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Some Problems of Feedback Control Strategies and It's Treatment

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Abstract

This paper extends and improves the feedback control strategies. In detailed, the ordinary feedback, dislocated feedback, speed feedback and enhancing feedback control for a several dynamical systems are discussed here. It is noticed that there some problems by these strategies. For this reason, this Letter proposes a novel approach for treating these problems. The results obtained in this paper show that the strategies with positive feedback coefficients can be controlled in two cases and failed in another two cases. Theoretical and numerical simulations are given to illustrate and verify the results.

Keywords: Modified hyperchaotic Pan system, hyperchaotic Lorenz system, hyperchaotic Liu system feedback control strategies, Routh-Hurwitz method.

1. Introduction

Chaos control, an important topic in nonlinear dynamical science (Dou, Sun, Duan, & Lü, 2009; Tao & Yang, 2008; Yang, Tao & Wang, 2010) and is one of the main features of chaos applied in practical engineering (Yassen, 2003; Zhu, 2009; Zhu & Chen, 2008). and its play a very important role in the study of dynamical systems and has great significance in the application of chaos. Since Ott et al,1990, first introduced in the notation of chaos control (AL-Azzawi, 2012; Pang & Liu 2011).Various kinds of control schemes and techniques such as OGY method, time-delay feedback, Lyapunov method, impulsive control, sliding method control, differential geometric, H_∞ control, adaptive control, chaos suppression method, and so on have been successfully applied to achieve chaos control (Aziz & AL-Azzawi 2016; Tao, Yang, Luo, Xiong & Hu 2005). Among them, the feedback control is especially attractive and has been commonly applied to practical implementation due to its simplicity in configuration and implementation (Tao & Yang, 2008). Generally speaking, there are two main approaches for controlling chaos: feedback control and non-feedback control. The feedback control approach offers many advantages such as robustness and computational complexity over the non- feedback control method (Yang, Tao, & Wang, 2010; Zhu & Chen 2008).

On the other hand, there are some problems with this approach and one of these problems is that we get a negative feedback coefficient and the second problem is that when the feedback coefficient vanishes, Consequently, these strategies are failing. However, in a feedback control strategies the necessary condition for suppressing the dynamical system is that the feedback coefficient must be positive. Most of the previous work on chaos control was mostly focused on classical dynamical systems under this condition such as the Liu system (Dou, Sun, Duan, & Lü, 2009; Zhu & Chen 2008), Lü system (Pang & Liu 2011), Unified system (Tao, Yang, Luo, Xiong, & Hu 2005), Lorenz system (Zhu, 2010), Chen system (Yan, 2005), and et al. But in works (Aziz, & AL-Azzawi 2015; Zhu, 2010) founded some cases it can't be controlled, although with positive feedback coefficients.

This paper answer this equation and it focused on these problems and we suggest a new method which includes four cases: two cases have at least two positive feedback coefficients and other cases have only one positive feedback coefficient. Finally, we found the system can be controlled if there is an intersection between these coefficients and they can't be controlled if it there is not for the first two cases.

Briefly, this study presents three fundamental questions. First, when can we get a positive feedback coefficient and can suppress a system? Second, when can we get a positive feedback coefficient and can't suppress the system? And third, how can we distinguish between these two cases? This paper begins with the suggestion of a new method that will answer these questions.

2. Problem Formulation and Our Methodology Using Feedback Control Strategies

In this section, we describe the problem formulation for the chaos control for dynamical systems and our methodology using feedback control strategies.

Let us consider the dynamical system in the following form:

$$\dot{X} = AX + f(X) \tag{1}$$

where $X(t) = [x_i]^T = [x_1, x_2, \ldots\ldots, x_n]^T \in R^{n\times 1}$, $i = 1, 2, \ldots, n$, is the state of the system, $A = (a_{ij})_{n\times n}$ is the matrix of the system parameters and $f: R^n \to R^n$ is the nonlinear part of the system.

If we add the controller $U = [u_i]^T = [u_1, u_2, \ldots\ldots, u_n]^T \in R^{n\times 1}$ into the system (1), then the controlled system is given by:

$$\dot{X} = AX + f(X) + U \tag{2}$$

The purpose of the control problem is to choose a feedback controller U such that $\lim_{t\to\infty} \|X(t)\| = 0$.

As we know, there are four standard kinds of feedback control techniques: ordinary, dislocated, speed and enhancing feedback control. and the definition of each kind as:

Definition 1 Ordinary feedback control

The system's variable is often multiplied by a coefficient as the feedback gain, and the feedback gain is added to the right -hand of the corresponding equation (Aziz & AL-Azzawi, 2015; Zhuang, 2012; Zhuang & Chai, 2012).

Definition 2 Dislocated feedback control

If a system variable multiplied by a coefficient and its added to the right -hand of another equation, this strategy is called a dislocated feedback control (Zhuang & Chai, 2012; Pang & Liu 2011; Tao & Yang, 2008; Zhu, 2010).

Definition 3 Speed feedback control

The independent variable of a system function is often multiplied by a coefficient as the feedback gain, so the method is called displacement feedback control. Similarly, if the derivative of an independent variable is multiplied by a coefficient as the feedback gain(Tao, Yang, Luo, Xiong & Hu 2005; Zhuang, 2012; Yan, 2005; Zhu & Chen 2008).

Definition 4 Enhancing feedback control

It is difficult for a complex system to be controlled by only one feedback variable, and in such cases the feedback gain is always very large. So we consider using multiple variables multiplied by a proper coefficient as the feedback gain. This method is called enhanced feedback control(Aziz, & AL-Azzawi, 2015; Zhuang, 2012; Dou, Sun, Duan, & Lü, 2009; Zhu, & Chen, 2008; Pang & Liu 2011).

Obviously, from the above definitions, the three first feedback control strategies take just a single controller while in enhancing feedback control includes more than one feedback controller. Consequently, the controller U can represent for each kind as:

$$U = u_i = \begin{cases} -kx_i & ; \quad if \quad i = j \quad (\,ordinary) \\ -kx_j & ; \quad if \quad i \neq j \quad (dislocated) \\ -k\dot{x}_j & ; \quad if \quad i \neq j \quad (\ speed\ ) \\ -k[x_i]_{i=1}^n & ; \quad if \quad i = j \quad (enhancing) \end{cases} \tag{3}$$

where k is called a feedback coefficient, and $k > 0$. when substitute one of these formulations in Eq. 2 and in order to find the feedback coefficients, we can use the following formula:

$$\dot{X} = AX + U \tag{4}$$

i.e. deals with linear terms only of this strategy, but with other methods such as nonlinear feedback method, adaptive and active techniques etc. deals with linear and nonlinear terms.

At some time, by this strategy we got more than one positive feedback coefficient based on Routh-Hurwitz method and k is interval, in order to select a suitable feedback coefficient we used the following formulation

$$k = \cap_{i=1}^{n} k_i \ = k_1 \cap k_2 \cap \ldots\ldots.. \cap k_n \tag{5}$$

According to above these strategies, the system can control it if we have a positive feedback coefficient. This condition is necessary and sufficient for the controlled hyperchaotic systems. But in some cases, we establish that the system can't be controlled, although satisfied this condition. In order to overcome this problem and this weakness, we improve and extend these strategies by the following a novel approach.

Novel approach: The control problem by applying feedback control strategies for dynamical systems has posed four cases with a positive feedback coefficient, these cases as the following:

1. Control it if it has more than one positive feedback coefficients and there are intersection between them.

2. No control if it has more than one positive feedback coefficients and there are not an intersection

between them.

3. Control it if it has only one positive feedback coefficient and satisfied all Routh-Hurwitz conditions.
4. No control if it has only one positive feedback coefficient and it does not satisfy any one of Routh-Hurwitz conditions.

3. Applications

In this section, we take several four–dimensional hyperchaotic systems, for example to show how to use the results obtained in this paper to analyze the controlling a class of hyperchaotic systems.

3.1. Modified hyperchaotic Pan system

The modified hyperchaotic Pan system is described by the following dynamical system(Aziz & AL-Azzawi 2015; Aziz & AL-Azzawi 2016).

$$\begin{cases} \dot{x} = a(y - x) \\ \dot{y} = cx - xz + w \\ \dot{z} = xy - bz \\ \dot{w} = -dy \end{cases} \tag{6}$$

where $(x, y, z, w) \in R^4$, and $a, b, c, d \in R$ are constant parameters. When parameters $a = 10$, $b = 8/3$, $c = 28$ and $d = 10$, system (6) is hyperchaotic and has two positive Lyapunov exponents, i.e. $LE_1 = 0.24784$, $LE_2 = 0.08194$. The system (6) has only one equilibrium $O(0,0,0,0)$, and the equilibrium is an unstable under these parameters. This system is similar to the Lorenz system, but it is not topological equivalent.

Theorem 1. For system (6) with control $U = [0,0,0,u_4]^T$, i.e. $u_4 = -kw$ based on ordinary feedback control, where the feedback coefficients $k < 0.3571$ and $k > 12.1139$. then system (6) can't be control although we have two positive feedback.

Proof. According to the previous discussion, and based on formulation (4), system (6) with this control is

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} -10 & 10 & 0 & 0 \\ 28 & 0 & 0 & 1 \\ 0 & 0 & -8/3 & 0 \\ 0 & -10 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\boldsymbol{k}\mathbf{w} \end{bmatrix} \tag{7}$$

then the characteristic equation is

$$(\lambda + 8/3)(\lambda^3 + (k + 10)\lambda^2 + (10k - 270)\lambda + 100 - 280k) = 0 \tag{8}$$

the above equation can be rewritten as:

$$(\lambda + 8/3)(\lambda^3 + A\lambda^2 + B\lambda + C) = 0$$

where $A = k + 10$, $B = 10k - 270$, $C = 100 - 280k$

Now, according to the Routh-Hurwitz method, the Jacobian matrix (7) has four negative real part eigenvalues if satisfied the following three conditions

1) $A > 0$,

2) $C > 0$,

3) $AB - C > 0$.

Obviously $A = k + 10 > 0$, from $C = 100 - 280k$ we have a positive feedback coefficient such that $k < 0.3571$. Finally, from third condition, we have quadratic equation form:

$$10k^2 + 110k - 2800 > 0 \tag{9}$$

Solving above inequality (Eq. 9) we get another positive feedback coefficient as $k > 12.1139$.

Therefore, we find two positive feedback coefficients by this scheme. But, which feedback coefficient that effective in system? After testing, we noticed that both of a positive coefficients can't be effective and active on the system (6), Fig.1 with $k = 0.3$ and Fig.2 with $k = 12.5$ explain this result numerically. For this reason, we use our new approach to knowledge the cause of this problem.

Let us the two positive feedback coefficients as $k_1 < 0.3571$, $k_2 > 12.1139$.

Now, based on formulation (5) and a novel approach then

$$k = k_1 \cap k_2 = (0, 0.3571) \cap (12.1139, \infty) = \emptyset$$

Hence, there is no intersection between them, according to a second case of a new approach. So, this strategy fails to control the system (6). Thus the proof is complete.

Remark1. We can apply theoretical and numerical methods for of a positive feedback coefficient.

Theorem 2. For hyperchaotic system (6), let control $U = [0, u_2, 0,0]^T$, i.e $u_2 = -kw$ based on dislocated feedback control, where the feedback coefficients $k < 1$. Then the system (6) can't be controlled although we has a single positive feedback.

Proof. According to the formulation (4), system (6) with this control is

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} -10 & 10 & 0 & 0 \\ 28 & 0 & 0 & 1 \\ 0 & 0 & -8/3 & 0 \\ 0 & -10 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} + \begin{bmatrix} 0 \\ -\boldsymbol{k}\mathbf{w} \\ 0 \\ 0 \end{bmatrix} \tag{10}$$

then the characteristic equation is

$$(\lambda + 8/3)(\lambda^3 + 10\lambda^2 - (10k - 270)\lambda + 100(1 - k)) = 0 \tag{11}$$

Where $A = 10$, $B = -(10k - 270)$, $C = 100(1 - k)$

According to the Routh-Hurwitz method, system (10) has only one positive feedback coefficient as $k < 1$. To test this coefficient theoretically, let substitute the value of feedback coefficients $k = 1$ (critical value) in above Equation we get the following Equation as

$$(\lambda + 8/3)(\lambda^3 + 10\lambda^2 + 260\lambda) = 0 \tag{12}$$

and the roots of Equation (12) are $\lambda_1 = 0$, $\lambda_{2,3} = -5 \pm 15.3297i$ and $\lambda_4 = -8/3$, Consequently, the roots of Eq (12) don't contain a simple pair imaginary, So, we don't get a Hopf bifurcation, therefore, the feedback coefficients is not effective in a system (6). Also Fig.3 justifies this result numerical with positive feedback coefficient $k = 0.5$. In order to discover the main causes for this problem, we used new approach. Obviously, one condition of Routh-Hurwitz method is not satisfied (third condition of Routh-Hurwitz), Therefore, this strategy is failing to control system (6) according to the four case of the new approach, the proof is complete.

Remark 2. In the context of ordinary differential equations ODEs the word "bifurcation" has come to mean any marked change in the structure of the orbits of a system (usually nonlinear) as a parameter passes through a critical value [1], Bifurcation refers to qualitative changes in the solution structure of dynamical systems with slight variation in system parameters as well as one conditions of bifurcation that dynamical system has a simple pair of pure imaginary eigenvalues and no other eigenvalues with zero real parts.

Theorem 3. For system (6) with control $U = [0,0,0, u_4]^T$, i.e. $u_4 = -k\dot{x}$ (speed feedback control) where the feedback coefficients $k > 28$. Then the system (6) can be controlled.

Proof. System (6) with this control can be reformulated in the following form:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} -10 & 10 & 0 & 0 \\ 28 & 0 & 0 & 1 \\ 0 & 0 & -8/3 & 0 \\ 0 & -10 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \mathbf{10}\boldsymbol{k}(\boldsymbol{x} - \boldsymbol{y}) \end{bmatrix} \tag{13}$$

then the characteristic equation is

$$(\lambda + 8/3)(\lambda^3 + 10\lambda^2 + (10k - 270)\lambda + 100) = 0 \tag{14}$$

Where $A = 10$, $B = 10k - 270$, $C = 100$

Based on Routh-Hurwitz method, we Obvious $A, C > 0$, from the condition $AB > C$, we get inequality form $100k - 2800 > 0$ imply that $k > 28$ and Fig.4 show the convergence to equilibrium point with $k = 29$. Consequently, this strategy is success to control the system (6). Therefore, satisfied the three case of a new approach, the proof is complete.

Theorem 4. For system (6) with control $U = [u_1, u_2, 0, u_4]^T$, i.e. $u_1 = -kx$, $u_2 = -ky$ and $u_4 = -kw$ (enhancing feedback control) then system (6) can be controlled when $k > 11.9298$.

Proof. According to the formulation (4), The Jacobi matrix defined as:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} -10 & 10 & 0 & 0 \\ 28 & 0 & 0 & 1 \\ 0 & 0 & -8/3 & 0 \\ 0 & -10 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} + \begin{bmatrix} -\boldsymbol{k}\mathbf{x} \\ -\boldsymbol{k}\mathbf{y} \\ 0 \\ -\boldsymbol{k}\mathbf{w} \end{bmatrix} \tag{15}$$

then the characteristic equation is

$$(\lambda + 8/3)(\lambda^3 + (3k + 10)\lambda^2 + (10k - 270)\lambda + k^3 + 10k^2 - 270k + 100) = 0 \tag{16}$$

where $A = 3k + 10$, $B = 10k - 270$ and $C = k^3 + 10k^2 - 270k + 100$

According to the Routh-Hurwitz method, Obvious $A = 3k + 10 > 0$ and second condition contains inequality form cubic equation, Solving this inequality we have two appositive feedback coefficient $k > 11.9218$ and $k > 0.3758$. Finally, the third condition of Routh-Hurwitz gives the following inequality $8k^3 + 80\mathrm{k}^2 - 340\mathrm{k} - 2800 > 0$, solving this inequality we get another appositive feedback coefficient $k > 6.1528$. According to the new method, and based on formulation (5), Let us the three positive feedback coefficients as $k_1 > 6.1528$, $k_2 > 0.3758$ and $k_3 > 11.9298$.

then

$$k = k_1 \cap k_2 \cap k_3 = (11.9218, \infty) \cap (0.3758, \infty) \cap (6.1528, \infty) = (11.9218, \infty)$$

$$k \in (11.9218, \infty)$$

$$\therefore k_1 \cap k_2 \cap k_3 \neq \emptyset$$

Hence there is an intersection between them. Consequently, this strategy is success to control system (6). Satisfied first case of the new method. The proof is complete.

In the light of this study, the necessary and sufficient condition for the control hyperchaotic systems to be asymptotically stable is we get a positive feedback coefficient. But in theorem 1 and theorem 2 we get a positive feedback coefficient and can't control it. And a new approach given causes for each case. These cases are one problem of feedback control strategies.

In adding, there are other problems when that vanished the feedback coefficient as the following dislocated feedback controller:

$$U = \begin{bmatrix} -\boldsymbol{kz} \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{17}$$

Consequently, this strategy has failed to control the system (6). According to the above discussion, we found some problems and weakness of feedback control strategies. In table 1, we can briefly describe all cases that we have a positive feedback coefficient via these strategies.

3.2 Hyperchaotic Lorenz system

The nonlinear differential equations that describe the hyperchaotic Lorenz system are

$$\begin{cases} \dot{x} = a(y - x) \\ \dot{y} = cx - y - xz + w \\ \dot{z} = xy - bz \\ \dot{w} = -dx \end{cases} \tag{18}$$

Where a, b, c, d are conctant parameters. and the system (18) is hyperchaotic when parameter takes the values $a = 10$, $b = 8/3$, $c = 28$ and $d = 5$ and has two positive Lyapunov exponents, i.e. $LE_1 = 0.3997$ and $LE_2 = 0.3113$. And it has a unique unstable equilibrium $O(0,0,0,0)$ (Dou, Sun, Duan, & L$\ddot{u}$, 2009).

In order to focus on the goal and the main idea of this search will be shortened and we are going to reduce some mathematical steps.

Theorem 5. Let the dislocated feedback control strategy be defined as $U = [u_1, 0,0,0]^T$, i.e $u_1 = -ky$, then the system (18) can asymptotically converge to the unstable equilibrium when $k \in (9.6485,10)$.

Proof. System (18) with a new control can be rewritten as

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} -10 & 10 & 0 & 0 \\ 28 & -1 & 0 & 1 \\ 0 & 0 & -8/3 & 0 \\ -5 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} + \begin{bmatrix} -\boldsymbol{ky} \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{19}$$

the characteristic equation is

$$(\lambda + 8/3)(\lambda^3 + 11\lambda^2 + (28k - 270)\lambda + 50 - 5k) = 0 \tag{20}$$

then we have $k_1 < 10$, $k_2 > 9.6485$ based on Routh-Hurwitz method.

Now, According to the novel approach,

$$k = k_1 \cap k_2 = (0,10) \cap (9.6485, \infty) = (9.6485,10)$$
$$\therefore k \in (9.6485,10)$$
$$k = k_1 \cap k_2 \neq \emptyset$$

So, this strategy satisfy the first case of a novel approach. Therefore the system can be controlled by this strategy. The proof is now complete.

In addition, if we make simple changes into above control $U = [u_1, 0,0,0]^T$ i.e. $u_1 = -kw$,then the system (18) can't asymptotically converge to the unstable equilibrium although we have one a positive feedback s.t. $k < 10$ and the characteristic equation is

$$(\lambda + 8/3)(\lambda^3 + 11\lambda^2 - (270 + 5k)\lambda + 50 - 5k) = 0 \tag{21}$$

because one condition of Routh-Hurwitz is not satisfied. Consequently, this a positive feedback coefficient is not active and the system can't be controlled by this strategy based on four case of a novel approach ,The proof is now complete.

Similarly, if we make another simple change in to above control based on the speed feedback control $U = [u_1, 0,0,0]^T$,i.e. $u_1 = -k\dot{w}$,then the system (18) can't be controlled, although we have two positive feedbacks such that $k_1 < 2.2$ and $k_2 > 2.2355$. where the characteristic equation is

$$(\lambda + 8/3)(\lambda^3 + (11 - 5k)\lambda^2 - (270 + 5k)\lambda + 50) = 0 \tag{22}$$

and according to the novel approach, second case, there are no intersection between these two positive feedback coefficients

$$k = k_1 \cap k_2 = (0,2.2) \cap (2.2355, \infty) = \emptyset$$
$$k = k_1 \cap k_2 = \emptyset$$

Consequently, the system can't be controlled by this strategy.

Also, if we take the control based on enhancing feedback as $U = [u_1, 0, u_3, 0]^T$ i.e. $u_1 = -kx$ and $u_3 = -kz$, then the system (18) can be control with single a positive feedback s.t. $k < 270.1778$ according to the third case of a novel approach

3.2 Hyperchaotic Liu System

The four-dimensional autonomous Liu system is described by

$$\begin{cases} \dot{x} = a(y - x) \\ \dot{y} = cx - gxz + w \\ \dot{z} = hx^2 - bz \\ \dot{w} = -dx \end{cases}$$

Where a, b, c, d, g, h are conctant parameters, when parameters $a = 10$, $b = 2.5$, $c = 40$, $d = 10.6$, $g = 1$ and $h = 4$ system (23) is hyperchaotic and has only equilibrium $O(0,0,0,0)$, and the equilibrium is an unstable saddle node under these parameters (Zhu, 2010).

Theorem 6. If the ordinary feedback control is designed as $U = [, 0, u_2, 0,0]^T$, i.e. $u_2 = -ky$, Then the zero solution of the hyperchaotic system (23) is globally asymptotically stable when $k > 40.211$.

Proof. System (23) with a new control is

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} -10 & 10 & 0 & 0 \\ 40 & 0 & 0 & 1 \\ 0 & 0 & -2.5 & 0 \\ -10.6 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} + \begin{bmatrix} 0 \\ -\boldsymbol{ky} \\ 0 \\ 0 \end{bmatrix} \tag{24}$$

the characteristic equation is

$$(\lambda + 8/3)(\lambda^3 + (k + 10)\lambda^2 + (10k - 400)\lambda + 106) = 0 \tag{25}$$

Here, we get only one positive feedback coefficient i.e. $k > 40.211$ based on Routh-Hurwitz method and satisfied all conditions of this method. Consequently, the third case of a novel approach is satisfied by this strategy.

On the other hand, if design control based on dislocated feedback control as $U = [u_1, 0,0,0]^T$, i.e. $u_1 = -ky$, then the zero solution of the hyperchaotic system(23) can't be control when $k_1 < 10$ and $k_2 > 10$. where the characteristic equation is

$$(\lambda + 8/3)(\lambda^3 + 10\lambda^2 + (40k - 400)\lambda + 106 - 106\mathrm{k}) = 0$$

The cause in this case, by basing novel approach, second case, we find there are no intersection between these two positive feedbacks

$$k = k_1 \cap k_2 = (0{,}10) \cap (10, \infty)$$

$$k = k_1 \cap k_2 \neq \emptyset$$

Therefore the system (23) can't be controlled by this strategy. The proof is now complete.

In addition, if we make a simple change in the dislocated feedback control such that $U = [0, u_2, 0,0]^T$ i.e. $u_2 = -kw$,then the system (23) can't asymptotically converge to the unstable equilibrium although we have one a positive feedback s.t. $k < 10.6$. and the characteristic equation is

$$(\lambda + 8/3)(\lambda^3 + 10\lambda^2 - 400\lambda + 10k - 106) = 0 \tag{26}$$

because one condition of Routh-Hurwitz is not satisfied. Consequently, this a positive feedback coefficient is not active on a system, So this strategy failed to control this system based on four case of a novel approach, The proof is now complete.

Finally, if design control based on enhancing feedback control as $U = -[kx, ky, 0, kw]^T$, then system (23) can be control it with three positive feedback $(k_1 > 7.8576)$, $(k_2 > 15.4484)$ and $(k_3 > 0.2668)$, where the characteristic equation is

$$(\lambda + 8/3)(\lambda^3 + (3k + 10)\lambda^2 + (3k^2 + 20k - 400)\lambda + k^3 + 10k^2 - 400k + 106) = 0 \tag{27}$$

And the active value of positive feedback calculated based on formulation (4) as

$$k = k_1 \cap k_2 \cap k_3 = (7.8576, \infty) \cap (15.4484, \infty) \cap (0.2668, \infty) = (15.4484, \infty)$$

$$\therefore k \in (15.4484, \infty)$$

$$k = k_1 \cap k_2 \cap k_3 \neq \emptyset$$

Hence, there are intersection between three positive feedback coefficients, So, this strategy satisfy fist case of a novel approach. Therefore the system can be controlled by this strategy. The proof is now complete.

Remark 3. Most of previous works used the following formulation $k = max\{k_i\}$ to find the a positive, active feedback coefficient when we get more than one positive, but this formulation failed in some time and theorem 5 with control $U = [-ky, 0,0,0]^T$ is example, in this case, So prefer using the formulation (5) always.

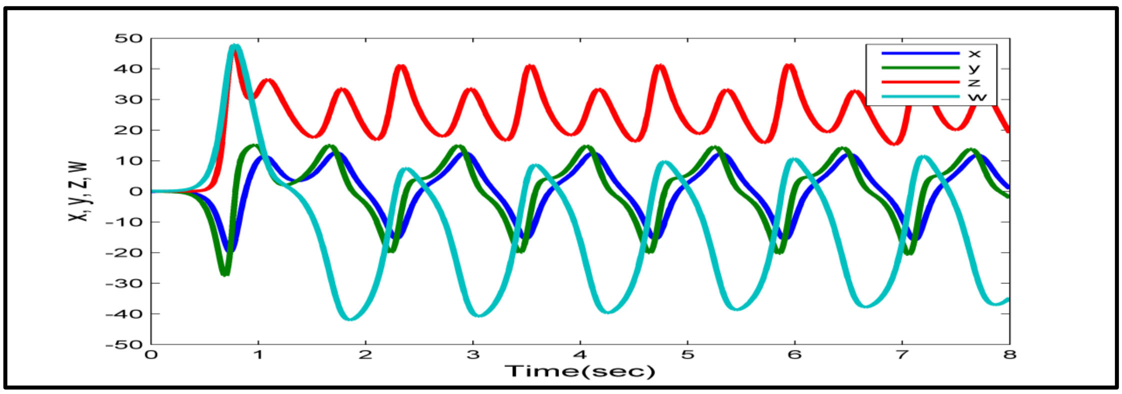

Figure 1. system (6) with control $\boldsymbol{U} = [\mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{0.3}\boldsymbol{w}]$

Table 1. Relationship between appositive feedback controller and suppressed for system (6) according to the new method

Type of feedback control	Control $U = [u_i]^T$	Number of a positive feedback	Feedback controller	Intersection $k = \cap_{i=1}^{n} k_i$	Satisfied all Routh-Hurwitz	Result
ordinary	$U = -[0, ky, 0,0]^T$	one	$k > 27.2683$		yes	control
ordinary	$U = -[0,0,0, kw]^T$	two	$k_1 < 0.3571$ $k_2 > 12.1139$	$k = k_1 \cap k_2$ $= \emptyset$	yes	not control
dislocated	$U = -[ky, 0,0,0]^T$	one	$k > 10$		yes	control
dislocated	$U = -[kw, 0,0,0]^T$	two	$k_1 > 10$ $k_2 < 0.3571$	$\emptyset$	yes	not control
dislocated	$U = -[0, kx, 0,0]^T$	one	$k > 28$		yes	control
dislocated	$U = -[0, kw, 0,0]^T$	one	$k < 1$		no	not control
speed	$U = -[0, k\dot{w}, 0,0]^T$	two	$k_1 > 1.0268$ $k_2 < 1$	$\emptyset$	yes	not control
speed	$U = -[0,0,0, k\dot{x}]^T$	one	$k > 28$		yes	control
speed	$U = -[0,0,0, k\dot{y}]^T$	one	$k > 27.2683$		yes	control
enhancing	$U = -k[x, y, 0,0]^T$	one	$k > 12.3620$		yes	control
enhancing	$U = -k[x, 0,0, w]^T$	two	$k_1 > 9.4923$ $k_2 < 0.3703$	$\emptyset$	yes	not control
enhancing	$U = -k[0, y, z, 0]^T$	one	$k > 27.2683$	……	yes	control
enhancing	$U = -k[0, y, 0, w]^T$	two	$k_1 > 27.6381$ $k_2 > 7.6751$	$= (27.6381, \infty)$ $\neq \emptyset$	yes	control
enhancing	$U = -k[0,0, z, w]^T$	two	$k_1 > 12.1139$ $k_2 < 0.3571$	$\emptyset$	yes	not control
enhancing	$U = -k[x, y, z, 0]^T$	one	$k > 12.3620$		yes	control
enhancing	$U = -k[x, 0, z, w]^T$	two	$k_1 > 9.4923$ $k_2 < 0.3703$	$\emptyset$	yes	not control
enhancing	$U = -k[0, y, z, w]^T$	two	$k_1 > 27.6381$ $k_2 < 7.6751$	$(27.6381, \infty)$ $\neq \emptyset$	yes	control
enhancing	$U = -k[x, y, 0, w]^T$	three	$k_1 > 6.1528$ $k_2 > 0.3758$ $k_3 > 11.9298$	$(11.9298, \infty)$ $\neq \emptyset$	yes	control
enhancing	$U = -k[x, y, z, w]^T$	three	$k_1 > 6.1528$ $k_2 > 0.3758$ $k_3 > 11.9298$	$(11.9298, \infty)$ $\neq \emptyset$	yes	control

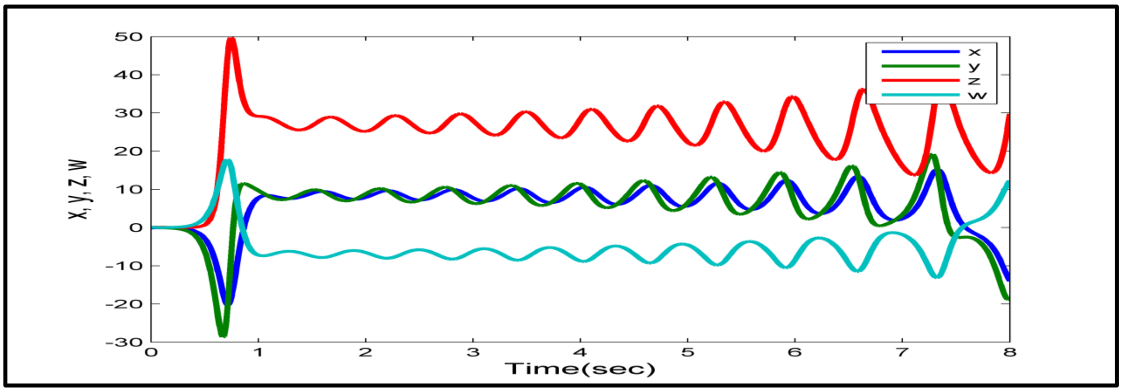

Figure 2. system (6) with control $\boldsymbol{U} = [\mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{12.5}\boldsymbol{w}]$

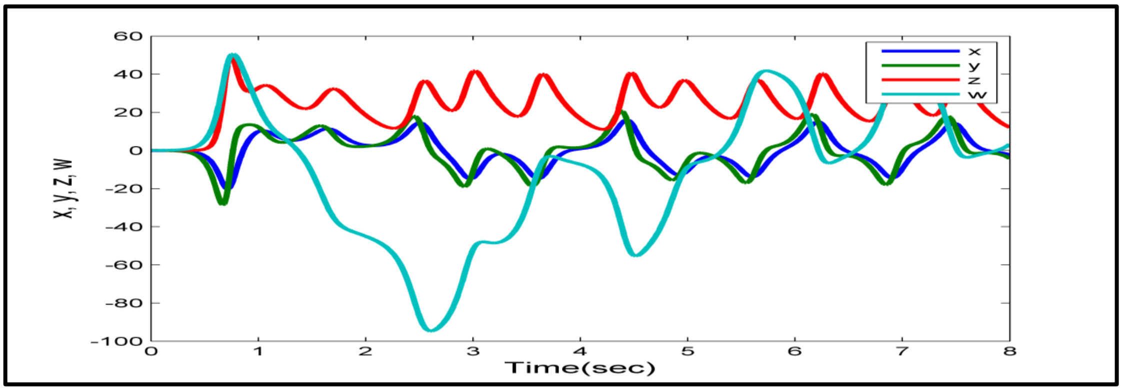

Figure 3. system (6) with control $\boldsymbol{U} = [\mathbf{0}, \mathbf{0.5}\boldsymbol{w}, \mathbf{0}, \mathbf{0}]$

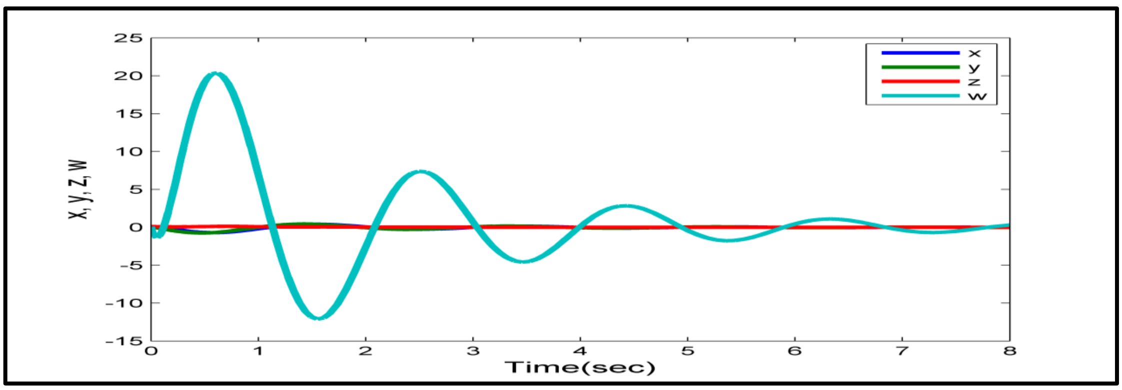

Figure 4. system (6) with control $\boldsymbol{U} = [\mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{290}(\boldsymbol{x} - \boldsymbol{y})]$

The following flow chart explains briefly the new method

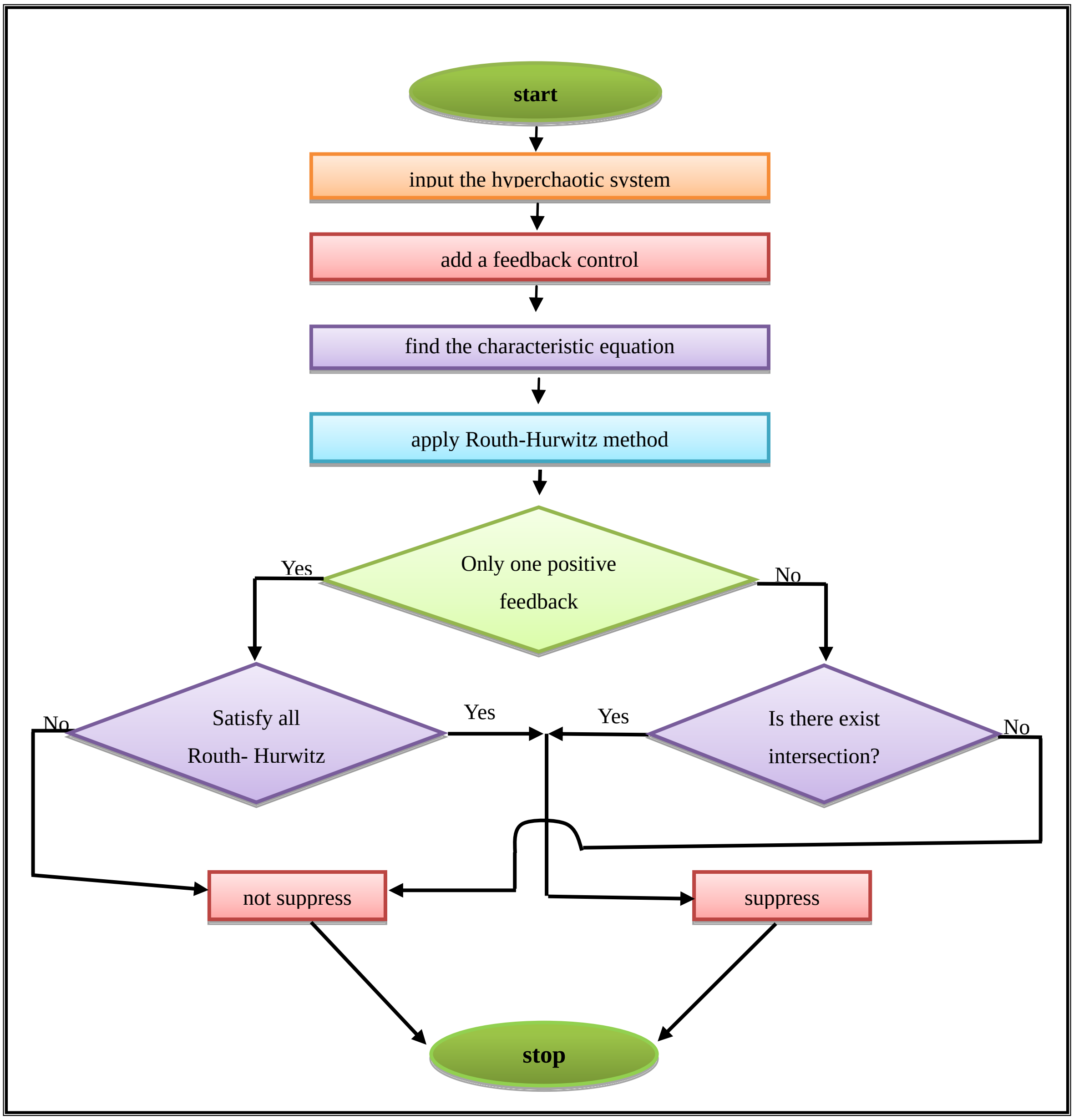

Figure 5. Flow chart which described briefly a new method

4. Conclusions

The results obtained in this paper show that the strategies with positive feedback coefficients can be controlled in two cases and failed in another two cases, which are performed in the above theorems.

Also we consider the weakness of feedback control strategies, when we get positive feedback coefficients and can't suppress the system. This is a try to understand this problem, when it happen and what is that reason? So, we suggest a new method which includes answers to all these questions. Explain this method, theoretical, numerical and justify the results. Finally, we can apply this method for another chaotic and hyper chaotic systems to check the control of these systems.

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