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Super Lehmer-3 Mean Labeling

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Abstract

Let f:V(G)→{1,2,…..p+q} be an injective function .The induced edge labeling f*(e=uv) is defined by ,f*(e)=$\left\lceil\frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2}\right\rceil$ (or) $\left\lfloor\frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2}\right\rfloor$, then f is called Super Lehmer-3 mean labeling, if {f (V(G))} U {f(e)/e Є E(G)} ={1,2,3,…..p+q}, A graph which admits Super Lehmer-3 Mean labeling is called Super Lehmer-3 Mean graph.

In this paper we prove that Path, Comb, Ladder, Crown are Super Lehmer-3 mean graphs.

Keywords: graph, Lehmer-3 mean graph, Super Lehmer-3 mean graph, Path, Comb, Ladder, Kite, Crown.

1. Introduction

A graph considered here are finite, undirected and simple. The vertex set and the edge set of a graph is denoted by V(G) and E(G) respectively. Lehmer mean is another type of generalized mean. A path of length n is denoted by P_n. For standard terminology and notations we follow Harary (1988) and for the detailed survey of graph labeling we follow J.A. Gallian (2010). S.Somasundaram, S.S Sandhya and R.Ponraj introduced the concept of Harmonic Mean Labeling of Graphs in (Somasundaram, Ponraj, & Sandhya) and its basic results was proved in (Somasundaram, Ponraj, & Sandhya). We will provide a brief summary of other in formations which are necessary for our present investigation.

Definition 1.1

A graph G=(V,E) with p vertices and q edges is called **Lehmer-3 mean graph**. If it is possible to label vertices x Є V with distinct labels f(x) from 1, 2, 3,……q+1 in such a way that when each edge e=uv is labeled with f(e=uv)=$\left\lceil\frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2}\right\rceil$ (or) $\left\lfloor\frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2}\right\rfloor$, then the edge labels are distinct. In this case "f " is called Lehmer-3 mean labeling of G.

Definition 1.2

A **Path** P_n is obtained by joining u_i to the consecutive vertices u_{i+1} for $1\le i\le n$

Definition 1.3

A graph obtained by joining a single pendant edge to each vertex of a path is called a **comb**

Definition 1.4

A product graph $P_m x P_n$ is called a planar grid $P_2 x P_n$ is called a **Ladder**.

Definition 1.5

Crown is a graph obtained by joining a single pendant edge to each vertex of a cycle.

Definition 1.6

The **corona** of two graphs G_1 and G_2 is the graph G = $G_1 \Theta G_2$ formed from one copy of G_1 and $|V(G_1)|$ copies of G_2 where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2.

2. Main Results

Theorem:2.1

A Path P_n is a Super Lehmer-3 mean graph.

Proof:

Let P_n be a Path $v_1,v_2,\ldots,v_n$ with edge set $E=\{v_iv_{i+1}/1\leq i\leq n-1\}$

Define a function $f:V(P_n)\rightarrow\{1,2,\ldots..p+q\}$ by

$$f(v_i)=2i-1;\ 1\leq i\leq n.$$

Then the induced edge labels are

$$f^*(v_iv_{i+1})=2i;\ 1\leq i\leq n-1$$

Therefore $f(V(P_n)Uf(e))=\{1,2,3,\ldots..p+q\}$

Hence P_n is a Super Lehmer-3 mean graph

Example:2.2

A Super Lehmer-3 mean labeling of P_6 is given below.

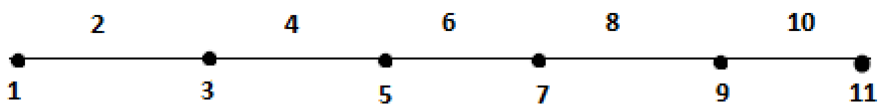

Figure 1.

Theorem: 2.3

$(P_n\Theta K_1)$ is a Super Lehmer-3 mean graph.

Proof:

Let G be a Comb obtained from a path $P_n= v_1,v_2,\ldots,v_n$ by joining the vertex v_i to u_i where $1\leq i\leq n$ and hence the edge set is $E=\{u_iu_{i+1}/1\leq i\leq n-1\}U\ \{u_iv_{i+1}/1\leq i\leq n\}$

Define a function $f:V(G)\rightarrow\{1,2,\ldots..p+q\}$ by

$$f(u_i)=4i-3;\ 1\leq i\leq n$$

$$f(v_i)=4i-1;\ 1\leq i\leq n$$

Thus the edges are labeled with

$$f^*(u_iu_{i+1})=4i;\ 1\leq i\leq n-1$$

$$f^*(u_iv_i)=4i-2;\ 1\leq i\leq n$$

Therefore $f(V(G))\ U\ \{f(e)/e\in E(G)\}=\{1,2,3,\ldots..p+q\}$

Thus f is a Super Lehmer-3 mean graph

Example: 2.4

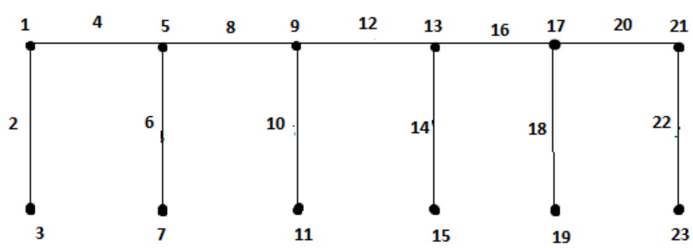

Figure 2.

A Super Lehmer-3 mean labeling of $P_6\Theta\ K_1$ is drawn above

Theorem: 2.5

A Ladder is a Super Lehmer-3 mean graph.

Proof:

Let G be a ladder L_n obtained from a path $P_n=v_1,v_2,\ldots\ldots v_n$ and $u_1,u_2,\ldots\ldots u_n$ joining u_i to v_i and u_i to u_{i+1} ,v_i to v_{i+1}.

Define a function $f:V(G)\rightarrow\{1,2,\ldots..p+q\}$ by

$$f(u_i)=2i-1;\ 1\leq i\leq n$$

$$f(v_1)=u_n+2;$$

here u_n denote the last vertex label of path u_i

$$f(v_i)=v_1+(3i-3);\ 2\leq i\leq n,$$

where v_1 denote the first vertex label of path v_i

Thus we get distinct edge labels.

Therefore f (V(G) U {f(e)/e Є E(G)) ={1,2,3,…..p+q}

Hence f is a Super Lehmer-3 mean graph.

Example: 2.6

L_5 is a Super Lehmer-3 mean graph

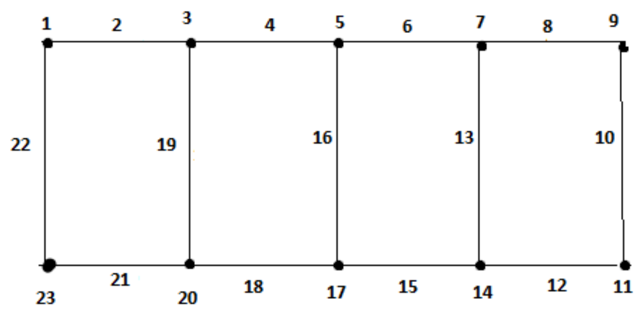

Figure 3.

Theorem: 2.7

Let G be a graph obtained by identifying a pendant vertex P_n and an end vertex C_3. Then G admits a Super Lehmer-3 mean labeling.

Proof:

Let P_n be a path $u_1,u_2,\ldots.u_n$ and uvw be a cycle C_3. Identify u with u_n. Then the resultant graph is G.

Define a function $f: V(G)\rightarrow\{1,2,\ldots..p+q\}$ by

$$f(u_i)=2i-1;\ 1\leq i\leq n$$

$$f(v)=2n+1$$

$$f(w)=2n+4$$

Thus the edges are labeled with

$$f^*(u_iu_{i+1})=2i;\ 1\leq i\leq n-1$$

$$f^*(u_nv)=2n$$

$$f^*(u_nw)=2n+2$$

$$f^*(vw)=2n+3$$

hence by the above labeling pattern { f V(G) U f(e)/e Є E(G)} ={1,2,3,…..p+q}

Thus G admits a Super Lehmer-3 mean labeling.

Example: 2.8

A Super Lehmer-3 mean labeling of G when n=6 is given below

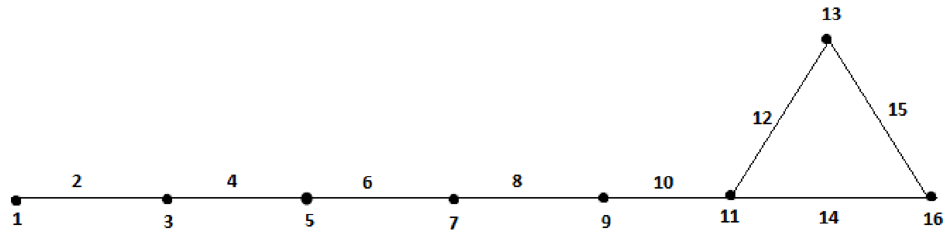

Figure 4.

Theorem: 2.9

$C_n \Theta K_1$ is a Super Lehmer-3 mean graph

Proof:

Let $u_1, u_2, u_3, \ldots u_n, u_1$ be a cycle of n vertices . Add a new vertices v_i such that v_i is adjacent to u_i, $1 \leq i \leq n$. Then define a function $f: V(C_n \Theta K_1) \rightarrow \{1,2,\ldots p+q\}$ by

$$f(u_1)=3$$

$$f(u_i)=4i-3;\ 2 \leq i \leq n-1$$

$$f(u_n)=4n-2$$

$$f(v_1)=1$$

$$f(v_i)=4i-1;\ 2 \leq i \leq n-1$$

$$f(v_n)=4n$$

Then the edges are labeled with

$$f^*(u_i u_{i+1})=4i;\ 1 \leq i \leq n-1$$

$$f^*(u_1 u_n)=4n-3$$

$$f^*(u_i v_i)=4i-2;\ 1 \leq i \leq n-1$$

$$f^*(u_n v_n)=4n-1$$

Thus vertices and edges together get distinct labels from $\{1,2,3,\ldots p+q\}$

Hence $C_n \Theta K_1$ is a Super Lehmer-3 mean graph

Example: 2.10

The labeling pattern of $C_6 \Theta K_1$ is

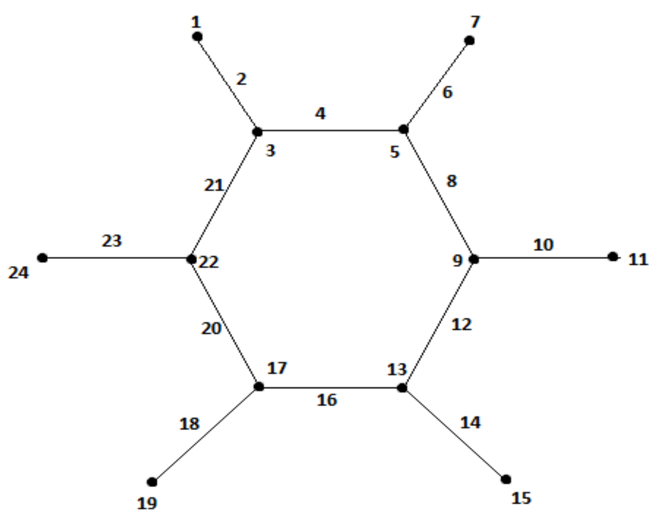

Figure 5.

Theorem: 2.11

nP_m is a Super Lehmer-3 mean graph

Proof:

Let v_{ij}, $1 \le i \le n$, $1 \le j \le m$ be the vertices of nP_m

Then the edge set is $E=\{v_{i,j}v_{i,j+1} / 1 \le i \le n, 1 \le j \le m-1\}$

Define a function $f: V(nP_m) \to \{1,2,\ldots..p+q\}$ by

$$f^*(v_{i,j})=(2m-1)(i-1)+(2j-1);\ 1 \le i \le n,\ 1 \le j \le m$$

Thus the induced edge labels are

$$f(v_{i,j}\, v_{i,j+1})=(2m-1)(i-1)+(2j);\ 1 \le i \le n,\ 1 \le j \le m-1$$

Thus f provides a Super Lehmer-3 mean labeling of nP_m

Example: 2.12

A Super Lehmer-3 mean labeling of $5P_6$ is given below

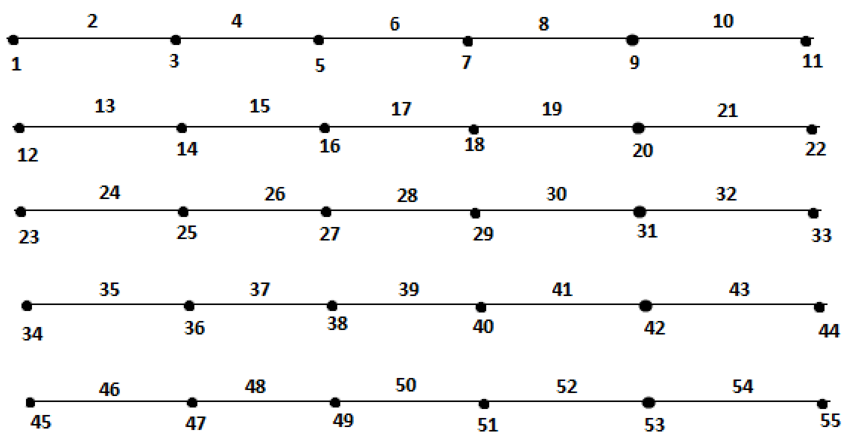

Figure 6.

Theorem: 2.13

$(P_n \Theta K_1) \cup P_m$ is a Super Lehmer-3 mean graph.

Proof:-

Let G be a graph obtained by the union of $(P_n \Theta K_1)$ and P_m

Let $(P_n \Theta K_1)$ be a graph with n vertices $u_1, u_2, \ldots..u_n$ and $v_1, v_2, \ldots.v_n$ respectively.

Let the vertices of P_m be $w_1, w_2, \ldots w_m$

Define a function $f: V(G) \to \{1,2,\ldots..p+q\}$ by

$$f(u_i)=4i-3;\ 1 \le i \le n$$

$$f(v_i)=4n-1;\ 1 \le i \le n$$

$$f(w_j)=v_n+(2j-1);\ 1 \le j \le m$$

Then the induced edge labels are

$$f^*(u_i\, u_{i+1});\ 1 \le i \le n$$

$$f^*(u_i\, v_i)=4i-2;\ 1 \le i \le n$$

$$f^*(w_j\, w_{j+1})=f(u_n v_n)+(2j+1);\ 1 \le j \le m-1$$

Thus the vertices and edges together get distinct labels from $\{1, 2, \ldots..p+q\}$.

This provides a Super Lehmer-3 mean labeling for $(P_n \Theta K_1) \cup P_m$

Example: 2.14

The Super Lehmer-3 mean labeling of $(P_6 \Theta K_1) \cup P_5$ is

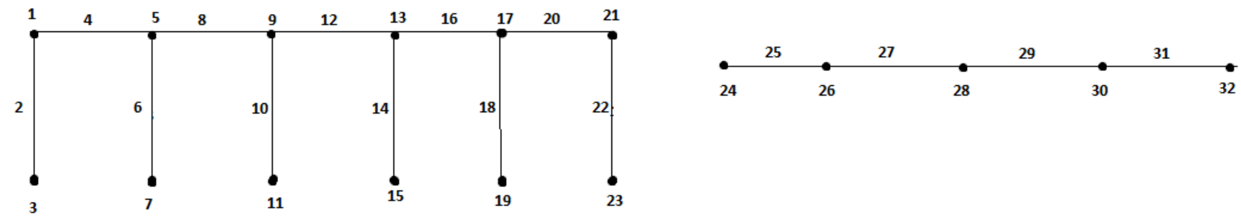

Figure 7.

Theorem: 2.15

(Kite)$\cup$ P_m is a Super Lehmer-3 mean graph.

Proof:-

Let G be a graph obtained from the union of kite and path

The vertices of kite be u_1, u_2, …….u_n and uvw. Identify u with u_n, uvw be a cycle

Let P_m be a graph with m vertices. Then the resultant graph is G

Define a function f: V(G)→{1,2,…..p+q} by

$$f(u_i)=2i-1;\ 1\le i\le n$$

$$f(v)=2n+1$$

$$f(w)=2n+4$$

$$f(x_j)=f(w)+(2j-1);\ 1\le j\le m$$

Thus the edges are labeled with

$$f^*(u_iu_{i+1})=2i;\ 1\le i\le n-1$$

$$f^*(u_nv)=2n$$

$$f^*(u_nw)=2n+2$$

$$f^*(vw)=2n+3$$

$$f^*(x_jx_{j+1})=f(vw)+(2j-1);\ 1\le j\le n$$

By the above labeling pattern {f V(G) U f(e)/e Є E(G)}={1,2,3,…..p+q}

Hence G admits a Super Lehmer-3 mean labeling.

Example: 2.16

The Super Lehmer-3 mean labeling pattern is given below

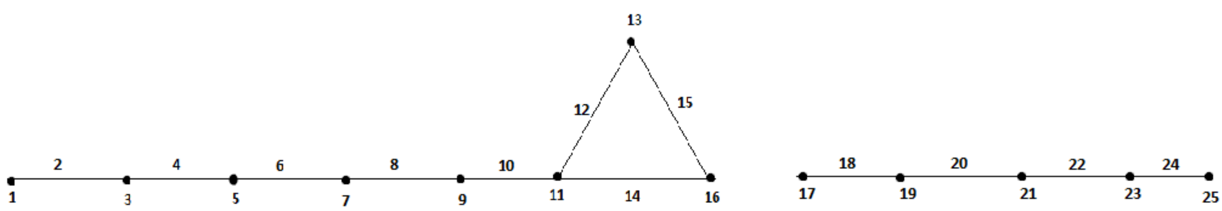

Figure 8.

Theorem: 2.17

($C_n\Theta K_1$)$\cup$ P_m is a Super Lehmer-3 mean graph

Proof:

Let $u_1,u_2,u_3,\ldots..u_n,u_1$ be the vertices of a cycle C_n. Add a new vertices v_i such that v_i is adjacent to u_i, $1\le i\le n$.

Let P_m be a path with m vertices

Define a function f : V(($C_n\Theta K_1$)$\cup$$P_m$)→{1,2,…..p+q} by

$$f(u_1)=3$$

$$f(u_i)=4i-3;\ 2\le i\le n-1$$

$$f(u_n)=4n-2$$
$$f(v_1)=1$$
$$f(v_i)=4i-1;\ 2\leq i\leq n-1$$
$$f(v_n)=4n$$
$$f(w_j)=f(v_n)+(2j-1);\ 1\leq j\leq m$$

Thus vertices and edges together get distinct labels from {1, 2, 3….. p+q}

Hence $(C_n \Theta K_1) \cup P_m$ is a Super Lehmer-3 mean graph

Example: 2.18

A Super Lehmer-3 mean labeling of $(C_6 \Theta K_1) \cup P_5$ is

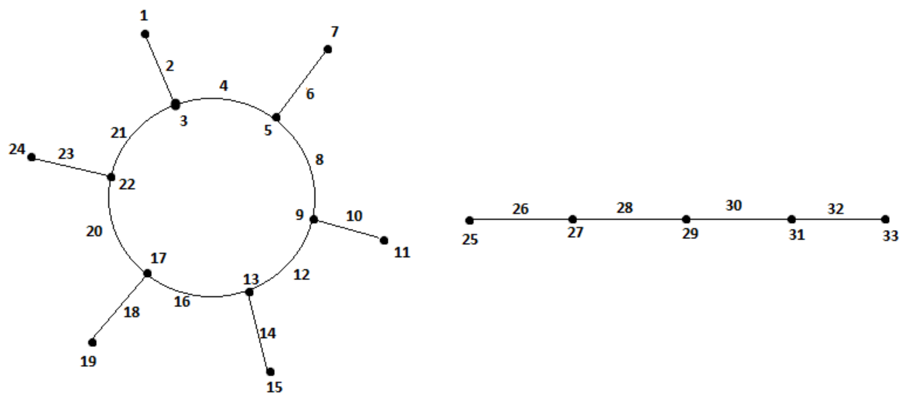

Figure 9.

Theorem: 2.19

$(C_n \Theta K_1) \cup (P_m \Theta K_1)$ is a Super Lehmer-3 mean graph

Proof:

Let $u_1, u_2, u_3, \ldots u_n, u_1$ be a cycle C_n. Add a new vertices v_i such that v_i is adjacent to u_i, $1\leq i\leq n$.

Let $(P_m \Theta K_1)$ is a comb of m vertices $w_1, w_2 \ldots w_m$; $x_1, x_2, \ldots x_m$ respectively

The graph G is defined by a function $f : V(G) \rightarrow \{1,2,\ldots p+q\}$ by

$$f(u_1)=3$$
$$f(u_i)=4i-3;\ 2\leq i\leq n-1$$
$$f(u_n)=4n-2$$
$$f(v_1)=1$$
$$f(v_i)=4i-1;\ 2\leq i\leq n-1$$
$$f(v_n)=4n$$
$$f(w_j)=f(v_n)+(4j-3);\ 1\leq j\leq m$$
$$f(x_j)=f(v_n)+(4j-1);\ 1\leq j\leq m$$

Then the vertices and edges together get distinct labels from {1, 2, 3….. p+q}

Hence $(C_n \Theta K_1) \cup (P_m \Theta K_1)$ forms a Super Lehmer-3 mean graph

Example: 2.20

A Super Lehmer-3 mean labeling of $(C_6 \Theta K_1) \cup P_5$ is

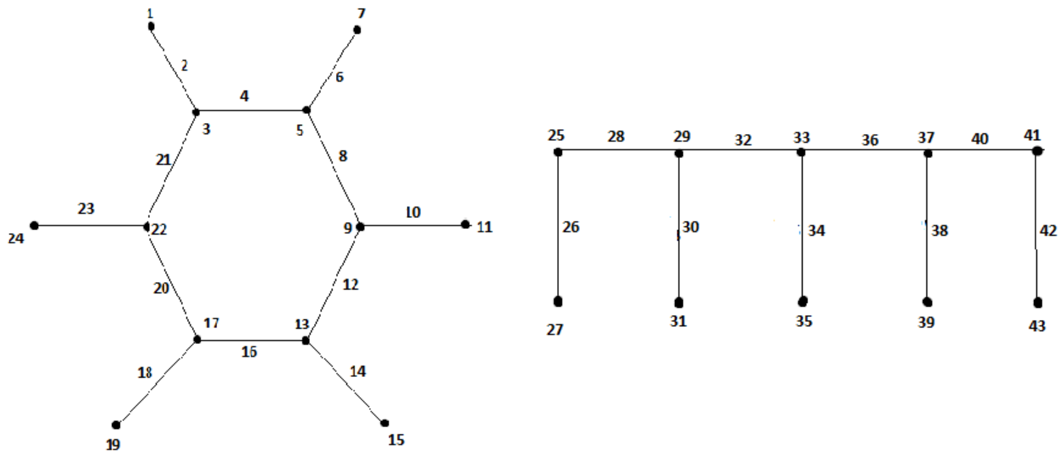

Figure 10.

3. Conclusion

Hence the union of two Super Lehmer-3 mean graph is again a Super Lehmer-3 mean graph.

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