde{\Gamma}$. Then the weighted abstract flow and weighted abstract cut problems in auxiliary abstract network (AAN) have totally dual integer optimum solutions.*

To get solution for MACF, we combine the concepts of path reversal criterion presented in (Pyakurel *et al.*, 2017) and MAF algorithm provided in (McCormick, 1996).

Lemma 1. *The MACF does not decrease the flow value after contraflow configuration.*

Proof. By definition, $C \subseteq E$ is the collection of disconnecting and saturated elements such that every path connects source and sink, and contains only one element from $\tilde{\gamma}$. By construction, $\tilde{b}(\tilde{C}) = \sum_{e \in \tilde{C}} \tilde{b}_e \geq \sum_{e \in C} b_e$, where $\tilde{C} \subseteq \tilde{E}$ represents the cut in auxiliary network. Following Theorem 1, we have

$$val_{max}(\tilde{f}) = max \sum_{\tilde{\gamma} \in \tilde{\Gamma}} f(\tilde{\gamma}) = min\{\sum_{\tilde{e} \in \tilde{C}} \tilde{b}_{\tilde{e}} : \tilde{C} \subseteq \tilde{E}\} \geq min\{\sum_{e \in C} b_e : C \subseteq E\} = max \sum_{\gamma \in \Gamma} f(\gamma) = val_{max}(f).$$

Thus the claim follows. □

Lemma 2. *(Pyakurel et al., 2017) The abstract contraflow doubles the flow value after contraflow configuration whenever each element in a minimum abstract cut has symmetric capacity.*

Algorithm 1. *Maximum Abstract Contraflow Algorithm*

1. *Given path reversible abstract network $N = (G, b, s, z)$, where $G = (E, \Gamma)$.*
2. *Construct the AAN, $\tilde{N} = (\tilde{G}, \tilde{b}, \tilde{s}, \tilde{z})$ with new capacity $\tilde{b}(\tilde{\gamma}) = b(\overrightarrow{\gamma}) + b(\overleftarrow{\gamma})$.*
3. *Solve the maximum abstract network flow problem in $\tilde{N} = (\tilde{G}, \tilde{b}_e, \tilde{s}, \tilde{z})$ using (McCormick, 1996) as follows:*
   (a) *Initialize $\tilde{f} = \tilde{f}_0$, if an initial solution is given, otherwise initialize as the zero flow.*
   (b) *While $\tilde{f}$ is not optimal:*
      i. *Compute an augmenting structure. If no such structure exists, return $\tilde{f}$.*
      ii. *Determine $\delta \in \tilde{N}$ so that all paths in augmenting structure can be augmented by δ.*
      iii. *For each path $\tilde{\gamma}^+$ in augmenting structure, set $\tilde{f}_{\tilde{\gamma}^+} = \tilde{f}_{\tilde{\gamma}^+} + \delta$.*
      iv. *For each path $\tilde{\gamma}^-$ in augmenting structure, set $\tilde{f}_{\tilde{\gamma}^-} = \tilde{f}_{\tilde{\gamma}^-} - \delta$.*
4. *A path $\overleftarrow{\gamma} \in \Gamma$ is reversed if and only if the flow along $\overrightarrow{\gamma} \in \Gamma$ is greater than $b(\overrightarrow{\gamma})$ or there is a non-negative flow along the path $\overrightarrow{\gamma} \notin \Gamma$.*

Theorem 2. *Algorithm 1 solves Problem 1 optimally.*

Proof. The Steps 2 and 3 are feasible by definition. Step 4 is well defined; i.e. not both paths $\overrightarrow{\gamma}$ and $\overleftarrow{\gamma}$ have to be switched at a time, this is ensured by the solution of the abstract flow in auxiliary network, (Hoffman, 1974). Switching property cancels cycle flows so that there is flow along $\overrightarrow{\gamma}$ or $\overleftarrow{\gamma}$ but not in both directions at the same time. Hence, the resulting flow from Step 4 is a feasible flow with path reversals in N.

Weights are supermodular and abstract path system satisfies switching property so that the MAF is totally dual integral with minimum cut, (Hoffman, 1974). By equivalence of optimization and separation problems of (Grötschel *et al.*, 1988), the oracle solves the separation problem for weighted abstract flow problem. The abstract maximum flow algorithm maintains a candidate set for an abstract minimum cut as solution of dual problem. The algorithm then calls oracle to verify the dual feasible solution, infact this is the case, the primal solution is a MAF, (McCormick, 1996). Otherwise, the oracle returns a violating path. The returned violated paths are then combined to an augmenting structure which allows to improve the flow value. In fact, any optimal solution to the maximum flow problem with path reversals on N is also a feasible solution to the maximum flow problem on $\tilde{N}$. As the amount of flow sent from s to z in Steps 3 is not changed in Step 4, the resulting flow is an optimal solution. □

Corollary 1. *(Pyakurel et al., 2017) Algorithm 1 computes MACF solution in polynomial time.*

Proof. The direction of paths can be reversed using Step 4 in linear time. Construction of AAN takes linear time. Thus, the complexity of Algorithm 1 depends on the complexity of Step 3 that takes $O(|\tilde{E}|logB)$ time in $\tilde{N}$, where B is the maximum capacity of any path $\tilde{\gamma} \in \tilde{\Gamma}$. □

Example 3. *Consider the abstract network of Figure 2 (i), where* 4 *units flow through* $(s; b; z)$*,* 2 *units flow through* $(s; a; z)$ *and* 1 *unit flow through* $(s; b; a; z)$ *can be send from source to sink. Here, the MAF is* 7 *in the network. But it is* 13 *for the network of Figure 2 (iii) after contraflow configuration with following path flows:* 6 *units through* $(s; b; z)$*,* 6 *units through* $(s; a; z)$ *and* 1 *units through* $(s; b; a; z)$*.*

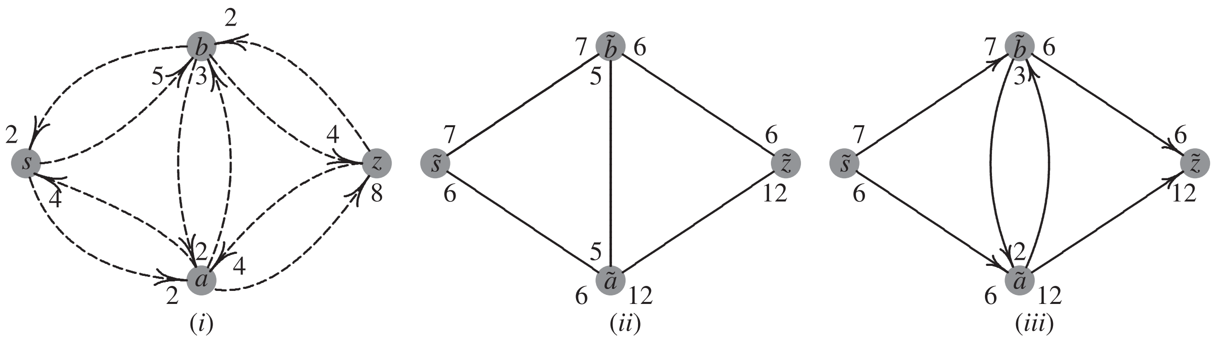

Figure 2. Abstract evacuation network, auxiliary network and network after contraflow reconfiguration, respectively

3.2 Maximum Dynamic Abstract Contraflow

The MDAF problem has been introduced and solved in (Kappmeier *et al.*, 2014). The maximum dynamic contraflow problem with arc reversal capability and its strongly polynomial time solution procedure are presented by (Rebennack *et al.*, 2010). We introduce and solve the maximum dynamic abstract contraflow (MDACF) with path reversal capability (cf. Problem 2).

Problem 2. *Given an abstract network* $N = (E, \Gamma, b, \tau, s, z, T)$*, the MDACF problem is to find the MDAF with path reversal capability at time zero.*

The abstract path system is allowed to be asymmetric with respect to the path capacities but the transit times are symmetric. In auxiliary network only capacities of the paths change but the transit times remain the same. As the abstract cut C_{dyn} contains an element of every temporally repeated paths, the capacity constraints are satisfied at each point of time. Thus for a subset $C_{dyn} \subseteq E_T$, the set $\gamma_t \cap C_{dyn}$ is nonempty to each $\gamma_t \in \Gamma_T$, (Kappmeier *et al.*, 2014). This implies that $\sum_{\gamma_t \in \Gamma_T} f_{dyn}(\gamma_t) \leq \sum_{(e,\, \theta) \in C_{dyn}} b_e$. The number of paths created by applying the time expansion is linear in T and thus exponential in the size of input. We combine the concept of path reversal capability of (Pyakurel *et al.*, 2017) and solution method of MDAF problem of (Kappmeier *et al.*, 2014). Algorithm 2 works in ATEN of an AAN that consists of a (static) abstract network. For each interval, a copy of the element set $\tilde{E}$, the element set $\tilde{E}_T := \tilde{E} \times \mathbf{T}$ will be constructed in ATEN is constructed by $\tilde{E}_T = \{(\tilde{e}, \theta) | \tilde{e} \in \tilde{E}, \theta \in \{1, 2, \cdots, T\}\}$.

Example 4. *The ATEN of an AAN can destroy the switching property. Let* $(\tilde{E}, \tilde{\Gamma})$ *be an auxiliary network with* $\tilde{\Gamma} = \{\gamma^1, \gamma^2, \gamma^3, \gamma^4\}$ *and* $\tilde{E} = \{s, a, b, z\}$*, where* $\gamma^1 = (s;\ a;\ b;\ z)$*,* $\gamma^2 = (s;\ b;\ a;\ z)$*,* $\gamma^3 = (s;\ a;\ z)$*,* $\gamma^4 = (s;\ b;\ z)$ *together with their reversals. The set* $\tilde{\Gamma}$ *satisfies the switching and order properties. For* $T = 4$*, we have* $\Gamma_T = \{\gamma_0^1, \gamma_2^1, \gamma_0^2, \gamma_1^2, \gamma_0^3, \gamma_1^3, \gamma_2^3, \gamma_0^4, \gamma_1^4, \gamma_2^4\}$ *with* $\gamma_0^1 = \{(s, 0), (a, 1,), (b, 2), (z, 3)\}$ *and* $\gamma_1^2 = \{(s, 1), (b, 2), (a, 3), (z, 4)\}$*. But time expansion of given network does not contain* $\gamma_0^1 \times_{(b,2)} \gamma_1^2$ *(cf. Figure 3).*

As in the abstract dynamic network, we can construct time-expanded abstract network $(\tilde{E}_T, \tilde{\Gamma}_T^{\sigma})$ of the auxiliary abstract dynamic network $(\tilde{E}, \tilde{\Gamma})$ with time horizon T. In this case, internal waiting does not make difference in optimality, (Kappmeier *et al.*, 2014).

Lemma 3. *The MDACF in* $N = (G, b, \tau, s, z, T)$ *is not more than the optimal flow in the MACF problem for the corresponding time expanded graph.*

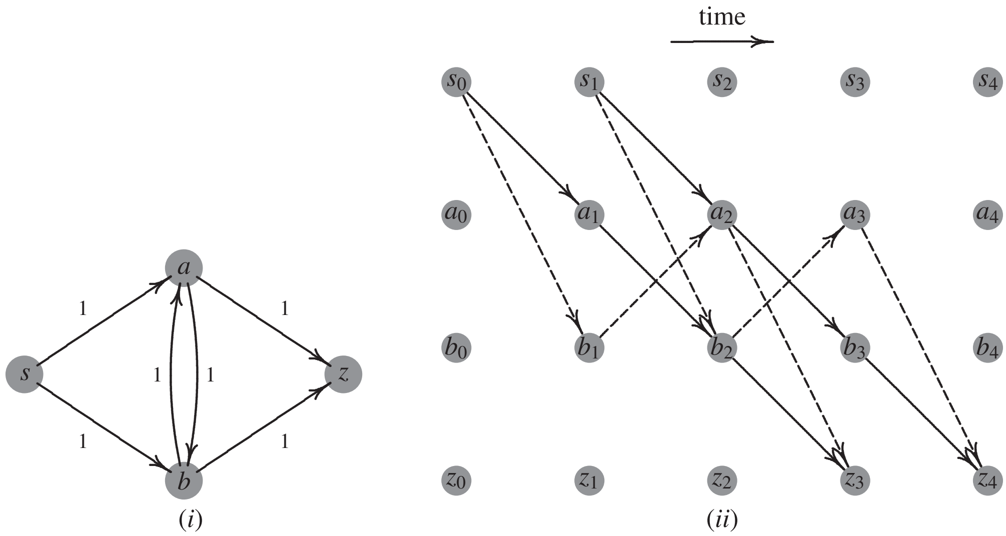

Figure 3. Dynamic abstract network and time expansions of the abstract network, respectively

Proof. The maximum amount of flow in the single source and single sink dynamic contraflow problem is less than the optimal flow in the maximum contraflow problem for the corresponding time expanded graph, (Rebennack *et al.*, 2010). The ATEN can be constructed by allowing intermediate waiting that does not make difference in optimality of MDAF, (Kappmeier *et al.*, 2014). Then we can conclude that every feasible flow to the MDACF problem has an equivalent feasible flow to the maximum contraflow problem of the time expanded graph. Hence, the claim follows. □

Lemma 4. *The dynamic abstract contraflow does not decrease the flow value after contraflow.*

Proof. By definition of minimum dynamic cut, $C_{dyn} \subseteq E_T$ is the collection of disconnecting and saturated elements with each path connecting source and sink in ATEN. The element set $\tilde{E}$ is copied for each point of time to construct the element set $\tilde{E}_T := \tilde{E} \times T$ in ATEN. The network $(\tilde{E}, \tilde{\Gamma})$ can destroy switching property. To construct abstract network, waiting is allowed in intermediate nodes but there is not any difference in optimality, (Kappmeier *et al.*, 2014). Every path passes through the cut set exactly onces on every time interval, i.e., $\tilde{\Gamma}^{\sigma} \cap \tilde{C}_{dyn} \neq \emptyset$, (Kappmeier *et al.*, 2014). The $\tilde{C}_{dyn}$ contains only one element from $\tilde{\gamma}$. The minimum cut capacity becomes $\tilde{b}(C^*_{dyn}) = \sum_{(\tilde{e},\theta)\in C^*} \tilde{b}_e\}$, where $\tilde{C}^*_{dyn}$ is the minimum cut. If each element $(e, \theta) \in C^*_{dyn}$ has capacity in both directions, the contraflow reconfiguration of the abstract network increases the capacity of cut but the capacity of minimum cut will not decrease even if there is not capacity in both directions. Then

$$\tilde{b}(\tilde{C}^*_{dyn}) = \sum_{(\tilde{e},\theta)\in\tilde{C}^*_{dyn}} \tilde{b}_{(\tilde{e},\theta)} \geq \sum_{(e,\theta)\in C^*_{dyn}} b_{(e,\theta)}.$$

Every MDAF is equal to the minimum dynamic abstract cut in auxiliary network, (Kappmeier *et al.*, 2014). Then,

$$\begin{aligned} val_{max}(\tilde{f}_{dyn}) &= max \sum_{\tilde{\gamma}_t\in\tilde{\Gamma}^{\sigma}_T} \tilde{f}(\tilde{\gamma}_t) = min\{\sum_{\tilde{e}\in\tilde{C}} \tilde{b}_{\tilde{e}} : \tilde{C}_{dyn} \subseteq \tilde{E}_T\} \\ &\geq min\{\sum_{e\in C} b_e : C_{dyn} \subseteq E_T\} = max \sum_{\gamma_t\in\Gamma^{\sigma}_T} f(\gamma_t) = val_{max}(f_{dyn}). \end{aligned}$$

The MDAF in AAN $\tilde{N} = (\tilde{G}, \tilde{b}_e, \tilde{\tau}, \tilde{s}, \tilde{z}, T)$ is a feasible dynamic abstract contraflow in original abstract network $N = (G, b_e, \tau, \tilde{s}, \tilde{z}, T)$ with path reversal capability. Hence the claim follows. □

Lemma 5. *The contraflow reconfiguration of an abstract contraflow network increases the flow value two times if each element in a minimum abstract dynamic cut has symmetric capacity.*

Algorithm 2. *Maximum Dynamic Abstract Contraflow Algorithm*

1. *Given abstract network $N = (G, b, \tau, \tilde{s}, \tilde{z}, T)$, where paths can be reverse without any cost.*
2. *Construct AAN, $\tilde{N} = (\tilde{G}, \tilde{b}_e, \tilde{\tau}, \tilde{s}, \tilde{z}, T)$ with new capacity and transit time functions $\tilde{b}(\tilde{\gamma}) = b(\overrightarrow{\gamma}) + b(\overleftarrow{\gamma})$ and $\tilde{\tau}(\tilde{\gamma}) = \sum_{\tilde{e}\in\tilde{\gamma}} \tilde{\tau}(\tilde{e})$.*
3. *Generate a temporally repeated dynamic flows $\tilde{G}$ with capacity $\tilde{b}(\tilde{\gamma})$ and transit time $\tilde{\tau}(\tilde{\gamma})$, (Kappmeier et al., 2014).*
4. *A path $\overleftarrow{\gamma} \in \Gamma$ is reversed if and only if the flow along $\overrightarrow{\gamma} \in \Gamma$ is greater than $b(\overrightarrow{\gamma})$ or there is a non negative flow along path $\overrightarrow{\gamma} \notin \Gamma$.*

Theorem 3. *Algorithm 2 solves Problem 2 optimally.*

Proof. The Steps 2 and 3 are feasible. Constructed flows from Step 3 are temporally repeated, there is only a flow in one direction of two elements at the same time as well as at different time periods, this ensures that the flow is less than the reversed capacities on all the paths at all time units and also ensures the feasibility of Step 4. Since every feasible flow of the DAF problem in $\tilde{N}$ is feasible to the MDACF problem in N, the MDACF in N is not more than the MDAF in $\tilde{N}$. The MACF in ATEN is not less than the MDACF in N, Lemma 3. Hence, the optimal dynamic contraflow in N is not greater than the MACF in N_T. The MACF problem in N_T is equivalent to the maximum flow problem in $\tilde{N}_T$, Theorem 2. Thus, the MACF in N_T is equal to the maximum abstract flow (MAF) in $\tilde{N}_T$. The temporally repeated abstract flow f^*_{dyn} is a MDAF, and C_{dyn} is a minimum abstract cut over time whose capacity is equal to the flow value, (Kappmeier *et al.*, 2014). Thus the maximum flow can be obtained by a temporally repeating a path flow of a static graph $\tilde{N}$. Hence the MAF in $\tilde{N}_T$ is equal to the MDAF in $\tilde{N}$. □

Corollary 2. *Algorithm 2 computes the MDACF in polynomial time.*

Proof. Steps 2 and Step 4 can be completed in linear time, so that the complexity is dominated by Step 3 that computes the MDAF problem in AAN. Step 3 can be computed in polynomial time, (Kappmeier *et al.*, 2014). □

Example 5. *Consider the symmetric transit time $\tau_e = 1$ for each $e \in E$ in Figure 2 (i). Using the algorithm, 6 units flow at time 4 through the paths $(s; a; z)$ and $(s; b; z)$, 7 units flow at time 5 through the paths $(s; a; z)$, $(s; b; z)$ and $(s; b; a; z)$ can be send to the sink. By the same idea, dynamic flows can be calculated in the network after contraflow configuration. The MDAF at time $T = 5$ are 13 and 25 before and after contraflow configuration, respectively, showing the significant increment with contraflow.*

3.3 Lexicographically Maximum Abstract Contraflow

If we have a given rank on the terminals with priorities, the flow is compared by its value in their rank ordering, referred to as lexicographically maximum flow. An existence and a polynomial solution of LMAF problem have been presented in (Kappmeier, 2015). In his model, the order of terminals has to fulfill compatible property if more than one terminal node is contained in a path.

Problem 3. *Let $N = (G, b, S, D)$ be an abstract network, where $G = (E, \Gamma)$ contains compatible sets of sources S and sinks D. The lexicographically maximum abstract contraflow (LMACF) problem is to find a LMAF where paths can be reversed without any cost.*

Based on the LMACF algorithm, (Kappmeier, 2015) and the lexicographically maximum contraflow algorithm (Pyakurel & Dhamala, 2015), we propose LMACF Algorithm 3, which solves the Problem 3 in polynomial time.

Algorithm 3. *Lexicographically Maximum Abstract Contraflow Algorithm*

1. *Given abstract network $N = (G, b, S, D, \omega \equiv 1)$ with a compatible sequence of sources $s_1, s_2, \cdots, s_k$ or sinks $z_1, z_2, \cdots, z_k$ in S and D, respectively.*
2. *Construct AAN, $\tilde{N} = (\tilde{G}, \tilde{b_e}, \tilde{S}, \tilde{Z}, \omega \equiv 1)$ with capacity $\tilde{b}(\tilde{\gamma}) = b(\overrightarrow{\gamma}) + b(\overleftarrow{\gamma})$.*

3. *Find solution in auxiliary network using Abstract Lexicographically Maximum Flow Algorithm, (Kappmeier, 2015):*
   (a) *Set* $i = 0$ *and initialize* $\tilde{f}^0 = 0$ *as the zero flow on all paths.*
   (b) *Set* $i = i + 1$ *and define the abstract networks* $(\tilde{E}, \tilde{\Gamma}^i_{\tilde{s}})$, *(or* $(\tilde{E}, \tilde{\Gamma}^i_{\tilde{z}})$ *w. r. t. the sinks).*
   (c) *Compute a flow* $\tilde{f}^i$ *using Step 3 of Algorithm 1 in* $(\tilde{E}, \tilde{\Gamma}^i_{\tilde{s}})$ *starting with solution* $\tilde{f}^{i-1}$.
   (d) *If* $i = k$ *return* $\tilde{f}^k$, *otherwise continue with Step 3b.*
4. *A path* $\overleftarrow{\gamma} \in \Gamma$ *is reversed if and only if the flow along* $\overrightarrow{\gamma} \in \Gamma$ *is greater than* $b(\overrightarrow{\gamma})$ *or there is a non negative flow along path* $\overrightarrow{\gamma} \notin \Gamma$.

Lemma 6. *Let* $\tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_k$ *be a compatible sequence of sources and let* $\tilde{f}^i$ *be a MAF in the auxiliary network* $(\tilde{A}, \tilde{\Gamma}^i_{\tilde{s}})$. *If LMAF algorithm is executed in* $(\tilde{A}, \tilde{\Gamma}^{i+1}_{\tilde{s}})$ *with initial flow* $\tilde{f}^i$ *as input, the flow value* $|f|^+_{\tilde{s}_j}$ *does not decrease for* $j = 1, \cdots, i$ *during the execution. Moreover, the same is true for sinks.*

Theorem 4. *Algorithm 3 solves Problem 3 in* $N = (G, b, S, D, \omega \equiv 1)$ *optimally.*

Proof. The feasibility can be proved as in Theorem 2. The path capacity is increased by adding the capacity of paths and either direction of them is allowed in auxiliary network with unaltered priority ordering. Step 3 works for source and sink element sequences in auxiliary network. We prove the optimality by induction on i. The first iteration computes a MAF $\tilde{f}^1$ in $(\tilde{E}, \tilde{\Gamma}^1_{\tilde{s}})$ with single source. Let $\tilde{f}^i$ be a LMAF in $(\tilde{E}, \tilde{\Gamma}^i_{\tilde{s}})$. By Lemma 6, flow $\tilde{f}^{i+1}$ does not reduce the inflow of source element $\tilde{s}_j$ for some $j \in \{1, 2, \cdots, i\}$ and the flow is maximum. Assume now that $\tilde{f}^{i+1}$ is not a LMAF in the abstract network $(\tilde{E}, \tilde{\Gamma}^{i+1}_{\tilde{s}})$. Then there is a flow $\tilde{f}'$ that sends more flow out of the source $\tilde{s}_j$ for some $j \in \{1, 2, \cdots, i\}$. Define the restricted $\tilde{f}^r$ by setting $\tilde{f}^r = \tilde{f}'$ for each path $\gamma \in \Gamma^i$. The outflow of source element $\tilde{s}_j$ is the same for $\tilde{f}'$ and $\tilde{f}^r$, and $\tilde{f}^r$ is a feasible abstract flow in $(\tilde{E}, \tilde{\Gamma}^i_{\tilde{s}})$ that sends more flow out of $\tilde{s}_j$ than does $\tilde{f}^i$ which contradicts that $\tilde{f}^i$ is a lexicographically maximum. Hence, $\tilde{f}^{i+1}$ is a LMAF in $(\tilde{E}, \tilde{\Gamma}^{i+1}_{\tilde{s}})$. Any optimal solution to the LMAF problem with path reversal in graph (E, Γ) is also a feasible solution to the maximum flow problem in $\tilde{N}$. As the amount of flow sent from $\tilde{S}$ to $\tilde{D}$ in Step 3 is not changed in Step 4, the resulting flow is optimal for the Problem 3. □

Corollary 3. *Algorithm 3 solves the LMACF problem in polynomial time complexity.*

Proof. The construction of auxiliary network and Step 4 can be completed in linear time. As $N = (G, b_e, S, D, \omega \equiv 1)$ is an abstract network with a compatible sequence of sources, the complexity depends upon the running time of Step 3. Moreover, a separation oracle is given in the problem. The LMAF can be computed in polynomial time which is dominated by $|\tilde{E}|$, (Kappmeier, 2015). □

3.4 Earliest Arrival Abstract Contraflow

The existence of earliest arrival abstract flow (EAAF) described by (Kappmeier, 2015) generalizes the earliest arrival flow which maximizes the DAF at each possible time point.

Problem 4. *Let* $N = (G, b, \tau, s, z, T)$ *be an abstract dynamic network, where* $G = (E, \Gamma)$. *The earliest arrival abstract contraflow (EAACF) problem is to find the EAAF with path reversal capability at time zero.*

Based on the earlier results of earliest arrival contraflow problem with arc reversals in (Pyakurel & Dhamala, 2015) and EAAF in (Kappmeier, 2015), we introduce EAACF problem and propose Algorithm 4 to solve it. (Minieka, 1973) proved that using the successive shortest path algorithm in the original network and sending flow along the generalized temporally repeated paths leads to an earliest arrival flow. This approach does not require waiting in intermediate nodes and sends flow only along temporal copies of the original paths. A similar algorithm to the successive shortest path algorithm is in (Martens and McCormick, 2008). Their algorithm computes a maximum weighted abstract flow by using augmenting structure of decreasing total reward, where the shortest path has the most reward.

Algorithm 4. *Earliest Arrival Abstract Contraflow Algorithm*

1. *Given abstract network* $N = (G, b, \tau, s, z, T)$ *where paths can be reverse without any cost.*
2. *Construct AAN,* $\tilde{N} = (\tilde{G}, \tilde{b}_e, \tilde{\tau}_e, \tilde{s}, \tilde{z})$ *with new capacity and transit functions* $\tilde{b}(\tilde{\gamma}) = b(\overrightarrow{\gamma}) + b(\overleftarrow{\gamma})$ *and* $\tilde{\tau}(\tilde{\gamma}) = \sum_{\tilde{e} \in \tilde{\gamma}} \tilde{\tau}(\tilde{e})$, *respectively.*

3. *Solve the problem in the auxiliary network using Step 3 of Algorithm 3 in corresponding* $(\tilde{E}_T, \tilde{\Gamma}_T^{\sigma})$ *of* $\tilde{N}$.

4. *A path* $\overleftarrow{\gamma} \in \Gamma$ *is reversed if and only if the flow along* $\overrightarrow{\gamma} \in \Gamma$ *is greater than* $b(\overrightarrow{\gamma})$ *or there is a non negative flow along path* $\overrightarrow{\gamma} \notin \Gamma$.

Theorem 5. *Algorithm 4 solves Problem 4 optimally.*

Proof. The feasibility can be ensured as in Theorem 2. Let $(\tilde{E}_T, \tilde{\Gamma}_T^{\sigma})$ be the corresponding increasing ATEN of $\tilde{N}$ with intermediate waiting, (Kappmeier *et al.*, 2014), where T be a significantly large time horizon. Step 3 is executed in $(\tilde{E}_T, \tilde{\Gamma}_T^{\sigma})$ that avoids reduction of flow sent in earlier time steps by removing backward paths in the residual network, Lemma 6. For an abstract network $(\tilde{E}, \tilde{\Gamma}, T)$, there exists an EAAF flow $\tilde{f}$ in $(\tilde{E}, \tilde{\Gamma}, T)$, (Kappmeier, 2015). The resulting flow $\tilde{f}$ from Step 3 signifies that the obtained flow is optimal in $(\tilde{E}, \tilde{\Gamma}, T)$. Any optimal solution to the earliest arrival flow problem with path reversal in network N is also a feasible solution to the EAAF problem in the auxiliary network $\tilde{N}$. As the amount of flow sent from s to z in Step 3 is not changed in Step 4, the resulting flow is an optimal solution to the EAACF problem in N. □

Example 6. *Consider the network of Example 5. In this example, MDACF satisfies the earliest arrival flow property. Here, the EAAF values before contraflow configuration at time* $T = 4$ *and* 5 *are* 6 *and* 13*, respectively. Also, the EAACF values at these times are* 12 *and* 25*, respectively.*

4. Conclusions

In this paper, we studied both abstract flow and contraflow models from litrature. Integrating these models, we introduced abstract contraflow approach with discrete time settings for the first time. Through these models we came to know that the switching property is the most essential force behind abstract flow. In abstract flow model, some structural results of classical network are also valid such as, the MAF and minimum abstract cut are strong dual to each other. We proposed efficient algorithms for maximum dynamic and EAACF problems in two-terminal abstract networks. We also proposed polynomial time algorithm for lexicographically maximum abstract contraflow. Our results increase the flow values by reducing crossing conflicts with arc reversals toward the safe destinations in evacuation planning.

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References

Altay, N., & Green III, W. G. (2006). Or/ms research in disaster operations management. *European Journal of Operational Research, 17*, 475-493.

Arulselvan, A. (2009). Network model for disaster management, PhD thesis, University of Florida.

Dhamala, T. N. (2015). A survey models algorithms for discrete evacuation planning network problems. *Journal of Industrial and Management Optimization, 11(1)*, 265-289.

Dhamala, T. N., & Pyakurel, U. (2013). Earliest arrival contraflow problem on series parallel graph. *International Journal of Operations Research, 10(1)*, 1-13.

Dhungana, R. C., Pyakurel, U., Khadka, S. R., & Dhamala, T. N. (2015). Universally maximum contraflow on for evacuation planning. *International Journal of Operations Research Nepal, 4(1)*, 67-78.

Fleischer, L., & Skutella, M. (2003). The quickest multicommodity flow problem. *Lecture Notes in Computer Science*, 36-53.

Ford, L. R., & Fulkerson, D. R. (1956). Maximal flow through a network. *Canadian Mathematical Society, 8*, 399-404.

Ford, L. R., & Fulkerson, D. R. (1958). Constructing maximal dynamic flows from static network. *Operations Research, 6*, 416-433.

Grötschel, M., Lovasz, L., & Schrijver, A. (1988). Geometric algorithms and combinatorial optimization. Vol. 2, Springer, Berlin, Heidelberg.

Hamacher, H. W., & Tjandra, S. A. (2001). Mathematical modeling of evacuation problems: a state of the art, in pedestrian and evacuation dynamics, (eds. M. Schreckenberger and S.D. Sharma), Springer.

Hoffman, A. (1974). A generalization of max flow-min cut. *Mathematical Programming, 6(1)*, 352-359.

Hoppe, B., & Tardos, E. (1994). Polynomial time algorithm for some evacuation problems. *Annual ACM-SIAM symposium on Discrete Algorithms, 5*, 433-441.

Hoppe, B., & Tardos, E. (2000). The quickest transshipment problem. *Mathematics of Operations Research, 25*, 36-62.

Hua, J., Ren, G., Cheng, Y., & Ran, B. (2014). An integrated contraflow strategy for multi modal evacuation. *Hindawi Publishing Corporation Mathematical Problems in Engineering*. Article ID 159473.

Jarvis, J. J., & Ratliff, H. D. (1982). Some equivalent objectives for dynamic network flow problems. *Management Science, 28(1)*, 106-109.

Kappmeier, J.-P. W. (2015). Generalizations of flows over time with application in evacuation optization, PhD thesis, Technical University, Berlin, Germany.

Kappmeier, J.-P. W., Matuschke, J., & Peis, B. (2014). Abstract flows over time: a first step towards solving dynamic packing problems. *Theoretical Computer Science, Algorithms and Computation, 544*, 74-83.

Kim, S., Shekhar, S., & Min, M. (2008). Contraflow transportation network recofiguration for evacuation route planning. *IEEE Transactions on Knowledge and Data Engineering, 20*, 1115-1129.

Martens, M., & McCormick, S. T. (2008). A polynomial algorithm for weighted abstract flow. *Integer Programming and Combinatorial Optimization*, 97-111.

McCormick, S. T. (1996). A polynomial algorithm for abstract maximum flow. *In: Proceedings of the seventh annual ACM-SIAM symposium on Discrete algorithms*, 490-497.

Megiddo, N. (1974). Optimal flows in networks with multiple sources and sinks. *Mathematical programming, 7(1)*, 97-107.

Minieka, E. (1973). Maximal lexicographic and dynamic network flows. *Operations Research, 21*, 517-527.

Moriarty, K., Ni, D., & Collura, J. (2007). Modeling traffic flow under emergency evacuation situations: current practice and future directions. *In: 86th Transportation Research Board Annual Meeting, Washington, D.C., 07-0745*.

Pel, A. J., Bliemer, M. C. J., & Hoogendoorn, S. P. (2012). A review on travel behavior modeling in dynamic traffic simulation models for evacuations. *Transportation, 39*, 97-123.

Pyakurel, U., & Dhamala, T. N. (2015). Models and algorithms on contraflow evacuation planning network problems. *International Journal of Operations Research, 12(2)*, 36-46.

Pyakurel, U. (2016). Evacuation planning problem with contraflow approach, PhD thesis, Tribhuvan University, Kathmandu, Nepal.

Pyakurel, U., & Dhamala, T. N. (2016). Continuous time dynamic contraflow models and algorithms. *Hindawi Publishing Corporation, Advances in Operations Research*. Article ID 7902460.

Pyakurel, U., & Dhamala, T. N. (2017a). Continuous dynamic contraflow approach for evacuation planning. *Annals of Operations Research, Springer, 253(1)*, 573-598.

Pyakurel, U., & Dhamala, T. N. (2017b). Evacuation planning by earliest arrival contraflow. *Journal of Industrial and Management Optimization, 13*(1), 489-503.

Pyakurel, U., Dhamala, T. N., & Dempe, S. (2017). Efficient continuous contraflow algorithms for evacuation planning problems. *Annals of Operation Research, 254(1-2)*, 335-364.

Pyakurel, U., Hamacher, H. W., & Dhamala, T. N. (2014). Generalized maximal dynamic contraflow on lossy network. *International Journal of Operations Research Nepal, 3(1)*, 27-44.

Pyakurel, U., Nath, H. N., & Dhamala, T. N. (2018). Effcient contraflow algorithms for quickest evacuation planning. *Science China Mathematics*. Doi 10.1007/s11425-017-9264-3.

Rebennack, S., Arulselvan, A., Elefteriadou, L., & Pardolas, P. M. (2010). Complexity analysis of maximum flow problem with arc reversal. *Journal of Combinatorial Optimization, 29*, 200-216.

Schadschneider, A., Klingsch, W., Klüpfel, H., Kretz, T., Rogsch, C., & Seyfried, A. (2009). Evacuation dynamics: Empirical results, modeling and applications. *In Encyclopedia of Complexity and Systems Science*, 3142-3176.

Wang, J., Wang, H., Zhang, W., Ip, W. H., & Furuta, K. (2013). Evacuation planning based on the contraflow technique with consideration of evacuation priorities and traffic setup time. *IEEE Transactions on Intelligent Transportation Systems, 14*, 480-485.

Xie, C., Lin, D. Y., & Waller, S. (2010). A dynamic evacuation network optimization problem with lane reversal and crossing elimination strategies. *Transportation Research Part E, 46*, 295-316.

Xie, C., & Turnquist, M. A. (2011). Lane-based evacuation network optimization: An integrated lagrangian relaxation and tabu search approach. *Transportation Research Part C, 19*, 40-63.

Zhao, X., Feng, Z.-Y., Li, Y., & Bernard, A. (2016). Evacuation network optimization model with lane-based reversal and routing. *Hindawi Publishing Corporation, Advances in Operations Research*. Article ID 1273508.