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Degree Splitting of Heronian Mean Graphs

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Abstract

In this paper, we prove Heronian Mean labeling of some degree splitting graphs. Already we have proved Heronian Mean labeling for some standard graphs. Here we prove that degree splitting of Path P_3, Path P_4, $P_3 \odot K_1$, $P_2 \odot K_{1,2}$, $P_2 \odot K_{1,3}$, $P_2 \odot K_3$ are Heronian Mean graphs.

Keywords: Heronian Mean graph, degree Splitting graphs, union of graphs, Path.

AMS Subject Classification: 05C78

1. Introduction

By a graph we mean a finite undirected graph without loops or parallel edges. For all detailed survey of graph labeling, we refer to J. A. Gallian (Gallian, 2013). For all other standard terminology and notations we follow Harary (Harary, 1988). The concept of Mean labeling was introduced in (Somasundaram & Ponraj, 2003). The concept of Harmonic Mean labeling was introduced in (Somasundaram, Ponraj, & Sandhya). The concept of Harmonic Mean labeling on Degree Splitting graph was introduced in (Sandhya, Jeyasekharan, & David). Motivated by the above results and by the motivation of the authors we study the Heronian Mean labeling on Degree Splitting graphs. Heronian Mean labeling was introduced in (Sandhya, Merly, & Deepa) and the Heronian Mean labeling of some standard graphs was proved in (Sandhya, Merly, & Deepa).

We shall make frequent references to the definitions and theorems that are useful for our present study. **A Path $\mathbf{P_n}$** is a walk in which all the vertices are distinct.

Definition 1.1:

A graph **G=(V,E)** with p vertices and q edges is said to be a **Heronian Mean graph** if it is possible to label the vertices **x∈V** with distinct labels $\mathbf{f(x)}$ from **1,2,…,q+1** in such a way that when each edge $\mathbf{e = uv}$ is labeled with,

$$\mathbf{f(e=uv)} = \left\lceil \frac{\mathbf{f(u)} + \sqrt{\mathbf{f(u)f(v)}} + \mathbf{f(v)}}{\mathbf{3}} \right\rceil (\mathbf{OR}) \left\lfloor \frac{\mathbf{f(u)} + \sqrt{\mathbf{f(u)f(v)}} + \mathbf{f(v)}}{\mathbf{3}} \right\rfloor$$

then the edge labels are distinct. In this case $\mathbf{f}$ is called a **Heronian Mean labeling** of G.

Definition 1.2:

Let G=(V,E) be a graph with $V = S_1 \cup S_2 \cup \dots . \cup S_t \cup T$, Where each S_i is a set of vertices having atleast two vertices and $T = V - \cup S_i$. The degree splitting graph of G is denoted by DS(G) and is obtained from G by adding vertices $w_1, w_2, \dots . w_t$ and joining w_i to each vertex of S_i ($1 \leq i \leq t$). The graph G and its degree splitting graph DS(G) are given in figure:1.

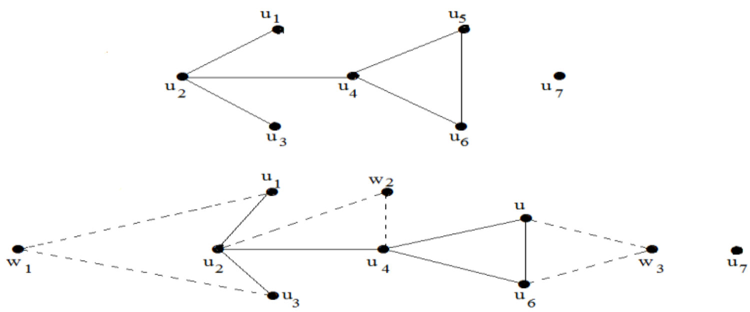

Figure 1.

Definition 1.3:

The union of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a graph $G = G_1 \cup G_2$ with vertex set $V = V_1 \cup V_2$ and the edge set $E = E_1 \cup E_2$.

Theorem 1.4: Any Path P_n is a Heronian mean graph.

Remark 1.5:

Any graph G is a subgraph of DS(G). If G has atleast two vertices, then G contains atleast two vertices of the same degree. Hence $G = K_1$ is the only graph such that G=DS(G).

Remark 1.6:

If G is regular, then DS(G)= $G + K_1$.

2. Main Results

Theorem 2.1:

$nDS(P_3)$ is a Heronian mean graph.

Proof:

The graph $DS(P_3)$ is shown in figure:2

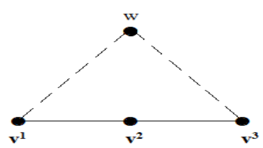

Figure 2.

Let $G = nDS(P_3)$. Let the vertex set of G be $V = V_1 \cup V_2 \cup \ldots \cup V_n$,

Where $V = \{V_i^1, V_i^2, V_i^3, w_i / 1 \le i \le n\}$ } is the vertex set of i^{th} copy of $DS(P_3)$

Define a function $f: V(G) \to \{1,2, \ldots\ldots q+1\}$ by

$$f(V_i^1) = 4i - 3, 1 \le i \le n$$

$$f(V_i^2) = 4i - 2, 1 \le i \le n$$

$$f(V_i^3) = 4i - 1, 1 \le i \le n$$

$$f(w_i) = 4i, 1 \le i \le n$$

Then the edges are labeled with

$$f(V_i^1 V_i^2) = 4i - 3, 1 \le i \le n$$

$$f(V_i^2 V_i^3) = 4i - 1, 1 \le i \le n$$

$$f(V_i^1 w_i) = 4i - 2, 1 \le i \le n$$

$$f(V_i^3 w_i) = 4i, 1 \le i \le n$$

Hence by definition 1.1, G is a Heronian mean graph.

Example 2.2: Heronian mean labeling of $4DS(P_3)$ is shown in figure 3.

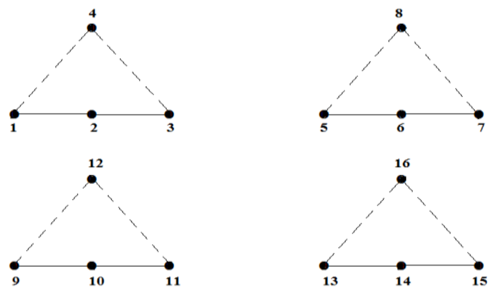

Figure 3.

Theorem 2.3:

$nDS(P_4)$ is a Heronian mean graph.

Proof:

The graph $DS(P_4)$ is shown in figure 4.

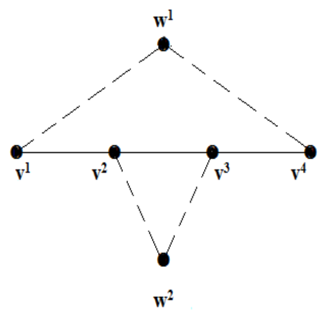

Figure 4.

Let $G = nDS(P_4)$. Let the vertex set of G be $V = V_1 \cup V_2 \cup \ldots \cup V_n$,

Where $V = \{V_i^1, V_i^2, V_i^3, V_i^4, w_i^1, w_i^2 / 1 \le i \le n\}$ is the vertex set of i^{th} copy of $DS(P_4)$

Define a function $f: V(G) \to \{1,2, \ldots\ldots q+1\}$ by

$$f(V_i^1) = 7i - 5, 1 \le i \le n$$

$$f(V_i^2) = 7i - 3, 1 \le i \le n$$

$$f(V_i^3) = 7i - 1, 1 \le i \le n$$

$$f(V_i^4) = 7i - 4, 1 \le i \le n$$

$$f(w_i^1) = 7i - 6, 1 \le i \le n$$

$$f(w_i^2) = 7i, 1 \le i \le n$$

Then the edges are labeled with

$$f(V_i^1 V_i^2) = 7i - 4, 1 \le i \le n$$

$$f(V_i^2 V_i^3) = 7i - 2, 1 \le i \le n$$

$$f(V_i^3 V_i^4) = 7i - 3, 1 \le i \le n$$

$$f(V_i^1 w_i^1) = 7i - 6, 1 \leq i \leq n$$

$$f(V_i^2 w_i^2) = 7i - 1, 1 \leq i \leq n$$

$$f(V_i^3 w_i^2) = 7i, 1 \leq i \leq n$$

$$f(V_i^4 w_i^1) = 7i - 5, 1 \leq i \leq n$$

Hence by definition 1.1, G is a Heronian mean graph.

Example 2.4: Heronian mean labeling of $4DS(P_4)$ is shown in figure 5.

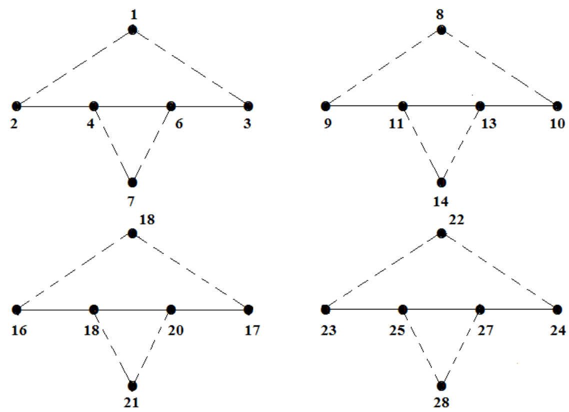

Figure 5.

Remark 2.5:

We know that $P_2 \odot K_1 = P_4$. Hence using theorem:2.3, $nDS(P_2 \odot K_1)$ is a Heronian mean graph.

Theorem 2.6:

$nDS(P_3 \odot K_1)$ is a Heronian mean graph.

Proof:

The graph $DS(P_3 \odot K_1)$ is shown in figure:6

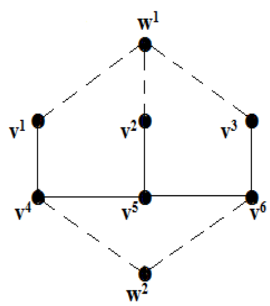

Figure 6.

Let $G = nDS(P_3 \odot K_1)$. Let the vertex set of G be $V = V_1 \cup V_2 \cup \ldots .\cup V_n$,

Where $V = \{V_i^1, V_i^2, V_i^3, V_i^4, V_i^5, V_i^6, w_i^1, w_i^2 / 1 \leq i \leq n\}$ is the vertex set of i^{th} copy of $DS(P_3 \odot K_1)$

Define a function $f: V(G) \rightarrow \{1,2, \ldots\ldots q+1\}$ by

$$f(V_i^1) = 10i - 3, 1 \leq i \leq n$$

$$f(V_i^2) = 10i - 2, 1 \leq i \leq n$$

$$f(V_i^3) = 10i - 1, 1 \leq i \leq n$$

$$f(V_i^{\,4})= 10i - 8\,,1 \le i \le n$$

$$f(V_i^{\,5})= 10i - 5\,,1 \le i \le n$$

$$f(V_i^{\,6})= 10i - 6\,,1 \le i \le n$$

$$f(w_i^{\,1})= 10i - 6,1 \le i \le n$$

$$f(w_i^{\,2})= 10i - 9\,,1 \le i \le n$$

Then we get distinct edge labels from {**1,2,…,q**} Hence G is a Heronian mean graph.

Example 2.7: Heronian mean labeling of $4DS(P_3 \odot K_1)$ is shown in figure 7.

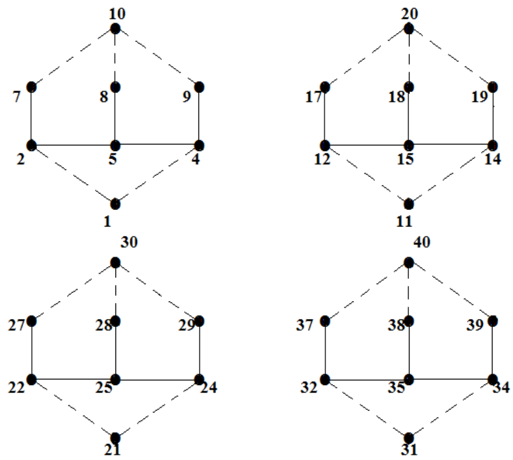

Figure 7.

Theorem 2.8:

$nDS(P_2 \odot K_{1,2})$ is a Heronian mean graph.

Proof:

The graph $DS(P_2 \odot K_{1,2})$ is shown in figure:8

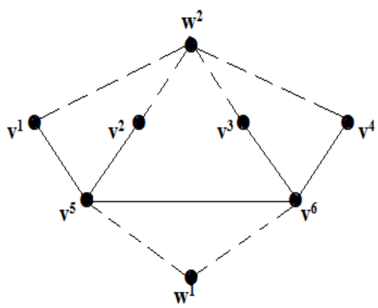

Figure 8.

Let $G = nDS(P_2 \odot K_{1,2})$. Let the vertex set of G be $V = V_1 \cup V_2 \cup \ldots . \cup V_n$, Where $V = \{V_i^{\,1}, V_i^{\,2}, V_i^{\,3}, V_i^{\,4}, V_i^{\,5}, V_i^{\,6}, w_i^{\,1}, w_i^{\,2} / 1 \le i \le n\}$ is the vertex set of i^{th} copy of $DS(P_2 \odot K_{1,2})$

Define a function$f: V(G) \rightarrow \{1,2, \ldots \ldots q+1\}$ by $f(V_i^{\,1})= 11i - 5\,,1 \le i \le n$

$$f(V_i^{\,2})= 11i - 3\,,1 \le i \le n$$

$$f(V_i^{\,3})= 11i - 2\,,1 \le i \le n$$

$$f(V_i^{\,4}) = 11i - 1, 1 \le i \le n$$

$$f(V_i^{\,5}) = 11i - 8, 1 \le i \le n$$

$$f(V_i^{\,6}) = 11i - 6, 1 \le i \le n$$

$$f(w_i^{\,1}) = 11i, 1 \le i \le n$$

$$f(w_i^{\,2}) = 11i - 10, 1 \le i \le n$$

Then we get distinct edge labels from {**1,2,…,q**} Hence G is a Heronian mean graph.

Example 2.9: Heronian mean labeling of $4DS(P_2 \odot K_{1,2})$ is shown in figure 9.

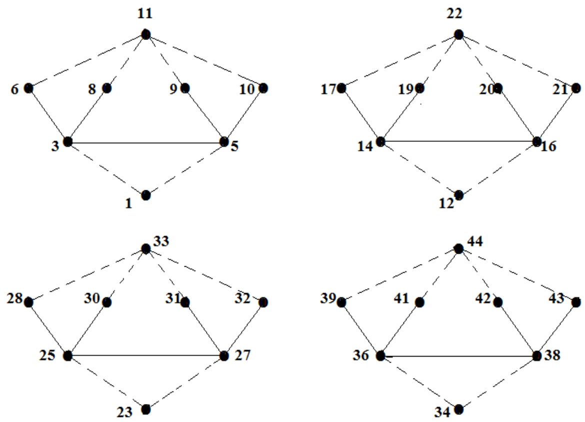

Figure 9.

Theorem 2.10:

$nDS(P_2 \odot K_{1,3})$ is a Heronian mean graph.

Proof:

The graph $DS(P_2 \odot K_{1,3})$ is shown in figure:10

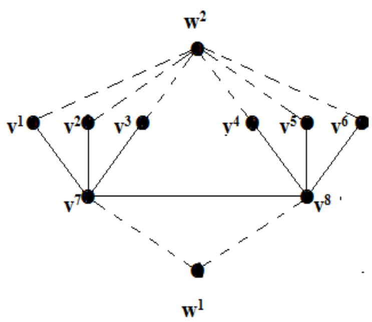

Figure 10.

Let $G = nDS(P_2 \odot K_{1,3})$. Let the vertex set of G be $V = V_1 \cup V_2 \cup \ldots . \cup V_n$,

Where $V = \{V_i^{\,1}, V_i^{\,2}, V_i^{\,3}, V_i^{\,4}, V_i^{\,5}, V_i^{\,6}, V_i^{\,7}, V_i^{\,8}, w_i^{\,1}, w_i^{\,2} / 1 \le i \le n\}$ is the vertex set of i^{th} copy of $DS(P_2 \odot K_{1,3})$

Define a function $f: V(G) \rightarrow \{1,2, \ldots \ldots q+1\}$ by

$$f(V_i^{\,1}) = 15i - 9, 1 \le i \le n$$

$$f(V_i^{\,2}) = 15i - 7, 1 \le i \le n$$

$$f(V_i^{\,3}) = 15i - 6, 1 \le i \le n$$

$$f(V_i^4)= 15i-5 ,1 \le i \le n$$

$$f(V_i^5)= 15i-3 ,1 \le i \le n$$

$$f(V_i^6)= 15i-1 ,1 \le i \le n$$

$$f(V_i^7)= 15i-2 ,1 \le i \le n$$

$$f(V_i^8)= 15i-10 ,1 \le i \le n$$

$$f(w_i^1)= 15i-14,1 \le i \le n$$

$$f(w_i^2)= 15i ,1 \le i \le n$$

Then we get distinct edge labels from {**1,2,…,q**} Hence G is a Heronian mean graph.

Example 2.11: Heronian mean labeling of $4DS(P_2 \odot K_{1,3})$ is shown in figure 11.

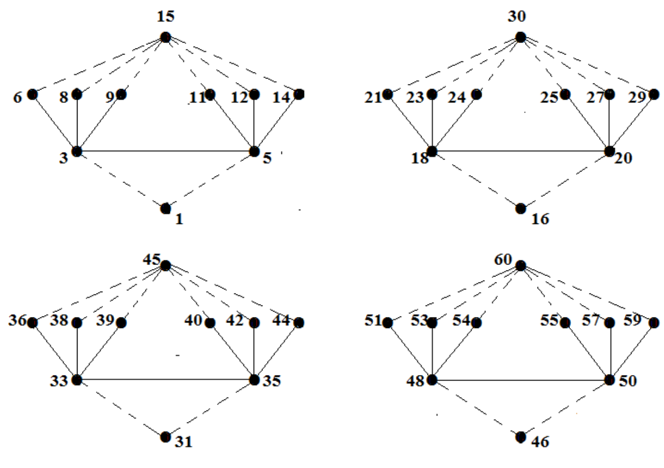

Figure 11.

Theorem 2.12:

$nDS(P_2 \odot K_3)$ is a Heronian mean graph.

Proof:

The graph $DS(P_2 \odot K_3)$ is shown in figure:12

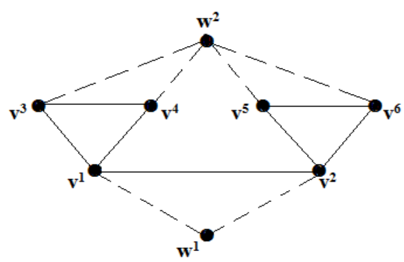

Figure 12.

Let $G = nDS(P_2 \odot K_3)$. Let the vertex set of G be $V = V_1 \cup V_2 \cup \dots \cup V_n$,

Where $V = \{V_i^1, V_i^2, V_i^3, V_i^4, V_i^5, V_i^6, w_i^1, w_i^2 / 1 \le i \le n\}$ is the vertex set of i^{th} copy of $DS(P_2 \odot K_3)$

Define a function $f: V(G) \to \{1,2, \dots . . q+1\}$ by $f(V_i^1)= 13i-10 ,1 \le i \le n$

$$f(V_i^2)= 13i-8 ,1 \le i \le n$$

$$f(V_i^3)= 13i-7 ,1 \le i \le n$$

$$f(V_i^{4}) = 13i - 5, 1 \leq i \leq n$$

$$f(V_i^{5}) = 13i - 3, 1 \leq i \leq n$$

$$f(V_i^{6}) = 13i - 1, 1 \leq i \leq n$$

$$f(w_i^{1}) = 13i - 12, 1 \leq i \leq n$$

$$f(w_i^{2}) = 13i, 1 \leq i \leq n$$

Then we get distinct edge labels from $\{\mathbf{1,2,\ldots,q}\}$ Hence G is a Heronian mean graph.

Example 2.13: Heronian mean labeling of $4DS(P_2 \odot K_3)$ is shown in figure 13.

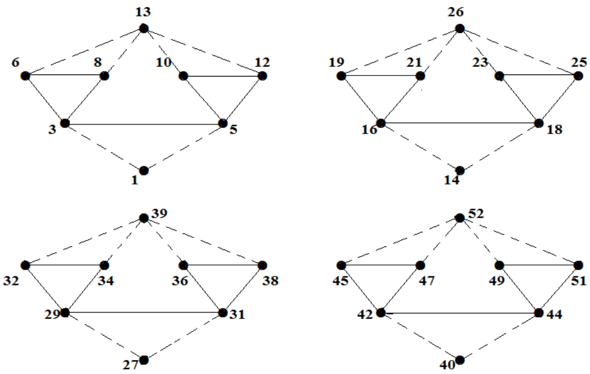

Figure 13.

3. Conclusion

In this paper, we studied the degree splitting behavior of some standard Heronian mean graphs. The authors are of the opinion that the study of Heronian mean labeling of degree splitting graphs will lead to newer and different results.

Acknowledgement

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