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An Approached Solution of Wave Equation with Cubic Damping by Homotopy Perturbation Method (HPM), Regular Pertubation Method (RPM) and Adomian Decomposition Method (ADM)

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Abstract

In this study, we consider the wave equation with cubic damping with its initial conditions. Homotopy Perturbation Method (HPM), Regular Pertubation Method (RPM) and Adomian decomposition Method (ADM) are applied to this equation. Then, the solution yielding the given initial conditions is gained. Finally, the solutions obtained by each method are compared.

Keywords: Wave equation, cubic damping, Homotopy Perturbation Method (HPM),Regular Pertubation Method (RPM), Adomian decomposition Method (ADM)

1. Introduction

Over the last decades, several analytical/approximate methods have been developed to solve ordinary and partial differential equations. Some of these techniques include Homotopy Perturbation Method (He, J. H., 1999; He, J. H., 2000; He, J. H., 2003; He, J. H., 2004; Gupta, S. & et al., 2013), Regular Pertubation Method (Ghazanfari, B., 2011) and Adomian decomposition Method (ABBAOUI, K., 1995; ABBAOUI, K. & CHERRUAULT, Y., 1994; ABBAOUI, K. & CHERRUAULT, Y., 1999; NGARHASTA, N. & et al., 2002; Oke, M. O., 2015; Ghoreishi, M. & et al, 2010), etc.

Linear and nonlinear phenomena are of fundamental importance in various fields of science and engineering. Most models of real-life problems, however, are still very difficult to solve. Therefore, approximate analytical solutions such as HPM, RPM and ADM were introduced, which are effective and convenient for both linear and nonlinear equations.

In this paper, we consider the following nonlinearly damped wave equation

$$\begin{cases} \dfrac{\partial^2 u}{\partial t^2} - \dfrac{\partial^2 u}{\partial x^2} = -\varepsilon \left(\dfrac{\partial u}{\partial t}\right)^3, \ t \geq 0, \ x \in \mathbb{R} \\ \\ u(0,x) = \cos x, \ \dfrac{\partial u}{\partial t}(0,x) = 0 \end{cases} \tag{1}$$

Where ε is perturbation parameter which , $u(t,x)$ is some physical quantity, x the space variable and t stands for time.

These types of equations are of considerable significance in various fields of applied sciences, mathematical physics, nonlinear hydrodynamics, engineering physics, biophysics, human movement sciences, astrophysics and plasma physics. The paper is organised as follows : in section 1, we start with the solving (1) by HPM. In Section 2 and section 3, we construct the solution of (1) respectively by RPM (He, J. H., 2004) and ADM. Section 4 contains the comparison of the solutions obtained by the different methods.

2. Homotopy Perturbation Method

2.1 Basic Idea of the Homotopy Perturbation Method

To illustrate the basic idea of HPM, consider the following nonlinear differential equation

$$A(u) = f(r), \quad r \in \Omega \tag{2}$$

with boundary conditions

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0, \quad r \in \Gamma$$

where A is a general differential operator, B is a boundary operator, $f(r)$ is a known analytic function, and Γ is the boundary of the domain Ω. Generally speaking, the operator A can be decomposed into two parts L and N, where L is a linear and N is a nonlinear operator. Equation (2), therefore, can be rewritten as follows :

$$L(u) + N(u) - f(r) = 0$$

We construct a homotopy $v(r, p)\ :\ \Omega \times [0,\ 1] \to \mathbb{R}$, that satisfies

$$H(v,\ p) = (1-p)\left[L(v) - L(u_0)\right] + p\left[A(v) - f(r)\right] = 0,\ \ p \in [0,\ 1],\ \ r \in \Omega \tag{3}$$

or, equivalently,

$$H(v,\ p) = L(v) - L(u_0) + pL(u_0) + p\left[N(v) - f(r)\right] = 0$$

where u_0 is an initial approximation to the solution of Equation (2). In this method, we use the homotopy parameter p to expand v as a power series

$$v = v_0 + pv_1 + p^2v_2 + p^3v_3 + p^4v_4 + p^5v_5 + \cdots$$

The approximate solution can be obtained by setting $p = 1$,

$$u = \lim p \to 1v = v_0 + v_1 + v_2 + \cdots \tag{4}$$

The convergence of the series of (4) has been proved in (He, J. H., 1999; He, J. H., 2000).

2.2 Application of HPM Wave Equation with Cubic Damping

According to the HPM (He, J. H., 1999; He, J. H., 2003; He, J. H., 2004; Gupta, S. & et al., 2013), we can construct the homotopie $H(v, p)$ for equation (1) which satisfies :

$$H(v, p) = (1-p)\left[\frac{\partial^2 v}{\partial t^2} - \frac{\partial^2 u_0}{\partial t^2}\right] + p\left[\frac{\partial^2 v}{\partial t^2} - \frac{\partial^2 v}{\partial x^2} + \varepsilon\left(\frac{\partial v}{\partial t}\right)^3\right]$$

As $H(v, p) = 0$, then we have :

$$\frac{\partial^2 v}{\partial t^2} - \frac{\partial^2 u_0}{\partial t^2} + p\frac{\partial^2 u_0}{\partial t^2} - p\frac{\partial^2 v}{\partial x^2} + p\varepsilon\left(\frac{\partial v}{\partial t}\right)^3 = 0$$

Let as choose the initial approximation as $u_0 = \cos x$, thus $\dfrac{\partial^2 u_0}{\partial t^2} = \dfrac{\partial u_0}{\partial t} = 0$

We have,

$$\frac{\partial^2 v}{\partial t^2} - p\frac{\partial^2 v}{\partial x^2} + p\varepsilon\left(\frac{\partial v}{\partial t}\right)^3 = 0 \tag{5}$$

Assume the solution of (1) to be in the form :

$$v = v_0 + pv_1 + p^2v_2 + p^3v_3 + p^4v_4 + p^5v_5 + \cdots \tag{6}$$

Substituting (6) into (5) and equating the coefficients of like powers p, we get the following set of differential equations :

$$p^0\ \ :\ \ \begin{cases} \dfrac{\partial^2 v_0}{\partial t^2} - \dfrac{\partial^2 u_0}{\partial t^2} = 0 \\ \\ v_0(0, x) = \cos x,\ \dfrac{\partial v_0}{\partial t}(0, x) = 0 \end{cases} \tag{7}$$

$$p^1\ \ :\ \ \begin{cases} \dfrac{\partial^2 v_1}{\partial t^2} - \dfrac{\partial^2 v_0}{\partial x^2} + 3\varepsilon\left(\dfrac{\partial v_0}{\partial t}\right)^2\left(\dfrac{\partial^2 v_1}{\partial t}\right) = 0 \\ \\ v_1(0, x) = 0,\ \dfrac{\partial v_1}{\partial t}(0, x) = 0 \end{cases} \tag{8}$$

$$p^2\ \ :\ \ \begin{cases} \dfrac{\partial^2 v_2}{\partial t^2} - \dfrac{\partial^2 v_1}{\partial x^2} + 3\varepsilon\left(\dfrac{\partial v_0}{\partial t}\right)^2\left(\dfrac{\partial v_2}{\partial t}\right) + 3\varepsilon\left(\dfrac{\partial v_0}{\partial t}\right)\left(\dfrac{\partial v_1}{\partial t}\right)^2 = 0 \\ \\ v_2(0, x) = 0,\ \dfrac{\partial v_2}{\partial t}(0, x) = 0 \end{cases} \tag{9}$$

$$p^3 \quad : \quad \begin{cases} \dfrac{\partial^2 v_3}{\partial t^2} - \dfrac{\partial^2 v_2}{\partial x^2} + 3\varepsilon\left(\dfrac{\partial v_0}{\partial t}\right)^2\left(\dfrac{\partial v_3}{\partial t}\right) + 6\varepsilon\left(\dfrac{\partial v_0}{\partial t}\dfrac{\partial v_1}{\partial t}\dfrac{\partial v_2}{\partial t}\right) + \varepsilon\left(\dfrac{\partial v_1}{\partial t}\right)^3 = 0 \\ v_3(0,x) = 0,\ \dfrac{\partial v_3}{\partial t}(0,x) = 0 \end{cases} \tag{10}$$

$$\vdots \tag{11}$$

From the above equations, we can obtain

$$\begin{aligned}
v_0(t,x) &= \cos x \\
v_1(t,x) &= -\frac{1}{2}t^2\cos x \\
v_2(t,x) &= \frac{1}{24}t^4\cos x \\
v_3(t,x) &= -\frac{1}{720}t^6\cos x + \frac{\varepsilon}{20}t^5\ \cos^3 x \\
v_4(t,x) &= \frac{\varepsilon}{140}t^7\ \cos x\ \sin^2 x - \frac{13\,\varepsilon}{840}\,t^7\ \cos^3 x + \frac{1}{40320}t^8\ \cos x \\
v_5(t,x) &= -\frac{1}{3628800}\,t^{10}\cos x - \frac{\varepsilon}{504}\,t^9\cos x\ \sin^2 x + \frac{71\,\varepsilon}{30240}\,t^9\ \cos^3 x - \frac{3\,\varepsilon^2}{224}\,t^8\ \cos^5 x \\
&\vdots
\end{aligned}$$

In principle, it is possible to calculate more components in the expansion series to enhance the approximation. Therefore, we get the tenth-order approximation,

$$\begin{aligned}
u(t,x) = \; & \left[\cos x\left(1 - \frac{t^2}{2!} + \frac{t^4}{4!} - \frac{t^6}{6!} + \frac{t^8}{8!} - \frac{t^{10}}{10!} + \frac{t^{12}}{12!} - \frac{t^{14}}{14!} + \frac{t^{16}}{16!} - \frac{t^{18}}{18!} + \frac{t^{20}}{20!}\right)\right] + \\
& \varepsilon\left[\cos^3 x\left(\frac{t^5}{20} - \frac{13\,t^7}{840} + \frac{71\,t^9}{30240} - \frac{491\,t^{11}}{3326400} + \frac{13711\,t^{13}}{1037836800} - \frac{28607\,t^{15}}{43589145600}\right.\right. \\
& + \frac{66811\,t^{17}}{2694601728000} - \frac{7198319\,t^{19}}{10137091700736000} + \frac{t^{21}}{13502619648000} \\
& \left. - \frac{61\,t^{23}}{47826278793216000} + \frac{547\,t^{25}}{2869576727592960000}\right) \\
& + \cos x\ \sin^2 x\left(\frac{t^7}{140} - \frac{t^9}{504} + \frac{47\,t^{11}}{184800} - \frac{739\,t^{13}}{43243200} + \frac{34403\,t^{15}}{36324288000} - \frac{727\,t^{17}}{18712512000}\right. \\
& \left.\left. + \frac{188947\,t^{19}}{153592298496000} - \frac{t^{21}}{4725916876800} + \frac{13\,t^{23}}{3416162770944000} - \frac{41\,t^{25}}{717394181898240000}\right)\right] \\
& + \varepsilon^2\left[\cos^5 x\left(-\frac{3\,t^8}{224} + \frac{439\,t^{10}}{100800} - \frac{23801\,t^{12}}{13305600} + \frac{743201\,t^{14}}{2421619200} - \frac{38239\,t^{16}}{931392000} + \frac{78121361\,t^{18}}{22230464256000}\right.\right. \\
& \left. - \frac{1903\,t^{20}}{482236416000} + \frac{5921\,t^{22}}{31191051386880} - \frac{297239\,t^{24}}{31304473391923200}\right) \\
& + \cos^3 x\ \sin^2 x\left(-\frac{13\,t^{10}}{2800} + \frac{3397\,t^{12}}{1663200} - \frac{352421\,t^{14}}{605404800} + \frac{83371\,t^{16}}{825552000} - \frac{26118227\,t^{18}}{2223046425600}\right. \\
& \left. + \frac{1247\,t^{20}}{42195686400} - \frac{1191041\,t^{22}}{779776284672000} + \frac{11675131\,t^{24}}{215218254569472000} + \frac{12517\,t^{25}}{1016715110400000}\right) \\
& + \cos x\ \sin^4 x\left(-\frac{13\,t^{12}}{61600} + \frac{1663\,t^{14}}{20180160} - \frac{16447\,t^{16}}{864864000} + \frac{573889\,t^{18}}{205837632000} - \frac{2903\,t^{20}}{281304576000}\right. \\
& \left.\left. + \frac{12517\,t^{23}}{20334302208000} - \frac{1191041\,t^{24}}{717394181898240000} - \frac{162721\,t^{25}}{122005813248000000}\right)\right] + \\
& \varepsilon^3\left[\cos^7 x\left(\frac{57\,t^{11}}{12320} - \frac{1319\,t^{13}}{443520} + \frac{54919\,t^{14}}{47297250} + \frac{83\,,t^{15}}{470400} - \frac{54919\,t^{16}}{1621620000} - \frac{227253749\,t^{17}}{823350528000}\right.\right.
\end{aligned}$$

$$+\frac{1901771\,t^{19}}{1969132032000}-\frac{37259\,t^{21}}{170031960000}+\frac{215576267\,t^{23}}{760872617164800 0}\Big)+\cos^5 x\,\sin^2 x\Big(\frac{839\,t^{13}}{320320}$$
$$-\frac{165827\,t^{15}}{84084000}+\frac{54919\,t^{16}}{270270000}+\frac{76842539\,t^{17}}{137225088000}-\frac{947\,t^{19}}{202585600}+\frac{5031053\,t^{21}}{7254696960000}-\frac{34493909\,t^{23}}{333716060160000}\Big)$$
$$+\cos^3 x\,\sin^4 x\Big(+\frac{193\,t^{15}}{600600}-\frac{65171\,t^{17}}{285885600}+\frac{129\,t^{19}}{40517120}-\frac{5819\,t^{21}}{12155136000}+\frac{277663\,t^{23}}{4359166035840}\Big)+$$
$$\cos x\,\sin^6 x\Big(\frac{193\,t^{17}}{27227200}-\frac{t^{19}}{6077568}+\frac{193\,t^{21}}{3646540800}-\frac{35353\,t^{23}}{4612874112000}\Big)\Big]$$
$$+\varepsilon^4\Big[\cos^9 x\Big(-\frac{1097\,t^{14}}{407680}+\frac{557813\,t^{16}}{358758400}-\frac{2383\,t^{18}}{83166720}+\frac{1830301\,t^{20}}{210689024000}-\frac{12469598887\,t^{22}}{7884404656128000}\Big)+$$
$$\cos^7 x\,\sin^2 x\Big(-\frac{76059\,t^{16}}{44844800}+\frac{29\,t^{18}}{495040}-\frac{183721\,t^{20}}{8103424000}+\frac{2080131\,t^{22}}{331082752000}\Big)$$
$$+\cos^5 x\,\sin^4 x\Big(-\frac{9\,t^{18}}{247520}+\frac{6543\,t^{20}}{526722560}-\frac{103197433\,t^{22}}{24334582272000}\Big)+\cos^3 x\,\sin^6 x\Big(-\frac{9\,t^{20}}{4702880}$$
$$+\frac{647\,t^{22}}{869092224}\Big)-\cos x\,\sin^8 x\frac{9\,t^{22}}{362121760}\Big]$$
$$+\varepsilon^5\Big[\cos^{11} x\Big(\frac{27\,t^{17}}{198016}-\frac{11899\,t^{19}}{263361280}+\frac{440053\,t^{21}}{22122347520}\Big)+\cos^9 x\,\sin^2 x\Big(\frac{10671\,t^{19}}{131680640}-\frac{156787\,t^{21}}{3456616800}\Big)$$
$$+\cos^7 x\,\sin^4 x\frac{21393\,t^{21}}{1152205600}\Big]+\varepsilon^6\Big[-\cos^{13} x\frac{297\,t^{20}}{4426240}\Big]$$

This solution can be written in the form :

$$\begin{aligned}u(t,x)\ \simeq\ &\cos t\cos x+\varepsilon\left[k_1(t)\cos^3 x+k_2(t)\cos x\sin^2 x\right]\\&+\varepsilon^2\left[k_3(t)\cos^5 x+k_4(t)\cos^3 x\sin^2 x+k_5(t)\cos x\sin^4 x\right]\\&+\varepsilon^3\left[k_6(t)\cos^7 x+k_7(t)\cos^5 x\sin^2 x+k_8(t)\cos^3 x\sin^4 x+k_9(t)\cos x\sin^6 x\right]\\&+\cdots\end{aligned}$$

Where

$$\begin{aligned}k_1(t)\ =\ &\frac{t^5}{20}-\frac{13\,t^7}{840}+\frac{71\,t^9}{30240}-\frac{491\,t^{11}}{3326400}+\frac{13711\,t^{13}}{1037836800}-\frac{28607\,t^{15}}{43589145600}+\frac{66811\,t^{17}}{2694601728000}\\&-\frac{7198319\,t^{19}}{10137091700736000}+\frac{t^{21}}{13502619648000}-\frac{61\,t^{23}}{47826278793216000}+\frac{547\,t^{25}}{28695767275929600000}\end{aligned}$$

$$\begin{aligned}k_2(t)\ =\ &\frac{t^7}{140}-\frac{t^9}{504}+\frac{47\,t^{11}}{184800}-\frac{739\,t^{13}}{43243200}+\frac{34403\,t^{15}}{36324288000}-\frac{727\,t^{17}}{18712512000}+\frac{188947\,t^{19}}{153592298496000}\\&-\frac{t^{21}}{4725916876800}+\frac{13\,t^{23}}{3416162770944000}-\frac{41\,t^{25}}{717394181898240000}\end{aligned}$$

$$\begin{aligned}k_3(t)\ =\ &-\frac{3\,t^8}{224}+\frac{439\,t^{10}}{100800}-\frac{23801\,t^{12}}{13305600}+\frac{743201\,t^{14}}{2421619200}-\frac{38239\,t^{16}}{931392000}+\frac{78121361\,t^{18}}{22230464256000}-\frac{1903\,t^{20}}{482236416000}\\&+\frac{5921\,t^{22}}{31191051386880}-\frac{297239\,t^{24}}{31304473391923200}\end{aligned}$$

$$\begin{aligned}k_4(t)\ =\ &-\frac{13\,t^{10}}{2800}+\frac{3397\,t^{12}}{1663200}-\frac{352421\,t^{14}}{605404800}+\frac{83371\,t^{16}}{825552000}-\frac{26118227\,t^{18}}{2223046425600}+\frac{1247\,t^{20}}{42195686400}\\&-\frac{1191041\,t^{22}}{779776284672000}+\frac{11675131\,t^{24}}{215218254569472000}+\frac{12517\,t^{25}}{1016715110400000}\end{aligned}$$

$$k_5(t) = -\frac{13\,t^{12}}{61600} + \frac{1663\,t^{14}}{20180160} - \frac{16447\,t^{16}}{864864000} + \frac{573889\,t^{18}}{205837632000} - \frac{2903\,t^{20}}{281304576000} + \frac{12517\,t^{23}}{20334302208000} - \frac{1191041\,t^{24}}{71739418189824000} - \frac{162721\,t^{25}}{12200581324800000}$$

$$k_6(t) = \frac{57\,t^{11}}{12320} - \frac{1319\,t^{13}}{443520} + \frac{54919\,t^{14}}{47297250} + \frac{83\,,t^{15}}{470400} - \frac{54919\,t^{16}}{1621620000} - \frac{227253749\,t^{17}}{823350528000} + \frac{1901771\,t^{19}}{1969132032000} - \frac{37259\,t^{21}}{170031960000} + \frac{215576267\,t^{23}}{7608726171648000}$$

$$k_7(t) = \frac{839\,t^{13}}{320320} - \frac{165827\,t^{15}}{84084000} + \frac{54919\,t^{16}}{270270000} + \frac{76842539\,t^{17}}{137225088000} - \frac{947\,t^{19}}{202585600} + \frac{5031053\,t^{21}}{7254696960000} - \frac{34493909\,t^{23}}{333716060160000}$$

$$k_8(t) = \frac{193\,t^{15}}{600600} - \frac{65171\,t^{17}}{285885600} + \frac{129\,t^{19}}{40517120} - \frac{5819\,t^{21}}{12155136000} + \frac{277663\,t^{23}}{4359166035840}$$

$$k_9(t) = \frac{193\,t^{17}}{27227200} - \frac{t^{19}}{6077568} + \frac{193\,t^{21}}{3646540800} - \frac{35353\,t^{23}}{4612874112000}$$

3. The Regular Perturbation Method

3.1 RPM Description

In order to show, the basic idea of RPM, consider the following differential equation

$$L_\varepsilon\,[u_\varepsilon(x)] = 0, \quad x = (x_1,\, x_2,\, \cdots,\, x_n) \in \Omega \tag{12}$$

with boundary conditions

$$B_\varepsilon\,[u_\varepsilon(x)] = 0, \quad x \in \partial\Omega \tag{13}$$

where L_ε is a genaral differential operator, B_ε is a boundary operator, and $\partial\Omega$

In general, the equations (12)-(13) contain a very small parameter ε. In this method, we use the parameter u_ε as a power series ,

$$u_\varepsilon(x) = \sum_{n=0}^{+\infty} \varepsilon^n\, u_n(x) \tag{14}$$

Substituting (14) in (12)-(13), and collecting the coefficient of like powers of ε yields and equating the coefficient of each power of ε to zero. We obtain systems of recurrent boundary problems, easy to solve.

The approximate solution is given by

$$u(x) = u_0(x) + \varepsilon u_1(x) + \varepsilon^2 u_2(x) + \cdots \tag{15}$$

3.2 Application of RPM

Let us suppose that the solution $u(t,x)$ of the initial value problem (1) has the following form (JAGER, DE. E. M., & JIANG, FU RU, 1996) :

$$u(t,x) = \sum_{n=0}^{+\infty} \varepsilon^n u_n(t,x) \tag{16}$$

Putting (16) into (1), and collecting equal powers of ε we obtain a system of recurrent initial value problems

$$\varepsilon^0 \quad : \quad \begin{cases} \dfrac{\partial^2 u_0}{\partial t^2} - \dfrac{\partial^2 u_0}{\partial x^2} = 0 \\ u_0(0,x) = \cos x, \ \dfrac{\partial u_0}{\partial t}(0,x) = 0 \end{cases} \tag{17}$$

$$\varepsilon^1 \quad : \quad \begin{cases} \dfrac{\partial^2 u_1}{\partial t^2} - \dfrac{\partial^2 u_1}{\partial x^2} + \left(\dfrac{\partial u_0}{\partial t}\right)^3 = 0 \\ u_1(0,x) = 0, \ \dfrac{\partial u_1}{\partial t}(0,x) = 0 \end{cases} \tag{18}$$

$$\varepsilon^2 \quad : \quad \begin{cases} \dfrac{\partial^2 u_2}{\partial t^2} - \dfrac{\partial^2 u_2}{\partial x^2} + 3\left(\dfrac{\partial u_0}{\partial t}\right)^2 \left(\dfrac{\partial u_1}{\partial t}\right) = 0 \\ u_2(0,x) = 0, \ \dfrac{\partial u_2}{\partial t}(0,x) = 0 \end{cases} \tag{19}$$

$$\varepsilon^3 \quad : \quad \begin{cases} \dfrac{\partial^2 u_3}{\partial t^2} - \dfrac{\partial^2 u_3}{\partial x^2} + 3\left(\dfrac{\partial u_0}{\partial t}\right)^2 \left(\dfrac{\partial u_2}{\partial t}\right) + 3\left(\dfrac{\partial u_0}{\partial t}\right)\left(\dfrac{\partial u_1}{\partial t}\right)^2 = 0 \\ u_3(0,x) = 0, \ \dfrac{\partial u_3}{\partial t}(0,x) = 0 \end{cases} \tag{20}$$

$$\vdots$$

To solve (17, 18, 19, 20), we use ADM and we obtain :

$$\begin{aligned}
u_0(t,x) &= \cos t \cos x \\
u_1(t,x) &= \cos^3 x \left(\frac{43}{108} \sin(3t) - \frac{75}{4} \sin t\right) + \cos x \sin^2 x \left(\frac{99}{2} \sin t - \frac{19}{54} \sin(3t)\right) \\
u_2(t,x) &= \cos^5 x \left(-\frac{43}{12} \cos^2 t \cos(3t) - \frac{3914}{625} \cos^5 t + \frac{9701909}{40500} \cos^3 t + \frac{26472442}{3375} \cos t\right) + \\
&\quad \cos^3 x \sin^2 x \left(\frac{19}{6} \cos^2 t \cos(3t) + \frac{23936}{625} \cos^5 t - \frac{31274533}{20250} \cos^3 t - \frac{216658108}{3375} \cos t\right) \\
&\quad + \cos x \sin^4 x \left(-\frac{236}{25} \cos^5 t + \frac{200564}{405} \cos^3 t + \frac{3396328}{135} \cos t\right) \\
u_3(t,x) &= \cos^5 x \sin^2 x \left(\frac{57}{2} \cos^4 t \sin(3t) + \frac{817}{108} \cos t \cos^2(3t) + 19 \cos^3 t \sin t \cos(3t)\right. \\
&\quad -\frac{947}{2} \cos^2 t \cos(3t) + \frac{71808}{125} \cos^6 t \sin t - \frac{31274533}{2250} \cos^4 t \sin t - \frac{216658108}{1125} \cos^2 t \sin t \\
&\quad \left.+\frac{22275}{4} \cos^3 t\right) + \cos^7 x \left(-\frac{129}{4} \cos^4 t \sin(3t) - \frac{1849}{432} \cos t \cos^2(3t) - \frac{43}{2} \cos^3 t \sin t \cos(3t)\right. \\
&\quad +\frac{1075}{8} \cos^2 t \cos(3t) - \frac{11742}{125} \cos^6 t \sin t + \frac{9701909}{4500} \cos^4 t \sin t + \frac{26472442}{1125} \cos^2 t \sin t \\
&\quad \left.-\frac{16875}{16} \cos^3 t\right) + \cos^3 x \sin^4 x \left(-\frac{361}{108} \cos t \cos^2(3t) + \frac{627}{2} \cos^2 t \cos(3t) - \frac{708}{5} \cos^6 t \sin t + \right. \\
&\quad \left.\frac{200564}{45} \cos^4 t \sin t + \frac{3396328}{45} \cos^2 t \sin t - \frac{29403}{4} \cos^3 t\right) + \cos x \sin^6 x \left(\frac{285 \sin(7t)}{4802}\right.
\end{aligned}$$

$$+\frac{52079\cos(7t)}{172872}+\frac{57\sin(5t)}{125}+\frac{8683\cos(5t)}{9000}-\frac{87932299816\sin^5 t}{16078125}+\frac{169628431490024\sin^3 t}{260465625}$$
$$\left.-\frac{2154223646101879\sin t}{173643750}-\frac{158316\cos^5 t}{125}+\frac{51050891\cos^3 t}{1350}+\frac{1911052939\cos t}{900}\right)$$

Hence, the approximate solution of (1) is given by :

$$u(t,x) \simeq u_0(t,x)+\varepsilon u_1(t,x)+\varepsilon^2 u_2(t,x)+\varepsilon^2 u_3(t,x)$$

Writting this solution in the form,

$$\begin{aligned} u(t,x) \simeq\ & \cos t\cos x+\varepsilon\left[q_1(t)\cos^3 x+q_2(t)\cos x\sin^2 x\right] \\ & +\varepsilon^2\left[q_3(t)\cos^5 x+q_4(t)\cos^3 x\sin^2 x+q_5(t)\cos x\sin^4 x\right] \\ & +\varepsilon^3\left[q_6(t)\cos^7 x+q_7(t)\cos^5 x\sin^2 x+q_8(t)\cos^3 x\sin^4 x+q_9(t)\cos x\sin^6 x\right] \\ & +\cdots \end{aligned}$$

Where

$$q_1(t) = \frac{43}{108}\sin(3t)-\frac{75}{4}\sin t$$

$$q_2(t) = \frac{99}{2}\sin t-\frac{19}{54}\sin(3t)$$

$$q_3(t) = -\frac{43}{12}\cos^2 t\cos(3t)-\frac{3914}{625}\cos^5 t+\frac{9701909}{40500}\cos^3 t+\frac{26472442}{3375}\cos t$$

$$q_4(t) = \frac{19}{6}\cos^2 t\cos(3t)+\frac{23936}{625}\cos^5 t-\frac{31274533}{20250}\cos^3 t-\frac{216658108}{3375}\cos t$$

$$q_5(t) = \frac{236}{25}\cos^5 t+\frac{200564}{405}\cos^3 t+\frac{3396328}{135}\cos t$$

$$\begin{aligned} q_6(t) = & -\frac{129}{4}\cos^4 t\sin(3t)-\frac{1849}{432}\cos t\cos^2(3t)-\frac{43}{2}\cos^3 t\sin t\cos(3t)+\frac{1075}{8}\cos^2 t\cos(3t) \\ & -\frac{11742}{125}\cos^6 t\sin t+\frac{9701909}{4500}\cos^4 t\sin t+\frac{26472442}{1125}\cos^2 t\sin t-\frac{16875}{16}\cos^3 t \end{aligned}$$

$$\begin{aligned} q_7(t) = & \frac{57}{2}\cos^4 t\sin(3t)+\frac{817}{108}\cos t\cos^2(3t)+19\cos^3 t\sin t\cos(3t)-\frac{947}{2}\cos^2 t\cos(3t) \\ & +\frac{71808}{125}\cos^6 t\sin t-\frac{31274533}{2250}\cos^4 t\sin t-\frac{216658108}{1125}\cos^2 t\sin t+\frac{22275}{4}\cos^3 t \end{aligned}$$

$$\begin{aligned} q_8(t) = & -\frac{361}{108}\cos t\cos^2(3t)+\frac{627}{2}\cos^2 t\cos(3t)-\frac{708}{5}\cos^6 t\sin t\frac{200564}{45}\cos^4 t\sin t \\ & +\frac{3396328}{45}\cos^2 t\sin t-\frac{29403}{4}\cos^3 t \end{aligned}$$

$$\begin{aligned} q_9(t) = & \frac{285\sin(7t)}{4802}+\frac{52079\cos(7t)}{172872}+\frac{57\sin(5t)}{125}+\frac{8683\cos(5t)}{9000}-\frac{87932299816\sin^5 t}{16078125} \\ & +\frac{169628431490024\sin^3 t}{260465625}-\frac{2154223646101879\sin t}{173643750}-\frac{158316\cos^5 t}{125} \\ & +\frac{51050891\cos^3 t}{1350}+\frac{1911052939\cos t}{900} \end{aligned}$$

4. The Adomian Decomposition Method

4.1 Generalities

General properties of ADM and its application can be found in (ABBAOUI, K., 1995; ABBAOUI, K., & CHERRUAULT, Y., 1994; ABBAOUI, K., & CHERRUAULT, Y., 1999; NGARHASTA, N. & et al., 2002). Some of these are outlined as follows. Suppose that we need to solve the following equation

$$Au = f \tag{21}$$

in a real Hilbert space H, where $A : H \to H$ is a linear or a nonlinear operator, $f \in H$ are given; and $u \in H$ is the unknown. The principle of the ADM is based on the decomposition of the nonlinear operator A in the following form:

$$A = L + R + N$$

where $L + R$ is linear, N nonlinear, L invertible with L^{-1} as inverse. Using that decomposition, equation (21) is equivalent to

$$u = \theta + L^{-1}f - L^{-1}Ru - L^{-1}Nu \tag{22}$$

where θ satisfies $L\theta = 0$. Equation (22) is called the Adomian fundamental equation or Adomian canonical form. We look for the solution of (21) in a series expansion form $u = \sum\limits_{n=0}^{+\infty} u_n$ and we consider $Nu = \sum\limits_{n=0}^{+\infty} A_n$ where A_n are special polynomials of variables $u_0, u_1, ..., u_n$ called Adomian polynomials and defined by (ABBAOUI, K., 1995; ABBAOUI, K., & CHERRUAULT, Y., 1994; ABBAOUI, K., & CHERRUAULT, Y., 1999; NGARHASTA, N. & et al., 2002):

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{i=0}^{+\infty}\lambda^i u_i\right)\right]_{\lambda=0} \qquad n = 0, 1, 2, ...$$

where λ is a parameter used by ”convenience ”. Thus (22) can be rewritten as follows :

$$\sum_{n=0}^{+\infty} u_n = \theta + L^{-1}f - L^{-1}R(\sum_{n=0}^{+\infty} u_n) - L^{-1}(\sum_{n=0}^{+\infty} A_n).$$

We suppose that the series $\sum\limits_{n=0}^{+\infty} u_n$ and $\sum\limits_{n=0}^{+\infty} A_n$ are convergent, and obtained by identification the Adomian algorithm :

$$\begin{cases} u_0 = \theta + L^{-1}f \\ u_1 = -L^{-1}(Ru_0) - L^{-1}A_0 \\ \quad\quad . \\ \quad\quad . \\ \quad\quad . \\ u_{n+1} = -L^{-1}(Ru_n) - L^{-1}A_n \end{cases} . \tag{23}$$

In practice it is often difficult to calculate all the terms of an Adomian series; so we approach the series solution by the truncated series $u = \sum\limits_{i=0}^{n} u_i$, where the choice of n depends on error requirements.

4.2 Application of ADM

Defining the operators :

$$L_t(\bullet) = \frac{\partial^2}{\partial t^2}(\bullet), \quad L_x(\bullet) = \frac{\partial^2}{\partial x^2}(\bullet), \quad Nu = \left(\frac{\partial u}{\partial t}\right)^3 \quad \text{and} \quad L_t^{-1}(\bullet) = \int_0^t \int_0^z (\bullet) ds dz$$

Equation (1) can be written as :

$$L_t u - L_x u \quad = \quad -\varepsilon Nu \tag{24}$$

Applying L_t^{-1} to (24), we obtain :

$$u(t, x) = u(0, x) + t\frac{\partial u}{\partial t}(0, x) + L_t^{-1}\left[L_x u(t, x)\right] - \varepsilon L_t^{-1}\left[Nu(t, x)\right] \tag{25}$$

Assuming that the solution of (1) can be given by :

$$u(t,x) = \sum_{n=0}^{+\infty} u_n(t,x) \tag{26}$$

and

$$Nu(t,x) = \sum_{n=0}^{+\infty} A_n(t,x) \tag{27}$$

where A_n are the Adomian's polynomials with

$$A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n}\left(N\left(\sum_{i=0}^{+\infty}\lambda^i u_i\right)\right)\right]_{\lambda=0}$$

By substituting (26) and (27) into (25), we obtain the Adomian algorithm

$$\begin{cases} u_0(t,x) = u(0,x) + t\dfrac{\partial u}{\partial t}(0,x) = \cos x \\ u_{n+1}(t,x) = L_t^{-1}\left[L_x u_n(t,x)\right] - \varepsilon L_t^{-1}\left[A_n(t,x)\right], \qquad n \geq 0 \end{cases}$$

with

$$\begin{aligned}
A_0 &= \left(\frac{\partial u_0}{\partial t}\right)^3 \\
A_1 &= 3\varepsilon\left(\frac{\partial u_0}{\partial t}\right)^2\left(\frac{\partial^2 u_1}{\partial t}\right) \\
A_2 &= 3\varepsilon\left(\frac{\partial u_0}{\partial t}\right)^2\left(\frac{\partial u_2}{\partial t}\right) + 3\varepsilon\left(\frac{\partial u_0}{\partial t}\right)\left(\frac{\partial u_1}{\partial t}\right)^2 \\
A_3 &= 3\varepsilon\left(\frac{\partial u_0}{\partial t}\right)^2\left(\frac{\partial u_3}{\partial t}\right) + 6\varepsilon\left(\frac{\partial u_0}{\partial t}\frac{\partial u_1}{\partial t}\frac{\partial u_2}{\partial t}\right) + \varepsilon\left(\frac{\partial u_1}{\partial t}\right)^3 \\
A_4 &= 3\varepsilon\left(\frac{\partial u_0}{\partial t}\right)^2\left(\frac{\partial u_4}{\partial t}\right) + 6\varepsilon\left(\frac{\partial u_0}{\partial t}\frac{\partial u_1}{\partial t}\frac{\partial u_3}{\partial t}\right) + 3\varepsilon\left(\frac{\partial u_0}{\partial t}\right)\left(\frac{\partial u_2}{\partial t}\right)^2 + 3\varepsilon\left(\frac{\partial u_1}{\partial t}\right)^2\left(\frac{\partial u_2}{\partial t}\right) \\
A_5 &= 3\varepsilon\left(\frac{\partial u_0}{\partial t}\right)^2\left(\frac{\partial u_5}{\partial t}\right) + 6\varepsilon\left(\frac{\partial u_0}{\partial t}\frac{\partial u_1}{\partial t}\frac{\partial u_4}{\partial t}\right) + 6\varepsilon\left(\frac{\partial u_0}{\partial t}\frac{\partial u_2}{\partial t}\frac{\partial u_3}{\partial t}\right) + 3\varepsilon\left(\frac{\partial u_1}{\partial t}\right)^2\left(\frac{\partial u_3}{\partial t}\right) \\
&\quad +3\varepsilon\left(\frac{\partial u_1}{\partial t}\right)\left(\frac{\partial u_2}{\partial t}\right)^2 \\
&\ \vdots
\end{aligned}$$

Using the algorithm, we have :

$$\begin{aligned}
u_0(t,x) &= \cos x \\
u_1(t,x) &= -\frac{1}{2}t^2\cos x \\
u_2(t,x) &= \frac{1}{24}t^4\cos x \\
u_3(t,x) &= -\frac{1}{720}t^6\cos x + \frac{\varepsilon}{20}t^5\cos^3 x \\
u_4(t,x) &= \frac{\varepsilon}{140}t^7\cos x\sin^2 x - \frac{13\,\varepsilon}{840}t^7\cos^3 x + \frac{1}{40320}t^8\cos x \\
u_5(t,x) &= -\frac{1}{3628800}t^{10}\cos x - \frac{\varepsilon}{504}t^9\cos x\sin^2 x + \frac{71\,\varepsilon}{30240}t^9\cos^3 x - \frac{3\,\varepsilon^2}{224}t^8\cos^5 x \\
&\ \vdots
\end{aligned}$$

Finally, the approximate solution of (1) is given by :

$$
\begin{aligned}
u(t,x) &\simeq u_0(t,x)+u_1(t,x)+u_2(t,x)+\cdots+u_{10}(t,x)\\
&\simeq \left[\cos x\left(1-\frac{t^2}{2!}+\frac{t^4}{4!}-\frac{t^6}{6!}+\frac{t^8}{8!}-\frac{t^{10}}{10!}+\frac{t^{12}}{12!}-\frac{t^{14}}{14!}+\frac{t^{16}}{16!}-\frac{t^{18}}{18!}+\frac{t^{20}}{20!}\right)\right]+\\
&\quad \varepsilon\left[\cos^3 x\left(\frac{t^5}{20}-\frac{13\,t^7}{840}+\frac{71\,t^9}{30240}-\frac{491\,t^{11}}{3326400}+\frac{13711\,t^{13}}{1037836800}-\frac{28607\,t^{15}}{43589145600}\right.\right.\\
&\quad +\frac{66811\,t^{17}}{2694601728000}-\frac{7198319\,t^{19}}{10137091700736000}+\frac{t^{21}}{13502619648000}\\
&\quad \left.-\frac{61\,t^{23}}{47826278793216000}+\frac{547\,t^{25}}{28695767275929600000}\right)\\
&\quad +\cos x\,\sin^2 x\left(\frac{t^7}{140}-\frac{t^9}{504}+\frac{47\,t^{11}}{184800}-\frac{739\,t^{13}}{43243200}+\frac{34403\,t^{15}}{36324288000}-\frac{727\,t^{17}}{18712512000}\right.\\
&\quad \left.\left.+\frac{188947\,t^{19}}{153592298496000}-\frac{t^{21}}{4725916876800}+\frac{13\,t^{23}}{3416162770944000}-\frac{41\,t^{25}}{717394181898240000}\right)\right]\\
&\quad +\varepsilon^2\left[\cos^5 x\left(-\frac{3\,t^8}{224}+\frac{439\,t^{10}}{100800}-\frac{23801\,t^{12}}{13305600}+\frac{743201\,t^{14}}{2421619200}-\frac{38239\,t^{16}}{931392000}+\frac{78121361\,t^{18}}{22230464256000}\right.\right.\\
&\quad \left.-\frac{1903\,t^{20}}{482236416000}+\frac{5921\,t^{22}}{31191051386880}-\frac{297239\,t^{24}}{31304473391923200}\right)\\
&\quad +\cos^3 x\,\sin^2 x\left(-\frac{13\,t^{10}}{2800}+\frac{3397\,t^{12}}{1663200}-\frac{352421\,t^{14}}{605404800}+\frac{83371\,t^{16}}{825552000}-\frac{26118227\,t^{18}}{2223046425600}\right.\\
&\quad \left.+\frac{1247\,t^{20}}{42195686400}-\frac{1191041\,t^{22}}{779776284672000}+\frac{11675131\,t^{24}}{215218254569472000}+\frac{12517\,t^{25}}{1016715110400000}\right)\\
&\quad +\cos x\,\sin^4 x\left(-\frac{13\,t^{12}}{61600}+\frac{1663\,t^{14}}{20180160}-\frac{16447\,t^{16}}{864864000}+\frac{573889\,t^{18}}{205837632000}-\frac{2903\,t^{20}}{281304576000}\right.\\
&\quad \left.\left.+\frac{12517\,t^{23}}{20334302208000}-\frac{1191041\,t^{24}}{71739418189824000}-\frac{162721\,t^{25}}{12200581324800000}\right)\right]+\\
&\quad \varepsilon^3\left[\cos^7 x\left(\frac{57\,t^{11}}{12320}-\frac{1319\,t^{13}}{443520}+\frac{54919\,t^{14}}{47297250}+\frac{83\,,t^{15}}{470400}-\frac{54919\,t^{16}}{1621620000}-\frac{227253749\,t^{17}}{823350528000}\right.\right.\\
&\quad \left.+\frac{1901771\,t^{19}}{1969132032000}-\frac{37259\,t^{21}}{170031960000}+\frac{215576267\,t^{23}}{7608726171648000}\right)+\cos^5 x\,\sin^2 x\left(\frac{839\,t^{13}}{320320}\right.\\
&\quad \left.-\frac{165827\,t^{15}}{84084000}+\frac{54919\,t^{16}}{270270000}+\frac{76842539\,t^{17}}{137225088000}-\frac{947\,t^{19}}{202585600}+\frac{5031053\,t^{21}}{7254696960000}-\frac{34493909\,t^{23}}{333716060160000}\right)\\
&\quad +\cos^3 x\,\sin^4 x\left(+\frac{193\,t^{15}}{600600}-\frac{65171\,t^{17}}{285885600}+\frac{129\,t^{19}}{40517120}-\frac{5819\,t^{21}}{12155136000}+\frac{277663\,t^{23}}{4359166035840}\right)+\\
&\quad \left.\cos x\,\sin^6 x\left(\frac{193\,t^{17}}{27227200}-\frac{t^{19}}{6077568}+\frac{193\,t^{21}}{3646540800}-\frac{35353\,t^{23}}{4612874112000}\right)\right]\\
&\quad +\cdots
\end{aligned}
$$

This solution can be written in the form :

$$
\begin{aligned}
u(t,x) &\simeq \cos t\cos x+\varepsilon\left[l_1(t)\cos^3 x+l_2(t)\cos x\sin^2 x\right]\\
&\quad +\varepsilon^2\left[l_3(t)\cos^5 x+l_4(t)\cos^3 x\sin^2 x+l_5(t)\cos x\sin^4 x\right]\\
&\quad +\varepsilon^3\left[l_6(t)\cos^7 x+l_7(t)\cos^5 x\sin^2 x+l_8(t)\cos^3 x\sin^4 x+l_9(t)\cos x\sin^6 x\right]\\
&\quad +\cdots
\end{aligned}
$$

$$
\begin{aligned}
l_1(t) &= \frac{t^5}{20}-\frac{13\,t^7}{840}+\frac{71\,t^9}{30240}-\frac{491\,t^{11}}{3326400}+\frac{13711\,t^{13}}{1037836800}-\frac{28607\,t^{15}}{43589145600}+\frac{66811\,t^{17}}{2694601728000}\\
&\quad -\frac{7198319\,t^{19}}{10137091700736000}+\frac{t^{21}}{13502619648000}-\frac{61\,t^{23}}{47826278793216000}+\frac{547\,t^{25}}{28695767275929600000}
\end{aligned}
$$

$$
\begin{aligned}
l_2(t) &= \frac{t^7}{140}-\frac{t^9}{504}+\frac{47\,t^{11}}{184800}-\frac{739\,t^{13}}{43243200}+\frac{34403\,t^{15}}{36324288000}-\frac{727\,t^{17}}{18712512000}+\frac{188947\,t^{19}}{153592298496000}\\
&\quad -\frac{t^{21}}{4725916876800}+\frac{13\,t^{23}}{3416162770944000}-\frac{41\,t^{25}}{717394181898240000}
\end{aligned}
$$

$$
\begin{aligned}
l_3(t) &= -\frac{3\,t^8}{224}+\frac{439\,t^{10}}{100800}-\frac{23801\,t^{12}}{13305600}+\frac{743201\,t^{14}}{2421619200}-\frac{38239\,t^{16}}{931392000}+\frac{78121361\,t^{18}}{22230464256000}-\frac{1903\,t^{20}}{482236416000}\\
&\quad +\frac{5921\,t^{22}}{31191051386880}-\frac{297239\,t^{24}}{31304473391923200}
\end{aligned}
$$

$$
\begin{aligned}
l_4(t) &= -\frac{13\,t^{10}}{2800}+\frac{3397\,t^{12}}{1663200}-\frac{352421\,t^{14}}{605404800}+\frac{83371\,t^{16}}{825552000}-\frac{26118227\,t^{18}}{2223046425600}+\frac{1247\,t^{20}}{42195686400}\\
&\quad -\frac{1191041\,t^{22}}{779776284672000}+\frac{11675131\,t^{24}}{215218254569472000}+\frac{12517\,t^{25}}{1016715110400000}
\end{aligned}
$$

$$
\begin{aligned}
l_5(t) &= -\frac{13\,t^{12}}{61600}+\frac{1663\,t^{14}}{20180160}-\frac{16447\,t^{16}}{864864000}+\frac{573889\,t^{18}}{205837632000}-\frac{2903\,t^{20}}{281304576000}+\frac{12517\,t^{23}}{20334302208000}\\
&\quad -\frac{1191041\,t^{24}}{71739418189824000}-\frac{162721\,t^{25}}{12200581324800000}
\end{aligned}
$$

$$
\begin{aligned}
l_6(t) &= \frac{57\,t^{11}}{12320}-\frac{1319\,t^{13}}{443520}+\frac{54919\,t^{14}}{47297250}+\frac{83\,,t^{15}}{470400}-\frac{54919\,t^{16}}{1621620000}-\frac{227253749\,t^{17}}{823350528000}+\frac{1901771\,t^{19}}{1969132032000}\\
&\quad -\frac{37259\,t^{21}}{170031960000}+\frac{215576267\,t^{23}}{7608726171648000}
\end{aligned}
$$

$$
\begin{aligned}
l_7(t) &= \frac{839\,t^{13}}{320320}-\frac{165827\,t^{15}}{84084000}+\frac{54919\,t^{16}}{270270000}+\frac{76842539\,t^{17}}{137225088000}-\frac{947\,t^{19}}{202585600}+\frac{5031053\,t^{21}}{7254696960000}\\
&\quad -\frac{34493909\,t^{23}}{333716060160000}
\end{aligned}
$$

$$
l_8(t) = \frac{193\,t^{15}}{600600}-\frac{65171\,t^{17}}{285885600}+\frac{129\,t^{19}}{40517120}-\frac{5819\,t^{21}}{12155136000}+\frac{277663\,t^{23}}{4359166035840}
$$

$$
l_9(t) = \frac{193\,t^{17}}{27227200}-\frac{t^{19}}{6077568}+\frac{193\,t^{21}}{3646540800}-\frac{35353\,t^{23}}{4612874112000}
$$

5. Comparison of the Approximate Solutions

In this section we analyze the approximate solutions of (1) are obtained by the three numerical methods (ADM, RPM and ADM).

The solution in equation (1) which obtained by HPM is absolutely same as that of the solution obtained by ADM. Furthermore, the main advantage in using the HPM for solving the considered model is that the approximate solutions obtained successfully without requiring a small parameter in the equation and without calculating the complicated Adomian's polynomials. The approximate solution obtained by RPM, differs from the one obtained by ADM and HPM.

The tables (1), (3) and (5) give some values of the approximate solutions obtained by the three methods. One notes a significant variation of the values obtained by RPM when ε becomes increasingly large.

The tables (2), (4) and (6) show absolute error between the various approximate solutions. It is noticed that the absolute error between the solutions of RPM and ADM just as RPM and HPM increases when ε becomes increasingly large.

Table 1. Approximate solutions by ADM, RPM and HPM, for $t = 0.2$ and $\varepsilon = 0.001$

x	u_{adm}	u_{rpm}	u_{hpm}
0	0.9801	0.9850	0.9801
0.1	0.9752	0.9793	0.9752
0.2	0.9605	0.9626	0.9605
0.3	0.9363	0.9355	0.9363
0.4	0.9027	0.8992	0.9027
0.5	0.8601	0.8546	0.8601
0.6	0.8089	0.8027	0.8089
0.7	0.7496	0.7443	0.7496
0.8	0.6828	0.6798	0.6828
0.9	0.6092	0.6093	0.6092
1	0.5295	0.5328	0.5295

Table 2. Absolute error for variables x from 0 to 1 and $t = 0.2$ and $\varepsilon = 0.001$

x	$\lvert u_{adm} - u_{rpm}\rvert$	$\lvert u_{adm} - u_{hpm}\rvert$	$\lvert u_{hpm} - u_{rpm}\rvert$
0	0.0049	0	0.0049
0.1	0.0041	0	0.0041
0.2	0.0021	0	0.0021
0.3	0.0008	0	0.0008
0.4	0.0035	0	0.0035
0.5	0.0055	0	0.0055
0.6	0.0062	0	0.0062
0.7	0.0053	0	0.0053
0.8	0.0030	0	0.0030
0.9	0.0001	0	0.0001
1	0.0033	0	0.0033

Table 3. Approximate solutions by ADM, RPM and HPM, for $t = 0.5$ and $\varepsilon = 0.002$

x	u_{adm}	u_{rpm}	u_{hpm}
0	0.8776	0.9040	0.8776
0.1	0.8732	0.8946	0.8732
0.2	0.8601	0.8681	0.8601
0.3	0.8384	0.8681	0.8384
0.4	0.8083	0.7843	0.8083
0.5	0.7702	0.7382	0.7702
0.6	0.7243	0.6936	0.7243
0.7	0.6712	0.6505	0.6712
0.8	0.6114	0.6058	0.6114
0.9	0.5455	0.5558	0.5455
1	0.4742	0.4969	0.4742

Table 4. Absolute error for variables x from 0 to 1 and $t = 0.5$ and $\varepsilon = 0.002$

x	$\lvert u_{adm} - u_{rpm}\rvert$	$\lvert u_{adm} - u_{hpm}\rvert$	$\lvert u_{hpm} - u_{rpm}\rvert$
0	0.0264	0	0.0264
0.1	0.0214	0	0.0214
0.2	0.0080	0	0.0080
0.3	0.0090	0	0.0090
0.4	0.0240	0	0.0240
0.5	0.0320	0	0.0320
0.6	0.0307	0	0.0307
0.7	0.0207	0	0.0207
0.8	0.0056	0	0.0056
0.9	0.0103	0	0.0103
1	0.0227	0	0.0227

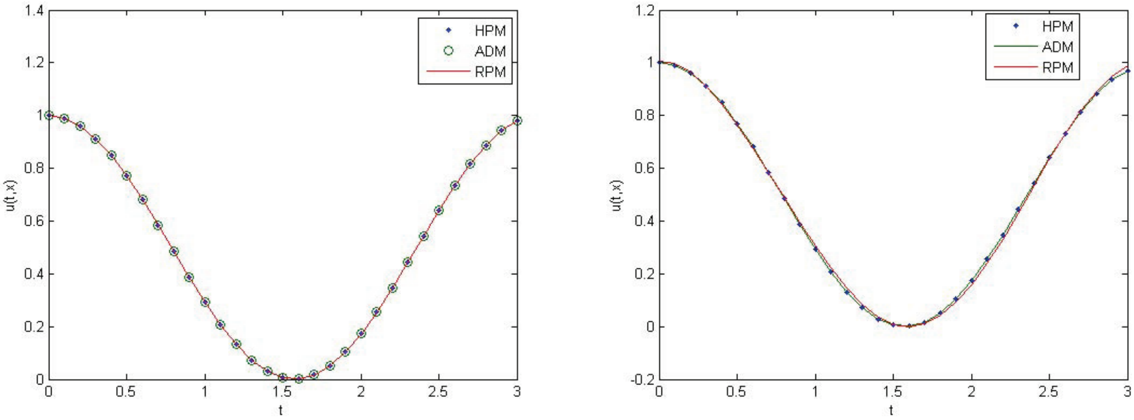
Table 5. Approximate solutions by ADM, RPM and HPM, for $t = 0.8$ and $\varepsilon = 0.003$

x	u_{adm}	u_{rpm}	u_{hpm}
0	0.6967	0.7921	0.6967
0.1	0.6933	0.7694	0.6933
0.2	0.6933	0.7086	0.6933
0.3	0.6656	0.6293	0.6656
0.4	0.6417	0.5550	0.6417
0.5	0.6114	0.5042	0.6114
0.6	0.5750	0.4831	0.5750
0.7	0.5329	0.4844	0.5329
0.8	0.4854	0.4913	0.4854
0.9	0.4331	0.4854	0.4331
1	0.3764	0.4537	0.3764

Table 6. Absolute error for variables x from 0 to 1 and $t = 0.8$ and $\varepsilon = 0.003$

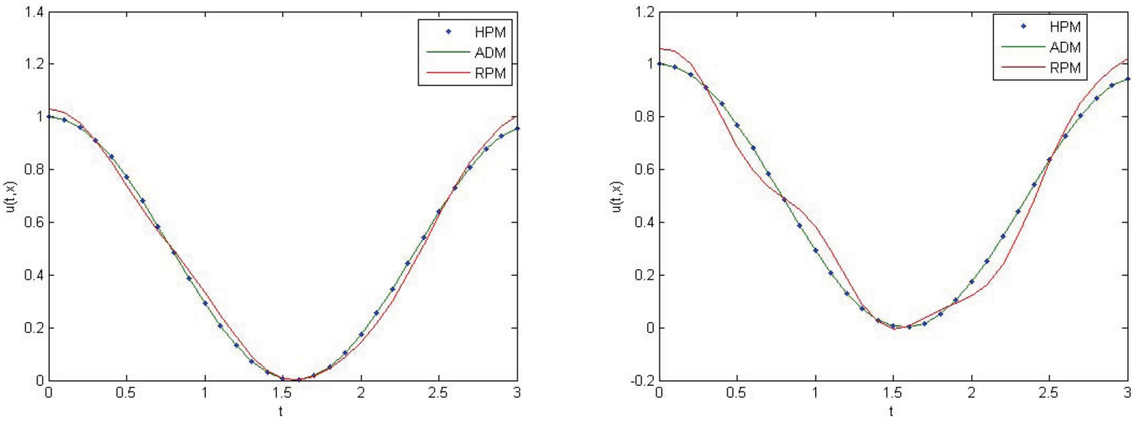
x	$\lvert u_{adm} - u_{rpm}\rvert$	$\lvert u_{adm} - u_{hpm}\rvert$	$\lvert u_{hpm} - u_{rpm}\rvert$
0	0.0954	0	0.0954
0.1	0.0761	0	0.0761
0.2	0.0257	0	0.0257
0.3	0.0363	0	0.0363
0.4	0.0867	0	0.0867
0.5	0.1072	0	0.1072
0.6	0.0919	0	0.0919
0.7	0.0485	0	0.0485
0.8	0.0059	0	0.0059
0.9	0.0523	0	0.0523
1	0.0773	0	0.0773

The figures (1) and (2) give the comparison of the approximate solutions in dimension 2, obtained by the three methods. In dimension 3, we obtain the figures (3), (4),(5) and (6).



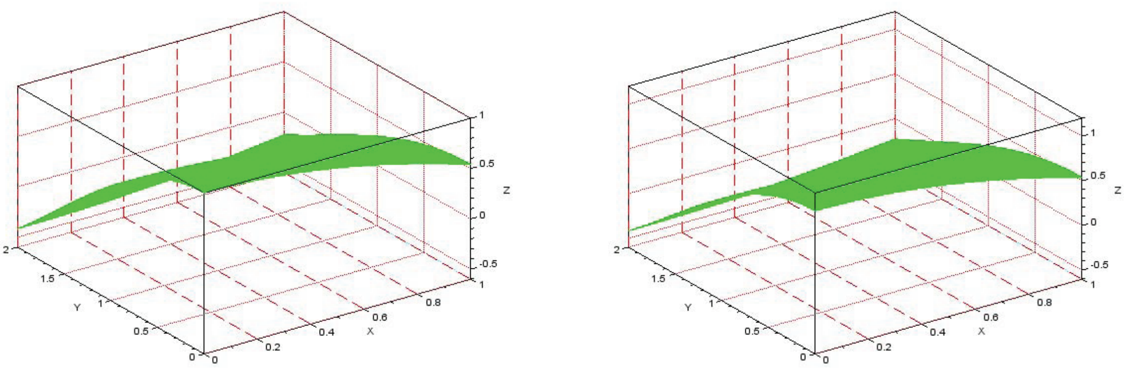
(a) $\varepsilon = 0.0001$ (b) $\varepsilon = 0.001$

Figure 1. Comparison of the HPM solution, ADM solution and RPM solution



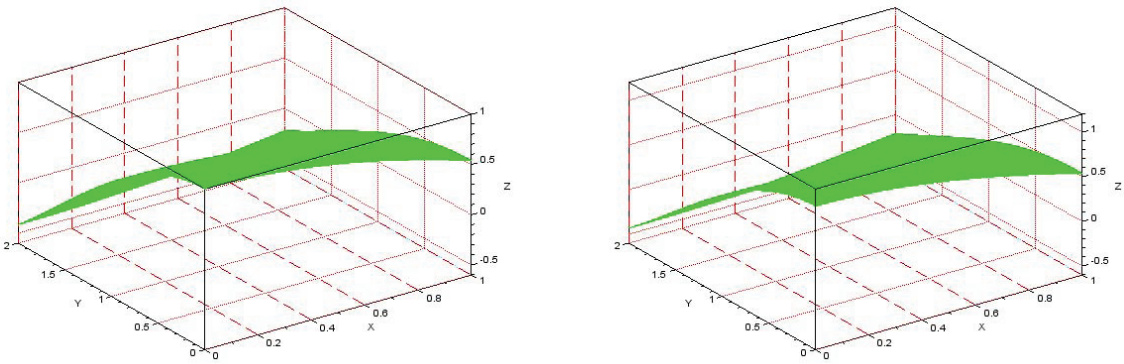
(a) $\varepsilon = 0.002$ (b) $\varepsilon = 0.003$

Figure 2. Comparison of the HPM solution, ADM solution and RPM solution



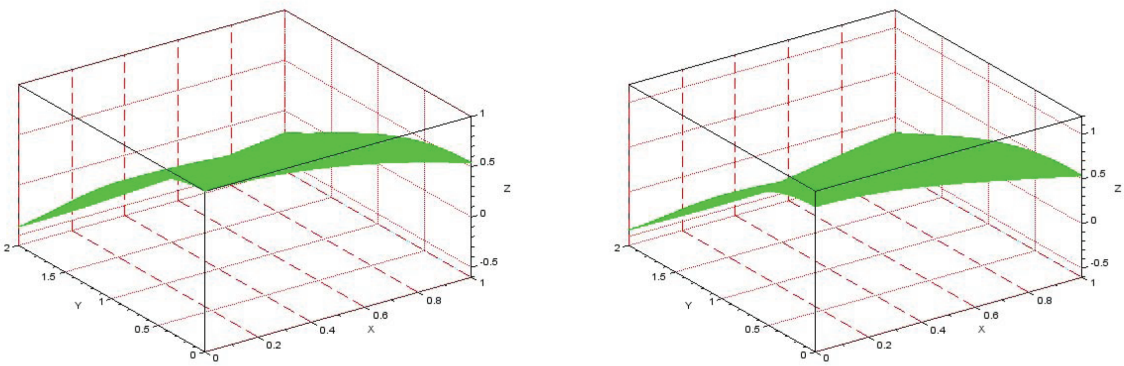
(a) HPM solution (b) RPM solution

Figure 3. Comparison of the HPM solution with RPM solution for $\varepsilon = 0.0001$



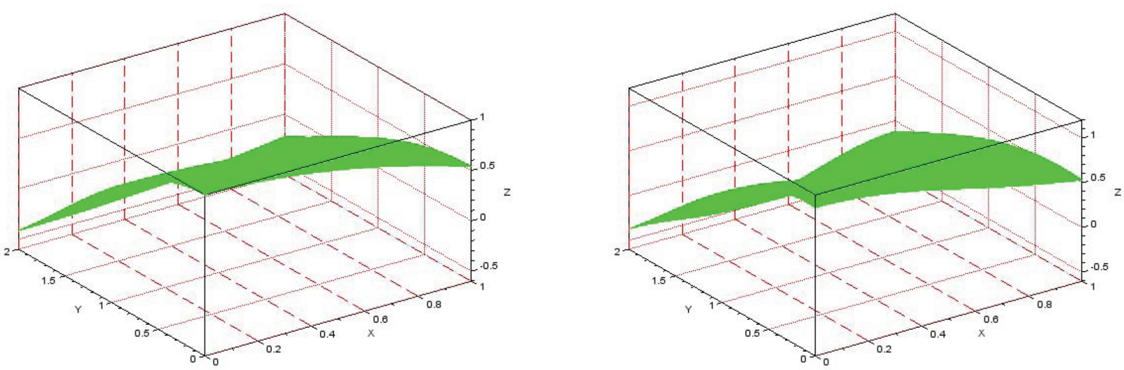
(a) ADM solution (b) RPM solution

Figure 4. Comparison of the HPM solution with RPM solution for $\varepsilon = 0.001$



(a) HPM solution (b) RPM solution

Figure 5. Comparison of the HPM solution with RPM solution for $\varepsilon = 0.002$



(a) ADM solution (b) RPM solution

Figure 6. Comparison of the HPM solution with RPM solution for $\varepsilon = 0.003$

6. Conclusion

In this paper, the HPM, ADM and RPM have been successfully employed to obtain the approximate analytical solutions of the wave equation with cubic damping. Finally, the results obtained by three methods were comapared. RPM solution differs from the ADM and HPM solution, but ADM and HPM give the same approximate solution. This research reveals that although the obtained results by HPM and ADM are the same, HPM are much easier, more convenient, and efficient in comparison. Different from ADM, where specific algorithms are usually used to determine the Adomian polynomials, HPM handle linear and nonlinear problem in simple manner by deforming a difficult problem into a simple one.

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