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Even Star Decomposition of Complete Bipartite Graphs

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Abstract

A decomposition $(G_1, G_2, G_3, \cdots, G_n)$ of a graph G is an Arithmetic Decomposition(AD) if $|E(G_i)| = a + (i-1)d$ for all $i = 1, 2, \cdots, n$ and $a, d \in Z^+$. Clearly $q = \frac{n}{2}[2a + (n-1)d]$. The AD is a CMD if a = 1 and d = 1. In this paper we introduced the new concept Even Decomposition of graphs. If a = 2 and d = 2 in AD, then q = n(n + 1). That is, the number of edges of G is the sum of first n even numbers $2, 4, 6, \cdots, 2n$. Thus we call the AD with a = 2 and d = 2 as Even Decomposition. Since the number of edges of each subgraph of G is even, we denote the Even Decomposition as $(G_2, G_4, \cdots, G_{2n})$.

Keywords: Continuous Monotonic Decomposition, Decomposition of graph, Even Decomposition, Even Star Decomposition (ESD)

1. Introduction

All basic terminologies from Graph Theory are used in this paper in the sense of Frank Harary. Gnanadhas. N and Paulraj Joseph. J discussed on Continuous Monotonic Decomposition (CMD) of graphs. Ebin Raja Merly. E and Gnanadhas. N introduced Arithmetic Odd Decomposition (AOD). In this paper we investigate Even Star decomposition (ESD) of Complete Bipartite Graphs. Throughout this paper Sn denotes the star graph of size n.

The definitions which are useful for the present investigation are given below.

1.1 Definition (Gnanadhas & Joseph, 2000)

A graph G = (V, E) be a simple connected graph with p vertices and q edges. If $G_1, G_2, \cdots, G_n$ are connected edge-disjoint subgraphs of G with $E(G) = E(G_1) \cup E(G_2) \cup \ldots \cup E(G_n)$, then $(G_1, G_2, \ldots, G_n)$ is a Decomposition of G.

1.2 Definition (Harary, 1969)

A bigraph or bipartite graph G is a graph whose vertex set V can be partitioned into two subsets V_1 and V_2 such that every edge of G joins V_1 and V_2. If G contains every edge joining V_1 and V_2 then **G** is a complete bigraph. If V_1 and V_2 have m and n vertices, we write $G= K_{m,n} =K(m,n)$. A star is a complete bipartite graph of the form $K_{1,n}$ and is denoted by S_n. Clearly $K_{m,n}$ has mn edges.

2. Even Decomposition of Graphs

2.1 Definition (Merly & Gnanadhas, 2011)

A decomposition $(G_1, G_2, G_3, \cdots, G_n)$ of G is said to be an Arithmetic Decomposition (AD) if $|E(G_i)| = a+(i-1)d$ for all $i = 1, 2, \cdots, n$ and $a, d \in Z^+$. Clearly $q = \frac{n}{2}[2a + (n-1)d]$. If a = 1 and d = 1, then AD is a CMD. If a = 1 and d = 2, then AD is an Arithmetic Odd Decomposition (AOD).

If a = 2 and d = 2, then q = n(n+1). Clearly n(n+1) is the sum of first n even numbers $2, 4, 6, \cdots, 2n$. Thus we call this Decomposition as an Even Decomposition denoted by $(G_2, G_4, G_6, \cdots, G_{2n})$.

The following theorem is a necessary and sufficient condition for a graph G admits Even Decomposition.

2.2 Theorem

Any graph G admits Even Decomposition ($G_2, G_4, G_6, \ldots, G_{2n}$), where $G_{2i} = (V_{2i}, E_{2i})$ and $|E(G_{2i})| = 2i$, (i = 1, 2, 3, 4…., n) if and only if q = n(n+1) for each $n \in Z^+$.

Proof:

Suppose q = n (n+1) for each $n \in Z^+$. Applying induction on ‘n’. The result is obvious when n = 1 and n = 2.

Suppose the result is true when n = k. Let G be any connected graph with q = k (k + 1), then G can be decomposed into ($G_2, G_4, G_6, \ldots, G_{2k}$).

We prove that the result is true for n = k + 1. Let G' be any connected graph with (k + 1) [(k + 1)+1] edges. We prove that G′ admits ($G_2, G_4, G_6, \ldots, G_{2k}, G_{2(k+1)}$). Now (k + 1) (k + 2) = k (k + 1) + 2 (k + 1). Thus q(G′)=k(k + 1)+ 2(k + 1).

Let G* and $G_{2(k+1)}$ be two subgraphs of G' with k (k + 1) and 2(k + 1) edges respectively.

By our induction hypothesis G * can be decomposed into k subgraphs ($G_2, G_4, G_6, \ldots G_{2k}$).

Therefore G' can be decomposed into ($G_2, G_4, G_6, \ldots, G_{2k}$) and $G_{2(k+1)}$. Hence G admits Even Decomposition. Conversely, suppose G admits Even Decomposition ($G_2, G_4, G_6, \ldots, G_{2n}$).

Then obviously q(G) = 2 + 4 + 6 +… + 2n = n(n + 1), $n \in Z^+$. Hence the proof is finished.

2.3 Example

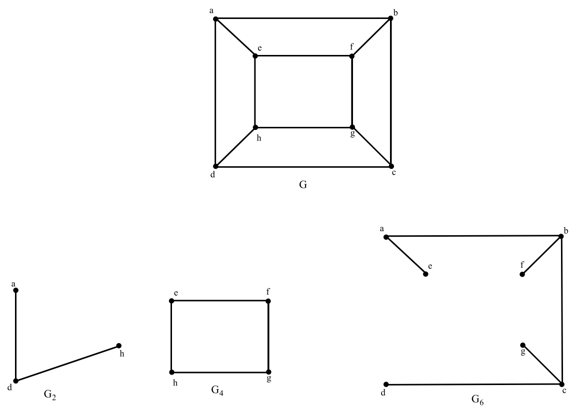


Figure 1. G with Even Decomposition (G_2, G_4, G_6)

3. Even Star Decomposition of Complete Bipartite Graph

3.1 Definition (Merly & Gnanadhas, 2012)

An Even Decomposition ($S_2, S_4, S_6, \ldots, S_{2n}$) of G is called an Even Star Decomposition(ESD).

A graph G with q = 12 having an ESD (S_2, S_4, S_6,) is shown in Figure 2.

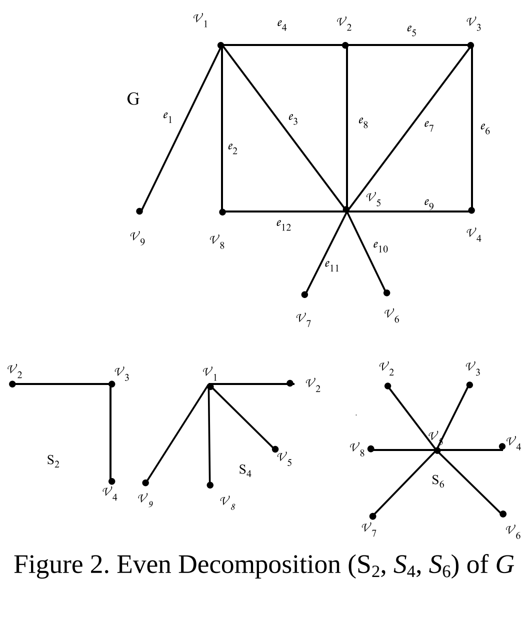


Figure 2. Even Decomposition (S_2, S_4, S_6) of G

3.2 Remark

1. $K_{2,1}$ admits ESD

2. $K_{2,3}$ admits AED, but not ESD. It is shown in the Figure 3.

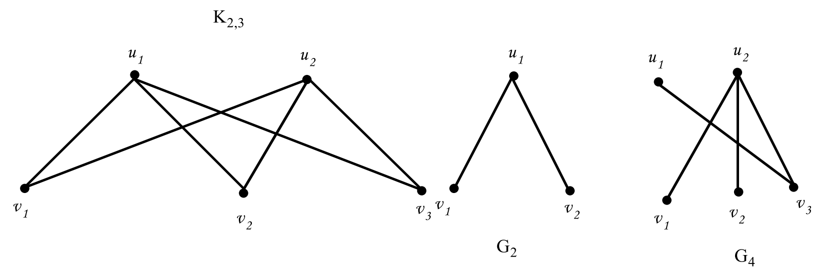


Figure 3. Even Decomposition of $K_{2,3}$

3. ESD (S_2, S_4, S_6,) of $K_{2,6}$ is shown in Figure 4.

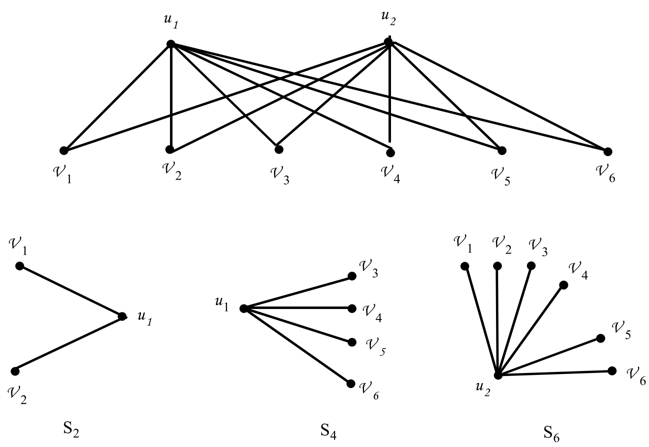


Figure 4. ESD (S_2, S_4, S_6,) of $K_{2,6}$

4. ESD(S_2, S_4, S_6,S_8) of $K_{2,10}$ is shown in Figure 5.

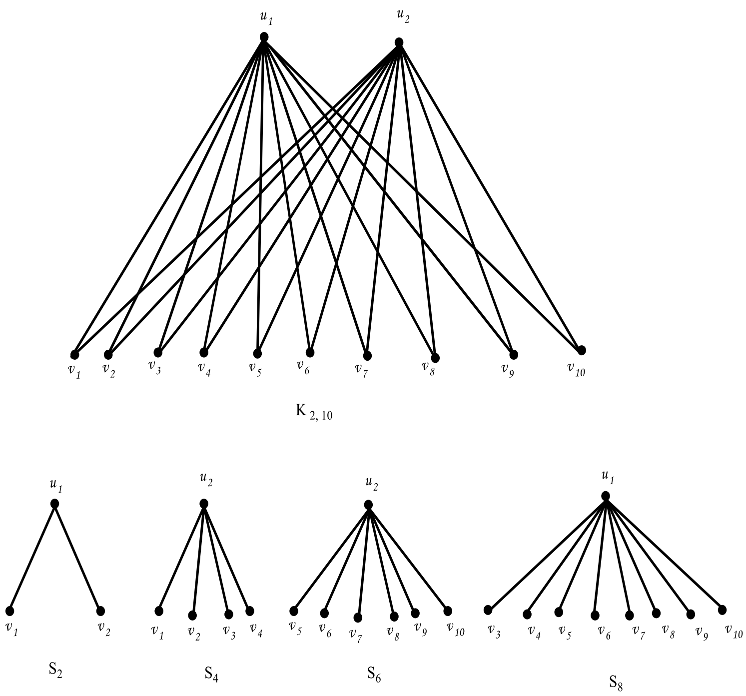


Figure 5. ESD (S_2, S_4, S_6,S_8) of $K_{2,10}$

3.3 Theorem

A complete bipartite graph K_{2^t,s_t} admits Even Star Decomposition $(S_2, S_4, \ldots, S_{k2^{t+2}-2})$ if and only if

$s_t = 2k\,(k\,2^{t+1} - 1)$,where $n = k2^{t+1} - 1$, t, $k(\neq 1) \in N$.

Proof:

Assume K_{2^t,s_t} admits ESD $(S_2, S_4, \ldots, S_{k2^{t+2}-2})$, we know that $q(K_{2^t,s_t}) = 2^t s_t$

Therefore , $2^t s_t = n(n+1)$. This implies $s_t = 2k(k\,2^{t+1} - 1)$, where $n = k2^{t+1} - 1$, $k \neq 1$

Conversely, assume $s_t = 2k(k\,2^{t+1} - 1)$, to prove K_{2^t,s_t} admits ESD $(S_2, S_4, \ldots, S_{k2^{t+2}-2})$, applying induction on 't' the result is obvious when t = 1.

Suppose the result is true when t = g. That is K_{2^g,s_g} admits ESD $(S_2, S_4, \ldots, S_{k2^{g+2}-2})$.

We prove that the result is true for t = g +1, that is to prove $K_{2^{g+1},s_{g+1}}$ admits ESD.

We have

$$q(K_{2^{g+1},s_{g+1}}) = 2^{g+1}s_{g+1} = 2^{g+1}(k^2\,2^{g+3} - 2k) = k^2\,2^{2g+4} - k2^{g+2}.$$

Also,

$$q(K_{2^g,s_g}) = 2^g s_g = 2^g\,(k^2\,2^{g+2} - 2k) = k^2\,2^{2g+2} - k2^{g+1}.$$

Therefore,

$$q(K_{2^{g+1},s_{g+1}}) - q(K_{2^g,s_g}) = 3k^2\,2^{2g+2} - k2^{g+1} \quad (1)$$

Now,

$$q(S_{k2^{g+2}}) + q(S_{k2^{g+2}+2}) + \cdots + q(S_{k2^{g+2}-2}).$$

Equal to

$$q(S_{2n+2}) + q(S_{2n+4}) + \ldots + q(S_{4n+2}) = 3n^2 + 5n + 2, = 3k^2\,2^{2g+2} - k2^{g+1} \quad (2)$$

From (1) and (2) we have proved that

$$q(S_{2^{g+1},s_{g+1}}) - q(K_{2^g,s_g}) = q(S_{k2^{g+2}}) + q(S_{k2^{g+2}+2}) + \cdots + q(S_{k2^{g+2}-2}) .$$

Therefore,

$$q(S_{2^{g+1},s_{g+1}}) = q(K_{2^g,s_g}) + q(S_{k2^{g+2}}) + q(S_{k2^{g+2}+2}) + \cdots + q(S_{k2^{g+2}-2}).$$

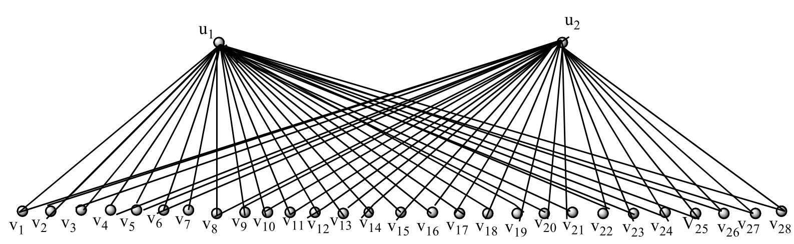
Therefore, $K_{2^{g+1},s_{g+1}}$ admits ESD $(S_2, S_4, \ldots, S_{k2^{g+3}-2})$.

Therefore the result is true for t = g+1.

Hence, K_{2^t,s_t}admits ESD $(S_2, S_4, \ldots, S_{k2^{t+2}-2})$, where $n = k2^{t+1} - 1$, t, $k(\neq 1) \in N$.

3.4 Example

$K_{2,28}$ admits ESD (S_2, S_4, S_6, S_8, S_{10}, S_{12}, S_{14})



$K_{2,28}$

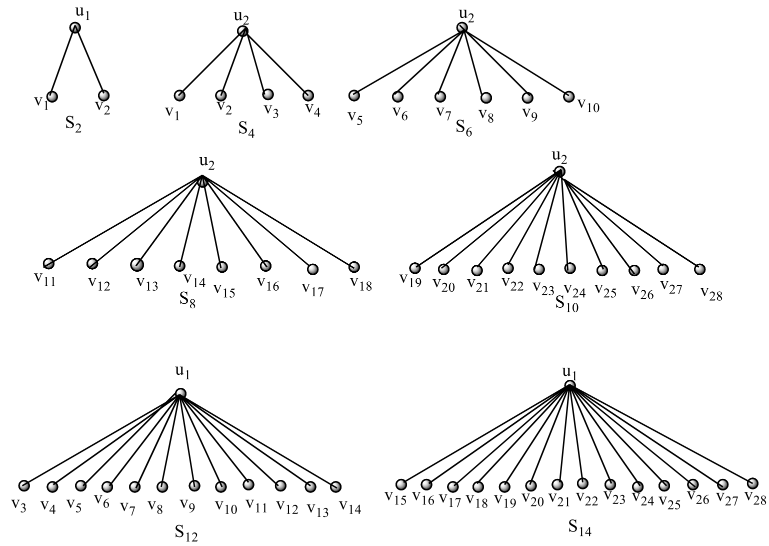


Figure 6. ESD (S_2, S_4, S_6, S_8, S_{10}, S_{12}, S_{14}) of $K_{2,28}$

3.5 Theorem

A complete bipartite graph K_{2^t,s_t} admits Even Star Decomposition$(S_2, S_4, \dots, S_{k2^{t+2}})$ if and only if $s_t = 2k(k\,2^{t+1} + 1)$, where $n = k2^{t+1}$, t, k $\in$ N

Proof:

Assume K_{2^t,s_t} admits ESD $(S_2, S_4, \dots, S_{k2^{t+2}})$,. We know that $q(K_{2^t,s_t}) = 2^t s_t$.

Therefore $2^t s_t = n(n+1)$. Implies $s_t = 2k(k\,2^{t+1} + 1)$, where $n = k2^{t+1}$.

Conversely, Assume $s_t = 2k(k\,2^{t+1}+1)$, to prove K_{2^t,s_t} admits ESD $(S_2, S_4, \dots, S_{k2^{t+2}})$.

Applying induction on 't' the result is obvious when t = 1.

Suppose the result is true when t = g. That is K_{2^g,s_g} admits ESD $(S_2, S_4, \dots, S_{k2^{g+2}})$.

To prove the result is true for t = g +1, That is to prove $K_{2^{g+1},s_{g+1}}$ admits ESD.

We have

$$q(K_{2^{g+1},s_{g+1}}) = 2^{g+1}s_{g+1}, = 2^{g+1}(k^2\,2^{g+3} + 2k) = k^2\,2^{2g+4} + k2^{g+2}.$$

Also,

$$q(K_{2^g,s_g}) = 2^g s_g = 2^g\,(k^2\,2^{g+2} + 2k) = k^2\,2^{2g+2} + k2^{g+1}.$$

Therefore,

$$q(K_{2^{g+1},s_{g+1}}) - q(K_{2^g,s_g}) = 3k^2\,2^{2g+2} + k2^{g+1} \qquad (3)$$

Now,

$$q(S_{k2^{g+2}+2}) + q(S_{k2^{g+2}+4}) + \dots + q(S_{k2^{g+3}}).$$

That is

$$q(S_{2n+2}) + q(S_{2n+4}) + \dots + q(S_{4n}) = 3n^2 + n = 3k^2\,2^{2g+2} + k2^{g+1} \qquad (4)$$

From (3) and (4) We have proved that

$$q(K_{2^{g+1},s_{g+1}}) - q(K_{2^g,s_g}) = q(S_{k2^{g+2}+2}) + q(S_{k2^{g+2}+4}) + \dots\dots\dots + q(S_{k2^{g+3}}).$$

Therefore,

$$q(K_{2^{g+1},s_{g+1}}) = q(K_{2^g,s_g}) + q(S_{k2^{g+2}+2}) + q(S_{k2^{g+2}+4}) + \dots\dots\dots + q(S_{k2^{g+3}}).$$

Therefore $K_{2^{g+1},s_{g+1}}$ admits ESD $(S_2, S_4, \dots, S_{k2^{g+3}})$.

Therefore the result is true for t = g+1.

Hence, $K_2{}^t,s_t$ admits ESD $(S_2, S_4, \dots, S_{k2^{t+2}})$,$n = 2^{t+1}k + 1$, t, k $\in$ N.

3.6 Example

$K_{4,18}$ admits ESD (S_2, S_4, …, S_{16}).

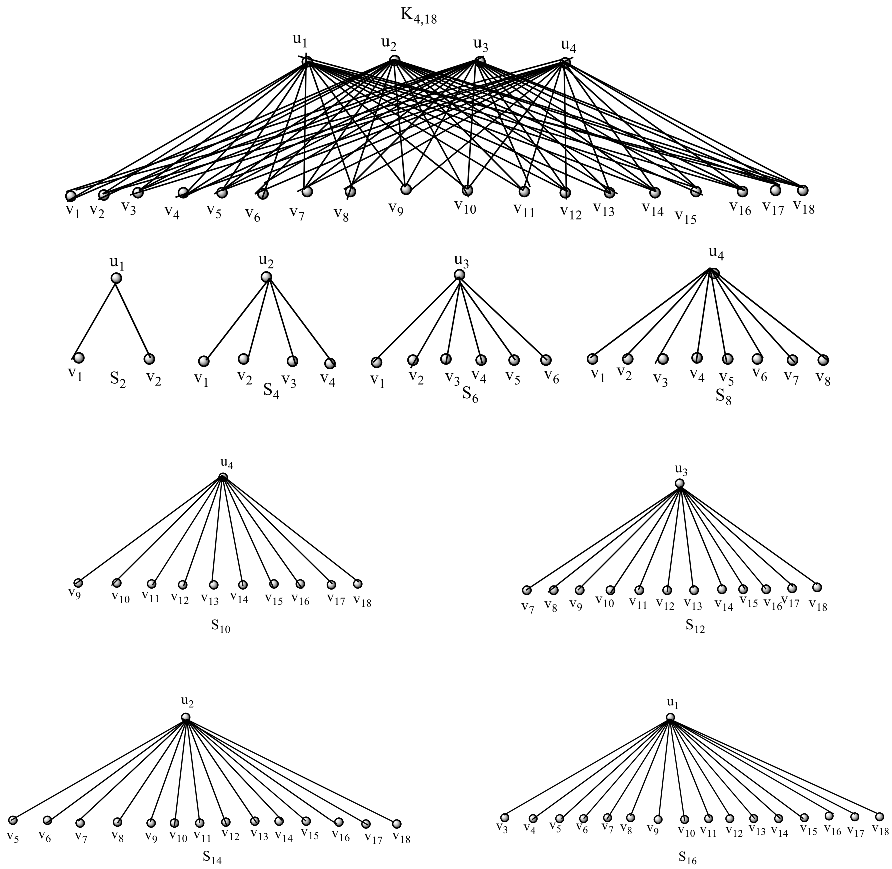


Figure 7. ESD (S_2, S_4, …, S_{16}) of $K_{4,18}$

3.7 Remark

Complete bipartite graph $K_{3,s}$, $K_{6,s}$,…, $K_{w,s}$ does not admit AESD where w is odd or odd multiples.

References

Merly, E. E. R., & Gnanadhas, N. (2011). Linear Path Decomposition of Lobster. *International Journal of Mathematics Research,3*(5), 447-455.

Merly, E. E. R., & Gnanadhas, N. (2012). Linear Star Decomposition of Lobster. *Int. J. Contemp. Math. Sciences,7*(6), 251-261.

Frank, H. (1969). *Graph Theory.* Addison-Wesley Publishing Company.

Gnanadhas, N., & Paulraj, J. J. (2000). Continuous Monotonic Decomposition of Graphs. *International Journal of Management and Systems, 16*(3), 333-344.

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