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# Strong Goldbach Number in Goldbach's Problem

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**Abstract**

Based on the data we get, $n \log n$ is acceptable approximation to $A(P_n)$ for small $P_n$ ( most of relative errors are on the order of 1%, 1‰ or 0.1‰ for $P_n$ less than 2000 ), where $P_n$ is the $n$-th prime for $n \geq 2$ and $A(P_n) = L_n - P_n$ but $L_n$ is the largest strong Goldbach number generated by $P_n$. Thus we propose a proposition that $A(P_n) \approx n \log n$ for all $P_n$. To give indirect verification of the proposition for large $P_n$, we obtain an experimental formula for calculating the number of primes not greater than $P_n$ relying on existence of $A(P_n)$. Using the formula, our found relative errors are generally smaller than that arising from $x/\log x$, for example, there is a found relative error to be about 0.00935% for the 382465573492-th prime but the relative error is about 3.45305% by $x/\log x$, 0.16046% by $x/((\log x) - 1.08366)$, 0.02479% by $x/\log x + x/(\log x)^2 + 2x/(\log x)^3$. If the proposition is proven, then Goldbach's conjecture is true.

**Keywords:** prime, Goldbach number, strong Goldbach number sequence, Goldbach's conjecture

**2010 Mathematics Subject Classification:** 11A41, 11B99.

## 1. Introduction

Goldbach's conjecture is one of best-known unsolved problems in mathematics, and it states that every even number greater than 2 can be written as the sum of two primes, which is equivalent to the statement that every even number greater than 4 can be written as the sum of two odd primes. In 1973 Chen showed that every sufficiently large even integer is the sum of either two primes, or a prime and the product of two primes ( Chen, 1973 ). There are some researches on the exceptional set of Goldbach numbers to come close to Goldbach's conjecture ( Montgomery, & Vaughan, 1975; Chen, & Pan, 1980; Chen, 1983; Chen, & Liu, 1989; Li, 1999; Li, 2000; Lu, 2010). By Li ( Li, 1999; Li, 2000 ), Goldbach number is defined as a positive number to be a sum of two odd primes and the exceptional set of Goldbach numbers is usually written as $E(x)$ to denote the number of even numbers not exceeding $x$ which cannot be represented as the sum of two odd primes. Thus Goldbach's conjecture is equivalent to proving that $E(x) = 2$ for every $x \geq 4$ and also proving that all even numbers greater than 4 are Goldbach numbers. In this paper we built a mathematical framework in which every prime $P_n \geq 3$ will generate a corresponding strong Goldbach number sequence { 6, 8, …, $L_n$ }. Thus we will find an approach to prove Goldbach's conjecture by idea of strong Goldbach number.

## 2. Strong Goldbach Number Sequence

**Definition 2.1.** Let $P_n$ be the $n$-th prime for $n \geq 2$. Then $G_n = p + q$ is called *a Goldbach number generated by* $P_n$ if $p$ and $q$ are two ( same or distinct ) odd primes not greater than $P_n$.

**Definition 2.2.** Let $G_n$ be a Goldbach number generated by a given $P_n$. Then $G_n$ is called *a strong Goldbach number generated by* $P_n$ and written as $S_n$ if all even numbers from 6 to $G_n$ are Goldbach numbers generated by $P_n$.

**Definition 2.3.** Let $S_n$ be a strong Goldbach number generated by a given $P_n$. Then $S_n$ is called *the largest strong Goldbach number generated by* $P_n$ and written as $L_n$ if $S_n + 2$ is not a Goldbach number generated by $P_n$.

**Definition 2.4.** Write all strong Goldbach numbers generated by a given $P_n$ as a corresponding consecutive even number sequence { 6, 8, …, $L_n$ }. Then the sequence is called *strong Goldbach number sequence generated by* $P_n$.

The main results arising from the definitions are as follows. 6 = 3 + 3 is the smallest Goldbach number and also the smallest strong Goldbach number but $2P_n = P_n + P_n$ is the largest Goldbach number for a given $P_n$. Every term in { 6, 8, …, $L_n$ } is a strong Goldbach number and also a Goldbach number but every Goldbach number outside { 6, 8, …, $L_n$ } is not a strong Goldbach number for a given $P_n$. If $S_n$ is a strong Goldbach number greater than 6 generated by $P_n$, then all even numbers from 6 to $S_n$ are strong Goldbach numbers and also Goldbach numbers generated by $P_n$. If $S_n$ is a strong Goldbach number generated by $P_n$, then $S_n$ is also a strong Goldbach number for every $P_{n+k}$ for $k \geq 1$. It is obvious that $L_n \leq 2P_n$ for all $P_n$ and there are some even numbers between $L_n$ and $2P_n$ which are not Goldbach numbers generated by $P_n$ if

$L_n < 2P_n$ for a given $P_n$.

**Lemma 2.5.** *There is a strong Goldbach number sequence { 6, 8, …, $L_n$ } for any given $P_n$.*

*Proof.* Since { 6 } is strong Goldbach number sequence generated by $P_2 = 3$ ( the first odd prime ) but { 6, 8, …, $L_n$ } generated by $P_n$ will remain in { 6, 8, …, $L_{n+1}$ } generated by $P_{n+1}$ as the first part ( including complete sequence ) of { 6, 8, …, $L_{n+1}$ } generated by $P_{n+1}$. Hence any given $P_n$ will generate a corresponding strong Goldbach number sequence { 6, 8, …, $L_n$ }. Thus the lemma holds.

The lemma means that any given $P_n$ will generate a corresponding $L_n$.

**Observation 2.6.** *Status of { 6, 8, …, $L_n$ } generated by $P_n$ for $P_n$ less than 2000.*

By Lemma 2.5 we can give an observation for status of { 6, 8, …, $L_n$ } generated by $P_n$ for $P_n$ less than 2000. In the following observation, $L_n$ is the largest strong Goldbach number generated by $P_n$ but $A(P_n) = L_n - P_n$.

| $P_n$ | $L_n$ | $A(P_n)$ | $n$ | log $n$ | $n$ log $n$ | $\delta_n$(%) | $\delta_n'$(%) | $\delta_n'/\delta_n$ |
|---|---|---|---|---|---|---|---|---|
| 3 | 6 | 3 | 2 | 0.693147 | 1.386294 | 53.79 | 53.79 | 1.000 |
| 5 | 10 | 5 | 3 | 1.098612 | 3.295836 | 34.08 | 34.08 | 1.000 |
| 7 | 14 | 7 | 4 | 1.386294 | 5.545176 | 20.78 | 20.78 | 1.000 |
| 11 | 18 | 7 | 5 | 1.609437 | 8.047185 | 14.96 | 26.84 | 1.794 |
| 13 | 26 | 13 | 6 | 1.791759 | 10.75055 | 17.30 | 17.30 | 1.000 |
| 17 | 30 | 13 | 7 | 1.945910 | 13.62137 | 4.780 | 19.87 | 4.157 |
| 19 | 38 | 19 | 8 | 2.079441 | 16.63552 | 12.44 | 12.44 | 1.000 |
| 23 | 42 | 19 | 9 | 2.197224 | 19.77501 | 4.079 | 14.02 | 3.437 |
| 29 | 42 | 13 | 10 | 2.302585 | 23.02585 | 77.12 | 20.60 | 0.267 |
| 31 | 54 | 23 | 11 | 2.397895 | 26.37684 | 14.68 | 14.91 | 1.016 |
| 37 | 62 | 25 | 12 | 2.484906 | 29.81887 | 19.27 | 19.41 | 1.007 |
| 41 | 74 | 33 | 13 | 2.564949 | 33.34433 | 1.043 | 18.67 | 17.90 |
| 43 | 74 | 31 | 14 | 2.639057 | 36.94679 | 19.18 | 14.08 | 0.734 |
| 47 | 90 | 43 | 15 | 2.708050 | 40.62075 | 5.533 | 13.57 | 2.453 |
| 53 | 90 | 37 | 16 | 2.772588 | 44.36140 | 19.90 | 16.30 | 0.819 |
| 59 | 90 | 31 | 17 | 2.833213 | 48.16462 | 55.37 | 18.37 | 0.332 |
| 61 | 108 | 47 | 18 | 2.890371 | 52.02667 | 10.70 | 14.71 | 1.375 |
| 67 | 114 | 47 | 19 | 2.944438 | 55.94432 | 19.03 | 16.50 | 0.867 |
| 71 | 114 | 43 | 20 | 2.995732 | 59.91464 | 39.34 | 15.61 | 0.397 |
| 73 | 134 | 61 | 21 | 3.044522 | 63.93496 | 4.811 | 12.45 | 2.580 |
| 79 | 134 | 55 | 22 | 3.091042 | 68.00292 | 23.64 | 13.92 | 0.589 |
| 83 | 146 | 63 | 23 | 3.135494 | 72.11636 | 14.47 | 13.11 | 0.906 |
| 89 | 162 | 73 | 24 | 3.178053 | 76.27327 | 4.484 | 14.29 | 3.187 |
| 97 | 172 | 75 | 25 | 3.218875 | 80.47187 | 7.296 | 17.04 | 2.336 |
| 101 | 180 | 79 | 26 | 3.258096 | 84.71049 | 7.229 | 16.12 | 2.230 |
| 103 | 186 | 83 | 27 | 3.295836 | 88.98757 | 7.214 | 13.60 | 1.885 |
| 107 | 186 | 79 | 28 | 3.332204 | 93.30171 | 18.10 | 12.80 | 0.707 |
| 109 | 218 | 109 | 29 | 3.367295 | 97.65155 | 10.41 | 10.41 | 1.000 |
| 113 | 222 | 109 | 30 | 3.401197 | 102.0359 | 6.389 | 9.703 | 1.517 |
| 127 | 230 | 103 | 31 | 3.433987 | 106.4535 | 3.353 | 16.18 | 4.826 |
| 131 | 240 | 109 | 32 | 3.465735 | 110.9035 | 1.746 | 15.34 | 8.786 |
| 137 | 240 | 103 | 33 | 3.496507 | 115.3847 | 12.02 | 15.78 | 1.313 |
| 139 | 254 | 115 | 34 | 3.526360 | 119.8962 | 4.258 | 13.74 | 3.227 |
| 149 | 258 | 109 | 35 | 3.555348 | 124.4371 | 14.16 | 16.49 | 1.165 |
| 151 | 270 | 119 | 36 | 3.583518 | 129.0066 | 8.409 | 14.57 | 1.733 |
| 157 | 270 | 113 | 37 | 3.610917 | 133.6039 | 18.23 | 14.90 | 0.817 |
| 163 | 290 | 127 | 38 | 3.637586 | 138.2282 | 8.841 | 15.20 | 1.719 |
| 167 | 290 | 123 | 39 | 3.663561 | 142.8788 | 16.16 | 14.44 | 0.894 |
| 173 | 290 | 117 | 40 | 3.688879 | 147.5551 | 26.12 | 14.71 | 0.563 |
| 179 | 330 | 151 | 41 | 3.713572 | 152.2564 | 0.832 | 14.94 | 17.96 |
| 181 | 348 | 167 | 42 | 3.737669 | 156.9820 | 5.999 | 13.27 | 2.212 |
| 191 | 348 | 157 | 43 | 3.761200 | 161.7316 | 3.014 | 15.32 | 5.083 |
| 193 | 366 | 173 | 44 | 3.784189 | 166.5043 | 3.755 | 13.73 | 3.656 |

| $P_n$ | $L_n$ | $A(P_n)$ | $n$ | $\log n$ | $n \log n$ | $\delta_n(\%)$ | $\delta_n'(\%)$ | $\delta_n'/\delta_n$ |
|---|---|---|---|---|---|---|---|---|
| 197 | 366 | 169 | 45 | 3.806662 | 171.2997 | 1.361 | 13.05 | 9.589 |
| 199 | 366 | 167 | 46 | 3.828641 | 176.1174 | 5.460 | 11.50 | 2.106 |
| 211 | 398 | 187 | 47 | 3.850147 | 180.9569 | 3.232 | 14.24 | 4.406 |
| 223 | 398 | 175 | 48 | 3.871201 | 185.8176 | 6.181 | 16.67 | 2.697 |
| 227 | 410 | 183 | 49 | 3.891820 | 190.6991 | 4.207 | 15.99 | 3.800 |
| 229 | 410 | 181 | 50 | 3.912023 | 195.6011 | 8.067 | 14.58 | 1.807 |
| 233 | 434 | 201 | 51 | 3.931825 | 200.5230 | 0.237 | 13.94 | 58.82 |
| 239 | 440 | 201 | 52 | 3.951243 | 205.4646 | 2.221 | 14.03 | 6.317 |
| 241 | 440 | 199 | 53 | 3.970291 | 210.4254 | 5.741 | 12.69 | 2.210 |
| 251 | 474 | 223 | 54 | 3.988984 | 215.4051 | 3.406 | 14.18 | 4.163 |
| 257 | 474 | 217 | 55 | 4.007333 | 220.4033 | 1.568 | 14.24 | 9.082 |
| 263 | 474 | 211 | 56 | 4.025351 | 225.4196 | 6.834 | 14.23 | 2.082 |
| 269 | 474 | 205 | 57 | 4.043051 | 230.4539 | 12.42 | 14.33 | 1.154 |
| 271 | 474 | 203 | 58 | 4.060443 | 235.5056 | 16.01 | 13.10 | 0.818 |
| 277 | 522 | 245 | 59 | 4.077537 | 240.5746 | 1.806 | 13.15 | 7.281 |
| 281 | 522 | 241 | 60 | 4.094344 | 245.6606 | 1.934 | 12.58 | 6.505 |
| 283 | 528 | 245 | 61 | 4.110873 | 250.7632 | 2.352 | 11.39 | 4.843 |
| 293 | 528 | 235 | 62 | 4.127134 | 255.8823 | 8.886 | 12.67 | 1.426 |
| 307 | 566 | 259 | 63 | 4.143134 | 261.0174 | 0.779 | 14.98 | 19.23 |
| 311 | 570 | 259 | 64 | 4.158883 | 266.1685 | 2.768 | 14.42 | 5.210 |
| 313 | 570 | 257 | 65 | 4.174387 | 271.3351 | 5.578 | 13.31 | 2.386 |
| 317 | 570 | 253 | 66 | 4.189654 | 276.5171 | 9.295 | 12.77 | 1.374 |
| 331 | 614 | 283 | 67 | 4.204692 | 281.7143 | 0.454 | 14.89 | 32.80 |
| 337 | 614 | 277 | 68 | 4.219507 | 286.9264 | 3.584 | 14.86 | 4.146 |
| 347 | 630 | 283 | 69 | 4.234106 | 292.1533 | 3.234 | 15.81 | 4.889 |
| 349 | 634 | 285 | 70 | 4.248495 | 297.3946 | 4.349 | 14.79 | 3.401 |
| 353 | 650 | 297 | 71 | 4.262679 | 302.6502 | 1.902 | 14.26 | 7.497 |
| 359 | 680 | 321 | 72 | 4.276666 | 307.9199 | 4.075 | 14.23 | 3.492 |
| 367 | 680 | 313 | 73 | 4.290459 | 313.2035 | 0.065 | 14.66 | 225.5 |
| 373 | 680 | 307 | 74 | 4.304065 | 318.5008 | 3.746 | 14.61 | 3.900 |
| 379 | 680 | 301 | 75 | 4.317488 | 323.8116 | 7.579 | 14.56 | 1.921 |
| 383 | 680 | 297 | 76 | 4.330733 | 329.1357 | 10.82 | 14.06 | 1.299 |
| 389 | 686 | 297 | 77 | 4.343805 | 334.4729 | 12.62 | 14.02 | 1.111 |
| 397 | 686 | 289 | 78 | 4.356708 | 339.8232 | 17.59 | 14.40 | 0.819 |
| 401 | 686 | 285 | 79 | 4.369447 | 345.1863 | 21.12 | 13.92 | 0.659 |
| 409 | 686 | 277 | 80 | 4.382026 | 350.5621 | 26.56 | 14.29 | 0.538 |
| 419 | 722 | 303 | 81 | 4.394449 | 355.9503 | 17.48 | 15.05 | 0.861 |
| 421 | 722 | 301 | 82 | 4.406719 | 361.3509 | 20.05 | 14.17 | 0.707 |
| 431 | 794 | 363 | 83 | 4.418840 | 366.7638 | 1.037 | 14.90 | 14.37 |
| 433 | 794 | 361 | 84 | 4.430816 | 372.1885 | 3.099 | 14.04 | 4.530 |
| 439 | 794 | 355 | 85 | 4.442651 | 377.6253 | 6.373 | 13.98 | 2.194 |
| 443 | 822 | 379 | 86 | 4.454347 | 383.0738 | 1.075 | 13.53 | 12.59 |
| 449 | 822 | 373 | 87 | 4.465908 | 388.5339 | 4.165 | 13.47 | 3.234 |
| 457 | 854 | 397 | 88 | 4.477336 | 394.0055 | 0.754 | 13.78 | 18.28 |
| 461 | 854 | 393 | 89 | 4.488636 | 399.4886 | 1.651 | 13.34 | 8.080 |
| 463 | 854 | 391 | 90 | 4.499809 | 404.9829 | 3.576 | 12.53 | 3.504 |
| 467 | 906 | 439 | 91 | 4.510859 | 410.4881 | 6.495 | 12.10 | 1.863 |
| 479 | 906 | 427 | 92 | 4.521788 | 416.0044 | 2.575 | 13.15 | 5.107 |
| 487 | 930 | 443 | 93 | 4.532599 | 421.5317 | 4.846 | 13.44 | 2.773 |
| 491 | 930 | 439 | 94 | 4.543294 | 427.0696 | 2.718 | 13.02 | 4.790 |
| 499 | 962 | 463 | 95 | 4.553876 | 432.6182 | 6.562 | 13.30 | 2.027 |
| 503 | 966 | 463 | 96 | 4.564348 | 438.1774 | 5.361 | 12.89 | 2.404 |
| 509 | 966 | 457 | 97 | 4.574710 | 443.7468 | 2.900 | 12.82 | 4.421 |
| 521 | 966 | 445 | 98 | 4.584967 | 449.3267 | 0.972 | 13.76 | 14.16 |
| 523 | 966 | 443 | 99 | 4.595119 | 454.4916 | 2.690 | 13.02 | 4.840 |

| $P_n$ | $L_n$ | $A(P_n)$ | $n$ | $\log n$ | $n\log n$ | $\delta_n$(%) | $\delta_n'$(%) | $\delta_n'/\delta_n$ |
|---|---|---|---|---|---|---|---|---|
| 541 | 966 | 425 | 100 | 4.605170 | 460.5170 | 8.357 | 14.88 | 1.781 |
| 547 | 990 | 443 | 101 | 4.615120 | 466.1271 | 5.221 | 14.78 | 2.831 |
| 557 | 990 | 433 | 102 | 4.624972 | 471.7471 | 8.949 | 15.31 | 1.711 |
| 563 | 990 | 427 | 103 | 4.634728 | 477.3769 | 11.80 | 15.21 | 1.289 |
| 569 | 990 | 421 | 104 | 4.644390 | 483.0165 | 14.73 | 15.11 | 1.026 |
| 571 | 1014 | 443 | 105 | 4.653960 | 488.6658 | 10.31 | 14.42 | 1.399 |
| 577 | 1050 | 473 | 106 | 4.663439 | 494.3245 | 4.508 | 14.33 | 3.179 |
| 587 | 1050 | 463 | 107 | 4.672828 | 499.9925 | 7.990 | 14.82 | 1.855 |
| 593 | 1050 | 457 | 108 | 4.682131 | 505.6701 | 10.65 | 14.73 | 1.383 |
| 599 | 1050 | 451 | 109 | 4.691347 | 511.3568 | 13.38 | 14.63 | 1.093 |
| 601 | 1050 | 449 | 110 | 4.700480 | 517.0528 | 15.16 | 13.97 | 0.922 |
| 607 | 1050 | 443 | 111 | 4.709530 | 522.7578 | 18.00 | 13.88 | 0.771 |
| 613 | 1150 | 537 | 112 | 4.718498 | 528.4717 | 1.588 | 13.79 | 8.684 |
| 617 | 1150 | 533 | 113 | 4.727387 | 534.1947 | 0.224 | 13.42 | 59.91 |
| 619 | 1150 | 531 | 114 | 4.736198 | 539.9265 | 1.681 | 12.77 | 7.597 |
| 631 | 1220 | 589 | 115 | 4.744932 | 545.6671 | 7.356 | 13.52 | 1.838 |
| 641 | 1220 | 579 | 116 | 4.753590 | 551.4164 | 4.764 | 13.98 | 2.935 |
| 643 | 1220 | 577 | 117 | 4.762173 | 557.1742 | 3.436 | 13.35 | 3.885 |
| 647 | 1220 | 573 | 118 | 4.770684 | 562.9407 | 1.756 | 12.99 | 7.397 |
| 653 | 1266 | 613 | 119 | 4.779123 | 568.7156 | 7.224 | 12.91 | 1.787 |
| 659 | 1266 | 607 | 120 | 4.787491 | 574.4998 | 5.354 | 12.82 | 2.394 |
| 661 | 1296 | 635 | 121 | 4.795790 | 580.2907 | 8.616 | 12.21 | 1.417 |
| 673 | 1296 | 623 | 122 | 4.804021 | 586.0905 | 5.924 | 12.91 | 2.179 |
| 677 | 1296 | 619 | 123 | 4.812184 | 591.8980 | 4.378 | 12.57 | 2.871 |
| 683 | 1296 | 613 | 124 | 4.820281 | 597.7148 | 2.494 | 12.49 | 5.008 |
| 691 | 1326 | 635 | 125 | 4.828313 | 603.5391 | 4.954 | 12.66 | 2.556 |
| 701 | 1326 | 625 | 126 | 4.836281 | 609.3714 | 2.501 | 13.07 | 5.226 |
| 709 | 1356 | 647 | 127 | 4.844187 | 615.2117 | 4.913 | 13.23 | 2.693 |
| 719 | 1356 | 637 | 128 | 4.852030 | 621.0598 | 2.502 | 13.62 | 5.444 |
| 727 | 1374 | 647 | 129 | 4.859812 | 626.9157 | 3.104 | 13.77 | 4.436 |
| 733 | 1388 | 655 | 130 | 4.867534 | 632.7794 | 3.392 | 13.67 | 4.030 |
| 739 | 1388 | 649 | 131 | 4.875197 | 638.6508 | 1.595 | 13.58 | 8.514 |
| 743 | 1406 | 663 | 132 | 4.882801 | 644.5292 | 2.786 | 13.25 | 4.756 |
| 751 | 1406 | 655 | 133 | 4.890349 | 650.4164 | 0.700 | 13.39 | 19.13 |
| 757 | 1406 | 649 | 134 | 4.897839 | 656.3104 | 1.126 | 13.30 | 11.81 |
| 761 | 1430 | 669 | 135 | 4.905274 | 662.2119 | 1.015 | 12.98 | 12.79 |
| 769 | 1430 | 661 | 136 | 4.912654 | 668.1209 | 1.077 | 13.12 | 12.18 |
| 773 | 1466 | 693 | 137 | 4.919981 | 674.0387 | 2.736 | 12.80 | 4.678 |
| 787 | 1466 | 679 | 138 | 4.927253 | 679.9609 | 0.142 | 13.60 | 95.77 |
| 797 | 1466 | 669 | 139 | 4.934473 | 685.8917 | 2.525 | 13.94 | 5.521 |
| 809 | 1530 | 721 | 140 | 4.941642 | 691.8298 | 4.046 | 14.48 | 3.579 |
| 811 | 1530 | 719 | 141 | 4.948759 | 697.7750 | 2.952 | 13.96 | 4.729 |
| 821 | 1530 | 709 | 142 | 4.955827 | 703.7274 | 0.744 | 14.28 | 19.19 |
| 823 | 1574 | 751 | 143 | 4.962844 | 709.6866 | 5.501 | 13.77 | 2.503 |
| 827 | 1574 | 747 | 144 | 4.969813 | 715.6530 | 4.196 | 13.46 | 3.208 |
| 829 | 1574 | 745 | 145 | 4.976733 | 721.6062 | 3.137 | 12.95 | 4.128 |
| 839 | 1574 | 735 | 146 | 4.983606 | 727.6064 | 1.006 | 13.28 | 13.20 |
| 853 | 1574 | 721 | 147 | 4.990432 | 733.5935 | 1.747 | 14.00 | 8.014 |
| 857 | 1626 | 769 | 148 | 4.997212 | 739.5877 | 3.825 | 13.70 | 3.582 |
| 859 | 1670 | 811 | 149 | 5.003946 | 745.5879 | 8.006 | 13.20 | 1.636 |
| 863 | 1692 | 829 | 150 | 5.010635 | 751.5952 | 9.337 | 12.91 | 1.383 |
| 877 | 1692 | 815 | 151 | 5.017279 | 757.6091 | 7.042 | 13.61 | 1.933 |
| 881 | 1692 | 811 | 152 | 5.023880 | 763.6297 | 5.841 | 13.32 | 2.280 |
| 883 | 1722 | 839 | 153 | 5.030437 | 769.6568 | 8.265 | 12.84 | 1.554 |
| 887 | 1722 | 835 | 154 | 5.036952 | 775.6906 | 7.103 | 12.55 | 1.767 |

| $P_n$ | $L_n$ | $A(P_n)$ | $n$ | $\log n$ | $n\log n$ | $\delta_n$(%) | $\delta_n'$(%) | $\delta_n'/\delta_n$ |
|---|---|---|---|---|---|---|---|---|
| 907 | 1722 | 815 | 155 | 5.043425 | 781.7308 | 4.082 | 13.81 | 3.383 |
| 911 | 1722 | 811 | 156 | 5.049856 | 787.7775 | 2.863 | 13.53 | 4.726 |
| 919 | 1722 | 803 | 157 | 5.056245 | 793.8304 | 1.142 | 13.62 | 11.93 |
| 929 | 1722 | 793 | 158 | 5.062595 | 799.8900 | 0.869 | 13.90 | 16.00 |
| 937 | 1778 | 841 | 159 | 5.068904 | 805.9557 | 4.167 | 13.99 | 3.357 |
| 941 | 1806 | 865 | 160 | 5.075173 | 812.0276 | 6.124 | 13.71 | 2.239 |
| 947 | 1806 | 859 | 161 | 5.081404 | 818.1060 | 4.761 | 13.61 | 2.859 |
| 953 | 1806 | 853 | 162 | 5.087596 | 824.1905 | 3.377 | 13.52 | 4.004 |
| 967 | 1806 | 839 | 163 | 5.093750 | 830.2812 | 1.039 | 14.14 | 13.61 |
| 971 | 1806 | 835 | 164 | 5.099866 | 836.3780 | 0.165 | 13.86 | 84.00 |
| 977 | 1806 | 829 | 165 | 5.105945 | 842.0809 | 1.626 | 13.77 | 8.469 |
| 983 | 1806 | 823 | 166 | 5.111987 | 848.5898 | 3.109 | 13.67 | 4.397 |
| 991 | 1806 | 815 | 167 | 5.117993 | 854.7048 | 4.872 | 13.75 | 2.822 |
| 997 | 1830 | 833 | 168 | 5.123963 | 860.8257 | 3.340 | 13.66 | 4.090 |
| 1009 | 1920 | 911 | 169 | 5.129898 | 866.9527 | 4.835 | 14.08 | 2.912 |
| 1013 | 1920 | 907 | 170 | 5.135798 | 873.0856 | 3.739 | 13.81 | 3.694 |
| 1019 | 1920 | 901 | 171 | 5.141663 | 879.2243 | 2.417 | 13.72 | 5.676 |
| 1021 | 1920 | 899 | 172 | 5.147494 | 885.3609 | 1.516 | 13.28 | 8.760 |
| 1031 | 1920 | 889 | 173 | 5.153291 | 891.5193 | 0.283 | 13.53 | 47.81 |
| 1033 | 1920 | 887 | 174 | 5.159055 | 897.6755 | 1.204 | 13.10 | 10.88 |
| 1039 | 2018 | 979 | 175 | 5.164785 | 903.8373 | 7.677 | 13.01 | 1.695 |
| 1049 | 2054 | 1005 | 176 | 5.170483 | 910.0050 | 9.452 | 13.25 | 1.402 |
| 1051 | 2054 | 1003 | 177 | 5.176149 | 916.1783 | 8.656 | 12.83 | 1.482 |
| 1061 | 2054 | 993 | 178 | 5.181783 | 922.3573 | 7.114 | 13.07 | 1.837 |
| 1063 | 2054 | 991 | 179 | 5.187385 | 928.5419 | 6.303 | 12.65 | 2.007 |
| 1069 | 2054 | 985 | 180 | 5.192956 | 934.7320 | 5.103 | 12.56 | 2.461 |
| 1087 | 2054 | 967 | 181 | 5.198497 | 940.9279 | 2.696 | 13.44 | 4.985 |
| 1091 | 2054 | 963 | 182 | 5.204006 | 947.1290 | 1.648 | 13.19 | 8.004 |
| 1093 | 2054 | 961 | 183 | 5.209486 | 953.3359 | 0.798 | 12.78 | 16.02 |
| 1097 | 2054 | 957 | 184 | 5.214935 | 959.5480 | 0.266 | 12.53 | 47.11 |
| 1103 | 2144 | 1041 | 185 | 5.220355 | 965.7656 | 7.227 | 12.44 | 1.721 |
| 1109 | 2144 | 1035 | 186 | 5.225746 | 971.9887 | 6.088 | 12.35 | 2.029 |
| 1117 | 2144 | 1027 | 187 | 5.231108 | 978.2171 | 4.750 | 12.42 | 2.615 |
| 1123 | 2144 | 1021 | 188 | 5.236441 | 984.4509 | 3.580 | 12.34 | 3.447 |
| 1129 | 2144 | 1015 | 189 | 5.241747 | 990.6901 | 2.395 | 12.25 | 5.115 |
| 1151 | 2144 | 993 | 190 | 5.247024 | 996.9345 | 0.396 | 13.39 | 33.81 |
| 1153 | 2144 | 991 | 191 | 5.252273 | 1003.184 | 1.229 | 12.99 | 10.57 |
| 1163 | 2226 | 1063 | 192 | 5.257495 | 1009.439 | 5.039 | 13.20 | 2.620 |
| 1171 | 2226 | 1055 | 193 | 5.262690 | 1015.699 | 3.725 | 13.26 | 3.560 |
| 1181 | 2226 | 1045 | 194 | 5.267858 | 1021.964 | 2.204 | 13.47 | 6.112 |
| 1187 | 2226 | 1039 | 195 | 5.272999 | 1028.234 | 1.036 | 13.38 | 12.92 |
| 1193 | 2226 | 1033 | 196 | 5.278114 | 1034.510 | 0.146 | 13.28 | 90.96 |
| 1201 | 2226 | 1025 | 197 | 5.283203 | 1040.790 | 1.540 | 13.34 | 8.657 |
| 1213 | 2226 | 1013 | 198 | 5.288267 | 1047.076 | 3.364 | 13.68 | 4.067 |
| 1217 | 2226 | 1009 | 199 | 5.293304 | 1053.367 | 4.397 | 13.45 | 3.059 |
| 1223 | 2226 | 1003 | 200 | 5.298317 | 1059.663 | 5.649 | 13.36 | 2.365 |
| 1229 | 2226 | 997 | 201 | 5.303304 | 1065.964 | 6.917 | 13.27 | 1.918 |
| 1231 | 2310 | 1079 | 202 | 5.308267 | 1072.269 | 0.624 | 12.89 | 20.66 |
| 1237 | 2310 | 1073 | 203 | 5.313205 | 1078.586 | 0.520 | 12.81 | 24.63 |
| 1249 | 2394 | 1145 | 204 | 5.318119 | 1084.896 | 5.249 | 13.14 | 2.503 |
| 1259 | 2394 | 1135 | 205 | 5.323009 | 1091.217 | 3.858 | 13.32 | 3.453 |
| 1277 | 2394 | 1117 | 206 | 5.327876 | 1097.542 | 1.742 | 14.05 | 8.065 |
| 1279 | 2454 | 1175 | 207 | 5.332718 | 1103.872 | 6.053 | 13.69 | 2.262 |
| 1283 | 2454 | 1171 | 208 | 5.337538 | 1110.207 | 5.192 | 13.47 | 2.595 |
| 1289 | 2454 | 1165 | 209 | 5.342334 | 1116.547 | 4.159 | 13.38 | 3.217 |

| $P_n$ | $L_n$ | $A(P_n)$ | $n$ | $\log n$ | $n\log n$ | $\delta_n(\%)$ | $\delta_n'(\%)$ | $\delta_n'/\delta_n$ |
|---|---|---|---|---|---|---|---|---|
| 1291 | 2454 | 1163 | 210 | 5.347107 | 1122.892 | 3.449 | 13.02 | 3.775 |
| 1297 | 2454 | 1157 | 211 | 5.351858 | 1129.242 | 2.399 | 12.93 | 5.390 |
| 1301 | 2454 | 1153 | 212 | 5.356586 | 1135.596 | 1.509 | 12.71 | 8.423 |
| 1303 | 2542 | 1239 | 213 | 5.361292 | 1141.955 | 7.833 | 12.36 | 1.578 |
| 1307 | 2562 | 1255 | 214 | 5.365976 | 1148.318 | 8.501 | 12.14 | 1.428 |
| 1319 | 2562 | 1243 | 215 | 5.370638 | 1154.657 | 7.105 | 12.46 | 1.754 |
| 1321 | 2562 | 1241 | 216 | 5.375278 | 1161.060 | 6.442 | 12.11 | 1.880 |
| 1327 | 2630 | 1303 | 217 | 5.379897 | 1167.437 | 10.40 | 12.02 | 1.156 |
| 1361 | 2630 | 1269 | 218 | 5.384495 | 1173.819 | 7.500 | 13.75 | 1.833 |
| 1367 | 2630 | 1263 | 219 | 5.389071 | 1180.206 | 6.555 | 13.66 | 2.084 |
| 1373 | 2634 | 1261 | 220 | 5.393627 | 1186.597 | 5.900 | 13.58 | 2.302 |
| 1381 | 2634 | 1253 | 221 | 5.398162 | 1192.993 | 4.789 | 13.61 | 2.842 |
| 1399 | 2664 | 1265 | 222 | 5.402677 | 1199.394 | 5.186 | 14.27 | 2.752 |
| 1409 | 2664 | 1255 | 223 | 5.407171 | 1025.799 | 3.920 | 14.42 | 3.679 |
| 1423 | 2664 | 1241 | 224 | 5.411646 | 1212.205 | 2.320 | 14.81 | 6.384 |
| 1427 | 2664 | 1237 | 225 | 5.416100 | 1218.622 | 1.486 | 14.60 | 9.825 |
| 1429 | 2736 | 1307 | 226 | 5.420534 | 1225.040 | 6.271 | 14.27 | 2.276 |
| 1433 | 2736 | 1303 | 227 | 5.424950 | 1231.463 | 5.490 | 14.06 | 2.561 |
| 1439 | 2736 | 1297 | 228 | 5.429345 | 1237.890 | 4.557 | 13.98 | 3.068 |
| 1447 | 2762 | 1315 | 229 | 5.433722 | 1244.322 | 5.375 | 14.01 | 2.607 |
| 1451 | 2762 | 1311 | 230 | 5.438079 | 1250.758 | 4.595 | 13.80 | 3.003 |
| 1453 | 2762 | 1309 | 231 | 5.442417 | 1257.198 | 3.958 | 13.48 | 3.406 |
| 1459 | 2762 | 1303 | 232 | 5.446737 | 1263.642 | 3.021 | 13.39 | 4.432 |
| 1471 | 2762 | 1291 | 233 | 5.451038 | 1270.091 | 1.620 | 13.66 | 8.432 |
| 1481 | 2814 | 1333 | 234 | 5.455321 | 1276.545 | 4.235 | 13.81 | 3.261 |
| 1483 | 2814 | 1331 | 235 | 5.459585 | 1283.002 | 3.606 | 13.49 | 3.741 |
| 1487 | 2814 | 1327 | 236 | 5.463831 | 1289.464 | 2.829 | 13.28 | 4.694 |
| 1489 | 2828 | 1339 | 237 | 5.468060 | 1295.930 | 3.217 | 12.97 | 4.032 |
| 1493 | 2828 | 1335 | 238 | 5.472275 | 1302.400 | 2.442 | 12.77 | 5.229 |
| 1499 | 2828 | 1329 | 239 | 5.476463 | 1308.874 | 1.514 | 12.68 | 8.375 |
| 1511 | 2954 | 1443 | 240 | 5.480638 | 1315.353 | 8.846 | 12.95 | 1.464 |
| 1523 | 2982 | 1459 | 241 | 5.484796 | 1321.835 | 9.401 | 13.21 | 1.405 |
| 1531 | 2994 | 1463 | 242 | 5.488937 | 1328.322 | 9.206 | 13.24 | 1.438 |
| 1543 | 3006 | 1463 | 243 | 5.493061 | 1334.813 | 8.762 | 13.49 | 1.540 |
| 1549 | 3026 | 1477 | 244 | 5.497168 | 1341.308 | 9.187 | 13.41 | 1.460 |
| 1553 | 3026 | 1473 | 245 | 5.501258 | 1347.808 | 8.499 | 13.21 | 1.554 |
| 1559 | 3026 | 1467 | 246 | 5.505331 | 1354.311 | 7.682 | 13.13 | 1.709 |
| 1567 | 3026 | 1459 | 247 | 5.509388 | 1360.818 | 6.729 | 13.16 | 1.956 |
| 1571 | 3026 | 1455 | 248 | 5.513428 | 1367.330 | 6.025 | 12.96 | 2.151 |
| 1579 | 3026 | 1447 | 249 | 5.517432 | 1373.845 | 5.056 | 12.99 | 2.569 |
| 1583 | 3026 | 1443 | 250 | 5.521460 | 1380.365 | 4.341 | 12.80 | 2.949 |
| 1597 | 3026 | 1429 | 251 | 5.525452 | 1386.888 | 2.947 | 13.16 | 4.466 |
| 1601 | 3102 | 1501 | 252 | 5.529429 | 1393.416 | 7.167 | 12.97 | 1.810 |
| 1607 | 3102 | 1495 | 253 | 5.533389 | 1399.947 | 6.358 | 12.88 | 2.026 |
| 1609 | 3102 | 1493 | 254 | 5.537334 | 1406.482 | 5.795 | 12.59 | 2.173 |
| 1613 | 3102 | 1489 | 255 | 5.541263 | 1413.022 | 5.103 | 12.40 | 2.430 |
| 1619 | 3120 | 1483 | 256 | 5.545177 | 1419.565 | 4.277 | 12.32 | 2.881 |
| 1621 | 3146 | 1525 | 257 | 5.549076 | 1426.112 | 6.484 | 12.02 | 1.854 |
| 1627 | 3146 | 1519 | 258 | 5.552959 | 1432.663 | 5.684 | 11.94 | 2.101 |
| 1637 | 3180 | 1543 | 259 | 5.556828 | 1439.218 | 6.726 | 12.08 | 1.796 |
| 1657 | 3180 | 1523 | 260 | 5.560681 | 1445.777 | 5.070 | 12.75 | 2.515 |
| 1663 | 3180 | 1517 | 261 | 5.564520 | 1452.339 | 4.262 | 12.67 | 2.973 |
| 1667 | 3180 | 1513 | 262 | 5.568344 | 1458.906 | 3.575 | 12.48 | 3.491 |
| 1669 | 3180 | 1511 | 263 | 5.572154 | 1465.476 | 3.013 | 12.19 | 4.046 |
| 1693 | 3260 | 1567 | 264 | 5.575949 | 1472.050 | 6.059 | 13.05 | 2.154 |

| $P_n$ | $L_n$ | $A(P_n)$ | $n$ | $\log n$ | $n\log n$ | $\delta_n$(%) | $\delta_n'$(%) | $\delta_n'/\delta_n$ |
|---|---|---|---|---|---|---|---|---|
| 1697 | 3260 | 1563 | 265 | 5.579729 | 1478.628 | 5.398 | 12.87 | 2.384 |
| 1699 | 3260 | 1561 | 266 | 5.583496 | 1485.209 | 4.859 | 12.58 | 2.591 |
| 1709 | 3338 | 1629 | 267 | 5.587248 | 1491.795 | 8.423 | 12.71 | 1.509 |
| 1721 | 3342 | 1621 | 268 | 5.590986 | 1498.384 | 7.564 | 12.94 | 1.711 |
| 1723 | 3350 | 1627 | 269 | 5.594711 | 1504.977 | 7.500 | 12.65 | 1.687 |
| 1733 | 3356 | 1623 | 270 | 5.598421 | 1511.573 | 6.865 | 12.78 | 1.862 |
| 1741 | 3356 | 1615 | 271 | 5.602118 | 1518.173 | 5.995 | 12.80 | 2.135 |
| 1747 | 3356 | 1609 | 272 | 5.605802 | 1524.778 | 5.234 | 12.72 | 2.430 |
| 1753 | 3356 | 1603 | 273 | 5.609471 | 1531.385 | 4.468 | 12.64 | 2.829 |
| 1759 | 3356 | 1597 | 274 | 5.613128 | 1537.997 | 3.695 | 12.56 | 3.399 |
| 1777 | 3356 | 1579 | 275 | 5.616771 | 1544.612 | 2.178 | 13.08 | 6.006 |
| 1783 | 3356 | 1573 | 276 | 5.620400 | 1551.230 | 1.384 | 13.00 | 9.393 |
| 1787 | 3380 | 1593 | 277 | 5.624017 | 1557.852 | 2.206 | 12.82 | 5.811 |
| 1789 | 3380 | 1591 | 278 | 5.627621 | 1564.478 | 1.667 | 12.55 | 7.528 |
| 1801 | 3380 | 1579 | 279 | 5.631211 | 1571.107 | 0.500 | 12.76 | 25.52 |
| 1811 | 3434 | 1623 | 280 | 5.634789 | 1577.740 | 2.789 | 12.88 | 4.618 |
| 1823 | 3470 | 1647 | 281 | 5.638354 | 1584.377 | 3.802 | 13.09 | 3.443 |
| 1831 | 3470 | 1639 | 282 | 5.641907 | 1591.017 | 2.928 | 13.11 | 4.477 |
| 1847 | 3470 | 1623 | 283 | 5.645446 | 1597.661 | 1.561 | 13.50 | 8.648 |
| 1861 | 3470 | 1609 | 284 | 5.648974 | 1604.308 | 0.292 | 13.79 | 47.23 |
| 1867 | 3470 | 1603 | 285 | 5.652489 | 1610.959 | 0.497 | 13.71 | 27.59 |
| 1871 | 3500 | 1629 | 286 | 5.655991 | 1617.613 | 0.699 | 13.54 | 19.37 |
| 1873 | 3500 | 1627 | 287 | 5.659482 | 1624.271 | 0.168 | 13.28 | 79.05 |
| 1877 | 3500 | 1623 | 288 | 5.662960 | 1630.932 | 0.489 | 13.11 | 26.81 |
| 1879 | 3500 | 1621 | 289 | 5.666426 | 1637.597 | 1.024 | 12.85 | 12.55 |
| 1889 | 3548 | 1659 | 290 | 5.669880 | 1644.265 | 0.888 | 12.96 | 14.59 |
| 1901 | 3548 | 1647 | 291 | 5.673323 | 1650.936 | 0.239 | 13.15 | 55.02 |
| 1907 | 3548 | 1641 | 292 | 5.676753 | 1657.611 | 1.012 | 13.08 | 12.92 |
| 1913 | 3560 | 1647 | 293 | 5.680172 | 1664.290 | 1.050 | 13.00 | 12.38 |
| 1931 | 3560 | 1629 | 294 | 5.683579 | 1670.972 | 2.577 | 13.47 | 5.227 |
| 1933 | 3560 | 1627 | 295 | 5.686975 | 1677.657 | 3.114 | 13.21 | 4.242 |
| 1949 | 3704 | 1755 | 296 | 5.690359 | 1684.346 | 4.026 | 13.58 | 3.373 |
| 1951 | 3704 | 1753 | 297 | 5.693732 | 1691.038 | 3.535 | 13.32 | 3.768 |
| 1973 | 3768 | 1795 | 298 | 5.697093 | 1697.733 | 5.419 | 13.95 | 2.574 |
| 1979 | 3768 | 1789 | 299 | 5.700443 | 1704.432 | 4.727 | 13.87 | 2.934 |
| 1987 | 3834 | 1847 | 300 | 5.703782 | 1711.134 | 7.356 | 13.88 | 1.887 |
| 1993 | 3834 | 1841 | 301 | 5.707110 | 1717.840 | 6.690 | 13.81 | 2.064 |
| 1997 | 3834 | 1837 | 302 | 5.710427 | 1724.548 | 6.122 | 13.64 | 2.228 |
| 1999 | 3834 | 1835 | 303 | 5.713732 | 1731.260 | 5.653 | 13.39 | 2.369 |

## 3. Approach to Prove Goldbach's Conjecture

**Lemma 3.1.** *$L_n$ generated by every $P_n \geq 3$ can be written as $P_n + A(P_n)$.*

*Proof.* By Definition 2.3 and Definition 2.1 $L_n$ is the largest strong Goldbach number and also a Goldbach number generated by $P_n$, therefore, there is at least an odd prime pair ( $p$, $q$ ) such that $L_n$ can be written as $L_n = p + q$, where $p$ and $q$ are two odd primes not greater than $P_n$. Since such odd prime pair ( $p$, $q$ ) can be written as $p + q = P_n + A(P_n)$ for any given $P_n$. Hence $L_n$ generated by every $P_n \geq 3$ can be written as $P_n + A(P_n)$. Thus the lemma holds.

The lemma means that any given $P_n$ will generate a corresponding $A(P_n)$, that is, $A(P_n) = L_n - P_n$. From the data in Observation 2.6 we see a link between $A(P_n)$ and $n \log n$ has been established. Hence we have the following proposition.

**Proposition 3.2.** *$A(P_n) \approx n \log n$ for all $P_n$.*

**Remark 3.3.** The proposition means $n \log n$ is approximate function form of $A(P_n)$. There are five extreme cases in Observation 2.6 such that $A(P_n)$ is accurately equal to integer part of $n \log n$ for $n$ = 7, 9, 13, 73, 138. The occurrence of such cases and many almost extreme cases ( $A(P_n)$ is very close to integer part of $n \log n$ ) strongly supports the proposition. By the proposition, one may use $A(P_n)$ to replace $n \log n$ or use $n \log n$ to replace $A(P_n)$ for all $P_n$ specially large $P_n$. For example, using $n\log n$ to replace $A(P_n)$, we may obtain $L_n \approx P_n + n\log n$, which means the $n$-th largest strong Goldbach number is about the sum of the $n$-th prime and $n\log n$ for $n \geq 2$.

In Observation 2.6, $\delta_n = (A(P_n) - n\log n)/A(P_n)$ denotes relative error ( absolute value ) using $n\log n$ as approximation to $A(P_n)$ and we see $\delta_n$ is very small in general for $P_n$ less than 2000 such that there is a relative error to be on the order of 0.1‰ ( $\delta_{73}$ = 0.65‰ for $P_{73}$ = 367 ) and there are 27 relative errors to be on the order of 1‰ ( $\delta_n$ = 8.32‰, 2.37‰, 7.79‰, 4.54‰, 7.54‰, 9.72‰, 2.24‰, 7.00‰, 1.42‰, 7.44‰, 8.69‰, 1.65‰, 2.83‰, 7.98‰, 2.66‰, 3.96‰, 1.46‰, 6.24‰, 5.20‰, 5.00‰, 2.92‰, 4.97‰, 6.99‰, 1.68‰, 4.89‰, 8.88‰, 2.39‰ for $P_n$ = 179, 233, 307, 331, 457, 521, 617, 751, 787, 821, 929, 971, 1031, 1093, 1097, 1151, 1193, 1231, 1237, 1801, 1861, 1867, 1871, 1873, 1877, 1889, 1901 ) but there are 233 relative errors to be on the order of 1%. Excepting $\delta_{217}$ = 10.40%, all $\delta_n$ are on the order of 1% or 1‰ for $n > 111$ in the observation. Further, there are 22 consecutive occurrences of $\delta_n$(%) from $P_{274}$ = 1759 to $P_{295}$ =1933, whose integer part is smaller than 4, but there are 8 such $\delta_n$(%) whose integer part is 0 and there are 6 such $\delta_n$(%) whose integer part is 1 among the 22 consecutive $\delta_n$(%). Therefore, we are almost certain that $A(P_n)$ will approximate $n\log n$ as much as possible for all $P_n$ specially large $P_n$. When $n\log n$ being used as approximation to $P_n$, $\delta_n' = (P_n - n\log n)/P_n$ is relative error of this approximation. We listed values of $\delta_n'$ and values of the ratio $\delta_n'/\delta_n$ in Observation 2.6. There are no very small values of $\delta_n'$. Excepting $\delta_{30}'$ = 9.703%, all $\delta_n'$ are on the order of 10% in the observation. Comparing status of $\delta_n'$ with that of $\delta_n$, we see $A(P_n)$ is much obviously closer to $n\log n$ than $P_n$ in general in the observation. On the other hand, Rosser's theorem states that the $n$-th prime is greater than $n\log n$ ( for $n \geq 1$ )( Rosser, 1939 ), and further, P. Dusart showed that the $n$-th prime is greater than $n\log n + n\log\log n - n$ for $n \geq 2$ ( Dusart, 1999 ). Hence we are also almost certain that $A(P_n)$ will be much obviously closer to $n\log n$ than $P_n$ for all $P_n$ specially large $P_n$ in general. Therefore, there is a corollary of Proposition 3.2 from which we will give a clear proof of Goldbach's conjecture.

**Corollary 3.4.** *$L_n \to \infty$ as $n \to \infty$.*

*Proof.* It is obvious that $n\log n \to \infty$ as $n \to \infty$. By Proposition 3.2 $n\log n$ is replaced by $A(P_n)$, we obtain $A(P_n) \to \infty$ as $n \to \infty$. It is obvious that $P_n \to \infty$ as $n \to \infty$. By Lemma 3.1 $L_n = P_n + A(P_n)$, we have $L_n = P_n + A(P_n) \to \infty$ as $n \to \infty$. Thus the corollary holds.

**Proposition 3.5.** *Corollary 3.4 is equivalent to Goldbach's conjecture.*

*Proof.* Let $n \to \infty$. Then $L_n \to \infty$ as $n \to \infty$ by Corollary 3.4. Since $L_n$ is the largest strong Goldbach number generated by $P_n$. Hence the strong Goldbach number sequence { 6, 8, 10, …, $L_n - 4$, $L_n - 2$, $L_n$ } generated by $P_n$ as $n \to \infty$ will become an infinite sequence by $L_n \to \infty$ as $n \to \infty$. Thus all even numbers greater than 4 will become strong Goldbach numbers and also Goldbach numbers generated by $P_n$ as $n \to \infty$, in which every even number is the sum of two odd primes not greater than $P_n$ such as 6 = 3 + 3. It implies every even number greater than 4 is the sum of two odd primes and Goldbach's conjecture is true. If Goldbach's conjecture is true, then every even number greater than 4 is the sum of two odd primes not greater than $P_n$ as $n \to \infty$, that is, all even numbers greater than 4 are Goldbach numbers and also strong Goldbach numbers generated by $P_n$ as $n \to \infty$. It implies $L_n \to \infty$ *as* $n \to \infty$. Thus the proposition holds.

**Remark 3.6.** By Corollary 3.4 and Proposition 3.5 it is true that Goldbach's conjecture would follow from Proposition 3.2. Therefore, if the proposition holds then Goldbach's conjecture is true.

## 4. An Experimental Calculation Formula for the Number of Primes not Greater than $P_n$

In order to give indirect verification of Proposition 3.2 for large $P_n$, we will get a formula for calculating the number of primes not greater than $P_n$ relying on existence of $A(P_n)$. Taking $x$ as $P_n$, $n = \pi(P_n)$ denotes the number of primes not greater than $P_n$ but $n' = P_n/\log P_n$ is a calculated result for the number, and $n - n'$ is absolute error but $(n - n')/n$ is relative error. By Proposition 3.2, one may try to find a calculation formula $n'' = n' + \omega_n$ to obtain a number $\omega_n$ which will embody existence of $A(P_n)$. Thus we have

$$n'' = n' + \omega_n. \tag{4.1}$$

It may be estimated that $\omega_n = (P_n - A(P_n))/\log(P_n - A(P_n))$ by reference to $n' = P_n/\log P_n$ but we say that the number is actually the number of primes among integers from 1 to $P_n - A(P_n)$, which will be larger than the number of primes among integers from $A(P_n) + 1$ to $P_n$ because the density of primes among integers from 1 to $P_n - A(P_n)$ is larger than that from $A(P_n) + 1$ to $P_n$. By Proposition 3.2, we use $n\log n$ to replace $A(P_n)$ and obtain

$$\omega_n = C_n(P_n - n\log n)/\log(P_n - n\log n) + \lambda_n, \tag{4.2}$$

where $C_n = (\log\log n)^2/\log n$ but $\lambda_n$ is the sum of infinite series as follows

$$\lambda_n = C_n^3(P_n - n\log n)/(\log(P_n - n\log n))^2 +$$
$$C_n^4(P_n - n\log n)/(\log(P_n - n\log n))^3 +$$

$$C_n^5(P_n - n\log n)/(\log(P_n - n\log n))^4 + \ldots. \tag{4.3}$$

Using the first three terms of (4.1) with (4.2) and (4.3), $n'' = P_n/\log P_n + C_n(P_n - n\log n)/\log(P_n - n\log n) + C_n^3(P_n - n\log n)/(\log(P_n - n\log n))^2$, we may calculate the number of primes not greater than $P_n$ as indirect verification of Proposition 3.2 if both values of $P_n$ and $n$ are known, because we may find more acceptable relative errors for large $P_n$ if it is acceptable to use $n\log n$ to replace $A(P_n)$ in calculating the number of primes not greater than $P_n$. Therefore, we chose the following typical examples for calculating the number, in which $\omega_n$ is strongly offsetting the absolute error $n - n'$ arising from $P_n/\log P_n$.

**Example 4.1.** *The largest prime less than $10^2$.*

$P_n = 97$, $n = 25$. $n' = P_n/\log P_n \approx 21$, $n - n' \approx 4$, $(n - n')/n \approx 16\%$.

$P_n - n\log n \approx 16.528104$, $\log(P_n - n\log n) \approx 2.805062$, $(P_n - n\log n)/\log(P_n - n\log n) \approx 5.892241$.

$\log n \approx 3.218875$, $(\log\log n)^2 \approx 1.366636$, $C_n \approx 0.424569$. $\omega_n \approx 2.5 + 0.2 = 2.7$, $n'' = n' + \omega_n \approx 23.7$, $n - n'' \approx 1.3$,

$(n - n'')/n \approx 5.2\%$. $\alpha \approx 3.08$, where $\alpha$ is the ratio of $(n - n')/n$ to $(n - n'')/n$.

**Remark 4.2.** The relative error is about 11.14% by $x/((\log x) - 1.08366)$ and is about 10.62% by $x/\log x + x/(\log x)^2 + 2x/(\log x)^3$ taking $x$ as $P_n$. $\beta \approx 0.49$, where $\beta$ is the ratio of $(n - n'')/n$ to relative error arising from $x/\log x + x/(\log x)^2 + 2x/(\log x)^3$.

**Example 4.3.** *The largest prime less than $10^3$.*

$P_n = 997$, $n = 168$. $n' = P_n/\log P_n \approx 144$, $n - n' \approx 24$, $(n - n')/n \approx 14.29\%$.

$P_n - n\log n \approx 136.174051$, $\log(P_n - n\log n) \approx 4.913933$, $(P_n - n\log n)/\log(P_n - n\log n) \approx 27.711824$.

$\log n \approx 5.123963$, $(\log\log n)^2 \approx 2.669721$, $C_n \approx 0.521026$. $\omega_n \approx 14 + 1 = 15$, $n'' = n' + \omega_n \approx 159$, $n - n'' \approx 9$,

$(n - n'')/n \approx 5.36\%$. $\alpha \approx 2.67$.

**Remark 4.4.** The relative error is about 1.79% by $x/((\log x) - 1.08366)$ and is about 1.79% by $x/\log x + x/(\log x)^2 + 2x/(\log x)^3$. $\beta \approx 2.99$.

**Example 4.5.** *The largest prime less than $10^4$.*

$P_n = 9973$, $n = 1229$. $n' = P_n/\log P_n \approx 1083$, $n - n' \approx 146$, $(n - n')/n \approx 11.88\%$.

$P_n - n\log n \approx 1229.947941$, $\log(P_n - n\log n) \approx 7.114727$, $(P_n - n\log n)/\log(P_n - n\log n) \approx 172.873525$.

$\log n \approx 7.113956$, $(\log\log n)^2 \approx 3.849673$, $C_n \approx 0.541143$. $\omega_n \approx 94 + 4 = 98$, $n'' = n' + \omega_n \approx 1181$, $n - n'' \approx 48$,

$(n - n'')/n \approx 3.91\%$. $\alpha \approx 3.04$.

**Remark 4.6.** The relative error is about 0.08% by $x/((\log x) - 1.08366)$ and is about 0.24% by $x/\log x + x/(\log x)^2 + 2x/(\log x)^3$. $\beta \approx 16.29$.

**Example 4.7.** *The largest prime less than $10^5$.*

$P_n = 99991$, $n = 9592$. $n' = P_n/\log P_n \approx 8685$, $n - n' \approx 907$, $(n - n')/n \approx 9.456\%$.

$P_n - n\log n \approx 12044.976390$, $\log(P_n - n\log n) \approx 9.396402$, $(P_n - n\log n)/\log(P_n - n\log n) \approx 1281.851967$.

$\log n \approx 9.168684$, $(\log\log n)^2 \approx 4.909742$, $C_n \approx 0.535490$. $\omega_n \approx 686 + 21 = 707$, $n'' = n' + \omega_n \approx 9392$,

$n - n'' \approx 200$, $(n - n'')/n \approx 2.085\%$. $\alpha \approx 4.54$.

**Remark 4.8.** The relative error is about 0.042% by $x/((\log x) - 1.08366)$ and is about 0.229% by $x/\log x + x/(\log x)^2 + 2x/(\log x)^3$. $\beta \approx 9.10$.

**Example 4.9.** *The largest prime less than $10^6$.*

$P_n = 999983$, $n = 78498$. $n' = P_n/\log P_n \approx 72381$, $n - n' \approx 6117$, $(n - n')/n \approx 7.793\%$.

$P_n - n\log n \approx 115245.510237$, $\log(P_n - n\log n) \approx 11.654820$, $(P_n - n\log n)/\log(P_n - n\log n) \approx 9888.227380$.

$\log n \approx 11.270828$, $(\log\log n)^2 \approx 5.867139$, $C_n \approx 0.520559$. $\omega_n \approx 5147 + 120 \approx 5267$, $n'' = n' + \omega_n \approx 77648$,

$n - n'' \approx 850$, $(n - n'')/n \approx 1.083\%$. $\alpha \approx 7.20$.

**Remark 4.10.** The relative error is about 0.056% by $x/((\log x) - 1.08366)$ and is about 0.153% by $x/\log x + x/(\log x)^2 + 2x/(\log x)^3$. $\beta \approx 7.08$.

**Example 4.11.** *A random large prime $P_n$ for $n = 55\ 762\ 149\ 072$.*

$P_n = 1\ 505\ 578\ 024\ 919$, $n = 55\ 762\ 149\ 072$[Prime Curios!: Carmody, July 2003].

$n' = P_n/\log P_n \approx 53\ 693\ 558\ 953$, $n - n' \approx 2\ 068\ 590\ 119$, $(n - n')/n \approx 3.70967\%$.

$P_n - n \log n \approx 125\ 779\ 270\ 109$, $\log(P_n - n \log n) \approx 25.557\ 794\ 383\ 127\ 1$,

$(P_n - n \log n)/\log(P_n - n \log n) \approx 4\ 921\ 366\ 383$.

$\log n \approx 24.744\ 361\ 144\ 119\ 4$, $(\log \log n)^2 \approx 10.295\ 098\ 754\ 425\ 2$, $C_n \approx 0.416\ 058\ 377\ 683$.

$\omega_n \approx 2\ 047\ 575\ 713 + 13\ 868\ 361 \approx 2\ 061\ 444\ 074$, $n'' = n' + \omega_n \approx 55\ 755\ 003\ 027$, $n - n'' \approx 7\ 146\ 045$,

$(n - n'')/n \approx 0.01282\%$. $\alpha \approx 289$.

**Remark 4.12.** The relative error is about 0.16123% by $x/((\log x) - 1.08366)$ and is about 0.03072% by $x/\log x + x/(\log x)^2 + 2x/(\log x)^3$. $\beta \approx 0.41732$.

**Example 4.13.** *A random large prime $P_n$ for $n = 382\ 465\ 573\ 492$.*

$P_n = 11\ 091\ 501\ 631\ 241$, $n = 382\ 465\ 573\ 492$[Prime Curios!: Carmody, July 2003].

$n' = P_n/\log P_n \approx 369\ 258\ 836\ 242$, $n - n' \approx 13\ 206\ 737\ 250$, $(n - n')/n \approx 3.45305\%$.

$P_n - n \log n \approx 891\ 181\ 318\ 472$, $\log(P_n - n \log n) \approx 27.515\ 813\ 743\ 682\ 5$,

$(P_n - n \log n)/\log(P_n - n \log n) \approx 32\ 387\ 968\ 852$.

$\log n \approx 26.669\ 904\ 482\ 219\ 9$, $(\log \log n)^2 \approx 10.781\ 607\ 065\ 648\ 3$, $C_n \approx 0.404\ 261\ 180\ 344\ 2$.

$\omega_n \approx 13\ 093\ 198\ 517 + 77\ 765\ 590 \approx 13\ 170\ 964\ 107$, $n'' = n' + \omega_n \approx 382\ 429\ 800\ 349$, $n - n'' \approx 35\ 773\ 143$,

$(n - n'')/n \approx 0.00935\%$. $\alpha \approx 369$.

**Remark 4.14.** The relative error is about 0.16046% by $x/((\log x) - 1.08366)$ and is about 0.02479% by $x/\log x + x/(\log x)^2 + 2x/(\log x)^3$. $\beta \approx 0.37717$.

From such data we see the relative error using this calculation formula is smaller than that using $P_n/\log P_n$ for every example but the relative errors arising from two random large primes mean Proposition 3.2 is more suitable to large prime. However, the formula is only used to find indirect verification of Proposition 3.2 as an experimental formula. Comparing calculated results using the experimental formula with that using some known methods in the prime number theorem such as $x/\log x$ (Gauss, 1792), $x/((\log x) - 1.08366)$ (Legendre, 1808) and the first three terms of $\mathrm{Li}(x)$ i. e. $x/\log x + x/(\log x)^2 + 2x/(\log x)^3$ (Derbyshire, 2004), we see that the experimental formula is acceptable in finding indirect verification of Proposition 3.2 for primes specially large primes.

## 5. Conclusion

In above discussion, we presented a new method relying on the existence of strong Goldbach number for proving Goldbach's conjecture, that is, if Proposition 3.2 is proven then Goldbach's conjecture is true. It means that if enough numerical evidence for a proper proof of Proposition 3.2 can be found then Goldbach's conjecture will be proven. Based on $n \log n$ to replace $A(P_n)$, we obtain an experimental formula for calculating the number of primes not greater than $P_n$, which may be identified as indirect verification of Proposition 3.2 for primes specially large primes.

## References

Chen, J. R. (1973). On the representation of a large even integer as the sum of a prime and the product of at most two primes. *Sci Sinica, 16,* 157-176.

Chen, J. R., & Pan, C. D. (1980). The exceptional set of Goldbach numbers. *Sci Sinica*, *23*, 416-430.

Chen, J. R. (1983). The exceptional set of Goldbach numbers (II). *Sci Sinica*, *26*, 714-731.

Chen, J. R., & Liu, J. M. (1989). The exceptional set of Goldbach numbers (III), *Chinese Quart J Math 4,* 1-15.

Dusart, P. (1999). The $k^{th}$ prime is greater than $k(\ln k + \ln \ln k - 1)$ for $k \geq 2$. *Math Comp, 68*(225), 411-415. https://doi.org/10.1090/S0025-5718-99-01037-6

Li, H. (1999). The Exceptional Set of Goldbach Numbers. *Quart J Math Oxford*(2)*50*, 471-482. https://doi.org/10.1093/qjmath/50.200.471

Li, H. (2000). The exceptional set of Goldbach numbers (II). *Acta Arith 92*(1), 71-88.

Lu, W. C. (2010). Exceptional set of Goldbach number. *J Number Theory 130*(10), 2359-2392. https://doi.org/10.1016/j.jnt.2010.03.017

Montgomery, H. L., & Vaughan, R. C. (1975). The exceptional set in Goldbach's problem. *Acta Arith, 27,* 353-370. https://doi.org/10.4064/aa-27-1-353-370

Rosser, J. B. (1939). The *n*-th prime is greater than *n* log *n*. *Proc London Math Soc*(2)*45*, 21-44.

https://doi.org/10.1112/plms/s2-45.1.21