

# Efficiency of MOMA-plus Method to Solve Some Fully Fuzzy L-R Triangular Multiobjective Linear Programs

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Received: December 25, 2017 Accepted: January 8, 2018 Online Published: March 5, 2018

doi:10.5539/jmr.v10n2p77 URL: <https://doi.org/10.5539/jmr.v10n2p77>

## Abstract

In this paper, we propose a novel approach for solving some fully fuzzy L-R triangular multiobjective linear optimization programs using MOMA-plus method (Kounhinir, 2017). This approach is composed of two relevant steps such as the converting of the fully fuzzy L-R triangular multiobjective linear optimization problem into a deterministic multiobjective linear optimization and the applying of the adapting MOMA-plus method. The initial version of MOMA-plus method is designed for multiobjective deterministic optimization (Kounhinir, 2017) and having already been tested on the single-objective fuzzy programs (Abdoulaye, 2017). Our new method allow to find all of the Pareto optimal solutions of a fully fuzzy L-R triangular multiobjective linear optimization problems obtained after conversion. For highlighting the efficiency of our approach a didactic numerical example is dealt with and obtained solutions are compared to Total Objective Segregation Method proposed by Jayalakslmi and Pandia (Jayalakslmi 2014).

**Keywords :** Fuzzy triangular numbers; fuzzy linear programming; MOMA-plus method.

## 1. Introduction

A fuzzy linear program is a linear program in which all of the coefficients and/or variables or a part of those elements are fuzzy numbers. This kind of linear program is defined in an environment where the available informations are indefinite, imprecise and incomplete. Let's notice that the notions about fuzzy number have been proposed first time by Zadeh (Zadeh, 1965) since 1965 and that of decision making in the fuzzy environment by Bellman and Zadeh in 1970 (Bellman, 1970). As to the notions of linear programming in the fuzzy environment, they have been introduced by Zimmerman (Zimmerman, 1978). Since then, several works have been done for the resolution of some fuzzy linear programs.

It should be noted that the modeling of most problems in the daily life leads to take into account of several objectives or criteria which can be known in the stochastic way. But in general, these objectives are conflicting and lead to change the resolution concept in which the notion of optimal solution is replaced with Pareto optimal solutions or best compromises. Therefore, there exists many approach for solving this kind of problems but among those method we pay intention for which try to convert the fuzzy program in deterministic program before total resolution. For the recent works on this class of methods one can find it in the literature (Ahmed, 2017; Priyadarsini, 2017; Babita, 2011; Rasha, 2016; Krishnapada, 2015; Kiruthiga, 2015; Hadi-Vencheh, 2014; Jayalakslmi, 2014).

In this work, we are particularly interested in the multiobjective linear programming of which the coefficients and the variables are fully fuzzy L-R triangular numbers. The method that we propose here is a combination of the conversion technical of the fuzzy program into deterministic program proposed by Hosseinzadeh and Edalatpanah (Hosseinzadeh, 2016) and our adapted version of MOMA-plus. In fact, on the one hand, for solving a fully fuzzy linear program, Hosseinzadeh and Edalatpanah have proposed a new approach consisting in first converting the fully fuzzy linear program into a deterministic multiobjective linear program. This multiobjective deterministic problem obtained is thereafter solved by the lexicographic method and also by the linear methods. Their method was very efficient and provided better solutions than those proposed by Ezzati and *al.* (Ezzati, 2015) and Kumar and *al.* (Kumar, 2011). However, it has never been tested in the multi-objective case. On the other hand, the MOMA-plus method is a metaheuristic for finding the good approximation or even exact solutions of a deterministic multiobjective optimization problem (Kounhinir, 2017). By already the MOMA-plus method has proved its capacity in the resolution of fuzzy triangular single objective linear programs fuzzy (Abdoulaye, 2017).

Given the efficiency of the two approaches mentioned previously, namely the conversion technique of Hosseinzadeh and Edalatpanah from the fuzzy program to a deterministic program and the adapted version of MOMA-plus to the mono-objective case, we propose here a hybridization of these algorithms to the resolution of fully fuzzy L-R triangular

multiobjective linear programs.

As for the rest of this paper it will be structured in five sections. Indeed, in the section two we will give the basic terminologies about fully fuzzy L-R triangular numbers. In the section three we will describe the technique of conversion of fully fuzzy L-R triangular mono-objective linear program into the deterministic multiobjective linear program like mentioned above. In section four, we will present fully fuzzy L-R multiobjective optimization concepts and our technique to convert it in deterministic multiobjective linear program. The algorithm of MOMA-plus and its adapted version will be presented in section five. The sixth section will be devoted to an implementation of our approach and a comparative study of our results to those given by the Total Objective Segregation method. And to finish a last and seventh section for the conclusion.

## 2. Basics Terminology

In this part, we make a non-exhaustive presentation of some important notions about fuzzy L-R triangular numbers. This is necessary for a good understanding of the rest of this work.

**Definition 1** (Zadeh, 1965) : Let  $X$  be a set, called universe, whose elements are denoted  $x$ . A fuzzy subset  $\tilde{A}$  of  $X$  is defined using a membership function  $\mu_{\tilde{A}}$  defined on  $X$  and taking its values in the interval  $[0, 1]$ .  $\tilde{A}$  is therefore characterized by

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X\}. \quad (1)$$

The membership function may represent a degree of possibility or a degree of preference depending on the situation.

**Definition 2** (Didier, 1978) : A fuzzy number  $\tilde{m}$  is said to be L-R if its membership function  $\mu_{\tilde{m}}$  is defined by

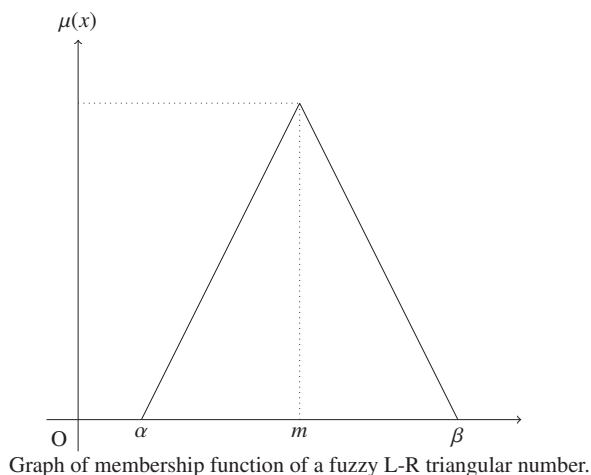
$$\mu_{\tilde{m}}(x) = \begin{cases} L(\frac{m-x}{\alpha}) & \text{if } x \leq m, \\ R(\frac{x-m}{\beta}) & \text{if } x \geq m \end{cases} \quad (2)$$

where :

- ✓  $\alpha > 0$  and  $\beta > 0$  are the left and the right deviation respectively, and  $m$  its average value,
- ✓  $L$  and  $R$  are said reference functions of the fuzzy number checking the following properties:
  - $L$  and  $R$  are non-increasing on  $[0, +\infty[$ .
  - $L$  and  $R$  are symmetrical :  $L(x) = L(-x)$ ;  $R(x) = R(-x)$
  - $L(0) = R(0) = 1$ .

**Remark 1** : If we add to this definition that  $L(1) = R(1) = 0$ , the support of the fuzzy number is finite. In the rest of this work we will assume to be in this condition.

Let there be a triangular fuzzy number  $\tilde{m}$  then there are reals  $m$ ,  $\alpha$  and  $\beta$  such that  $\tilde{m} = (m, \alpha, \beta)_{LR}$  to simplify. Then the associated membership function is represented as follows :



## 2.1 Operations

**Theorem 1** (Didier, 1978) : Let  $\tilde{m} = (m, \alpha, \beta)_{LR}$  and  $\tilde{n} = (n, \gamma, \delta)_{LR}$  two fuzzy L-R triangular numbers. So we have :

1.  $(m, \alpha, \beta)_{LR} \oplus (n, \gamma, \delta)_{LR} = (m + n, \alpha + \gamma, \beta + \delta)_{LR}$ ,
2.  $-(m, \alpha, \beta)_{LR} = (-m, \beta, \alpha)_{LR}$ ,
3.  $(m, \alpha, \beta)_{LR} \ominus (n, \gamma, \delta)_{LR} = (m - n, \alpha + \delta, \beta + \gamma)_{LR}$ .

An ordinary real number  $m$  is also a fuzzy L-R triangular number denoted by  $(m, 0, 0)_{LR}$  whose deviations are zero. According to Dubois and Prad, a number  $\tilde{m} = (m, \alpha, \beta)_{LR}$  is positive if  $m > 0$ , negative if  $m < 0$  and null if  $m = 0$ . Using the left and right deviations one can say that the number  $\tilde{m}$  is positive if  $m \geq 0, m - \alpha \geq 0$  et  $m + \beta \geq 0$  (Hossein-zadeh, 2016 ).

The multiplication of fuzzy L-R triangular number obeys the equalities proposed in the below theorem :

**Theorem 2** (Didier, 1978) : Let  $\tilde{m} = (m, \alpha, \beta)_{LR}$  and  $\tilde{n} = (n, \gamma, \delta)_{LR}$  two fuzzy L-R triangular numbers. So, we have:

1.  $(m, \alpha, \beta)_{LR} \odot (n, \gamma, \delta)_{LR} = (mn, m\gamma + n\alpha, m\delta + n\beta)_{LR}$  if  $\tilde{a}$  and  $\tilde{b}$  are positive,
2.  $(m, \alpha, \beta)_{LR} \odot (n, \gamma, \delta)_{LR} = (mn, n\alpha - m\delta, n\beta - m\gamma)_{LR}$  if  $\tilde{a}$  negative and  $\tilde{b}$  positive,
3.  $(m, \alpha, \beta)_{LR} \odot (n, \gamma, \delta)_{LR} = (mn, -n\beta - m\delta, -n\alpha - m\gamma)_{LR}$  if  $\tilde{a}$  and  $\tilde{b}$  are negative.

## 2.2 Comparison

To compare two fuzzy L-R triangular numbers, we have the definition and the following theorem :

**Definition 3** (Didier, 1978) : Let  $\tilde{m} = (m, \alpha, \beta)_{LR}$  and  $\tilde{n} = (n, \gamma, \delta)_{LR}$  two fuzzy L-R triangular numbers. We say that  $\tilde{m} = \tilde{n}$  if and only if  $m = n$ ;  $\alpha = \gamma$  and  $\beta = \delta$ .

**Theorem 3** (Krishnapada, 2015) : Let  $\tilde{m} = (m, \alpha, \beta)_{LR}$  and  $\tilde{n} = (n, \gamma, \delta)_{LR}$  two triangular fuzzy numbers of the L-R type. We say that  $\tilde{m} \leq \tilde{n}$  if and only if  $m \leq n$ ;  $m - \alpha \leq n - \gamma$  and  $m + \beta \leq n + \delta$ .

The following definition has been proposed for comparing two fuzzy L-R triangular numbers :

**Definition 4** (Hossein-zadeh, 2016) : Let  $\tilde{m} = (m, \alpha, \beta)_{LR}$  and  $\tilde{n} = (n, \gamma, \delta)_{LR}$  two fuzzy L-R triangular numbers. We say that  $\tilde{m}$  is relatively smaller than  $\tilde{n}$ , and we note  $\tilde{m} < \tilde{n}$ , if and only if :

1.  $m < n$ , or
2.  $m = n$  and  $\alpha + \beta > \gamma + \delta$ , or
3.  $m = n, \alpha + \beta = \gamma + \delta$  and  $(2m - \alpha + \beta) < 2n - \gamma + \delta$

**Remark 2** (Hossein-zadeh, 2016) : It is clear from the previous definition that

$$m = n, (\alpha + \beta) = (\gamma + \delta), \text{ and } (2m - \alpha + \beta) = (2n - \gamma + \delta) \quad (3)$$

if and only if

$$\tilde{m} = \tilde{n}.$$

## 2.3 Ranking Function

To compare two fuzzy L-R triangular numbers, the use of the ranking function is often necessary.

**Definition 5** (Hossein-zadeh, 2016) : Let  $\mathcal{R}$  be a ranking function. On can say that  $\mathcal{R}$  is a ranking function defined on  $F(R)$  to  $R$  if :

$$\mathcal{R} : F(R) \longrightarrow R$$

$$\tilde{m} \longmapsto \mathcal{R}(\tilde{m}) = \mathcal{R}[(m, \alpha, \beta)_{LR}] = m + \frac{\beta - \alpha}{4}$$

where  $F(R)$  is the set of fuzzy numbers defined on the set of real numbers  $R$ . In other words  $\mathcal{R}$  has a aim to transpose any fuzzy L-R triangular number on the straight line of real where the order is easy to establish.

**Theorem 4** (Hosseinzadeh, 2016) : If  $\tilde{m}$  and  $\tilde{n}$  are two triangular numbers and  $\mathcal{R}$  a ranking function then :

$$\tilde{m} \geq \tilde{n} \iff \mathcal{R}[\tilde{m}] \geq \mathcal{R}[\tilde{n}]. \quad (4)$$

### 3. Fully Fuzzy L-R Triangular Linear Program

#### 3.1 Formulation

Consider the next fully fuzzy linear program with  $m$  fuzzy equality constraints and  $n$  fuzzy variables :

$$\begin{aligned} \max(\min) \quad & \tilde{Z} = \tilde{C}^t \otimes \tilde{x} \\ \text{S.t :} \quad & \begin{cases} \tilde{A} \otimes \tilde{x} = \tilde{b} \\ \tilde{x} \geq 0 \end{cases} \end{aligned} \quad (5)$$

where  $\tilde{C} = [\tilde{c}_j]_{1 \times n}$ ,  $\tilde{x} = [\tilde{x}_j]_{n \times 1}$ ,  $\tilde{A} = [\tilde{a}_{ij}]_{m \times n}$ ,  $\tilde{b} = [\tilde{b}_i]_{m \times 1}$  are matrix whose the coefficients are L-R triangular numbers. Let :

$$\begin{aligned} \tilde{C}^t \tilde{x} &= ((c^t x)^m, (c^t x)^l, (c^t x)^u)_{LR}, \quad \tilde{A} \tilde{x} = ((Ax)^m, (Ax)^l, (Ax)^u)_{LR}, \quad \tilde{b} = ((b)^m, (b)^l, (b)^u)_{LR}; \\ \text{and } \tilde{x} &= ((x)^m, (x)^l, (x)^u)_{LR}. \end{aligned}$$

The following conversion method has been proposed by Hosseinzadeh and Edalatpanah.

#### 3.2 Conversion to Deterministic Program

Using the operations techniques on the fuzzy L-R triangular numbers presented in sub section 2.1, the problem (5) can be written as :

$$\begin{aligned} \max(\min) \quad & \tilde{Z} = ((c^t x)^m, (c^t x)^l, (c^t x)^u)_{LR}; \\ \text{S.t :} \quad & \begin{cases} ((Ax)^m, (Ax)^l, (Ax)^u)_{LR} = ((b)^m, (b)^l, (b)^u)_{LR} \\ (x)^m \geq 0 \\ (x)^m - (x)^l \geq 0 \\ (x)^m + (x)^u \geq 0 \end{cases} \end{aligned} \quad (6)$$

Using the definition 3 for the equality constraints, the problem (6) becomes :

$$\begin{aligned} \max(\min) \quad & \tilde{Z} = ((c^t x)^m, (c^t x)^l, (c^t x)^u)_{LR}; \\ \text{S.t :} \quad & \begin{cases} (Ax)^m = b^m \\ (Ax)^l = b^l \\ (Ax)^u = b^u \\ x^m \geq 0 \\ (x)^m - (x)^l \geq 0 \\ (x)^m + (x)^u \geq 0 \end{cases} \end{aligned} \quad (7)$$

Using the definition 4 and the remark 1 on the objectives functions, the problem (7) becomes a multiobjective deterministic problem as following :

$$\begin{aligned} \max(\min) \quad & Z^m = (c^t x)^m \\ \min(\max) \quad & Z^l = (c^t x)^l + (c^t x)^u \\ \max(\min) \quad & Z^u = 2(c^t x)^m - (c^t x)^l + (c^t x)^u \\ \text{S.t :} \quad & \begin{cases} (Ax)^m = b^m \\ (Ax)^l = b^l \\ (Ax)^u = b^u \\ x^m \geq 0 \\ (x)^m - (x)^l \geq 0 \\ (x)^m + (x)^u \geq 0 \end{cases} \end{aligned} \quad (8)$$

In the next section, we will generalize this technique of conversion by conceiving for the first time its application to the multiobjective linear programs with fully fuzzy triangular numbers of L-R type.

#### 4. Fully Fuzzy L-R Triangular Multiobjective Linear Program

##### 4.1 Definition

**Definition 6 :** A fully fuzzy L-R triangular multiobjective linear program is formulated as follows :

$$\begin{aligned} \max \quad & \bar{Z}^k = \bar{C}_k^t \otimes \bar{x}; \quad k = 1, \dots, K \\ \text{S.t :} \quad & \begin{cases} \bar{A} \otimes \bar{x} = \bar{b} \\ \bar{T} \otimes \bar{x} \leq \bar{t} \\ \bar{x} \geq 0 \end{cases} \end{aligned} \quad (9)$$

where  $\bar{Z}^k, k = 1, 2, \dots, K$  are objective functions which must be maximized.

##### 4.2 Optimality Concepts

**Definition 7** (Nasseri, 2013) :  $\bar{x}^*$  is called the complete optimal solution of the fuzzy multiobjective linear program if and only if

$$\bar{Z}^k(\bar{x}^*) \geq \bar{Z}^k(\bar{x}); \quad k = 1, \dots, K; \quad \forall \bar{x} \in X.$$

**Definition 8** (Nasseri, 2013) :  $\bar{x}^*$  is called Pareto optimal solution of the fuzzy multiobjective linear program, if and only if there does not exist  $\bar{x} \in X$ , such that

$$\bar{Z}^k(\bar{x}) \geq \bar{Z}^k(\bar{x}^*), \quad \forall k \in \{1, 2, \dots, K\}.$$

**Definition 9** (Nasseri, 2013) :  $\bar{x}^*$  is called a weak Pareto optimal solution of the fuzzy multiobjective linear program, if and only if there does not exist  $\bar{x} \in X$ , such as :

$$\bar{Z}^k(\bar{x}) > \bar{Z}^k(\bar{x}^*), \quad \forall k \in \{1, 2, \dots, K\}.$$

##### 4.3 Conversion into a Deterministic Program

Using the definitions of the fuzzy L-R triangular numbers of the sub-section 2.1, the problem (9) can be written in the following form :

$$\begin{aligned} \max(\min) \quad & \bar{Z}_k = ((c^t x)_k^m, (c^t x)_k^l, (c^t x)_k^u)_{LR}; \quad k = 1, \dots, K \\ \text{S.t :} \quad & \begin{cases} ((Ax)^m, (Ax)^l, (Ax)^u)_{LR} = ((b)^m, (b)^l, (b)^u)_{LR} \\ ((Tx)^m, (Tx)^l, (Tx)^u)_{LR} \leq ((t)^m, (t)^l, (t)^u)_{LR} \\ x^m \geq 0 \\ (x)^m - (x)^l \geq 0 \\ (x)^m + (x)^u \geq 0 \end{cases} \end{aligned} \quad (10)$$

In virtue of the definition 3 and the theorem 3, the constraints of the problem (10) turn into deterministic constraints as follows :

$$\begin{aligned} \max(\min) \quad & \bar{Z}_k = ((c^t x)_k^m, (c^t x)_k^l, (c^t x)_k^u)_{LR}; \quad k = 1, \dots, K \\ \text{S.t :} \quad & \begin{cases} (Ax)^m = b^m \\ (Ax)^l = b^l \\ (Ax)^u = b^u \\ (Tx)^m \leq (t)^m \\ (Tx)^m - (Tx)^l \leq (t)^m - (t)^l \\ (Tx)^m + (Tx)^u \leq (t)^m + (t)^u \\ x^m \geq 0 \\ (x)^m - (x)^l \geq 0 \\ (x)^m + (x)^u \geq 0 \end{cases} \end{aligned} \quad (11)$$

Using the conversion technique proposed by Hosseinzadeh and Edalatpanah to each objective function, the problem (11) becomes a deterministic multiobjective problem of  $3K$  objective functions as following :

$$\begin{aligned}
 & \max(\min) \quad Z^m = (c^t x)_k^m; \quad k = 1, \dots, K \\
 & \min(\max) \quad Z^l = (c^t x)_k^l + (c^t x)_k^u; \quad k = 1, \dots, K \\
 & \max(\min) \quad Z^u = 2(c^t x)_k^m - (c^t x)_k^l + (c^t x)_k^u; \quad k = 1, \dots, K \\
 & \text{S.t :} \quad \left\{ \begin{array}{l} (Ax)^m = b^m \\ (Ax)^l = b^l \\ (Ax)^u = b^u \\ (Tx)^m \leq (t)^m \\ (Tx)^m - (Tx)^l \leq (t)^m - (t)^l \\ (Tx)^m + (Tx)^u \leq (t)^m + (t)^u \\ x^m \geq 0 \\ (x)^m - (x)^l \geq 0 \\ (x)^m + (x)^u \geq 0 \end{array} \right. \quad (12)
 \end{aligned}$$

**Remark 3 :** Each fuzzy objective function split up into three deterministic objective functions. So, the deterministic program resulting from a fuzzy linear program of  $K$  objective functions has  $3K$  deterministic objective functions.

For the resolution of the obtained multiobjective program resulting from the conversion, we propose an adaptation of the MOMA-plus algorithm.

## 5. MOMA-Plus Method

The algorithm of this method allows to transform a multiobjective optimization problem with several variable into a mono-objective optimization problem with single variable and without constraint in order to make easy the reach of global optimum. The main steps of this method can be presented in five (Kounhinir, 2017).

For the best presentation of MOMA-plus algorithm let's consider the multiobjective optimization problem on the following form gives in equation (13) :

$$\begin{aligned}
 & \min \quad f_p(x), \quad p = 1, 2, \dots, K \\
 & \text{S.t:} \quad \left\{ \begin{array}{l} G_i(x) \leq 0, \quad i = 1, \dots, m \\ x \geq 0. \end{array} \right. \quad (13)
 \end{aligned}$$

where  $x = (x_1, x_2, \dots, x_n)$  is a positive real variable and  $K, m$  some known entire numbers.

### 5.1 Algorithm of MOMA-plus

**STEP I :** Consists in aggregation of objective functions. That allows to transform the multiple objectives into a single objective. The problem (13) owns  $K$  objective functions and for transforming this problem into single objective function we have used the weighted sum. It allows us to transform the problem as follows :

$$\begin{aligned}
 & \min \quad S(f, \lambda) = \sum_{p=1}^K \lambda_p f_p(x) \\
 & \text{S.t:} \quad \left\{ \begin{array}{l} G_i(x) \leq 0, \quad i = 1, \dots, m \\ x \geq 0. \end{array} \right. \quad (14)
 \end{aligned}$$

With  $\lambda_1 + \lambda_2 + \dots + \lambda_K = 1$ .

Let  $\mathcal{D} = \{x \geq 0 : G_i(x) \leq 0, i = \overline{1, m}\}$  be the set of admissible solutions defined by the deterministic constraints of (9). The obtaining of optimal Pareto solutions, relative to the method of weighted sum of the objective functions, is guaranteed by the following theorem :

**Theorem 5** (Jacques, 2003) : *Either the parametric problem*

$$\min_{x \in \mathcal{D}} s(f, \lambda) \quad (P)$$

with  $\lambda \in \Lambda = \left\{ \lambda_i \in [0, 1]; \sum_{i=1}^K \lambda_i = 1 \right\}$ .

- ✓ If  $x$  is an optimal solution of  $(P)$ ,  $x$  is an efficient solution.
- ✓ If  $x$  is an efficient solution and  $Z_D$  (image set of  $\mathcal{D}$ ) is a convex set, there exists  $\lambda \in \Lambda$  such as  $x$  is an optimal solution of  $(P)$ .

**STEP II :** Consists in penalization of the problem. That is the transformation of constraints optimization problem into a without constraint optimization problem. The penalization of the problem (14) leads to a problem without constraints. The function of penalization that we use is given in (Kounhinir, 2013) and makes it possible to obtain the problem in the following form :

$$\begin{cases} \text{Glob. min } L(x) \\ x \in \mathcal{D} \end{cases} \quad (15)$$

where

$$L(x) = S(f, \lambda) + \tau \sum_{p=1}^K (G_i(x) + |G_i(x)|)$$

and  $\tau$  is a positive real defined by :

$$\tau \geq \frac{M - S(f, \lambda)}{\sum_{i=1}^m G_i(x)} \quad \text{and} \quad M = \max_{x \in \mathcal{D}} S(f, \lambda).$$

**Theorem 6** (Balira, 2005) *Let  $x^*$  be the global minimum of  $L(x)$ , then  $x^*$  is the global minimum of  $\min_{x \in \mathcal{D}} S(f, \lambda)$ .*

**STEP III :** Consists in variables number reduction. That is the transformation of a function with several variables into a function a single variable by using the Alienor transformation. The application of a Alienor transformation of the Konf-Cherruault (Balira, 2005) which is in the form :

$$x_j = h_j(\theta) = \frac{1}{2}[(b_j - a_j) \cos(\omega_j \theta + \varphi_j) + b_j + a_j]; \quad j = 1, \dots, n \text{ and } \theta \in [0, 2\pi]$$

with :

- ✓  $(\omega_j)_{j=1;n}$  and  $(\varphi_j)_{j=1;n}$  are slowly increasing,
- ✓  $a_j$  and  $b_j$  are the extreme values of  $x_j$ , in other words,  $x_j \in [a_j; b_j]$ ,

on the problem (15) makes it possible to obtain the problem in the form :

$$\begin{cases} \text{Glob. min } L(\theta) \\ \theta \in [0, \theta_{\max}] \end{cases} \quad (16)$$

where

$$L^*(\theta) = L(h_1(\theta); h_2(\theta); \dots; h_n(\theta)) \quad \text{with } \theta_{\max} \text{ the largest value of the variable } \theta.$$

**Theorem 7** (Kounhinir, 2011 & 2013) : *If  $\theta^*$  is global minimum of  $L(\theta)$  then  $x^* = h(\theta^*)$  is global minimum of  $L(x)$ .*

**STEP IV :** Consists in global optimization. That is the using of the Nelder-Mead simplex procedure to reach the optimum of the resulting function from the last step. As the problem (16) is the minimization of a function of a single variable without constraint, we use the simplex algorithm of Nelder Mead, known as "fminsearch" in MATLAB software, to determine its overall minimum.

**STEP V :** Consists in the configuration of the obtained solution. That is transformation of the solution obtained in above step which in one-dimension into the solution of  $n$ -dimension for the initial problem. So, the solutions of the Problem (13) are deduced by using the previous reductive transformation. So, we have :

$$x_j^* = h_j(\theta^*).$$

## 5.2 Adapted MOMA-plus Method

The adapted version of the MOMA-plus method to the resolution of the fuzzy multiobjective program integrates in initial procedure three other steps which are : first the conversion of the fuzzy program into deterministic presented in paragraph 4.3, the conversion of the solutions provided by MOMA-plus in fuzzy numbers and finally the selection of non-dominated solutions. Therefore adapted MOMA-plus version can be presented as follows:

**STEP I :** Conversion of fuzzy multiobjective linear program into deterministic multiobjective linear program;

**STEP II :** Application of MOMA-plus;

**STEP III :** Deduction of the solutions of the initial fuzzy multiobjective linear program from those obtained in step 2:

**STEP IV :** Selection of non-dominated solutions or Pareto optimal solutions of the fuzzy triangular multiobjective linear program.

To highlight the performance of our new algorithm, we will propose its implementation through an example already treated by Jayalakslmi and Pandian (Jayalakslmi, 2014) by the *total objective-segregation method*.

## 6. Example Didactic

Let's consider the following fuzzy L-R triangular multiobjective optimization program extracted from (Jayalakslmi, 2014):

$$\begin{aligned} \max \quad & \widetilde{Z}_1 = (1, 2, 3)\widetilde{x}_1 + (2, 4, 5)\widetilde{x}_2 \\ \max \quad & \widetilde{Z}_2 = (2, 3, 4)\widetilde{x}_1 + (3, 4, 5)\widetilde{x}_2 \\ \text{S.t :} \quad & \begin{cases} (0, 1, 2)\widetilde{x}_1 + (1, 2, 3)\widetilde{x}_2 \leq (1, 10, 27) \\ (1, 2, 3)\widetilde{x}_1 + (0, 1, 2)\widetilde{x}_2 \leq (2, 11, 28) \\ \widetilde{x}_1; \widetilde{x}_2 \in TLR. \end{cases} \end{aligned} \quad (17)$$

where *TLR*, here, is for L-R Triangular L-R no-negative. By transforming this problem into fully fuzzy L-R triangular problem, we obtain :

$$\begin{aligned} \max \quad & \widetilde{Z}_1 = (2, 1, 1)_{LR}\widetilde{y}_1 \oplus (4, 2, 1)_{LR}\widetilde{y}_2 \\ \max \quad & \widetilde{Z}_2 = (3, 1, 1)_{LR}\widetilde{y}_1 \oplus (4, 1, 1)_{LR}\widetilde{y}_2 \\ \text{S.t :} \quad & \begin{cases} (1, 1, 1)_{LR}\widetilde{y}_1 \oplus (2, 1, 1)_{LR}\widetilde{y}_2 \leq (10, 9, 17)_{LR} \\ (2, 1, 1)_{LR}\widetilde{y}_1 \oplus (1, 1, 1)_{LR}\widetilde{y}_2 \leq (11, 9, 17)_{LR} \\ \widetilde{y}_1; \widetilde{y}_2 \in TLR. \end{cases} \end{aligned} \quad (18)$$

### 6.1 Adapted MOMA-plus Method

By setting  $\widetilde{y}_1 = (y_1^m, y_1^l, y_1^u)_{LR}$  and  $\widetilde{y}_2 = (y_2^m, y_2^l, y_2^u)_{LR}$ , we obtain :

$$\begin{aligned} \max \quad & \widetilde{Z}_1 = (2, 1, 1)_{LR} \odot (y_1^m, y_1^l, y_1^u)_{LR} \oplus (4, 2, 1)_{LR} \odot (y_2^m, y_2^l, y_2^u)_{LR} \\ \max \quad & \widetilde{Z}_2 = (3, 1, 1)_{LR} \odot (y_1^m, y_1^l, y_1^u)_{LR} \oplus (4, 1, 1)_{LR} \odot (y_2^m, y_2^l, y_2^u)_{LR} \\ \text{S.t :} \quad & \begin{cases} (1, 1, 1)_{LR} \odot (y_1^m, y_1^l, y_1^u)_{LR} \oplus (2, 1, 1)_{LR} \odot (y_2^m, y_2^l, y_2^u)_{LR} \leq (10, 9, 17)_{LR} \\ (2, 1, 1)_{LR} \odot (y_1^m, y_1^l, y_1^u)_{LR} \oplus (1, 1, 1)_{LR} \odot (y_2^m, y_2^l, y_2^u)_{LR} \leq (11, 9, 17)_{LR} \\ (y_1^m, y_1^l, y_1^u)_{LR}; (y_2^m, y_2^l, y_2^u)_{LR} \in TLR \end{cases} \end{aligned} \quad (19)$$

✓ By applying the techniques of conversion proposed above to the didactic example (17), we obtain :

$$\begin{aligned} \max \quad & z_{11} = 2y_1^m + 4y_2^m \\ \min \quad & z_{12} = 2y_1^l + 2y_1^m + 4y_2^l + 3y_2^m + 2y_1^u + 4y_2^u \\ \max \quad & z_{13} = 4y_1^m + 7y_2^m - 2y_1^l - 4y_2^l + 2y_1^u + 4y_2^u \\ \max \quad & z_{21} = 3y_1^m + 4y_2^m \\ \min \quad & z_{22} = 3y_1^l + 2y_1^m + 4y_2^l + 2y_2^m + 3y_1^u + 4y_2^u \\ \max \quad & z_{23} = 6y_1^m + 8y_2^m - 3y_1^l - 4y_2^l + 3y_1^u + 4y_2^u \\ \text{S.t :} \quad & \begin{cases} y_1^m + 2y_2^m \leq 10 \\ y_2^m - y_1^l - 2y_2^l \leq 1 \\ 2y_1^m + 3y_2^m + y_1^u + 2y_2^u \leq 27 \\ 2y_1^m + y_2^m \leq 11 \\ y_1^m - 2y_1^l - y_2^l \leq 2 \\ 3y_1^m + 2y_2^m + 2y_1^u + y_2^u \leq 28 \\ y_1^m - y_1^l \geq 0 \\ y_1^m + y_1^u \geq 0 \\ y_2^m - y_2^l \geq 0 \\ y_2^m + y_2^u \geq 0 \\ y_1^m \geq 0; y_2^m \geq 0; y_1^l \geq 0; y_2^l \geq 0; y_1^u \geq 0; y_2^u \geq 0 \end{cases} \end{aligned} \quad (20)$$

- ✓ By applying the adapted algorithm of MOMA-plus that we have proposed in this work, we find the Pareto optimal solutions. For our example some solutions in gives the following table.

We remind that the  $\lambda_i$ ,  $i = 1, \dots, 6$  are weights for the objectives functions which are generated automatically by the algorithm such as  $\sum_{i=1}^6 \lambda_i = 1$  and precisely seven sets of weights for the objective functions are used.

No	Weights	$\tilde{x}_1^*$	$\tilde{x}_2^*$
1	0.5, 0.1, 0.2, 0.2, 0, 0	(4.0000, 1.3820, 2.0702)	(3.0000, 0.3090, 3.9649)
2	0.5, 0.2, 0, 0, 0, 0.3	(4.0000, 0.6667, 3.3333)	(3.0000, 0.6667, 3.3333)
3	0.6, 0, 0, 0, 0, 0.4	(3.5794, 0.3162, 3.8240)	(3.2103, 0.9471, 3.1931)
4	0.6, 0, 0.1, 0.1, 0.1, 0.1	(4.0000, 0.6667, 1.4756)	(3.0000, 0.6667, 4.2622)
5	0.7, 0, 0, 0, 0, 0.3	(3.6561, 0.3801, 3.7345)	(3.1719, 0.8959, 3.2187)
6	0.8, 0, 0, 0, 0, 0.2	(3.7054, 0.4212, 3.6770)	(3.1473, 0.8630, 3.2351)
7	0.9, 0, 0, 0, 0, 0.1	(3.5678, 0.3065, 3.8376)	(3.2161, 0.9548, 3.1893)

That allows to deduce the objective functions  $\tilde{Z}_k$ ,  $k = 1, 2$  and also the ranking function values  $\mathcal{R}$ . All these results are presented in the below table :

No	$\tilde{Z}_1^*$	$\tilde{Z}_2^*$	$R[\tilde{Z}_1^*]$	$R[\tilde{Z}_2^*]$
1	(20.0000, 8.0000, 27.0000)	(24.0000, 12.3820, 29.0702)	24.7500	28.1721
2	(20.0000, 8.0000, 27.0000)	(24.0000, 11.6667, 30.3333)	24.7500	28.6667
3	(20.0000, 8.0000, 27.2103)	(23.5794, 11.5265, 31.0343)	24.8026	28.4564
4	(20.0000, 8.0000, 27.0000)	(24.0000, 11.6667, 28.4756)	24.7500	28.2022
5	(20.0000, 8.0000, 27.1719)	(23.6561, 11.5520, 30.9065)	24.7930	28.4947
6	(20.0000, 8.0000, 27.1473)	(23.7054, 11.5685, 30.8243)	24.7868	28.5194
7	(20.0000, 8.0000, 27.2161)	(23.5678, 11.5226, 31.0537)	24.8040	28.4506

By noting  $\mathcal{R}_M$  the ranking function associated to the MOMA-plus method we have :

$$\mathcal{R}_M[\tilde{Z}_1^*] \geq 24.75 \text{ and } \mathcal{R}_M[\tilde{Z}_2^*] \geq 28.1721$$

### 6.2 Total Objective-segregation Method

Let's note that for this example, the method of Jayalakslni and Pandian gives a unique solution which is completely optimal solution :

$$\tilde{Z}_1^* = (4, 20, 43) \text{ and } \tilde{Z}_2^* = (7, 24, 49).$$

By bring backing this result on the L-R triangular form we obtain :

$$\tilde{Z}_1^* = (20, 16, 23)_{LR} \text{ and } \tilde{Z}_2^* = (24, 17, 25)_{LR}.$$

Also, by setting  $\mathcal{R}_T$  the ranking function associated to the Total objective segregation method we get :

$$R_T[\tilde{Z}_1^*] = 21.75 \text{ and } R_T[\tilde{Z}_2^*] = 26.$$

### 6.3. Comparison

Considering the provided solutions by the two methods during the resolution of the didactic example (17) we have:

$$\begin{cases} R_M[\tilde{Z}_1^*] \geq R_T[\tilde{Z}_1^*] \\ R_M[\tilde{Z}_2^*] \geq R_T[\tilde{Z}_2^*] \end{cases} \implies R_M[\tilde{Z}] \geq R_T[\tilde{Z}].$$

Therefore, one the one hand the adapted version of MOMA-plus gives better compromise solutions than the Total objective segregation method. One the other hand we can add that the adapted version of MOMA-plus gives the possibility to a decision maker to make some choices according to his preferences which can be translated in our case by weights.

## 7. Conclusion

In this paper, we have proposed a new method for solving Fully fuzzy L-R triangular multiobjective linear programming problems. It has consisted of the conversion of the fuzzy basic problem into a deterministic multiobjective optimization problem in order to use our adapted version of the algorithm of the MOMA-plus method. That has allowed us to generate better solutions of compromise or Pareto optimal solutions compared to the Total objective segregation method as testifies the treated example. Therefore we can conclude that the extension we have proposed provides better results for the resolution of fully fuzzy L-R triangular multi-objective linear programs.

## Acknowledgments

The authors thank anonymous referees for their suggestions and comments to improve the earlier version of this paper. Also, the corresponding author would like to thank Germain Academic Exchange Service (DAAD) for its financing that allows to be in the best conditions in order to finish this work.

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