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Rigorous Proof for Riemann Hypothesis Using the Novel Sigma-power Laws and Concepts from the Hybrid Method of Integer Sequence Classification

John Y. C. Ting[1]

[1] Rural Generalist in Anesthesia and Emergency medicine, Dental and Medical Surgery, 729 Albany Creek Road, Albany Creek, Queensland 4035, Australia

Correspondence: Dr. John Y. C. Ting, 729 Albany Creek Road, Albany Creek, Queensland 4035, Australia. E-mail: jycting@hotmail.com

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Abstract

Proposed by Bernhard Riemann in 1859, Riemann hypothesis refers to the famous conjecture explicitly equivalent to the mathematical statement that the critical line in the critical strip of Riemann zeta function is the location for all non-trivial zeros. The Dirichlet eta function is the proxy for Riemann zeta function. We treat and closely analyze both functions as unique mathematical objects looking for key intrinsic properties and behaviors. We discovered our key formula (coined the Sigma-power law) which is based on our key Ratio (coined the Riemann-Dirichlet Ratio). We recognize and propose the Sigma-power laws (in both the Dirichlet and Riemann versions) and the Riemann-Dirichlet Ratio, together with their various underlying mathematically-consistent properties, in providing crucial *de novo* evidences for the most direct, basic and elementary mathematical proof for Riemann hypothesis. This overall proof is succinctly summarized for the reader by the sequential Theorem I to IV in the second paragraph of Introduction section. Concepts from the Hybrid method of Integer Sequence classification are important mathematical tools employed in this paper. We note the intuitively useful mental picture for the idea of the Hybrid integer sequence metaphorically becoming the non-Hybrid integer sequence with certain criteria obtained using Ratio study.

Keywords: Riemann hypothesis, Ratio study, Sigma-power laws, Dimensional analysis homogeneity

Subject classification: 11M26, 10H05

1. Introduction

In 1859, the famous German mathematician Bernhard Riemann proposed the Riemann hypothesis. Hilberts problems are a list of 23 mathematical problems published by the German mathematician David Hilbert in 1900. Millennium Prize Problems are 7 problems in mathematics that were stated by the Clay Mathematics Institute in 2000. The unsolved Riemann hypothesis problem belongs to Hilberts eighth problem and is one of the Millennium Prize Problems. The official statement of Riemann hypothesis as a Millennium Prize Problem was given by mathematician Enrico Bombieri. The proof or disproof of Riemann hypothesis would have far-reaching implications, for instance, in number theory especially for the distribution of prime numbers.

Riemann hypothesis refers to the famous conjecture explicitly equivalent to the mathematical statement that the critical line ($\sigma = \frac{1}{2}$) in the critical strip ($0 < \sigma < 1$) of Riemann zeta function is the location for all of its non-trivial zeros. The critical strip of Riemann zeta (ζ) function can be represented by the one and only one critical line in the middle and an infinite number of other parallel lines on either side of this critical line, with every single line mathematically described by this function with a particular designated sigma (σ) value comprising of real numbers. The contents of this paper provide all necessary evidence for, and are centered on, the following four theorems as the main theme.

Theorem I: The exact same Riemann-Dirichlet Ratio, directly derived from either the Riemann zeta or Dirichlet eta function, is an irrefutably accurate mathematical expression on the *de novo* criteria for the actual presence [but not the actual locations] of the complete set of (identical) infinite non-trivial zeros in both functions.

Theorem II: Both the near-identical (by proportionality factor-related) Riemann Sigma-power law and Dirichlet Sigma-power law with their derivations based on either the numerator or denominator of Riemann-Dirichlet Ratio have Dimensional analysis (DA) homogeneity only when their common and unknown σ variable has a value of $\frac{1}{2}$ as its solution.

Theorem III: The σ variable with value of $\frac{1}{2}$ derived using the Sigma-power law [from Theorem II above] is the exact same σ variable in Riemann hypothesis which conjectured σ to also have the value of $\frac{1}{2}$ (representing the critical line with $\sigma = \frac{1}{2}$ in the critical strip with $0 < \sigma < 1$) for the location of all non-trivial zeros of Riemann zeta function [and Dirichlet eta function by default], thus providing irrefutable evidence for this Riemann conjecture to be correct with further clarification from Theorem IV below.

Theorem IV: Condition 1. Any other values of σ apart from the $\frac{1}{2}$ value arising from $0 < \sigma < \frac{1}{2}$ and $\frac{1}{2} < \sigma < 1$ in the critical strip does not contain any non-trivial zeros ["the DA-wise mathematical impossibility argument" with resulting *de novo* DA non-homogeneity], together with Condition 2. The one and only one value of $\frac{1}{2}$ for σ in the critical strip contains all the non-trivial zeros ["the DA-wise one and only one mathematical possibility argument" with resulting *de novo* DA homogeneity] from Theorem III, fully support the rather mute, but nevertheless the whole, point of study in this paper that Riemann hypothesis is proven to be true when these two (mutually inclusive) conditions are met.

As the Dirichlet eta function is essentially the surrogate for Riemann zeta function, we treat and closely analyze both functions as unique mathematical objects looking for key intrinsic properties and behaviors. We discovered our key formula (coined the Sigma-power law) and our key Ratio (coined the Riemann-Dirichlet Ratio) with the aid of Dimensional analysis, Ratio study, Calculus, and concepts from the novel Hybrid method of Integer Sequence classification. In so doing we recognize that it is the (i) Sigma-power laws in both the Dirichlet and Riemann versions which are based on either the numerator or denominator part of (ii) Riemann-Dirichlet Ratio, together with their various underlying mathematically-consistent properties, that crucially provide hidden *de novo* evidences for the most direct, basic and elementary mathematical proof for Riemann hypothesis.

The derivation of the two Sigma(σ)-power laws has a qualitative Dimensional analysis component of equating either the numerator or denominator respective portions from each of the two sub-ratios of the Riemann-Dirichlet Ratio with their underlying complete mathematical expressions. The relevant σ-power laws derived using either the relevant parameter-{2n} numerator or shifted-by-one parameter-{2n-1} denominator is justifiably mathematically equivalent to each other and also being related by a common proportionality constant. Thus this paper could literally be summed up by the one concise sentence "The Dimensional analysis homogeneity property of the Sigma-power law provides the definitive mathematical proof for Riemann hypothesis to be true".

The Riemann zeta function, denoted by ζ(s) with s = $\sigma + \imath$t, has both its sum input $\Sigma ReIm\{s\}$[= $Re\{s\}$ depicted graphically on the x-axis + Im{s} depicted on the y-axis] and sum output ΣReIm{ζ(s)} [= Re{ζ (s)} depicted on the x-axis + Im{ζ(s)} depicted on the y-axis] constituted by complex numbers. The set of zeros, or roots, in ζ(s) consist of (the easily identifiable) trivial zeros and (the not-so-easily identifiable) non-trivial zeros both of infinite magnitude.

Like any other function, a key point of responsibility is tightly respecting the correct use of *Concept of a function* and various *Representations of that function* to validate its special properties. This will enable us to target our main goal of comprehending why all of the non-trivial zeros should lie on a particular vertical straight line called the critical line ($\sigma = \frac{1}{2}$). Manifesting as conjugate pairs of non-trivial zeros, they can succinctly be denoted by $\zeta(\frac{1}{2} \pm \imath t) = 0$. Gram points are the other conjugate pairs values on the critical line defined by $Im\{\zeta(\frac{1}{2} \pm \imath t)\} = 0$ whereby they obey Grams rule and Rossers rule with many other interesting characteristics needing a detailed treatise and constituting a separate topic on its own. In this research article, similar treatment on Gram points is not embarked upon - this will be carried out in our next planned publication after this paper (titled *Key role of Dimensional analysis homogeneity in proving Riemann hypothesis and providing explanations on the closely related Gram points*) thus establishing continuity and treating these two articles almost as one. In practice, the positive ($0 < t < +\infty$), and numerically equal to the negative ($-\infty < t < 0$), counterpart of the conjugate pairs for the zeros of $\zeta(s)$ and its Gram points is usually quoted, or employed for calculation purposes.

Finally as a bonus, we provide an intuitively useful mental picture for visualizing the idea of the Hybrid integer sequence metaphorically becoming the non-Hybrid (usual "garden-variety") integer sequence when, from Ratio study, the character for each of the numerator / denominator integer sequence in the selected Ratio change from being near-identical Class function [in Hybrid integer sequences, typified by A228186 from *The On-line Encyclopedia of Integer Sequences*] to identical Class function [in non-Hybrid integer sequences, typified in this paper by A100967 from The *On-line Encyclopedia of Integer Sequences*, and all infinite series from, or arising out of, the Riemann zeta and Dirichlet eta functions]. Here, the term Class function refers to the format of the integer sequences underlying mathematical expression. We stress from the outset that, for the purpose of this study, at least one mathematically designed Ratio (from Ratio study) having a cyclical nature to it must be present, and that this is a *sine-qua-non* prerequisite for the particular Ratio to be considered useful or relevant.

2. Riemann Conjecture and Riemann Hypothesis

In this section, the essential difference between a conjecture and a hypothesis is expounded below. We may risk being seen as pedantic by advocating that the traditionally-dubbed Riemann *hypothesis* should instead be previously labeled the Riemann *conjecture* as this entity was chronologically used in the era prior to a rigorous proof being obtained for the conjecture. In other words, once proven only then should a particular conjecture be strictly termed a hypothesis. A colloquial description of Riemann hypothesis and its broad consequences is now typified by us in the following imaginary conversation [adapted from a previous description by Princeton mathematician Peter Sarnak in page 222 of *The Riemann hypothesis: The greatest unsolved problem in mathematics* (Sabbagh, 2002)].

”There must be over five hundred previous papers which start with Assume the Riemann *conjecture* is true and the conclusion is fantastic. Those conclusions have now become theorems ever since this conjecture has been proven to be true. Riemann conjecture has at long last become Riemann hypothesis. With this one solution we have proven five hundred theorems or more at once.” A knowledgeable book planned for the general public entitled *Prime Numbers and the Riemann hypothesis* (Mazur & Stein, 2016) will have, as the title suggested, a large proportion of its content devoted to various wonderful direct or indirect relationships between prime numbers and the Riemann hypothesis - rather this term should contextually have been stated in the pre-proof era as Riemann *conjecture* (instead of Riemann *hypothesis*) with relationships here being its broad consequences or impacts on prime numbers, and vice versa.

3. Riemann zeta and Dirichlet eta Functions

The infinite series Riemann zeta function ζ(s) [represented by Eqs. (1), (2), and (3) below where n = 1, 2, 3,. . . , ∞] is regarded as one of the most influential mathematical objects with great importance in many branches of contemporary science and mathematics. Instead of the commonly or conventionally used z symbol for its complex variable, this variable is denoted traditionally in this paper by $s(=\sigma+\imath t)$ where $i=\sqrt{-1}$ is the imaginary number; σ (consisting of real numbers with values $-\infty<\sigma<+\infty$) refers to the argument for the real part of s [denoted by Re{s}]; and t (consisting of real numbers with values $-\infty<t<+\infty$) refers to the argument for the imaginary part of s [denoted by Im{s}].

$$\begin{aligned}\zeta(s) &= \sum_{n=1}^{\infty}\frac{1}{n^s} \\ &= \frac{1}{1^s}+\frac{1}{2^s}+\frac{1}{3^s}+\cdots\end{aligned}\tag{1}$$

Eq. (1) is defined only for $1<\sigma<\infty$ when $\zeta(s)$ is absolutely convergent.

$$\zeta(s)=2^s\pi^{s-1}\sin\left(\frac{\pi s}{2}\right).\Gamma(1-s).\zeta(1-s)\tag{2}$$

Eq. (2) is the Riemann’s functional equation satisfying $-\infty<\sigma<\infty$ and can be used to find all the trivial zeros on the horizontal line at $\imath t=0$ and σ = -2, -4, -6, . . . , ∞ [all negative even integer] whereby $\zeta(s)=0$ because the factor $\sin(\frac{\pi s}{2})$ vanishes. Γ is the gamma function, an extension of the factorial function [a product function denoted by the ! notation; n! = n(n-1)(n-2). . . (n-(n-1))] with its argument shifted down by 1, to real and complex numbers. That is, if n is a positive integer, $\Gamma(n)=(n-1)!$

$$\begin{aligned}\zeta(s) &= \frac{1}{(1-2^{1-s})}.\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^s} \\ &= \frac{1}{(1-2^{1-s})}.\left(\frac{1}{1^s}-\frac{1}{2^s}+\frac{1}{3^s}-\cdots\right)\end{aligned}\tag{3}$$

Eq. (3) is defined for all $\sigma>0$ except for a simple pole at $\sigma=1$. ζ(s) without the $\frac{1}{(1-2^{1-s})}$ proportionality factor, viz. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^s}$ is also known as Dirichlet eta (η) or alternating zeta function. This $\eta(s)$ function is a holomorphic function of s as defined by analytic continuation and can mathematically be seen to be defined at $\sigma=1$ whereby an analogous trivial zeros [with presence only] for η(s) [and not for ζ(s)] on the vertical straight line $\sigma=1$ are obtained at $s=1\pm i.\frac{2\pi k}{\log(2)}$ where k = 1, 2, 3, . . . , ∞. In this paper, unless stated otherwise, the symbol ’log’ will refer to natural logarithm. As ζ(s) is closely related by the proportionality factor to η(s), it can be seen that all non-trivial zeros of η(s) must be identical to those of $\zeta(s)$ - thus the statement that all the non-trivial zeros of η(s) in the critical strip are on the critical line ($\sigma=\frac{1}{2}$) is in accordance with [an alternative version of] the Riemann hypothesis is also true.

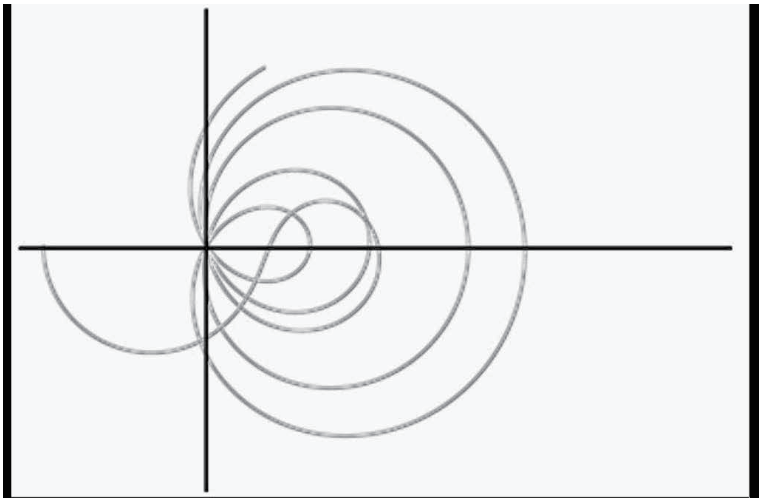

Figure 1a

Schematically depicted polar graph of $\zeta(\frac{1}{2}+\imath t)$ in Figure 3a with plot of $\zeta(s)$ along the critical line for real values of t running from 0 to 34, horizontal axis: $Re\{\zeta(\frac{1}{2}+\imath t)\}$, and vertical axis: $Im\{\zeta(\frac{1}{2}+\imath t)\}$. There is consistent *mathematical symmetry* about the horizontal axis and a plot of $\zeta(\frac{1}{2}+\imath t)$ would have revealed an identical mirror image graph reflected on this axis. The first 5 non-trivial zeros in the critical strip are geometrically visualized as the place where the spirals pass through the origin. This phenomenon should occur infinitely often as the real number values for t are also infinite. This has previously been checked for the first 10,000,000,000,000 non-trivial zeros solutions. Computationally this implies, but does not mathematically prove, that the complete set of non-trivial zeros occur at $\sigma = \frac{1}{2}$.

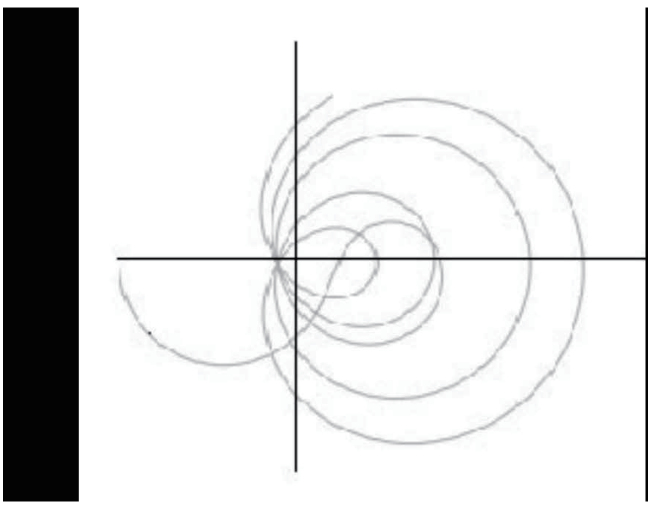

Figure 1b

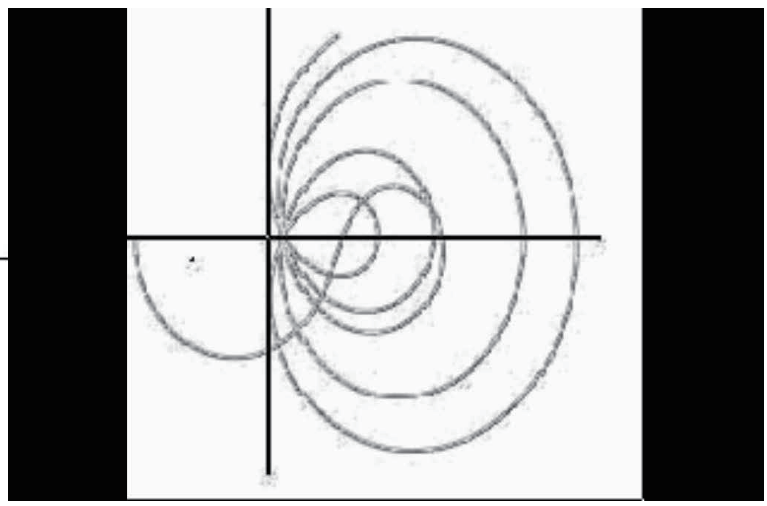

Figure 1c

Figure 1b $\zeta(\dfrac{2}{5}+\imath t)$ on the left and Figure 1c $\zeta(\dfrac{3}{5}+\imath t)$ on the right are schematically plotted with similar tactics that were used in Figure 1a. Note the relevant spirals with 'opposite' distortion now crossing the horizontal axis to the left of the origin in Figure 1b and to the right of the origin in Figure 1c. The pictorial representation of the first 5 non-trivial zeros in Figure 1a as the spirals passing through the origin do not occur anymore when the σ from $0 < \sigma < 1$ is of values other than $\frac{1}{2}$. Riemann hypothesis can also be stated as the condition of total absence of non-trivial zeros in the critical strip when σ satisfy $0 < \sigma < \frac{1}{2}$ and $\frac{1}{2} < \sigma < 1$.

4. Combinatorics Ratio

We initially referred to the proposed role of Combinatorics (in particular, the ubiquitous nature of Permutation with repetition) in Appendix 2 of our medical research paper *Supramaximal elevation in B-type natriuretic peptide and its N-terminal fragment levels in anephric patients with heart failure: a case series* (Ting & Pussell, 2012). Specifically, this role refers to its place in the mathematically-enriched arguments for the devised 'Blood Volume - B-type natriuretic peptide feedback control system' in providing plausible explanations for the study findings.

The topics of Permutations (P) and Combinations (C) come under Combinatorics. P is an ordered C. There are two types of P and two types of C with their relevant formulae given below, with n = 0, 1, 2,..., ∞. Note: (i) The variables n and k here for Combinatorics are chosen to start from 0 - whereas the n variable for the ζ(s) and η(s), and their related functions elsewhere in this paper starts from 1, (ii) The four equations below could be thought of having near-identical 'Class function' properties - belonging to either the P or C class under the Combinatorics umbrella, and apart from Eq. (4), all contains the discrete factorial (!) function which we have previously noted above to be intimately related to the continuous Γ function via $\Gamma(n) = (n-1)!$, and (iii) Binomial is C without repetition, often denoted as C_n^k.

$$\text{P with repetition:} \quad k^{n+2} \tag{4}$$

$$\text{P without repetition:} \quad \frac{k!}{(k-n-2)!} \tag{5}$$

$$\text{C with repetition:} \quad \frac{(k+n+1)!}{(n+2)!(k-1)!} \tag{6}$$

$$\text{C without repetition:} \quad \frac{k!}{(n+2)!(k-n-2)!} \tag{7}$$

Numerically, Eqs. (4) >(5) >(6) >(7) always holds true. Performing Ratio study, we define the selected ratio coined the *Combinatorics Ratio* = $\frac{\textbf{C with repetition}}{\textbf{C without repetition}}$. An important aspect of Combinatorics Ratio involves its novelty in supplying us with a Hybrid integer sequence (A228186) via the defined inequality relationship "Greatest k >n such that Combinatorics Ratio <2 is a maximum [= $\frac{(k+n+1)!(k-n-2)!}{k!(k-1)!}$ < 2 is a maximum]" as part of the Hybrid method of Integer Sequence classification. This was previously published by us on *The On-line Encyclopedia of Integer Sequences* website as A228186, https://oeis.org/A228186 (Ting, 2013). To the best of our knowledge, A228186 is the first ever Hybrid integer sequence artificially synthesized from the Combinatorics Ratio defined with the inequality criteria.

Hybrid integer sequence A228186 is equal to non-Hybrid integer sequence A100967 except for the 21 'exceptional' terms at positions 0, 11, 13, 19, 21, 28, 30, 37, 39, 45, 50, 51, 52, 55, 57, 62, 66, 70, 73, 77, and 81 with their values given by the relevant A100967 term plus 1. A100967 (Noe, 2004) in *The On-line Encyclopedia of Integer Sequences*, is defined by Greatest k such that Binomial (2k+1, k-n-1) ≥ Binomial (2k, k) whereby n = 0, 1, 2,...,∞, and was authored by Tony Noe. Note (i) the identical 'Class function' [Binomial, which is C without repetition] for numerator and denominator expressed in ratio $\frac{\textbf{C without repetition}}{\textbf{C without repetition}} = \frac{\text{Binomial (2k+1, k-n-1)}}{\text{Binomial(2k, k)}}$ or $\frac{(2k+1)!(k+2)!(k-2)!}{2k!(k+n)!(k-n+1)!} \geq 1$; and (ii) that A228186 can be seen to be a hybrid of two infinite series A100967(+0) and A100967(+1) - the convention employed here being A100967(+0) and A100967(+1) respectfully symbolizing add 0 and add 1 to every single A100967 term. The totally predictable 21 'exceptional' terms of A228186, possessing deterministic pseudo-randomness and self-similarity properties, makes A228186 a novel true pseudorandom infinite-length integer sequence with highly significant connections to the mathematical fields of Chaos and Fractals. Using logical deduction, the limited number of 21 'exceptional' terms can be explained by the following *mathematical constraint* - with progressively higher 'exceptional' terms, the fractional part of the Combinatorics Ratio using (k+1) values will overall be monotonously rising steadily approaching a value of 1 (boundary condition), which then limit the total number of possible exceptional terms to just 21.

5. Riemann-Dirichlet Ratio

Euler formula is commonly stated as $e^{\imath x} = \cos x + \imath.\sin x$. The magnificent Euler identity (where $x = \pi$) is $e^{\imath\pi} = \cos\pi + \imath.\sin\pi = -1 + 0$, commonly stated as $e^{\imath\pi} + 1 = 0$. The n^s of Riemann zeta function can be expanded to $n^s = n^{(\sigma+\imath t)} = n^{\sigma}.e^{t.\log(n).\imath}$ since $n^t = e^{t.\log(n)}$. Apply the Euler formula to n^s will result in $n^s = n^{\sigma}.(\cos(t.\log(n)) + \imath.\sin(t.\log(n))$ - designated here with the short-hand notation $n^s(Euler)$ - whereby n^{σ} is the modulus and $t.\log(n)$ is the polar angle.

Apply $n^s(Euler)$ to Eq. (1), we have $\zeta(s) = Re\{\zeta(s)\} + \imath.Im\{\zeta(s)\}$ whereby $Re\{\zeta(s)\} = \sum_{n=1}^{\infty} n^{-\sigma}.\cos(t.\log(n))$ and $Im\{\zeta(s)\}$

$= \mathrm{i}.\sum_{n=1}^{\infty} n^{-\sigma}. \sin(t. \log(n))$. As Eq. (1) is defined only for $\sigma > 1$ where zeros never occur, we will not carry out further treatment related to this subject area.

Apply $n^{s}(Euler)$ to Eq. (3), we have $\zeta(s) = \gamma.\eta(s) = \gamma.[Re\{\eta(s)\} + \imath.Im\{\eta(s)\}]$ whereby

$Re\{\eta(s)\} = \sum_{n=1}^{\infty}((2n-1)^{-\sigma}. \cos(t. \log(2n-1)) - (2n)^{-\sigma}. \cos(t. \log(2n)))$ and

$Im\{\eta(s)\} = \imath.\sum_{n=1}^{\infty}((2n)^{-\sigma}. \sin(t. \log(2n)) - (2n-1)^{-\sigma}. \sin(t. \log(2n)))$. Here γ is the proportionality factor $\frac{1}{(1-2^{1-s})}$.

Apply the trigonometry identity $\cos(x) - \sin(x) = \sqrt{2}. \sin\left(x + \frac{3}{4}\pi\right)$ to $\sum ReIm\{\eta(s)\} = Re\{\eta(s)\} + Im\{\eta(s)\}$. Then,

$$\begin{aligned}
\sum ReIm\{\eta(s)\} = &\sum_{n=1}^{\infty}[(2n-1)^{-\sigma}. \cos(t. \log(2n-1))\mathbf{TAG}: +\mathbf{cos2n-1(Re)} - (2n-1)^{-\sigma}. \sin(t. \log(2n-1)) \\
&\mathbf{TAG}: -\mathbf{\sin 2n-1(Im)} - (2n)^{-\sigma}. \cos(t. \log(2n))\mathbf{TAG}: -\mathbf{\cos 2n(Re)} + (2n)^{-\sigma}. \sin(t. \log(2n))\mathbf{TAG}: +\boldsymbol{sin}\ \mathbf{2n(Im)}] \\
&= \sqrt{2}\sum_{n=1}^{\infty}[(2n-1)^{-\sigma}. \sin(t. \log(2n-1) + \frac{3}{4}\pi)\mathbf{TAG}: \mathbf{+\cos 2n-1(Re)\& -\sin 2n-1(Im)} + -(2n)^{-\sigma}. \sin(t. \log(2n) \\
&+ \frac{3}{4}\pi)\mathbf{TAG}: \mathbf{-\cos 2n(Re)\& +\sin 2n(Im)}]
\end{aligned} \tag{8}$$

Note our self-explanatory **TAG** legend used to illustrate where each term in the equations above originated from. It can easily be seen that both terms in the final equation consist of a mixture of real and imaginary portions. As Riemann conjecture on non-trivial zeros based on $\zeta(s)$ is identical to that based on its proxy $\eta(s)$, then Riemann hypothesis is satisfied when

$$\sum ReIm\{\eta(s)\} = Re\{\eta(s)\} + Im\{\eta(s)\} = 0 \tag{9}$$

Ignoring the $\sqrt{2}$ term temporarily and with the application of Eq. (9), Eq. (8) becomes

$$\sum_{n=1}^{\infty}(2n-1)^{-\sigma}. \sin(t. \log(2n-1) + \frac{3}{4}\pi) = \sum_{n=1}^{\infty}(2n)^{-\sigma}. \sin(t. \log(2n) + \frac{3}{4}\pi) \tag{10}$$

Eq. (10) completely fulfill the 'presence of the complete set of non-trivial zeros' criteria. Rearranging its terms will result in our desired Riemann-Dirichlet Ratio given below.

$$\frac{\sum_{n=1}^{\infty} \sin(t. \log(2n) + \frac{3}{4}\pi)}{\sum_{n=1}^{\infty} \sin(t. \log(2n-1) + \frac{3}{4}\pi)} = \frac{\sum_{n=1}^{\infty}(2n)^{\sigma}}{\sum_{n=1}^{\infty}(2n-1)^{\sigma}} \tag{11}$$

Denote the left hand side ratio as Ratio R1 (of a 'cyclical' nature) and the right hand side ratio as Ratio R2 (of a 'non-cyclical' nature). Then the Riemann-Dirichlet Ratio can be deemed to be representing a more complicated 'dynamic' version of non-Hybrid integer sequence in that besides consisting of identical 'Class function' in each of the two functions when expressed in Ratio R1's numerator and denominator, this first Ratio R1 is again given as an equality to another seemingly different Ratio R2 whose numerator and denominator also consist of identical 'Class function'. One may intuitively think of a Hybrid integer sequence to metaphorically arise from a non-Hybrid integer sequence "in the limit" the non-identical 'Class function' in Hybrid integer sequence becomes the identical 'Class function' in the new non-Hybrid integer sequence. Note the absence and presence of σ variable in Ratio R1 and R2 respectively.

The Riemann-Dirichlet Ratio calculations, valid for all continuous real number values of t, would theoretically result in infinitely many non-Hybrid integer sequences [here arbitrarily] for the $0 < \sigma < 1$ critical strip region of interest with n = 1, 2, 3,..., ∞ being discrete integer number values, or n being continuous real numbers from 1 to ∞ with Riemann integral applied in the interval from 1 to ∞. This infinitely many integer sequences can geometrically be interpreted to representatively cover the entire plane of the critical strip bounded by σ values of 0 and 1, thus (at least) allowing our proposed proof to be of a 'complete' nature.

In Calculus, integration is defined as the reverse process of differentiation which is geometrically viewed as the area enclosed by the curve of the function and the axis. Using the definite integral I between the points a and b (i.e. in the interval [a, b] where a <b) and computing the value when $\Delta x \longrightarrow 0$, we get $I = \lim_{\Delta x \longrightarrow 0} \sum_{i=1}^{n} f(x_i)\Delta x_i = \int_a^b f(x)dx$ - this is the Riemann integral of the function f(x) in the interval [a, b]. We apply Riemann integral to the four continuous functions of Ratio R1 and Ratio R2 in Eq. (11) thus depicting the Riemann-Dirichlet Ratio in the integral forms - see the subsequent Eq. (16) in this section - and then display Ratio R1 and Ratio R2 physical characteristics for $\sigma = \frac{1}{2}$ at n = 1 and n = 2 situations in Fig. 2a and Fig. 2b respectively below.

Thereafter, step-by-step we derive the closely related Dirichlet σ-power law [expressed in real numbers] and the Riemann σ-power law [expressed in real and complex numbers] - this two laws are further elaborated in the following section of this paper. Due to the resemblance to various power-law functions in that the σ variable from s (= $\sigma + \imath t$) being the exponent of a power function n^{σ}, the log scale use, and the harmonic ζ(s) series connection in Zipf's law; we explain here why we have elected to endow our newly derived formula with the name Sigma-power law. Its Dirichlet and Riemann versions are directly related to each other via Dirichlet η(s) being the equivalence of Riemann ζ(s) but without the $\frac{1}{(1-2^{1-s})}$ proportionality factor. We stress that it is the main underlying mathematically-consistent properties of *symmetry* and *constraints* arising from this power law that also allowed our most direct, basic and elementary proof for the Riemann hypothesis to mature. An important characteristic to note of σ-power law is that its exact formula expression in the usual mathematical language [$y = f(x_1, x_2)$ format description for a 2-variable function] consists of $y = \{2n\}$ or $\{2n-1\} = f(t, \sigma)$ with n = 1, 2, 3,..., ∞ or n = 1 to ∞ with Riemann integral application; $-\infty < t < +\infty$; and σ being of real number values $0 < \sigma < 1$ corresponding to the [arbitrarily defined] critical strip of interest in this particular case scenario. Individual calculations using σ-power law for $\sigma = \frac{1}{2}$ and $\sigma = \frac{1}{2} \pm \delta$ (with δ being a real number between 0 and 0.5) showed an implied *mathematical symmetry* about the $\sigma = \frac{1}{2}$ value calculation - this is depicted graphically for $\delta = 0.3$ in Fig. 3a and Fig. 3b below for Dirichlet σ-power law expressed in the {2n} parameter format.

For the, initially, {2n} parameter integration of R1,

$\int_1^{\infty} \sin\left(t.\log(2n) + \frac{3}{4}\pi\right).dn$

Use integration by u-substitution technique to obtain $u = t.\log(2n) + \frac{3}{4}\pi$, $n = \frac{1}{2}e^{\frac{1}{t}(u-\frac{3}{4}\pi)}$, $\frac{du}{dn} = \frac{2t}{2n} = \frac{t}{n}, du = t.\frac{dn}{n}, dn = 2n.\frac{du}{2t} = n.\frac{du}{t}$

$\int_1^{\infty} \sin(u).\frac{n}{t}.du = \int_1^{\infty} \sin(u).\frac{1}{t}.\frac{1}{2}.e^{\frac{1}{t}(u-\frac{3}{4}\pi)}.du = \frac{1}{2t}.e^{\frac{3}{4}\pi} \int_1^{\infty} \sin(u).e^{\frac{1}{t}u}.du$

Use the Products of functions proportional to their second derivatives, namely the indefinite integral

$\int \sin(a.u).e^{b.u}du = \frac{e^{bu}}{a^2+b^2}(b.\sin(a.u) - a.\cos(a.u)) + C.$

Then a = 1, $b = \frac{1}{t}$, and temporarily ignore the $\frac{1}{2t}e^{\frac{3}{4}\pi}$ term, we have

$$\int_1^{\infty} \sin(u).e^{\frac{1}{t}u}.du$$
$$= [(e^{\frac{1}{t}u})/(1 + \frac{1}{t^2})).(\frac{1}{t}.\sin(u) - \cos(u)) + C]_1^{\infty}$$
$$= [(t^2.e^{\frac{1}{t}u})/(t^2 + 1).(\frac{1}{t}.\sin(u) - \cos(u)) + C]_1^{\infty}$$

Now apply the non-linear combination of sine and cosine functions identity, namely

$a.\sin(u) + b.\cos(u) = c.\sin(u + \varphi)$ where $c = \sqrt{(a^2 + b^2)}$ and $\varphi = atan2(b, a)$.

Here $a = \frac{1}{t}, b = -1, c = \sqrt{\left(\left(\frac{1}{t}\right)^2 + 1\right)} = \frac{\sqrt{(t^2+1)}}{t}$. Then we have

$$\int_1^{\infty} \sin(u).e^{\frac{1}{t}u}.du$$
$$= [(t^2.e^{\frac{1}{t}u})/(t^2 + 1).\frac{\sqrt{(t^2 + 1)}}{t}.\sin(u + atan2(b, a)) + C]_1^{\infty}$$
$$= [(t.e^{\frac{1}{t}u})/\sqrt{(t^2 + 1)}).\sin(u + \arctan(t)) + C]_1^{\infty}$$

But there was a $\frac{1}{2t}.e^{\frac{3}{4}\pi}$ term in front of this integral as can be seen above. Then after substituting this term and simplifying, the integral

$$\int_1^\infty \sin(u).e^{\frac{1}{t}u}.du$$
$$= [\left(e^{\frac{1}{t}u-\frac{3}{4}\pi}\right)/2\sqrt{(t^2+1)}.\sin(u-\arctan(t))+C]_1^\infty$$

But $u = t.\log(2n) + \frac{3}{4}\pi$. Reverting back to the n variable, and incorporating $\sqrt{2}$ originating from the beginning during Eq. (10) derivation, the equation for the {2n} parameter finally becomes

$$\sqrt{2}\int_1^\infty \sin(t.\log(2n)+\frac{3}{4}\pi).dn$$
$$= [\sqrt{2}.(\{2n\}.e^{\frac{1}{t}\cdot\frac{3}{4}\pi})/(2\sqrt{(t^2+1)}.e^{\frac{3}{4}\pi}.\sin(t.\log(2n)+\frac{3}{4}\pi-\arctan(t))+C)]_1^\infty \tag{12}$$

In a similar manner integration for the {2n-1} parameter, this equation becomes

$$[\sqrt{2}.(\{2n-1\}.e^{\frac{1}{t}\cdot\frac{3}{4}\pi})/(2\sqrt{(t^2+1)}.e^{\frac{3}{4}\pi}.\sin(t.\log(2n-1)+\frac{3}{4}\pi-\arctan(t))+C)]_1^\infty \tag{13}$$

In R2 using {2n} parameter,

$$\int_1^\infty (2n)^\sigma dn$$
$$= [1/(2(\sigma+1)).(2n)^{\sigma+1}+C)]_1^\infty$$
$$= [\frac{1}{3}\{2n\}(2n)^{\frac{1}{2}}+C)]_1^\infty \text{ when } \sigma=\frac{1}{2} \tag{14}$$

For the equivalent R2 based on {2n-1} parameter,

$$\int_1^\infty (2n-1)^\sigma dn$$
$$= [1/(2(\sigma+1)).(2n-1)^{\sigma+1}+C)]_1^\infty$$
$$= [\frac{1}{3}\{2n-1\}(2n-1)^{\frac{1}{2}}+C)]_1^\infty \text{ when } \sigma=\frac{1}{2} \tag{15}$$

The Ratio R1 and Ratio R2 of Riemann-Dirichlet Ratio (for $\sigma = \frac{1}{2}$) is defined by the integral

$$\frac{[(\{2n\}.(e^{\frac{1}{t}\cdot\frac{3}{4}\pi}/2\sqrt{(t^2+1)}.e^{\frac{3}{4}\pi}).\sin(t.\log(2n)+\frac{3}{4}\pi-\arctan(t))]_1^\infty}{[(\{2n-1\}.e^{\frac{1}{t}\cdot\frac{3}{4}\pi}/2\sqrt{(t^2+1)}.e^{\frac{3}{4}\pi}).\sin(t.\log(2n-1)+\frac{3}{4}\pi-\arctan(t))]_1^\infty} = \frac{[\frac{1}{3}\{2n\}(2n)^{\frac{1}{2}}]_1^\infty}{[\frac{1}{3}\{2n-1\}(2n-1)^{\frac{1}{2}}]_1^\infty}$$

Cancelling out the common parameter {2n} and {2n-1} terms,

$$\frac{[(e^{\frac{1}{t}\cdot\frac{3}{4}\pi}/2\sqrt{(t^2+1)}.e^{\frac{3}{4}\pi}).\sin(t.\log(2n)+\frac{3}{4}\pi-\arctan(t))]_1^\infty}{[(e^{\frac{1}{t}\cdot\frac{3}{4}\pi}/2\sqrt{(t^2+1)}.e^{\frac{3}{4}\pi}).\sin(t.\log(2n-1)+\frac{3}{4}\pi-\arctan(t))]_1^\infty} \leftarrow \textbf{this is R1}$$
$$= \frac{[\frac{1}{3}(2n)^{\frac{1}{2}}]_1^\infty}{[\frac{1}{3}(2n-1)^{\frac{1}{2}}]_1^\infty} \leftarrow \textbf{this is R2} \tag{16}$$

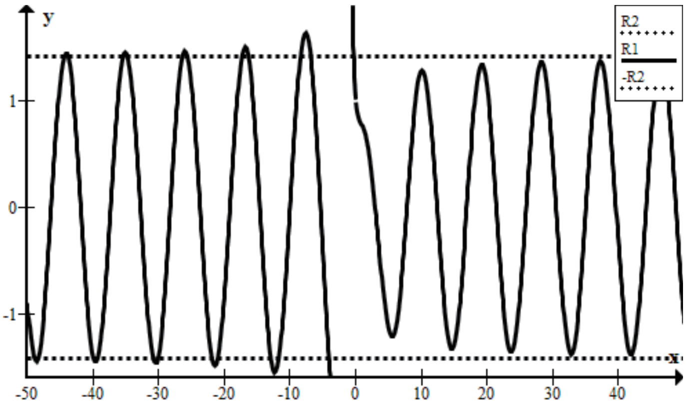

Figure 2a

Figure 2a. Riemann-Dirichlet Ratio displayed as Ratio R1 and Ratio R2 for $\sigma = \frac{1}{2}$ at n = 1 situation. x-axis = t, y-axis = R1 and R2 values. Note R2 lines seem to *mathematically constraint* the maxima and minima values of R1.

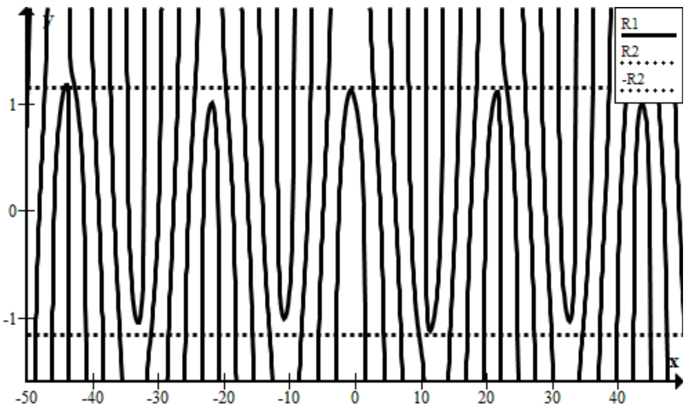

Figure 2b

Figure 2b. Riemann-Dirichlet Ratio displayed as Ratio R1 and Ratio R2 for $\sigma = \frac{1}{2}$ at n = 2 situation. x-axis = t, y-axis = R1 and R2 values. The R2 lines seem to *mathematically constraint* the maxima and minima values of R1 but R1 overall sinusoidal landscape is now vastly different to that of Figure 2a - we note here that n goes from 1, 2, 3. . . ,∞ and each individual n value could be graphically represented manifesting Fractals with self-similarity.

6. Sigma-power Law

The γ proportionality factor term in Riemann ζ function, viz. $\frac{1}{(1-2^{1-s})}$, can also be expressed with the aid of Euler formula as follows (with the formula for $\sigma = \frac{1}{2}$ substitution depicted last).

$$\begin{aligned}
&\frac{1}{(1-2^{1-s})} \\
&= \frac{(2^{\sigma}.2^{it})}{(2^{\sigma}.2^{it}-2)} \\
&= \frac{(2^{\sigma}.e^{t.\log(2\iota)})}{(2^{\sigma}.e^{t.\log(2\iota)}-2)} \\
&= \frac{(2^{\sigma}.(\cos(t.\log(2)+\iota.\sin(t.\log(2)))}{(2^{\sigma}.(\cos(t.\log(2)+\iota.\sin(t.\log(2)-2)} \\
&= \frac{(2^{\frac{1}{2}}.(\cos(t.\log(2)+\iota.\sin(t.\log(2)))}{(2^{\frac{1}{2}}.(\cos(t.\log(2)+\iota.\sin(t.\log(2)-2)}
\end{aligned} \tag{17}$$

The Dirichlet and Riemann σ-power laws are given by the exact formulae in Eqs. (18) to (21) below with ψ being the same proportionality constant valid for both power laws. We can now dispense with the constant of integration C. **Using Dimensional analysis approach we can easily conclude that the 'fundamental dimension' [Variable / Parameter**

/ Number X to the power of Number Y] has to be represented by the particular 'unit of measure' [Variable / Parameter / Number X to the power of Number Y whereby Number Y needs to be of the specific value $\frac{1}{2}$] for Dimensional analysis homogeneity to occur. This *de novo* Dimensional analysis homogeneity equates to the location of the complete set of non-trivial zeros and is crucially a fundamental property present in all laws of Physics. The 'unknown' σ variable, now endowed with the value of $\frac{1}{2}$, is treated as Number Y.

Dirichlet σ-power law using the {2n} parameter:

$$\left[2^{\frac{1}{2}}.\{2n\}.\frac{e^{\frac{1}{t}\cdot\frac{3}{4}\pi}}{2(t^2+1)^{\frac{1}{2}}.e^{\frac{3}{4}\pi}}.\sin(t.\log(2n)+\frac{3}{4}\pi-\arctan(t))\right]_1^{\infty}=\left[\psi.\frac{1}{3}\{2n\}(2n)^{\frac{1}{2}})\right]_1^{\infty}$$

With the common parameter {2n} cancelling out on both sides, the equation reduces to

$$\left[2^{\frac{1}{2}}.\frac{e^{\frac{1}{t}\cdot\frac{3}{4}\pi}}{2(t^2+1)^{\frac{1}{2}}.e^{\frac{3}{4}\pi}}.\sin(t.\log(2n)+\frac{3}{4}\pi-\arctan(t))-\psi.\frac{1}{3}(2n)^{\frac{1}{2}})\right]_1^{\infty}=0 \tag{18}$$

Similarly for the {2n-1} parameter, this equivalent equation is

$$\left[2^{\frac{1}{2}}.\frac{e^{\frac{1}{t}\cdot\frac{3}{4}\pi}}{2(t^2+1)^{\frac{1}{2}}.e^{\frac{3}{4}\pi}}.\sin(t.\log(2n-1)+\frac{3}{4}\pi-\arctan(t))-\psi.\frac{1}{3}(2n-1)^{\frac{1}{2}})\right]_1^{\infty}=0 \tag{19}$$

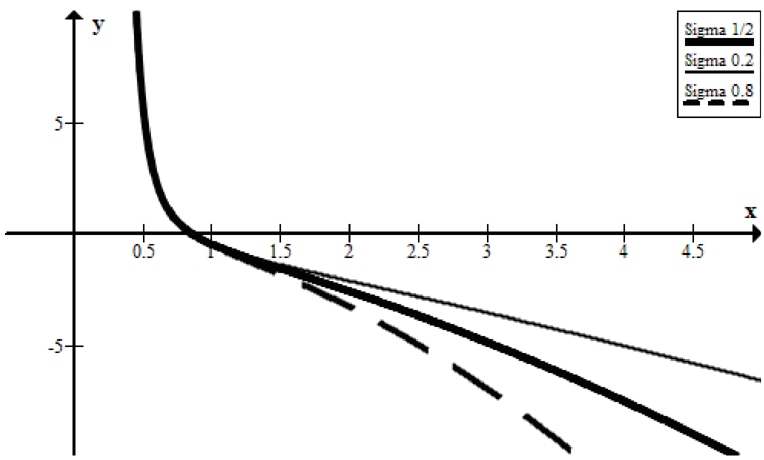

Figure 3a

Figure 3a. Dirichlet σ-power law: using {2n} parameter, n = 1 situation, x-axis = t, ψ arbitrarily defined with value 1, y-axis = σ-power law values obtained when σ = 0.2 (non-critical line), $\frac{1}{2}$ (critical line where all non-trivial zeros lie), and 0.8 (non-critical line) displayed on the usual linear-linear graph.

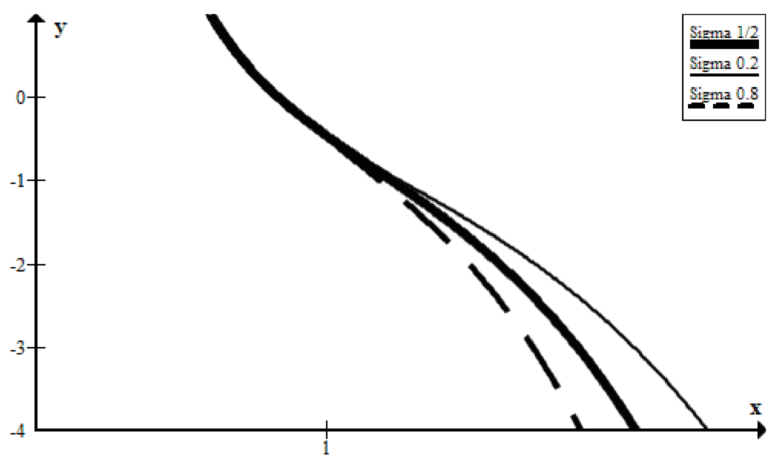

Figure 3b

Figure 3b. Dirichlet σ-power law: using the same criteria and data to that obtained in Figure 3a except that they are now displayed on the log-linear graph, with log here referring to logarithm in base 10. Note (i) the mathematical requirement of the y-axis in Figure 3b to be linear [and not with log scale] due to the mixture of positive and negative values obtained for y-axis and (ii) the implied *mathematical symmetry* about the $\sigma = \frac{1}{2}$ value in both Figure 3a and Figure 3b.

Finally, the Riemann σ-power law is given by the exact formulae using {2n} and {2n-1} parameters with the $\gamma = (2^{\frac{1}{2}}.(\cos(t.\log(2) + \imath.\sin(t.\log(2))/(2^{\frac{1}{2}}.(\cos(t.\log(2) + \imath.\sin(t.\log(2) - 2)$ substitution.

$$\left[2^{\frac{1}{2}}.\frac{e^{\frac{1}{t}\cdot\frac{3}{4}\pi}}{2(t^2+1)^{\frac{1}{2}}.e^{\frac{3}{4}\pi}}.\sin(t.\log(2n) + \frac{3}{4}\pi - \arctan(t)) - \psi.\gamma.\frac{1}{3}(2n)^{\frac{1}{2}})\right]_1^{\infty} = 0$$
$$\left[2^{\frac{1}{2}}.\frac{e^{\frac{1}{t}\cdot\frac{3}{4}\pi}}{2(t^2+1)^{\frac{1}{2}}.e^{\frac{3}{4}\pi}}.\sin(t.\log(2n) + \frac{3}{4}\pi - \arctan(t)) - \psi.\frac{2^{\frac{1}{2}}.(\cos(t.\log(2) + \imath.\sin(t.\log(2))))}{2^{\frac{1}{2}}.(\cos(t.\log(2) + \imath.\sin(t.\log(2) - 2)))}.\frac{1}{3}(2n)^{\frac{1}{2}})\right]_1^{\infty} = 0 \quad (20)$$

$$\left[2^{\frac{1}{2}}.\frac{e^{\frac{1}{t}\cdot\frac{3}{4}\pi}}{2(t^2+1)^{\frac{1}{2}}.e^{\frac{3}{4}\pi}}.\sin(t.\log(2n-1) + \frac{3}{4}\pi - \arctan(t)) - \psi.\gamma.\frac{1}{3}(2n-1)^{\frac{1}{2}})\right]_1^{\infty} = 0$$
$$\left[2^{\frac{1}{2}}.\frac{e^{\frac{1}{t}\cdot\frac{3}{4}\pi}}{2(t^2+1)^{\frac{1}{2}}.e^{\frac{3}{4}\pi}}.\sin(t.\log(2n-1) + \frac{3}{4}\pi - \arctan(t)) - \psi.\frac{2^{\frac{1}{2}}.(\cos(t.\log(2) + \imath.\sin(t.\log(2))))}{2^{\frac{1}{2}}.(\cos(t.\log(2) + \imath.\sin(t.\log(2) - 2)))}.\frac{1}{3}(2n-1)^{\frac{1}{2}})\right]_1^{\infty} = 0 \quad (21)$$

Note the γ proportionality factor given by Eq. (17) above when depicted with the $2^{\frac{1}{2}}$ constant numerical value (derived using $\sigma = \frac{1}{2}$ as conjectured in the original Riemann hypothesis) further allowing, and enabling, *de novo* Dimensional analysis homogeneity compliance in Riemann σ-power law.

We illustrate the Dimensional analysis non-homogeneity property for an $\sigma = \frac{1}{4}$ arbitrarily chosen value [clear-cut case with {2n}-parameter] of Riemann σ-power law lying on a non-critical line (with total absence of non-trivial zeros) in the following formula derived using Eqs. (17) and (21). **As Ratio R1 component of Riemann-Dirichlet Ratio is independent of σ variable, unlike the Ratio R2 component of Riemann-Dirichlet Ratio and the γ proportionality factor which are dependent on σ variable, we now note the mixture of $\frac{1}{4}$ and $\frac{1}{2}$ exponents subtly, but nonetheless, present in this formula indicating Dimensional analysis non-homogeneity.** Also the replacement of $\frac{1}{3}$ fraction with $\frac{2}{5}$ fraction [derived from substituting $\sigma = \frac{1}{4}$ into $\frac{1}{2(\sigma+1)}$] has occurred. Mathematically, this Dimensional analysis non-homogeneity property for any real number value of σ, when $\sigma \neq \frac{1}{2}$ and $0 < \sigma < 1$, will always be present.

$$[2^{\frac{1}{2}}.\frac{e^{\frac{1}{t}\cdot\frac{3}{4}\pi}}{2(t^2+1)^{\frac{1}{2}}.e^{\frac{3}{4}\pi}}.\sin(t.\log(2n) + \frac{3}{4}\pi - \arctan(t)) -$$
$$\psi.\frac{2^{\frac{1}{4}}.(\cos(t.\log(2) + \imath.\sin(t.\log(2))))}{2^{\frac{1}{4}}.(\cos(t.\log(2) + \imath.\sin(t.\log(2) - 2)))}.\frac{2}{5}(2n)^{\frac{1}{4}})]_1^{\infty} = 0 \quad (22)$$

7. Conclusions

The seemingly small but utterly essential mathematical step in recognizing and representing a 2-variable function with parameters {2n} or {2n-1} allows crucial moments where cancellation of the relevant "common" parameters in Riemann-Dirichlet Ratio and various Sigma-power laws can occur, further allowing the proper Dimensional analysis process to happen in the absolute correct way. These "common" parameters must be mathematically viewed as $(2n)^1$ or $(2n-1)^1$, viz. raised to a power (exponent) of 1 which will hamper proper Dimensional analysis if not serendipitously deleted - contrast this scenario with the presence of parameters $(2n)^{\frac{1}{2}}$ or $(2n-1)^{\frac{1}{2}}$, viz. raised to a power (exponent) of $\frac{1}{2}$ which will then enable proper Dimensional analysis (homogeneity) to proceed. The mathematical foot-prints (6 steps) of this paper are:

Step 1: Riemann zeta or Dirichlet eta function [for the critical strip $0 < \sigma < 1$] → *Step 2:* Riemann zeta or Dirichlet eta function [with Euler formula application] → *Step 3:* Riemann zeta or Dirichlet eta function [simplified and identical version specifically indicating the criteria for the presence of the complete set of non-trivial zeros] → *Step 4:* Riemann-Dirichlet Ratio [in discrete summation format] → *Step 5:* Riemann-Dirichlet Ratio [in continuous integral format] → *Step 6:* Riemann Sigma-power law and Dirichlet Sigma-power law [both with Dimensional analysis homogeneity].

Note that Riemann Sigma-power law and Dirichlet Sigma-power law, both with Dimensional analysis homogeneity indicative of the critical line when $\sigma = \frac{1}{2}$, coincide with the presence of the complete set of non-trivial zeros whereas Riemann Sigma-power law and Dirichlet Sigma-power law, both with Dimensional analysis non-homogeneity indicative of the infinite numbers of non-critical lines when $0 < \sigma < \frac{1}{2}$ and $\frac{1}{2} < \sigma < 1$, coincide with the total absence of non-trivial zeros.

Ratio study of the Riemann-Dirichlet Ratio indicated that it is directly derived from, and related to, either Riemann zeta or Dirichlet eta function via mathematically rearranging their respective 'Euler formula-enriched' infinite series, and then further transforming the original Ratio's 'discrete summation' format into the 'continuous integration' format using Riemann integral. This Ratio crucially incorporated the criteria for the presence of all (identical) non-trivial zeros, or roots, for Riemann zeta function [and Dirichlet eta function by default] in a *de novo* manner. Both the Riemann and Dirichlet Sigma power-laws are simply derivations of either the numerator or denominator portion of Riemann-Dirichlet Ratio.

The completeness of our proof is fully supported by the fact that critical strip ($0 < \sigma < 1$) is fully represented by the one and only one critical line ($\sigma = \frac{1}{2}$) in the middle and an infinite number of other lines ($0 < \sigma < 1$ and $\sigma \neq \frac{1}{2}$) on either side of this critical line, with every single line mathematically described by Riemann zeta function with a particular designated sigma value. The Dimensional analysis homogeneity property of Riemann and Dirichlet Sigma-power laws provides the definitive mathematical proof for Riemann hypothesis to be true - stated in this manner, this statement also "coincides" exactly with the critical line ($\sigma = \frac{1}{2}$) location of all non-trivial zeros [as conjectured by the original Riemann hypothesis]. All the other infinite numbers of [parallel] non-critical lines have Dimensional analysis with non-homogeneity property, and thus will not contain non-trivial zeros. In Dimensional analysis language, there is one, and only one, 'valid' equation at $\sigma = \frac{1}{2}$ complying with Dimensional analysis homogeneity property. Likewise all the non-critical lines consist of Dimensional analysis 'invalid' equations as they fail to comply with Dimensional analysis homogeneity property. It is an unconditional scientific principle that compliance and mathematical constraint of physical laws and mathematical equations with Dimensional analysis homogeneity enables fundamental laws of Physics and Mathematics to be correct.

The synopsis in the previous four paragraphs broadly summarizes the proposed findings of this study as outlined in the above sections of this paper, in particular via Theorem I to IV from our Introduction section. We theorized that with the correctness of those four theorems, although deceptively simplistic in nature, this *fait accompli* will respectfully supply the complete proof for Riemann hypothesis at the most fundamental level - this could also equivalently be stated as peer review's inability to falsify those four theorems. It is noted that neither the Riemann-Dirichlet Ratio nor the two Sigma-power laws by themselves have the means to be able to calculate the precise location of each non-trivial zeros. However, this is not at all essential for the purpose of our study.

There is one, and only one, exact mathematical equation of Riemann zeta function able to validly represent the critical strip, viz. $\zeta(s) = Re\{\zeta(s)\} + \imath.Im\{\zeta(s)\}$ - its real and imaginary components derived from $\frac{1}{(1-2^{1-s})}.\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^s}$, with implied non-trivial zeros 'location' signature capability able to be incorporated into its equivalently derived and expanded equation(s). One sense here that, intuitively, the original Formula X that does enable calculations for the precise locations of non-trivial zeros, by itself, will be totally resistant to being used alone [if at all] in actually proving the Riemann hypothesis as originally stated by Bernhard Riemann. Likely the added-on complexity afforded by the alternating positive and negative terms of Riemann zeta and Dirichlet eta functions, as a 'weak' principle, totally preclude this from happening. By aesthetic [but incorrect] reasoning alone for this case, this may simply be because the calculated values for the precise locations of particular non-trivial zeros *per se* from Formula X does not in principle guarantee that there is not another mathematically 'different' Formula Y out there that will also enable calculations for the precise locations of some other 'extra' non-trivial zeros located away from the critical line. Philosophically, we tantalized with a 'strong' principle that our Ratio study approach may well be the only option or 'lone' method to use when attempting to solve the Riemann hypothesis problem.

The refined explanation for Paragraph 1 above is our perceived mathematical preference to perform Dimensional analysis on Sigma-power laws instead of Riemann-Dirichlet Ratio because the former is more succinct in that it involves "only one solitary parameter {2n} or {2n-1} in equation format" while the later involves "both parameters {2n} and {2n-1} in ratio format". Ultimately the direct mathematical connection between the criteria for the presence of non-trivial zeros in Riemann zeta (or Dirichlet eta) function with the Dimensional analysis homogeneity criteria in Sigma-power laws, all occurring at the one $\sigma = \frac{1}{2}$ value, is in complete favor for our general argument on Riemann hypothesis to be true. This is simply because the solution for σ variable to be $\frac{1}{2}$ is to enable full compliance with Dimensional analysis homogeneity for Riemann Sigma-power law and Dirichlet Sigma-power law - now seen as Dimensional analysis-wise truly valid mathematical equations or physical laws [in Step 6 of our mathematical foot-prints] - is undeniably also the effective correct solution to Riemann zeta and Dirichlet eta function as simplified and identical versions specifically indicating the criteria for the presence of the complete set of non-trivial zeros [in Step 3 of our mathematical foot-prints].

Hadamard product:

$$\begin{aligned}
&\zeta(s)\\
&= \frac{e^{(\log(2\pi)-1-\frac{\gamma}{2}).s)}}{2(s-1).\Gamma(1+\frac{s}{2})}.\Pi_\rho\left(1-\frac{s}{\rho}\right).e^{\frac{s}{\rho}}\\
&= \pi^{\frac{s}{2}}.\frac{\Pi_\rho\left(1-\frac{s}{\rho}\right)}{2(s-1).\Gamma\left(1+\frac{s}{2}\right)}
\end{aligned}$$

Euler product formula:

$$\begin{aligned}
&\sum_{n=1}^{\infty}\frac{1}{n^s} \qquad \{\text{which is } \zeta(s)\}\\
&= \Pi_{p\ prime}\frac{1}{(1-p^{-s})}\\
&= \frac{1}{(1-2^{-s})}.\frac{1}{(1-3^{-s})}.\frac{1}{(1-5^{-s})}.\frac{1}{(1-7^{-s})}.\frac{1}{(1-11^{-s})}\cdots\frac{1}{(1-p^{-s})}\cdots
\end{aligned}$$

The wider scientific community clearly believes that Riemann hypothesis has numerous and widespread perceived impacts or consequences - we will not elaborate further on this important point. We leave behind a final note by briefly mentioning (i) the beautiful Hadamard product above - the infinite product expansion of $\zeta(s)$ based on Weierstrass's factorization theorem, and (ii) the beautiful Euler product formula above connecting Riemann zeta function and prime numbers discovered by Euler - this identity has, by definition, the left hand side being $\zeta(s)$ and the infinite product on the right hand side extends over all prime numbers p. The form of the Hadamard product clearly displays the simple pole at s = 1, the trivial zeros at all even negative integers due to the gamma function term in the denominator, and the non-trivial zeros at $s = \rho$; with the letter γ in the expansion here denoting the Euler-Mascheroni constant - note that this is different to the $\gamma[=\frac{1}{(1-2^{1-s})}]$ role being employed in the main content of this paper above. Note also that with the second simpler infinite product expansion formula of Hadamard, to ensure convergence, the product should be taken over "matching pairs" of zeroes, i.e. the factors for a pair of zeroes of the form ρ and $1-\rho$ should be combined.

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