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# Computing of Z- valued Characters for the Projective Special Linear Group $L_2$ ($2^m$) and the Conway Group $Co_3$

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**Abstract**

According to the main result of W. Feit and G. M. Seitz (see, Illinois J. Math. 33 (1), 103-131, 1988), the projective special linear group $L_2$ ($2^m$) for m = 3, 4, 5 and the smallest Conway group $Co_3$ are unmatured groups. In this paper, we continue our study on special finite groups (see Int. J. Theo. Physics, Group Theory, and Nonlinear Optics (17)1, 57-62, 2013) and the dominant classes and Q- conjugacy characters for the above groups are derived.

MSC Mathematics Subject Classification (2010): 20D05, 20C15

**Keywords:** projective special linear group, Sporadic Conway groups, Conjugacy class, Q-conjugacy character

## 1. Introduction

In recent years, the problems over group theory have drawn the wide attention of researchers in mathematics, physics and chemistry. Many problems of the computational group theory have been researched, such as the classification, the symmetry, the topological cycle index, etc. It is not only on the property of finite group, but also its wide-ranging connection with many applied sciences, such as Nanoscience, Chemical Physics and Quantum Chemistry, for instant see [Moghani, 2010].

S. Fujita suggested a new concept called the markaracter table, which enables us to discuss marks and characters for a finite group on a common basis, and then introduced tables of integer-valued characters and dominant classes, which are acquired for such groups. A dominant class is defined as a disjoint union of conjugacy classes corresponding the same cyclic subgroups, which is selected as a representative of conjugate cyclic subgroups. Moreover, the cyclic (dominant) subgroup selected from a non-redundant set of cyclic subgroups of G is used to compute the Q-conjugacy characters of G, as demonstrated in [Fujita, 1998].

The projective special linear groups $L_2$ (8), $L_2$ (16), $L_2$ (32) and the smallest Conway group $Co_3$ with orders 540, 4080, 32736 and 495766656000 respectively, are unmatured groups according to the main result of W. Feit and G. M. Seitz in [Feit et al., 1988]. The motivation for this study is outlined in [Safarisabet et al., 2013; Fujita, 1998; Moghani, 2009&2010; Aschbacher, 1997; Feit et al., 1988; Conway et al., 1985] and the reader is encouraged to consult these papers and [Moghani, 2009&2010; Aschbacher, 1997; Feit et al., 1988; Conway et al., 1985; GAP, 1995; Kerbe et al., 1982; Kerber, 1999] for background material as well as basic computational techniques.

This paper is organized as follows: In Section 2, we introduce some necessary concepts, such as the maturity and Q-conjugacy character of a finit group. In Section 3, we provide all the dominant classes and Q- conjugacy characters for the projective special linear group $L_2$ ($2^m$) for m = 3, 4, 5 and the Conway groups $Co_3$.

## 2. Preliminaries

Throughout this paper we adopt the same notations as in [Safarisabet et al., 2013; Conway, 1985]. For instance, we will use the ATLAS notations for conjugacy classes. Thus, nx, n is an integer and x = a, b, c…denotes an arbitrary conjugacy class of G of elements of order n.

**Definition 2.1:** Let G be an arbitrary finite group and $h_1, h_2 \in G$, we say $h_1$ and $h_2$ are Q-conjugate if $t \in G$ exists such that $t^{-1} < h_1 > t = < h_2 >$ which is an equivalence relation on group G and generates equivalence classes that are called dominant classes. Therefore, G is partitioned into dominant classes [Fujita, 1998].

**Definition 2.2:** Suppose H be a cyclic subgroup of order n of a finite group G. Then, the maturity discriminant of H denoted by m(H), is an integer number delineated by $|N_G(H):C_G(H)|$ in addition, the dominant class of $K \cap H$ in the normalizer $N_G(H)$ is the union of $t = \frac{m(H)}{\phi(|H|)}$ conjugacy classes of $G$ where $\varphi$ is Euler function, i.e. the maturity of G is clearly defined by examining how a dominant class corresponding to H contains conjugacy classes. The group G should be matured group if t = 1, but if t ≥ 2, the group G is an unmatured concerning subgroup H, see [Safarisabet et al., 2013; Fujita, 1998; Moghani, 2009&2010]. For some properties of the maturity see the following theorem which is introduced by the author in [Moghani, 2009]:

**Theorem 2.3:** The wreath products of the matured groups again is a matured group, but the wreath products of at least one unmatured group is an unmatured group.

**Definition 2.4:** Let $C_{u\times u}$ be a matrix of the character table for an arbitrary finite group G. Then, C is transformed into a more concise form called the Q-Conjugacy character table denoted by $C_G^Q$ containing integer-valued characters. By Theorem 4 in [Fujita, 1998], the dimension of a Q-conjugacy character table $C_G^Q$ is equal to its corresponding markaracter table denoted by $M_G^C$, i.e. $C_G^Q$ is a m × m –matrix where m ≤ u is the number of dominant classes or equivalently the number of non-conjugate cyclic subgroups denoted by denoted by $SCS_G$, see [Safarisabet et al., 2013; Fujita, 1998; Moghani, 2009&2010].

**Definition 2.5:** If $\chi_1$, ... , $\chi_k$ are all the irreducible characters of a finite group H, let Q(H) = Q($\chi_1$,..., $\chi_k$) be the field generated by all $\chi_i$ (x), x ∈H, 1 ≤ I ≤ k.

A character $\chi$ is rational if Q ($\chi$) = Q. A group H is a rational group if Q (H) = Q (e.g. every Weyl group is a rational group [Feit et al., 1988]).

**Theorem 2.6 [Feit et al., 1988]:** Let G be a non cyclic finite simple group. Then G is a composition factor of a rational group if and only if G is isomorphic to an alternating group or one of the following groups: $PSp_4(3)$, $Sp_6(2)$, $O_8^+(2)'$, $PSL_3(4)$, $PSU_4(3)$.

## 3. Conclusion

According to the Theorem 2.6, the projective special linear groups $L_2$ (8), $L_2$ (16), $L_2$ (32) and the Conway group $Co_3$ are unmatured groups. Now we are equipped to compute all the dominant classes and Q-conjugacy characters for the above groups with aid GAP program [GAP, 1995], http://www.gap–system.org.

**Theorem 3.1**

*(i) The projective special linear group $L_2$ (8) has two unmatured dominant classes with t = 3 in definition 2.2. Furthermore, there are five Q- conjugacy characters for $L_2$ (8) with the following degrees: 1, 7, 8, 21 and 27.*

*(ii) The projective special linear group $L_2$ (16) has three unmatured dominant classes with t= 2, 4 and 8. Furthermore, there are eight Q- conjugacy characters for $L_2$ (16) with the following degrees: 1, 16, 17, 34, 68 and 120.*

*(iii) The projective special linear group $L_2$ (32) has three unmatured dominant classes with t= 5, 15 and 10. Furthermore, there are six Q- conjugacy characters for $L_2$ (32) with the following degrees: 1, 31, 32, 155, 310 and 495.*

**Proof:** Here, because of similar discussions we verify via full discussions just (ii) for $L_2$ (16) of order 4050. To find all the number of dominant classes for $L_2$ (16) at first, we calculate the markaracter table for $L_2$ (16) via GAP system, see definition 2.2 and GAP programs in [Safarisabet et al., 2013; GAP, 1995] for more details.

Hence, see the markaracter table for $L_2$ (16) (i.e. $M_{L2(16)}^C$) in Table 1, corresponding to five non-conjugate cyclic subgroups ( i.e. $G_i \in SCS_{L2(16)}$) of orders 1, 2, 3, 5, 15 and 17 respectively, as follow:

$G_1$ = id, $G_2$ = < (2, 3)(4, 5)(6, 9)(7, 12)(8, 17)(10, 16)(11, 13)(14, 15) >, $G_3$ = < (3, 4, 5)(6, 10, 14)(7, 11, 15)(8, 12, 16)(9, 13, 17) >, $G_4$ = < (3, 8, 10, 13, 15)(4, 12, 14, 17, 7)(5, 16, 6, 9, 11) >, $G_5$ = < (3, 4, 5)(6, 10, 14)(7, 11, 15)(8, 12, 16)(9, 13, 17), (3, 8, 10, 13, 15)(4, 12, 14, 17, 7)(5, 16, 6, 9, 11) >, and $G_6$ = < (1, 2, 3, 6, 17, 11, 5, 13, 9, 10, 12, 8, 7, 4, 16, 14, 15) >.

Therefore, $\left|SCS_{L2(16)}\right|$ = 6 and its dominant classes are 1a, 2a, 3a, $K_5$= 5a ∪ 5b, $K_{15}$= 15a ∪ 15b ∪ 15c ∪ 15d and $K_{17}$ = 17a ∪ 17b ∪ 17c ∪ 17d ∪ 17e ∪ 17f ∪ 17g ∪ 17h, thus $L_2$ (16) has three unmatured dominant classes with t = 2, 4 and 8.

Furthermore, $L_2$ (16) has three unmatured Q-conjugacy characters $\varphi_2$, $\varphi_5$ and $\varphi_6$ which are the sum of eight, two and four irreducible characters respectively. Therefore, there are eight, two and four column-reductions respectively (similarly row-reductions) in the character table of $L_2$ (16). There are eight Q- conjugacy characters for $L_2$ (16) with the following degrees: 1, 16, 17, 34, 68 and 120, see Table 2.

Table 1. The markaracter Table of the projective special linear group $L_2$ (16)

| $\mathbf{M^{C}_{L2(16)}}$ | $\mathbf{G_1}$ | $\mathbf{G_2}$ | $\mathbf{G_3}$ | $\mathbf{G_4}$ | $\mathbf{G_5}$ | $\mathbf{G_6}$ |
|---|---|---|---|---|---|---|
| **($L_2$(16)/$G_1$)** | 4080 | 0 | 0 | 0 | 0 | 0 |
| **($L_2$(16)/$G_2$)** | 2040 | 8 | 0 | 0 | 0 | 0 |
| **($L_2$(16)/$G_3$)** | 1360 | 0 | 10 | 0 | 0 | 0 |
| **($L_2$(16)/$G_4$)** | 816 | 0 | 0 | 6 | 0 | 0 |
| **($L_2$(16)/$G_5$)** | 272 | 0 | 2 | 2 | 2 | 0 |
| **($L_2$(16)/$G_6$)** | 240 | 0 | 0 | 0 | 0 | 2 |

Besides, the dominant classes of $L_2$ (8) are 1a, 2a, 3a, $D_7$ = 7a ∪ 7b ∪ 7c and $D_9$ = 9a ∪ 9b ∪ 9c which has two unmatured dominant classes with t = 3. Similar discussions show that there are five Q- conjugacy characters for $L_2$ (8) with the following degrees: 1, 7, 8, 21 and 27.

$L_2$ (8) has two unmatured Q-conjugacy characters $\mu_3$ and $\mu_5$ which are the sum of three irreducible characters respectively, see Table 3.

Table 2. The Q-Conjugacy Character of the projective special linear group $L_2$ (16)

| $\mathbf{C^{Q}_{L2(16)}}$ | **1a** | **2a** | **3a** | $\mathbf{K_5}$ | $\mathbf{K_{15}}$ | $\mathbf{K_{17}}$ |
|---|---|---|---|---|---|---|
| $\phi_1$ | 1 | 1 | 1 | 1 | 1 | 1 |
| $\phi_2$ | 120 | -8 | 0 | 0 | 0 | 1 |
| $\phi_3$ | 16 | 0 | 1 | 1 | 1 | -1 |
| $\phi_4$ | 17 | 1 | -1 | 2 | -1 | 0 |
| $\phi_5$ | 34 | 2 | 4 | -1 | -1 | 0 |
| $\phi_6$ | 68 | 4 | -4 | -2 | 1 | 0 |

wherein $K_5$= 5a ∪ 5b, $K_{15}$= 15a ∪ 15b ∪ 15c ∪ 15d and $K_{17}$ = 17a ∪ 17b ∪ 17c ∪ 17d ∪ 17e ∪ 17f ∪ 17g ∪ 17h

The dominant classes of $L_2$ (32) are 1a, 2a, 3a, $L_{11}$ = 11a ∪ 11b ∪ 11c ∪ 11d ∪ 11e, $L_{31}$ = 31a ∪ 31b ∪ 31c ∪ 31d ∪ 31e ∪ 31f ∪ 31g ∪ 31h ∪ 31i ∪ 31j ∪ 31k ∪ 31l ∪ 31m ∪ 31n ∪ 31o and $L_{33}$ = 33a ∪ 33b ∪ 33c ∪ 33d ∪ 33e ∪ 33f ∪ 33g ∪ 33h ∪ 33i ∪ 33j which has three unmatured dominant classes with t = 5, 15 and 10.

Table 3. The Q-Conjugacy Character of the projective special linear group $L_2$ (8)

| $\mathbf{C^{Q}_{L2(8)}}$ | **1a** | **2a** | **3a** | $\mathbf{D_7}$ | $\mathbf{D_9}$ |
|---|---|---|---|---|---|
| $\mu_1$ | 1 | 1 | 1 | 1 | 1 |
| $\mu_2$ | 7 | -1 | -2 | 0 | 1 |
| $\mu_3$ | 21 | -3 | 3 | 0 | 0 |
| $\mu_4$ | 8 | 0 | -1 | 1 | -1 |
| $\mu_5$ | 27 | 3 | 0 | -1 | 0 |

Wherein, $D_7$ = 7a ∪ 7b ∪ 7c and $D_9$ = 9a ∪ 9b ∪ 9c

Table 4. The Q-Conjugacy Character of the projective special linear group $L_2$ (32)

| $\mathbf{C^{Q}_{L2(32)}}$ | **1a** | **2a** | **3a** | $\mathbf{L_{11}}$ | $\mathbf{L_{31}}$ | $\mathbf{L_{33}}$ |
|---|---|---|---|---|---|---|
| $\varsigma_1$ | 1 | 1 | 1 | 1 | 1 | 1 |
| $\varsigma_2$ | 120 | -8 | 0 | 0 | 0 | 1 |
| $\varsigma_3$ | 16 | 0 | 1 | 1 | 1 | -1 |
| $\varsigma_4$ | 17 | 1 | -1 | 2 | -1 | 0 |
| $\varsigma_5$ | 34 | 2 | 4 | -1 | -1 | 0 |
| $\varsigma_6$ | 68 | 4 | -4 | -2 | 1 | 0 |

Wherein $L_{11}$ = 11a ∪ 11b ∪ 11c ∪ 11d ∪ 11e, $L_{31}$ = 31a ∪ 31b ∪ 31c ∪ 31d ∪ 31e ∪ 31f ∪ 31g ∪ 31h ∪ 31i ∪ 31j ∪ 31k ∪ 31l ∪ 31m ∪ 31n ∪ 31o and $L_{33}$ = 33a ∪ 33b ∪ 33c ∪ 33d ∪ 33e ∪ 33f ∪ 33g ∪ 33h ∪ 33i ∪ 33j.

We afford all the Q-conjugacy characters of $L_2(2^m)$ for m = 3, 4, 5 in Tables 2-4.☐

**Theorem 3.2**

*The Conway groups $Co_3$ has six unmatured dominant classes with the t = 2.*

*Furthermore, there are thirty eight Q- conjugacy characters for $Co_3$ with the following degrees: 1, 23, 253, 275, 1771, 1792, 2024, 4025, 5544, 7040, 7084, 8855, 19250, 23000, 26082, 31625, 31878, 40250, 41216, 57960, 63250, 73600, 80960, 91125, 93312, 129536, 177100, 184437, 221375, 226688, 246400, 249480, 253000 and 255024.*

**Proof:** According to similar discussion in the previous theorem, it is enough to report the dominant classes of $Co_3$ as follow:

1a, 2a, 2b, 3a, 3b, 3c, 4a, 4b, 5a, 5b, 6a, 6b, 6c, 6d, 6e, 7a, 8a, 8b, 8c, 9a, 9b, 10a, 10b, $M_{11}$ = 11a ∪ 11b, 12a, 12b, 12c, 14a, 15a, 15b, 18a, $M_{20}$ = 20a ∪ 20b, 21a, $M_{22}$ = 22a ∪ 22b, $M_{23}$ = 23a ∪ 23b, 24a, 24b, 30a which has four unmatured dominant classes with t = 2.

Table 5. The Q-Conjugacy Character Table of the Conway group $Co_3$

| $C^Q_{Co3}$ | 1a | 2a | 2b | 3a | 3b | 3c | 4a | 4b | 5a | 5b | 6a | 6b |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\pi_1$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\pi_2$ | 23 | 7 | -1 | -4 | 5 | -1 | -5 | 3 | -2 | 3 | 4 | -2 |
| $\pi_3$ | 253 | 13 | -11 | 10 | 10 | 1 | 9 | 1 | 3 | 3 | 10 | 4 |
| $\pi_4$ | 253 | 29 | -11 | 10 | 10 | 1 | -11 | 5 | 3 | 3 | 2 | 2 |
| $\pi_5$ | 275 | 35 | 11 | 5 | 14 | -1 | 15 | 7 | 0 | 5 | 5 | -1 |
| $\pi_6$ | 1792 | 0 | 32 | 64 | -8 | -14 | 0 | 0 | -8 | 2 | 0 | 0 |
| $\pi_7$ | 1771 | -21 | 11 | -11 | 16 | 7 | -5 | -5 | -4 | 1 | 21 | -3 |
| $\pi_8$ | 2024 | 104 | 0 | -1 | 26 | 8 | -24 | 8 | -1 | 4 | -1 | 5 |
| $\pi_9$ | 7040 | -128 | 0 | -88 | 20 | -16 | 0 | 0 | -10 | 0 | -8 | 16 |
| $\pi_{10}$ | 4025 | 105 | 1 | -25 | 29 | -7 | -35 | 5 | 0 | 5 | 15 | -3 |
| $\pi_{11}$ | 5544 | 168 | 0 | -45 | 36 | 0 | 40 | 8 | -6 | 4 | 3 | -3 |
| $\pi_{12}$ | 7084 | -84 | 44 | 10 | 19 | -14 | -4 | -4 | 9 | -1 | 18 | 6 |
| $\pi_{13}$ | 8855 | 231 | 55 | -1 | 35 | -7 | 19 | 11 | 5 | 0 | -9 | -3 |
| $\pi_{14}$ | 19250 | 210 | -110 | 80 | -10 | 14 | 10 | -6 | 0 | 0 | 0 | 12 |
| $\pi_{15}$ | 41216 | 0 | -32 | -256 | -40 | 14 | 0 | 0 | 16 | 6 | 0 | 0 |
| $\pi_{16}$ | 23000 | 280 | 120 | 50 | 5 | 8 | 40 | 8 | 0 | 0 | 10 | 10 |
| $\pi_{17}$ | 26082 | -126 | -54 | 81 | 0 | 0 | -6 | 10 | 7 | -3 | 9 | 9 |
| $\pi_{18}$ | 31625 | 265 | -55 | 35 | 35 | -1 | -55 | 9 | 0 | 0 | -5 | -5 |
| $\pi_{19}$ | 31625 | -55 | -55 | 35 | 35 | -1 | 25 | -7 | 0 | 0 | 35 | -1 |
| $\pi_{20}$ | 31625 | 505 | -55 | 35 | 35 | -1 | -35 | 5 | 0 | 0 | -5 | 1 |
| $\pi_{21}$ | 31878 | 294 | -66 | 45 | 45 | 0 | 46 | -2 | 3 | 3 | -3 | -3 |
| $\pi_{22}$ | 40250 | -70 | 10 | -115 | -25 | 14 | 10 | 10 | 0 | 0 | 5 | -7 |
| $\pi_{23}$ | 57960 | 168 | 120 | 126 | 45 | 0 | -40 | -8 | 10 | 0 | 6 | 6 |
| $\pi_{24}$ | 63250 | 210 | -110 | -65 | -20 | 22 | -30 | 2 | 0 | 0 | 15 | 3 |
| $\pi_{25}$ | 73600 | 0 | 144 | 160 | 16 | 13 | 0 | 0 | 0 | -5 | 0 | 0 |
| $\pi_{26}$ | 80960 | -448 | 0 | 176 | 50 | 8 | 0 | 0 | 10 | 0 | -16 | -16 |
| $\pi_{27}$ | 91125 | 405 | 45 | 0 | 0 | 27 | 45 | -3 | 0 | 0 | 0 | 0 |
| $\pi_{28}$ | 93312 | 0 | -144 | 0 | 0 | 27 | 0 | 0 | 12 | -3 | 0 | 0 |
| $\pi_{29}$ | 129536 | -512 | 0 | -64 | 44 | 8 | 0 | 0 | -14 | -4 | 16 | -8 |
| $\pi_{30}$ | 129536 | 512 | 0 | -64 | 44 | 8 | 0 | 0 | -14 | 4 | -16 | 8 |
| $\pi_{31}$ | 177100 | 140 | 44 | -20 | -29 | -14 | -20 | 12 | 0 | -5 | 20 | -4 |
| $\pi_{32}$ | 184437 | 405 | -99 | 0 | 0 | -27 | 45 | -3 | 12 | -3 | 0 | 0 |
| $\pi_{33}$ | 221375 | 735 | 55 | -160 | -25 | -7 | -25 | -9 | 0 | 0 | 0 | -12 |
| $\pi_{34}$ | 226688 | 0 | -176 | 320 | -40 | -7 | 0 | 0 | -12 | 3 | 0 | 0 |
| $\pi_{35}$ | 246400 | 0 | 176 | 160 | -56 | 7 | 0 | 0 | 0 | 5 | 0 | 0 |
| $\pi_{36}$ | 249480 | -504 | 0 | -81 | 0 | 0 | -24 | 8 | 5 | 0 | -9 | 9 |
| $\pi_{37}$ | 253000 | -440 | 0 | -125 | 10 | -8 | 40 | 8 | 0 | 0 | -5 | 1 |
| $\pi_{38}$ | 255024 | -336 | 0 | -126 | 36 | 0 | -16 | -16 | -1 | 4 | -6 | 6 |

Table 5 (continued); wherein $M_{11}$= 11a $\cup$ 11b.

| $C^{Q}_{Co3}$ | 6c | 6d | 6e | 7a | 8a | 8b | 8c | 9a | 9b | 10a | 10b | $M_{11}$ | 12a |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\pi_1$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\pi_2$ | 1 | -1 | -1 | 2 | 1 | -3 | 1 | -1 | 2 | 2 | -1 | 1 | -2 |
| $\pi_3$ | -2 | -2 | 1 | 1 | -1 | 3 | -1 | 1 | 1 | 3 | -1 | 0 | 0 |
| $\pi_4$ | 2 | -2 | 1 | 1 | -3 | -3 | 1 | 1 | 1 | -1 | -1 | 0 | -2 |
| $\pi_5$ | 2 | 2 | -1 | 2 | 1 | 5 | 1 | -1 | 2 | 0 | 1 | 0 | 3 |
| $\pi_6$ | 0 | -4 | 2 | 0 | 0 | 0 | 0 | 4 | -2 | 0 | 2 | -1 | 0 |
| $\pi_7$ | 0 | 2 | -1 | 0 | -1 | -1 | -1 | -2 | -2 | 4 | 1 | 0 | 1 |
| $\pi_8$ | 2 | 0 | 0 | 1 | 4 | -4 | 0 | -1 | -1 | -1 | 0 | 0 | -3 |
| $\pi_9$ | 4 | 0 | 0 | -2 | 0 | 0 | 0 | 2 | 2 | 2 | 0 | 0 | 0 |
| $\pi_{10}$ | -3 | 1 | 1 | 0 | -1 | -5 | -1 | 2 | 2 | 0 | 1 | -1 | 1 |
| $\pi_{11}$ | 0 | 0 | 0 | 0 | -4 | 4 | 0 | 0 | 0 | -2 | 0 | 0 | 1 |
| $\pi_{12}$ | 3 | -1 | 2 | 0 | 0 | 0 | 0 | 4 | -2 | 1 | -1 | 0 | 2 |
| $\pi_{13}$ | 3 | 1 | 1 | 0 | 5 | 1 | 1 | 2 | -4 | 1 | 0 | 0 | 1 |
| $\pi_{14}$ | 6 | -2 | -2 | 0 | 6 | -2 | -2 | -4 | 2 | 0 | 0 | 0 | 4 |
| $\pi_{15}$ | 0 | 4 | -2 | 0 | 0 | 0 | 0 | -4 | -4 | 0 | -2 | -1 | 0 |
| $\pi_{16}$ | 1 | 3 | 0 | -2 | 0 | 0 | 0 | -1 | 2 | 0 | 0 | -1 | -2 |
| $\pi_{17}$ | 0 | 0 | 0 | 0 | 2 | 2 | -2 | 0 | 0 | -1 | 1 | 1 | -3 |
| $\pi_{18}$ | -5 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | 0 | 0 | 0 | -1 |
| $\pi_{19}$ | -1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | 0 | 0 | 0 | -5 |
| $\pi_{20}$ | 7 | -1 | -1 | -1 | -5 | -1 | -1 | -1 | -1 | 0 | 0 | 0 | 1 |
| $\pi_{21}$ | -3 | -3 | 0 | 0 | 2 | 2 | -2 | 0 | 0 | -1 | -1 | 0 | 1 |
| $\pi_{22}$ | -1 | 1 | -2 | 0 | -2 | -2 | -2 | 5 | -1 | 0 | 0 | 1 | 1 |
| $\pi_{23}$ | -3 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -2 | 0 | 1 | 2 |
| $\pi_{24}$ | 0 | -2 | -2 | -2 | 2 | 2 | 2 | 4 | 1 | 0 | 0 | 0 | 3 |
| $\pi_{25}$ | 0 | 0 | -3 | 2 | 0 | 0 | 0 | 4 | 1 | 0 | -1 | -1 | 0 |
| $\pi_{26}$ | 2 | 0 | 0 | -2 | 0 | 0 | 0 | -1 | 2 | 2 | 0 | 0 | 0 |
| $\pi_{27}$ | 0 | 0 | 3 | -1 | -3 | -3 | 1 | 0 | 0 | 0 | 0 | 1 | 0 |
| $\pi_{28}$ | 0 | 0 | 3 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | -1 | 0 |
| $\pi_{29}$ | 4 | 0 | 0 | 1 | 0 | 0 | 0 | -1 | -1 | -2 | 0 | 0 | 0 |
| $\pi_{30}$ | -4 | 0 | 0 | 1 | 0 | 0 | 0 | -1 | -1 | 2 | 0 | 0 | 0 |
| $\pi_{31}$ | -1 | -1 | 2 | 0 | 0 | 0 | 0 | -5 | 1 | 0 | -1 | 0 | 4 |
| $\pi_{32}$ | 0 | 0 | -3 | 1 | -3 | -3 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| $\pi_{33}$ | 3 | 1 | 1 | 0 | 3 | 3 | -1 | 2 | 2 | 0 | 0 | 0 | -4 |
| $\pi_{34}$ | 0 | 4 | 1 | 0 | 0 | 0 | 0 | 2 | -1 | 0 | -1 | 0 | 0 |
| $\pi_{35}$ | 0 | -4 | 1 | 0 | 0 | 0 | 0 | -2 | -2 | 0 | 1 | 0 | 0 |
| $\pi_{36}$ | 0 | 0 | 0 | 0 | -4 | 4 | 0 | 0 | 0 | 1 | 0 | 0 | -3 |
| $\pi_{37}$ | -2 | 0 | 0 | -1 | 4 | -4 | 0 | 1 | 1 | 0 | 0 | 0 | 1 |
| $\pi_{38}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | 0 | 2 |

Table 5 (continued); wherein $M_n$= na $\cup$ nb, for n= 20, 22, 23.

| $C_{Co3}^{Q}$ | **12b** | **12c** | **14a** | **15a** | **15b** | **18a** | $\mathbf{M_{20}}$ | **21a** | $\mathbf{M_{22}}$ | $\mathbf{M_{23}}$ | **24a** | **24b** | **30a** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\pi_1$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\pi_2$ | 0 | 1 | 0 | 1 | 0 | 1 | 0 | -1 | -1 | 0 | -2 | 0 | -1 |
| $\pi_3$ | -2 | 0 | -1 | 0 | 0 | 1 | -1 | 1 | 0 | 0 | 2 | 0 | 0 |
| $\pi_4$ | 2 | -2 | 1 | 0 | 0 | -1 | -1 | 1 | 0 | 0 | 0 | 0 | 2 |
| $\pi_5$ | 1 | 0 | 0 | 0 | -1 | -1 | 0 | -1 | 0 | -1 | 1 | -1 | 0 |
| $\pi_6$ | 0 | 0 | 0 | 4 | 2 | 0 | 0 | 0 | -1 | -2 | 0 | 0 | 0 |
| $\pi_7$ | 1 | -2 | 0 | -1 | 1 | 0 | 0 | 0 | 0 | 0 | -1 | -1 | 1 |
| $\pi_8$ | -1 | 0 | -1 | -1 | 1 | -1 | 1 | 1 | 0 | 0 | 1 | -1 | -1 |
| $\pi_9$ | 0 | 0 | -2 | 2 | 0 | -2 | 0 | -2 | 0 | 2 | 0 | 0 | 2 |
| $\pi_{10}$ | -1 | 1 | 0 | 0 | -1 | 0 | 0 | 0 | 1 | 0 | -1 | 1 | 0 |
| $\pi_{11}$ | -1 | -2 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | -1 | 1 | -2 |
| $\pi_{12}$ | 2 | -1 | 0 | 0 | -1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | -2 |
| $\pi_{13}$ | -1 | 1 | 0 | -1 | 0 | 0 | -1 | 0 | 0 | 0 | -1 | 1 | 1 |
| $\pi_{14}$ | 0 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | 4 | 0 |
| $\pi_{15}$ | 0 | 0 | 0 | 4 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| $\pi_{16}$ | 2 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | -1 | 0 | 0 | 0 | 0 |
| $\pi_{17}$ | 1 | 0 | 0 | 1 | 0 | 0 | -1 | 0 | 1 | 0 | -1 | -1 | -1 |
| $\pi_{18}$ | 3 | -1 | -1 | 0 | 0 | 1 | 0 | -1 | 0 | 0 | 1 | 1 | 0 |
| $\pi_{19}$ | -1 | 1 | 1 | 0 | 0 | -1 | 0 | -1 | 0 | 0 | 1 | 1 | 0 |
| $\pi_{20}$ | -1 | 1 | 1 | 0 | 0 | 1 | 0 | -1 | 0 | 0 | 1 | -1 | 0 |
| $\pi_{21}$ | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | -1 | -1 | 2 |
| $\pi_{22}$ | 1 | 1 | 0 | 0 | 0 | -1 | 0 | 0 | -1 | 0 | 1 | 1 | 0 |
| $\pi_{23}$ | -2 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | -1 | 0 | 0 | 0 | 1 |
| $\pi_{24}$ | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | -1 | -1 | 0 |
| $\pi_{25}$ | 0 | 0 | 0 | 0 | 1 | 0 | 0 | -1 | 1 | 0 | 0 | 0 | 0 |
| $\pi_{26}$ | 0 | 0 | 0 | 1 | 0 | -1 | 0 | 1 | 0 | 0 | 0 | 0 | -1 |
| $\pi_{27}$ | 0 | 0 | -1 | 0 | 0 | 0 | 0 | -1 | 1 | -1 | 0 | 0 | 0 |
| $\pi_{28}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -1 | 1 | 0 | 0 | 0 |
| $\pi_{29}$ | 0 | 0 | -1 | 1 | -1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| $\pi_{30}$ | 0 | 0 | 1 | 1 | -1 | -1 | 0 | 1 | 0 | 0 | 0 | 0 | -1 |
| $\pi_{31}$ | 0 | 1 | 0 | 0 | 1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\pi_{32}$ | 0 | 0 | -1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| $\pi_{33}$ | 0 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\pi_{34}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\pi_{35}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| $\pi_{36}$ | -1 | 0 | 0 | -1 | 0 | 0 | 1 | 0 | 0 | -1 | -1 | 1 | 1 |
| $\pi_{37}$ | -1 | -2 | 1 | 0 | 0 | 1 | 0 | -1 | 0 | 0 | 1 | -1 | 0 |
| $\pi_{38}$ | 2 | 2 | 0 | -1 | 1 | 0 | -1 | 0 | 0 | 0 | 0 | 0 | -1 |

Furthermore, $Co_3$ has four unmatured Q-conjugacy characters $\pi_6$, $\pi_9$, $\pi_{14}$ and $\pi_{15}$ which are the sum of two irreducible characters respectively. Therefore, there are two column-reductions (similarly row-reductions) in the character table of $Co_3$.

There are thirty eight Q- conjugacy characters for $Co_3$ with the following degrees: 1, 23, 253, 275, 1771, 1792, 2024, 4025, 5544, 7040, 7084, 8855, 19250, 23000, 26082, 31625, 31878, 40250, 41216, 57960, 63250, 73600, 80960, 91125, 93312, 129536, 177100, 184437, 221375, 226688, 246400, 249480, 253000 and 255024, see all the Q-conjugacy characters of $Co_3$ which are stored in Table5.☐

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