Journal of Mathematics Research; Vol. 10, No. 4; August 2018
ISSN 1916-9795 E-ISSN 1916-9809
Published by Canadian Center of Science and Education

Optimization of Time Slots for the Air-Traffic Management

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Received: January 22, 2018 Accepted: April 13, 2018 Online Published: May 28, 2018

doi:10.5539/jmr.v10n4p39 URL: https://doi.org/10.5539/jmr.v10n4p39

Abstract

The allocation of airport time slots depends on the configuration of the airport, particularly on that of the runways. Thus, in order to allocate the slots optimally in an airport platform, we proposed two optimization models in this article. The first model maximizes the airlines companies demands in the periods by taking into account the characteristics of the airport. This model assigns the flights' demands. It allows determining the number of demands which we can satisfy in a given period of amplitude of one hour. It also helps to incorporate dynamically the unmet demand of j period to the j+1 period.

The second model aims to schedule the confirmed requests by the first assigning model. We are interested in the optimal repartition of the confirmed requests, while minimizing the flights delays. These models are used to optimize the air-traffic management of the Diass airport. Therefore, we have developed and implemented two algorithms for the resolution of these two models. The ressults of experimentations in Cplex show that our algorithms are efficient comparing to those obtained by the reference works existing in this field. The data used are those of the International Blaise Diagne Airport (AIBD).

Keywords: Time Slot, Assigning problem, Scheduling problem, Airport capacity

1. Introduction

A time slot is an authorization given for the use of facilities in a coordinated airport. That is to say that, an airport where, in order to land or take off, an airline or any other operator of aircraft must be allocated a time slot by a coordinator at a specific date and time. Nevertheless, an exception is given to the State flights, urgency landings and humanitarian flights, according to the assignment made by the coordinators. Time slots are a valuable and desired resource for airlines companies since that, in case of an imbalance between the expected air traffic and the available capacity, generally the air-traffic controllers impose regulations.

However, several measures can be taken by the air-traffic controllers in order to respect the airport capacity. These measures can be: the delaying of the taking-off of an aircraft, the request pending in the hippodromes, the changing of the road used by the flight, even the alteration of the flight speed. The air-traffic controllers are in charge of solving regularly these problems. The regulation consists of imposing delays in the take-offs of certain flights. The delays are imposed on the basis of a fairness concept without privileging any airline company. The air-traffic controller assigns a departure or landing time slot to flights on the basis of First In, First Out policy (FIFO). In this article, we solve an optimization generic problem of air slots under two sub-problems namely, assignment and scheduling. For the flight assignment problem, we propose a linear model to optimize the demands at each interval. From a practical point of view, to solve such problem, it consists of allocating efficiently the flight requests in the periods for the whole horizon of the considered time. During these allocations, the requests that are not satisfied in a given period are automatically taken into account in the following period.

For the scheduling problem, we propose a second linear model which can allow us to minimize the flight expectations and delays on all demands confirmed by flight coordinators according to the first model. This model also ensures the optimal dispatch of flights by allocating a slot for each confirmed demand.

This paper is organized as following: after the introduction in section (1), we present a critical analyses of the different works related, in section (2). Section (3), is more centered on the description of the slot allocation problem in case of one single runway airport. Then, we are going to specify the principle determining factors of the slots. In section (4), we propose our linear models by decomposing the slot optimization problem into sub-problems namely: the allocation problem and the scheduling problem. In section (4.1), we give more details about the allocation model. It is about a model which can allow us to optimize the requests in their needed periods by respecting the airport characteristics particularly the runway capacity. In section (4.2), we present the solving algorithm linked to that model. In section (4.3), we give details of the second model which is a scheduling model. This model allows minimizing the flight delays when there is an imbalance between the flow of traffic and the available capacity. Section (4.4) presents the algorithm linked to this model. Section (5) presents the evaluation of our models performances by using the diagram data of the time slots of the International Blaise Diagne Airport of January the 6^{th} 2018. Finally, the last section sums up our work and gives more perspectives.

2. Critical Analysis of the Existing Referential Works

These last years, many researchers have put emphasis on the problem of air slots assignment and the airports capacity. We can name the results of JFRPLC[1] works presented in 2000 (Barnier, 2000) under the topic of lots assignment for the regulation of the air-traffic. We can also note the results of CMT[2] presented in 2003 (Conference, 2003) under the topic of challenges and promises of the liberalization. Finally, the works presented in 2004 by the CEDECE[3] (Lenoir & Nathalie 2004) which dealt with the topic of saturation and Air slots.

A first model from the waiting line was developed in 1975 by the European STBA[4] so as to determine the hourly rate in a case of one single runway (STBA, 1975). Gilbo and al (Gilbo, 1993) proposed in 1993 a dynamic programming model to determine the capacity of a runway. In 2003, they ameliorated their works by proposing method of approximation and estimation (Gilbo, 2003). In 2007, Alidou (Alidou, 2007) determined the movement area capacity of the Dakar Airport by an approximation and an evaluation method. In 2009, (Lorenzo & Castelli, 2009) proposed a mechanism for the selling of secondary slots, if it is question of a one single runway airport, so as to allocate those slots. In 2000, (Nicola & barnier, 2000) presented some modeling of the hourly capacity to regulate in real time the burden of the controllers. All these authors have contributed to the solving of the problem linked to the allocation of the airport slots by considering the runway capacity as the determining factor of the airport system. In order to overcome the limits linked to the runway capacity, (Ahamada and al, 2017), proposed in 2017 a model from the linear programming in integer number so as to determine the number of acceptable demands in a period. This model takes into account the runway capacity but also the passenger air terminal. An extension of this model has been achieved in that article using the dynamic programming in order to integrate the unmet requests of one period to another one.

In fact, the existing referential works which we have explored do not take into account the beginning and ending hour of a slot. Thus, in this article, we propose an allocation and scheduling model by taking into account the beginning and ending hour of a given slot. Indeed, these models can facilitate the central coordinators during the elaboration of the hourly planning.

3. Capacity and Airport Slots

The traffic rights, the airport capacity and the airport slots are quietly different questions but they complete each other. The coordinator plays the role of a leader by elaborating the slots diagrams and he oversees also the efficiency of the slots for the companies. The airport capacity is a determining factor for the allocation of the time slots. In this article, we assimilate this capacity[5] to the hourly rate K, which is the maximum number of flights per hour that can be accepted by the airport. Certainly, we consider an airport which has one single runway. The determination of this rate is based on the waiting line theory. In order to determine that rate we use the following formula (1) proposed by the European STBA (STBA in 1975:

$$K = \frac{1}{t_{AA}p^2 + (t_{AD} + t_{DA})p(1-p) + t_{DD}(1-p)^2} \quad (1)$$

Where p designs the landing proportion on the take-offs; t_{AA}: average times of occupation of the runway for a landing-landing operation; t_{AD}: average times of occupation of the runway for a taking-off-landing operation; t_{DA}: average times of occupation of the runway for a taking-off - landing operation; t_{DD}: average times of occupation of the runway for a taking-off - taking off operation.

[1] French Days of Logic Programming and Constraint Programming
[2] World Transport Conference
[3] Commission for the Study of the European Communities
[4] Technical Service of European Air Bases, July 1975
[5] Maximum number of operations (landings and take-offs) that can be carried out in a fluid time interval with a constant level of demand

4. Proposal of Slot Allocation Models

In this paper, we suggest two models so as to solve the allocation problem of slot, and they are as following:

First, the allocation model of flights in the periods second, the scheduling model of the flights

4.1 Allocation Model of Demands in the Periods

The allocation model that we propose allows us to maximize the flights requests in the considered periods. We consider the following mathematical notations:

$S = \{1, 2, 3, m\}$ all the periods of demands..

A_j: all flights that have formulated requests in the period $j \in S$.

0: fictitious flights and we pose:

$$A = A_j \cup \{0\}$$

K: hourly rate of the airport, maximum number of flights per hour which can be taken by the runway.

We designate by x_{jk} the variables of decisions $\forall j \in S$ and $k \in A_j$

$$x_{jk} = \begin{cases} 1 & \text{the aircraft demand } k \in A_j \text{ in the period } j \in S \text{ is confirmed.} \\ 0 & \text{if not} \end{cases}$$

X_j: Number of unmet demands in the period $j \in S$,

$$X_j = |A_j| - \sum_{k \in A_j} x_{jk}$$

The non-satisfied demands in $j \in S$ must be accepted in $j + 1 \in S$ period. Within this hypothesis we introduce some logical constraints (2) imposing that if the demand is not accepted in j, it must be in $j + 1 \in S$ period.

$$x_{jk} + x_{j+1\,k} = 1 \qquad \forall j \in S \quad and \quad k \in A \tag{2}$$

Certainly, these non-accepted demands in j are added dynamically with those which formulated their demands in $j + 1$ period. In order to determine the maximum number of accepted demands in $j \in S$ period, in that case, we must maximize the following objective function 3:

$$\max_{j \in S} \{X_{j-1} + \sum_{k \in A} x_{jk}\} \tag{3}$$

Under the constraints:

$$X_{j-1} + \sum_{k \in A} x_{jk} \leq K \qquad \forall j \in S \tag{4}$$

$$x_{jk} + x_{j+1\,k} = 1 \qquad \forall j \in S \tag{5}$$

$$X_j = |A| - \sum_{k \in A} x_{jk} \qquad \forall j \in S \tag{6}$$

$$x_{jk} \in \{0, 1\} \qquad \forall j \in S \tag{7}$$

$$X_j \geq 0, \quad \forall j \in S \tag{8}$$

The constraints (4) impose that the accepted demands in a period must not exceed the airport hourly capacity. They are capacity constraints in which K, is designated as the airport hourly capacity. The constraints (5) are logical constraints which impose that every demand must be accepted in one of two successive periods. The constraints 7 and 8 are constraints of integrity.

4.2 Algorithm for Resolving the Allocation Model

To resolve this model, we have elaborated the following Algorithm (1). We designate by V_j a request allocation sequence obtained in the period j, particularly a vector which components are the variables.

$$x_{jk} = 1, \qquad j \in S \quad and \quad k \in A$$

The vector V_j is an optimal solving of the assignment problem in the period $j \in S$. In other word, the number of slots of the period $j \in S$ given to the air company must be equal to the cardinal of the vector V_j. Following the rule of First in, First out, we introduce some variables which take into account the beginning and ending time of the slot so as to schedule the requests in the periods. Every request is assigned to a slot. In the following scheduling model, V_j stands for all the confirmed requests by the coordinator six months earlier.

Algorithm 1

Require:
S: all the interval of demands.
A_j: demand number of movement in the period $j \in S$.
X_j: number of unconfirmed demands in the period$j \in S$.
0: the fictitious flights.
We write: $A = A_j \cup \{0\}$.
Decision variables:
$x_{jk} = \begin{cases} 1 & \text{if the flight demand } k \in A \text{ in the period } j \in S \text{ is confirmed} \\ 0 & \text{if not} \end{cases}$

Ensure: $X_1 = 0$,
for $j \in S$ **do**
  for $k \in A$ **do**
    if $X_{i-1} + \sum_{k \in A} x_{ik} \leq K$ **then**
      $V_j = \max\{X_{j-1} + \sum_{k \in A} x_{jk}\}$
      $X_j = |A| - \sum_{k \in A} x_{jk}$
    end if
    $x_{jk} + x_{j+1\,k} = 1$
  end for
end for

4.3 Model for Scheduling Demands in Period

The scheduling model that we suggest allows minimizing the waiting and delay time of the flights. It allows us to allocate the slots to the confirmed flights by the above assignment model. We consider the following mathematical notations:

$V_j = \{1, 2, 3, ..., f, ...\}$ the entire confirmed flights in the period of $j \in S$.

C_j: the entire slots in a period $j \in S$.

R_j: the entire flights which the controllers have imposed delays in the period $j \in S$;

$\bar{R}_j$: the entire flights which the controllers have not imposed delays in the period $j \in S$;

In that case,

$$\forall j \in S \quad V_j = R_j \cup \bar{R}_j \qquad \forall f_i \in R_j \quad et \quad \forall \bar{f}_i \in \bar{R}_j$$

d_{ij}: start time of the slot $i \in C_j$ in the $j \in S$ period.

α: additional parameter corresponding to the duration which separates two beginning of successive time slots.

$$d_{i+1j} = d_{ij} + \alpha \tag{9}$$

t_{ij}: final time of the slot $i \in C_j$ in the $j \in S$ period;

δ: amplitude of slots and all the slots have the same amplitude

$$\delta = t_{ij} - d_{ij} \qquad \forall i \in C_j \tag{10}$$

Proposition 4.1. *The beginning hour of the i^{me} slot of the period $j \in S$ is given by the following relation*

$$d_{ij} = d_{1j} + (i-1)\alpha \qquad \forall j \in S \tag{11}$$

Proof. a slot in a period is defined by its initial hour and its final hour. The whole slots form the optimization horizon (C_j) as shown in the following figure1 according to: The above figure (1),

$$C_j = \{[d_{ij},\ d_{ij} + \alpha[, [d_{ij} + \alpha,\ d_{ij} + \alpha + \delta[,\ [d_{ij} + \alpha + \delta, d_{ij} + \alpha + 2\delta[, ...\}$$

Where, $d_{i+1j} = d_{ij} + \alpha$, it is a numeric series of ratio α and of the first term d_{1j}, $\forall j \in S$. We deduce by equation number

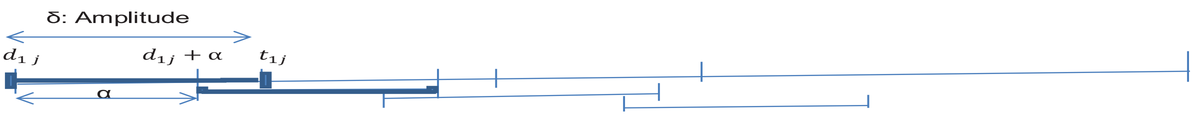

Figure 1. the entire slots

(9) that:

$$\begin{aligned} for \quad k=1, \quad d_{2j} &= d_{1j}+\alpha \\ for \quad k=2, \quad d_{3j} &= d_{2j}+\alpha = d_{1j}+2\alpha \\ for \quad k=3, \quad d_{4j} &= d_{3j}+\alpha = d_{1j}+3\alpha \\ \vdots \quad &= \vdots \quad \vdots \\ for \quad k=i-1 \quad &= d_{ij} = d_{i-1j}+\alpha = d_{1j}+(i-1)\alpha \end{aligned} \tag{12}$$

and, We can simplify the writing and we shall obtain $\forall i \in C_j$: $d_{ij} = d_{1j} + (i-1)\alpha \qquad \forall j \in S$ □

Proposition 4.2. *we designate δ as the amplitude of slot, we will have the final hour of the i^{me} slot by the following equation:*

$$t_{ij} = d_{1j} + (i-1)\alpha + \delta \qquad \forall j \in S \tag{13}$$

Proof. The number of slots in the period $j \in S$ are all equal amplitude δ for the whole optimization horizon. According to the equation (10), the amplitude is equal $\delta = t_{ij} - d_{ij}$, we will have in that case: $\delta = t_{ij} - d_{ij}$ we replace d_{ij} by his expressions we get that $t_{ij} = d_{1j} + (i-1)\alpha + \delta$ □

$E^{f_j}_{ij}$: Final hour of the flight $f_i \in R_j$ in the slot $i \in C_j$.

$\Delta^{f_j}_{ij}$: Delay of the flight of the period j in the slot $i \in C_j$.

$$\Delta^{f_i}_{ij} = E^{f_i}_{ij} - d^{f_i}_{ij} \qquad \forall f_i \in R_j \qquad \forall i \in C_j \tag{14}$$

The total waiting of the flight f_i of the period $j \in S$ is the sum of its own delay in the slot $i \in C_j$ and the waiting imposed by the air-traffic controllers, until it gets a free slot.

We designate by $h^{f_i}_{ij}$, the preferential hour of the flight f_i which is given the slot i of the period j, as agreed between the airlines company and the coordinators. This hour is always included in the beginning and ending hour of the slot. $h^{f_i}_{ij} \in [d^{f_i}_{ij};\ t^{f_i}_{ij}]$ but the flight's slot is:

$$\forall i \in C_j, \qquad i = [d^{f_i}_{ij}, \qquad t^{f_i}_{ij}] \qquad\qquad j \in S$$

Decisions Variables:

To control the air-traffic, the air-traffic controllers make some flights waiting $f_i \in R_j$. This operation decides the value of the decision variables $x^{f_i}_{ij}$ such as:

$$x^{f_i}_{ij} = \begin{cases} 1 & \text{if } f_i \in R_j \\ 0 & \text{if not} \end{cases}$$

The slot allocation policy selects the flights $f_i \in R_j$ in ascendant way according to the value of its waiting and then assigns every flight to the first free slot, that is to say, $x^{f_i}_{ij} = 1$ si $f_i \in R_j$ and i is not yet given.

Every flight must be given one slot in the period $j \in S$, that slot i' is defined by:

$$y^{f_i}_{i'} = \begin{cases} 1 & \text{if the flight } f_i \text{ is assigned to the slot free } i' \in C_j \\ 0 & \text{if not} \end{cases}$$

So as to minimize the flights waiting time, we propose the following linear model

$$\min \sum_{i \in C_j} \sum_{f_i \in V_j} \sum_{j \in S} \Delta_{ij}^{f_i} x_{ij}^{f_i} + \sum_{j \in S} \sum_{i \in C_j} \sum_{f_i \in V_j} (d_{i'j}^{f_i} - E_{ij}^{f_i}) y_{i'}^{f_i} \tag{15}$$

Under the constraints:

$$x_{ij}^{f_i} + x_{ij}^{\bar{f}_i} = 1 \quad \forall j \in S, \quad \forall i \in C_j \tag{16}$$

$$d_{ij}^{f_i} x_{ij}^{f_i} = (d_{1,j}^{f_1} + i'\alpha) y_{i'}^{f_i} \quad \forall f_i \in R_j \quad \forall i, i' \in C_j \tag{17}$$

$$t_{ij}^{f_i} x_{ij}^{f_i} = (d_{1j}^{f_1} + (i' - 1)\alpha + \delta) y_{i'}^{f_i} \qquad \forall f_i \in R_j \tag{18}$$

$$d_{ij}^{f_i} x_{ij}^{f_i} \leq d_{kj}^{f_k} x_{kj}^{f_k} \quad \forall i < k \quad \forall f_i,\ f_k \in R_j \tag{19}$$

$$\sum_{j \in S} \sum_{i \in C_j} y_{ij}^{f_i} \leq |C_j| \quad \forall j \in S \quad \forall f_i \in V_j \tag{20}$$

$$x_{ij}^{f_i}; \quad y_i^{f_i} \in \{0; 1\} \quad \forall f_i \in V_j \qquad \forall i \in C_j \tag{21}$$

In that model, the objective function (15) minimizes the flight delays and the time delays for the controler imposed. The constraints (16) show that all the flights assigned to the period $j \in S$ are whether imposed delays or not. Constraints (17) and (18) are allocation constraints. They show that for every flight made to wait in the slot i, the controllers give it the free slot i', which the beginning hour is $d_{i'j}^{f_i}$ and the ending hour is $t_{i'j}^{f_i}$ with $f_i \in R_j$ if the latter is free. The constraints (19) show that if the slot i comes before the slots k and when both flight f_i and f_k are waiting, we allocate the flight f_i before the flight f_k if there is a free slot. Nevertheless, the constraints (20) stipulate that the number of allocated slots in the period $j \in S$ must be small or equal to the number of movements per hour. The constraints (21) guarantee the integrity of the variable decisions $x_{ij}^{f_i}$ et $y_i^{f_i}$.

4.4 Algorithm of Resolution for the Scheduling Problem

For the resolution of this model, we have elaborated the following algorithm 2.

5. Performance Evaluation of our Models

In this section, we detail the performance evaluations of our two models.

5.1 Application Case: AIBD

In order to evaluate the performance of our models, we have used the International Blaise Diagne Airport (AIBD) data. It is indeed, the Senegalese new airport inaugurated December the 7^{th} 2017. All the experimentations have been done on a Dell computer with Intel (R) core (TM)i5-6200U CPU @2.30GHz 2,40GHz of microprocessor, 8.00Go of memory RAM, 1TB of hard disc drive and working on a windows 10 famille of 64 bits Edition. We have implemented our algorithms in the Cplex software, in order to experiment the scheduling method of requests in the periods.

5.2 Used Data

The used data during our experimentations are extracted from the time slots diagram of the International Blaise Diagne Airport the 6th January 2018, between 8pm to 9pm. The following table sums up the data; and this diagram is elaborated by the Senegalese National Agency for the Civil and Meteorology Aviation(ANACIM).

Table 1. Planning of 8h pm to 9h pm of Thursday January 2018

Periods	Num of Flights	Demands		Final hour of the flight
	1	KP97	10	00
	2	AEA13	15	04
	3	IST/NK	20	13
	4	SS990/4	28	12
$[8hpm, 9hpm[$	5	AF718	30	22
	6	TP14	36	33
	7	KP42	40	34
	8	SAA	47	38
	9	AF718	48	45
	10	IB69	58	66

Algorithm 2

Require:

V_j: the entire confirmed flight in the period $j \in S$

C_j: the entire slots in a period $j \in S$,

K: number of movements per hour which can be taken on the runway.

R_j: the entire flights imposed waiting in the period $j \in S$.

$\bar{R}_j$: the entire flights which are not imposed waiting in the period $j \in S$.

$E^{f_i}_{ij}$: final hour of the flight $f_i \in R_j$ in the slot $i \in C_j$.

δ: amplitude slot

α: parameter which separates two beginning of a successive slot;

sum of the beginning hour of the slots: $d_{ij} = d_{1j} + (i-1)\alpha \quad \forall j \in S$

Sum of the final hour of the slots: $t_{ij} = d_{1j} + (i-1)\alpha + \delta \quad \forall j \in S$

Decision variables:

$$x^{f_i}_{ij} = \begin{cases} 1 & \text{if } f_i \in R_j \\ 0 & \text{if not} \end{cases}$$

$$y^{f_i}_{i} = \begin{cases} 1 & \text{if the flight } f_i \text{ is assigned to the slot free} i \in C_j \\ 0 & \text{if not} \end{cases}$$

Ensure:

for $j \in S$ **do**

  for $i \in C_j$ **do**

    Calculate the value of $\Delta^{f_i}_{ij} := E^{f_i}_{ij} - d^{f_i}_{ij}$

    if $\Delta^{f_i}_{ij} \leq \alpha$ **then**

      The flight is allocated to the slot i which was booked to it.

    else

      The flight f_i is waiting

      if $y^{f_i}_{i'} = 1$ **then**

        Allocate to the flight $f_i \in R_j$ the beginning hour. $d^{f_i}_{i'j} = (d^{f_1}_{1,j} + i'\alpha)y^{f_i}_{i'}$

        Allocate to the flight $f_i \in R_j$ the final hour. $t^{f_i}_{i'j} = (d^{f_1}_{1j} + (i'-1)\alpha + \delta)y^{f_i}_{i'}$

      end if

      if the slot $i < k$ **then**

        The beginning hour of the flight f_i comes before the beginning hour of the f_k flight: $d^{f_i}_{ij}x^{f_i}_{ij} \leq d^{f_k}_{kj}x^{f_k}_{kj}$

        The final hour of the flight f_i comes before the final hour of the f_k flight:
        $t^{f_i}_{ij}x^{f_i}_{ij} \leq t^{f_k}_{kj}x^{f_k}_{kj}$

      end if

    end if

    The allocated slots in a period $j \in S$ is inferior or equal to $|C_j|$. $\sum_{j \in S} \sum_{i \in C_j} y^{f_i}_{ij} \leq |C_j|$

  end for

  Calculate, min $\sum_{i \in C_j} \sum_{f_i \in V_j} \sum_{j \in S} \Delta^{f_i}_{ij} x^{f_i}_{ij} + \sum_{i \in C_j} \sum_{f_i \in V_j} (d^{f_i}_{i'j} - E^{f_i}_{ij}) y^{f_i}_{i'}$

end for

Column (1) and (2) of the table represent the period and the i^{eme} flight allocated to the period. In the column (3) we have the requests composed of the flight number and its preferential hour of landing or taking-off from the airport.

5.3 Experimentation Results

For this application, the time of separation between two beginning of slots $\alpha = 4$ min. Proposition (4.1) and (4.2) allow us to obtain the hourly planning in the period [8h pm, 9h pm[as summed in the following table.

Table 2. Thursday proposal planning of January the 6^{th} 2018

Periods $j \in S$	slots	Flight	B.time of the slot	P.hour flight	End time of of the slot	F. hour of flight	Delay imposed to the flight
	$i = 1$	KP97	00	10	15	00	0
	$i = 2$	AEA13	04	15	19	04	0
	$i = 3$	IST/NK	08	20	23	13	5
	$i = 4$	SS990	12	28	27	12	0
$]8hpm - 9hpm[$	$i = 5$	–	16	–	31	–	–
	$i = 6$	AF718	20	30	35	22	2
	$i = 7$	–	24	–	39	–	–
	$i = 8$	TP14	28	36	43	33	5
	$i = 9$	KP28	32	40	47	34	2
	$i = 10$	SAA	36	47	51	38	2
	$i = 11$	AF718	40	48	55	45	5
	$i = 12$	–	44	–	59	–	–
	$i = 13$	–	48	–	63	–	–
	$i = 14$	–	52	–	67	–	–
	$i = 15$	IB69	56	58	71	66	10

We allocate to every flight a beginning hour, (column 4) and a final hour, (column 6), according to its preferential time (5). Thus, the i^{eme} flight (3) is allocated to the i^{eme} slot (column 2). The data of column (7) were taken from the site (www.flightradar24, Diass) and show the arrival and departure hours of the flights in the considered periods. In that case, we calculate the flight waiting Δ_{ij} of by the formula (14)). In real time, every flight which does not respect its slot will be put in waiting by the controllers until it gets a free slot. We obtain the results of the following table 2: In the experiment,

X- delayed flights		
N° slots	**flights**	**Value**
1	1	0
2	2	0
3	3	1
4	4	0
5		
6	6	0
7		
8	8	1
9	9	0
10	10	0
11	11	1
12		
13		
14		
15	15	1

Y- Affected slots		
N° Slots	**flights**	**Value**
1	1	1
2	2	1
3		0
4	4	1
5	3	1
6	6	1
7		
8		0
9	9	1
10	10	1
11		0
12	8	1
13	11	1
14		
15		0

Figure 2. Results of the simulation

we consider that the delay is significant if. We note that the flights (3, 8, 11 and 15) assigned to the slots (3, 8, 11 and 15) came in arrears. These flights must have a slot free to perform their movement. The Cplex model resolution assigns slots (5, 12, and 13) respectively to flights (3, 8, and 11). On the other hand the flight 15, was not allocated in the period. The total waiting time is equal to 57min, divided between the 4 flights.

With these results, we used Eviews 9.1 to graphically represent, in the Figur 3 the clouds of the points which symbolize all the niches of the period considered. The end times (HFINALE) are in function start times (HDEBUT) and we have:

$Hfinale = f(Hdebut) = Hdebut + \alpha$. The figure (4) shows the free and occupied slots in the period in the same period.

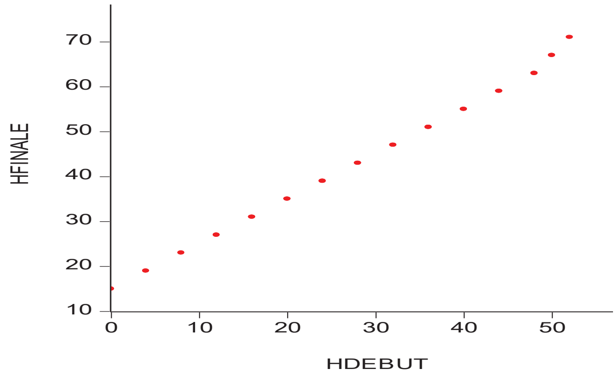

Figure 3. the entire slots in a period $j \in S$

The variable X represents the free slots of the period and Y represents the allocated slots. On this cloud of points, slots 07

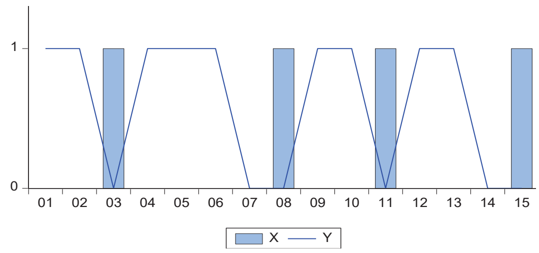

Figure 4. used and free slots in the period

and 14 are free. In contrast to the delays of the flights assigned to the slots 03, 08, 11 and 15 leave these slots free also. So in the period, we have a total of six unused slots. These results show that the more the flights make delays along the period, the slots become more and more unused. Penalty must be imposed on flights that have not respected their slots The slot development system at an airport must be applied at an airport where there is a very high demand for arrivals and departures in a period.

6. Conclusion and Perspectives

We have proposed in this paper two complementary models of a linear programming in integer numbers so as to solve the allocation problem of the airport time slots. The airport capacity is a determining factor for the allocation of slots. In this paper, we have assimilated that capacity to the hourly rate in order to regulate efficiently the air-traffic. From a practical point of view, the first model allows us to maximize the formulated requests by the airline companies six months earlier, by taking into account the runways capacity. It is all about solving an allocation problem with capacity constraints. Indeed, this model guarantees the allocation of every confirmed request to a time slot in the period of time horizon. In order to better regulate the air-traffic, the air-traffic controllers must impose in real time some delays to certain flights when there is a traffic imbalance.

So as to execute the flight plans, the second model allows us to minimize the delays imposed by the controllers. This model allows us also to ensure an optimal repartition of the flights by allocating one slot to every confirmed request. It is indeed a scheduling problem of the flights in the periods. We have evaluated our models performances by using the real data of the Senegalese AIBD. The obtained results show that the proposed model yield signification in terme of optimisation aireport time slots. These results show as well an efficient use of temporal resources during the airline traffic management in an airport which has constraint of capacity.

Some perspectives are still under study so as to ameliorate our works. First, in terms of reception capacity, we have considered a one single runway airport. We wish to extend our models in order to ensure the time slots management in case of two or more runways. We then plan to take into account the whole data of a day (24h) in order to elaborate a weekly, monthly even seasonal airport planning. We also wish to extend these models in a way that they will take into account the landings in one way and the taking-offs in the other way. We hope also to extend our models by imposing payment penalty to the flights which do not respect the allocated slots in order to guarantee a wonderful execution. Finally, we envisage to elaborate a multi-objective economic function. These same models simultaneously optimize several criteria related to the problem of air traffic management with constraints of capacity, time and costs.

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