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Kink Multi-soliton Solutions for the (2+1)-Dimensional Bogoyavlenskii-Schiff Equation

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Abstract

In this work, (2+1)-dimensional Bogoyavlenskii-Schiff equation, which cann't be converted to a complete form of Hirota's bilinear operator, is considered. By using modification of extended homoclinic test approach new kink breather soliton and kink multi-solitons solutions are obtained and exhibited, respectively.

Keywords: Bogoyavlenskii-Schiff equation, A modification of extended homoclinic test approach, kink breather soliton, kink multi-soliton

1. Introduction

As is well known that searching for exact solutions of nonlinear evolution equations arising in mathematical physics plays an important role in the study of nonlinear physical phenomena. Recently, the extended homoclinic test approach, a new method for seeking periodic soliton solutions for nonlinear evolution equation, was proposed by Dai and Wang. By means of the extended homoclinic test approach one can solve some nonlinear partial differential equations in their bilinear forms.

When an nonlinear partial differential equation has no a complete form of Hirota's bilinear operator we can not use this method. Recently, Darvishi and Najafi (2011) presented a modification of the the extended homoclinic test approach to solve some nonlinear partial differential equations which cann't be converted to a complete form of Hirota's bilinear operator. In this paper, upon using a modification of the the extended homoclinic test approach, we obtain some new kink breather soliton and kink multi-solitons solutions for (2+1)-D Bogoyavlenskii-Schiff (BS) equation.

We investigate explicit formulas of solution of the following (2+1)-D BS equation:

$$\varphi_{xt} + \varphi_{xxxy} + 4\varphi_x\varphi_{xy} + 2\varphi_{xx}\varphi_y = 0. \tag{1}$$

This equation was constructed by Bogoyavlenskii and Schiff in different ways. Namely, Bogoyavlenskii used the modified Lax formalism, whereas Schiff obtained the same equation by the reduction of the self-dual Yang-Mills equation (Schiff, 1992; Song et al., 1998). To solve Eq. (1), we introduce a new dependent variable φ by

$$\varphi = 2(lng)_x, \tag{2}$$

where $g = g(x, y, t)$ is unknown real function. Substituting Eq. (2) into Eq. (1), we can reduce Eq. (1) into the following equation

$$2(lng)_{xxt} + 2(lng)_{xxxxy} + 16(lng)_{xx}(lng)_{xxy} + 8(lng)_{xy}(lng)_{xxx} = 0, \tag{3}$$

which can be integrated once with resect to x to give

$$(lng)_{xt} + (lng)_{xxxy} + 6(lng)_{xx}(lng)_{xy} + 2\partial_x^{-1}((lng)_{xxy}(lng)_{xx} - (lng)_{xy}(lng)_{xxx}) = 0. \tag{4}$$

Therefore, Eq. (4) can be written as

$$(D_xD_t + D_x^3D_y)g \cdot g + 2g^2\partial_x^{-1}(D_x(lng)_{xy}(lng)_{xx}) = 0, \tag{5}$$

where the Hirota bilinear operator D is defined by (m, n ≥ 0)

$$D_x^m D_t^n f(x,t)\cdot g(x,t) = (\tfrac{\partial}{\partial x} - \tfrac{\partial}{\partial x'})^m (\tfrac{\partial}{\partial t} - \tfrac{\partial}{\partial t'})^n [f(x,t)g(x',t')]|_{x'=x,t'=t}. \tag{6}$$

Eq.(5) is a non-completed form of Hirota's bilinear operator equation. In this case we propose a new method, i.e. a modification of extended homoclinic test approach. We suppose

$$\begin{cases} (D_xD_t + D_x^3D_y)g\cdot g = 0 \\ \partial_x^{-1}(D_x(lng)_{xy}(lng)_{xx}) = 0. \end{cases} \tag{7}$$

2. Kink Breather Soliton Solutions

Now we choose extended homoclinic test funtion

$$g = e^{-\xi} + \delta_1\cos(\eta) + \delta_2 e^{\xi}, \tag{8}$$

where $\xi = \lambda_1 x + \mu_1 y + \nu_1 t, \eta = \lambda_2 x + \mu_2 y + \nu_2 t$, and $\lambda_i, \mu_i, \nu_i, \delta_i (i = 1, 2)$, are some constants to be determined later. Substituting Eq. (8) into Eqs. (7), and equating all the coefficients of different powers of $e^{\xi}, e^{-\xi}, \sin(\eta), \cos(\eta)$ and constant term to zero, we can obtain a set of algebraic equations for $\lambda_i, \mu_i, \nu_i, \delta_i (i = 1, 2)$.

$$\begin{cases} \delta_1\delta_2\mu_2\nu_1 + \delta_1\delta_2\lambda_1^3\mu_2 + \delta_1\delta_2\mu_1\nu_1 - 3\delta_1\delta_2\lambda_1\lambda_2^2\mu_2 + 3\delta_1\delta_2\lambda_1^2\lambda_2\mu_1 - \delta_1\delta_2\lambda_2^3\mu_1 = 0 \\ -3\delta_1\delta_2\lambda_1^2\lambda_2\mu_2 - \delta_1\delta_2\mu_2\nu_2 + \delta_1\delta_2\lambda_1^3\mu_1 + \delta_1\delta_2\mu_1\nu_1 - 3\delta_1\delta_2\lambda_1\lambda_2^2\mu_1 + \delta_1\delta_2\lambda_2^3\mu_2 = 0 \\ 16\delta_2\lambda_1^3\mu_1 + 4\delta_2\mu_1\nu_1 - \delta_1^2\mu_2\nu_2 + 4\delta_1\lambda_2^3\mu_2 = 0 \\ 3\delta_1\lambda_1\lambda_2^2\mu_2 - \delta_1\lambda_1^3\mu_2 + \delta_1\lambda_2^3 b_1 - \delta_1\mu_2\nu_1 - 3\delta_1\lambda_1^2\lambda_2\mu_1 - \delta_1\mu_1\nu_1 = 0 \\ \delta_1\lambda_1^3\mu_1 - 3\delta_1\lambda_1^2\lambda_2\mu_2 - 3\delta_1\lambda_1\lambda_2^2\mu_1 + \delta_1\mu_1\nu_1 + \delta_1\lambda_2^3\mu_2 - \delta_1\mu_2\nu_2 = 0 \\ -4\delta_1^2\delta_2\lambda_1^2\lambda_2^2\mu_1 + 4\delta_1^2\delta_2\lambda_1^3\lambda_2\mu_2 - 4\delta_1^2\delta_2\lambda_2^4\mu_1 + 4\delta_1^2\delta_2\lambda_1\lambda_2^3\mu_2 = 0 \\ 16\delta_1\delta_2\lambda_1^4\mu_2 - 16\delta_1\delta_2\lambda_1^3\lambda_2\mu_1 + 16\delta_1\delta_2\lambda_1^2\lambda_2^2\mu_2 - 16\delta_1\delta_2\lambda_1\lambda_2^3\mu_1 = 0 \\ 4\delta_1^2\lambda_1^2\lambda_2^2\mu_1 - 4\delta_1^2\lambda_1^3\lambda_2\mu_2 - 4\delta_1^2\lambda_1\lambda_2^3\mu_2 + 4\delta_1^2\lambda_2^4\mu_1 = 0. \end{cases} \tag{9}$$

Solving the system of Eqs. (9) with the aid of Maple, we get the following results:
Case(I):

$$\mu_1 = \tfrac{\lambda_1\mu_2}{\lambda_2}, \quad \nu_1 = \lambda_1(3\lambda_2^2 - \lambda_1^2), \quad \nu_2 = \lambda_2(\lambda_2^2 - 3\lambda_1^2), \quad \delta_2 = -\tfrac{\delta_1^2\lambda_2^2}{4\lambda_1^2}, \tag{10}$$

where $\lambda_1, \lambda_2, b_2, \delta_1$ are some free real constants. Substituting Eq. (10) into Eq. (8) and taking $\delta_2 < 0$, we have

$$g_1 = -2\sqrt{-\delta_2}\sinh(\xi + \tfrac{1}{2}\ln(-\delta_2)) + \delta_1\cos(\eta), \tag{11}$$

where $\xi = \lambda_1 x + \frac{\lambda_1\mu_2}{\lambda_2} y + \lambda_1(3\lambda_2^2 - \lambda_1^2)t, \eta = \lambda_2 x + \mu_2 y + \lambda_2(\lambda_2^2 - 3\lambda_1^2)t, \delta_2 = -\frac{\delta_1^2\lambda_2^2}{4\lambda_1^2}$. Substituting Eq. (11) into Eq. (2) yields the kink periodic soliton solutions for BS equation as follows:

$$\varphi_1 = \frac{2(2\lambda_1\sqrt{-\delta_2}\cosh(\xi + \frac{1}{2}\ln(-\delta_2)) + \lambda_2\delta_1\sin(\eta))}{2\sqrt{-\delta_2}\sinh(\xi + \frac{1}{2}\ln(-\delta_2)) - \delta_1\cos(\eta)}. \tag{12}$$

Case(II):

$$\lambda_1 = i\lambda_2 \quad \nu_1 = 4i\lambda_2^3, \quad \nu_2 = 4\lambda_2^3, \tag{13}$$

where $\mu_1, \delta_1, \delta_2$ are some free real constants, λ_2, μ_2 are free constants. Substituting Eq. (13) into Eq. (8), we have

$$g_2^* = e^{-\xi} + \delta_1\cos(\eta) + \delta_2 e^{\xi}, \tag{14}$$

where $\xi = i\lambda_2 x + \mu_1 y + 4i\lambda_2^3 t, \eta = \lambda_2 x + \mu_2 y + 4\lambda_2^3 t$. If taking $\lambda_2 = iA_2, \mu_2 = iB_2$ in Eq. (14) , then we have

$$g_2 = 2\sqrt{\delta_2}\cosh(\xi + \frac{1}{2}\ln(\delta_2)) + \delta_1\cosh(\eta), \tag{15}$$

where A_2, B_2 are real number, $\delta_2 > 0, \xi = -A_2 x + b_1 y + 4A_2^3 t, \eta = A_2 x + B_2 y - 4A_2^3 t$. Substituting Eq. (15) into Eq. (2) yields the kink breather soliton solutions of BS equation as follows:

$$\varphi_2 = \frac{2A_2(-2\sqrt{\delta_2}\sinh(\xi + \frac{1}{2}\ln(\delta_2)) + \delta_1\sinh(\eta))}{2\sqrt{\delta_2}\cosh(\xi + \frac{1}{2}\ln(\delta_2)) + \delta_1\cosh(\eta)}. \tag{16}$$

3. Kink multi-soliton solutions

If we choose extended homoclinic test funtion as

$$g = e^{-\xi} + \delta_1 \cos(\eta) + \delta_2 \sinh(\gamma) + \delta_3 e^{\xi}, \tag{17}$$

where $\xi = \lambda_1 x + \mu_1 y + \nu_1 t, \eta = \lambda_2 x + \mu_2 y + \nu_2 t, \gamma = \lambda_3 x + \mu_3 y + \nu_3 t$ and $\lambda_i, \mu_i, \nu_i, \delta_i (i = 1, 2, 3)$ are some constants to be determined later. Substituting Eq. (18) into Eqs. (8), and equating all the coefficients of different powers of $e^{\xi}, e^{-\xi}, \sin(\eta), \cos(\eta), \sinh(\gamma), \cosh(\gamma)$ and constant term to zero, we can obtain a set of algebraic equations for $\lambda_i, \mu_i, \nu_i, \delta_i (i = 1, 2, 3)$.

$$\begin{cases}
\delta_1\delta_3(-3\lambda_1\lambda_2^2\mu_2 + \lambda_1^3\mu_2 + \mu_1\nu_2 + 3\lambda_1^2\lambda_2\mu_1 + \nu_1\mu_2 - \lambda_2^3\mu_1) = 0 \\
\delta_1\delta_3(\lambda_2^3\mu_2 - \mu_2\nu_2 - 3\lambda_1^3\lambda_2\mu_2 + \mu_1\nu_1 - 3\lambda_1\lambda_2^2\mu_1 + \lambda_1^3\mu_1) = 0 \\
\delta_2\delta_3(\mu_3\nu_3 + \lambda_1^3\mu_1 + \lambda_3^3\mu_3 + 3\lambda_1\lambda_3^2\mu_1 + 3\lambda_1^2\lambda_3\mu_3 + \nu_1\mu_1) = 0 \\
\delta_2\delta_3(\lambda_3^3\mu_1 + \mu_1\nu_3 + 3\lambda_3^2\lambda_1\mu_3 + 3\lambda_1^2\lambda_3\mu_1 + \lambda_1^3\mu_3 + \mu_3\nu_1) = 0 \\
\delta_1\delta_2(\lambda_3^3\mu_2 - \lambda_2^3\mu_3 + 3\lambda_2\lambda_3^2\mu_3 + \mu_2\nu_3 - 3\lambda_2^2\lambda_3\mu_2 + \mu_3\nu_2) = 0 \\
\delta_1\delta_2(\lambda_3^3\mu_3 - 3\lambda_2\lambda_3^3 - 3\lambda_2^2\lambda_3\mu_3 + \lambda_2^3\mu_2 + \mu_3\nu_3 - \mu_2\nu_2) = 0 \\
-\delta_1^2\mu_2\nu_2 + 4\delta_1^2\lambda_2^3\mu_2 + 16\delta_3\lambda_1^3\mu_1 + 4\delta_3\mu_1\nu_1 + 4\delta_2^2\lambda_3^3\mu_3 + \delta_2^2\mu_3\nu_3 = 0 \\
\delta_1(\lambda_2^3\mu_1 - \lambda_1^3\mu_2 - \mu_1\nu_2 - \mu_2\nu_1 - 3\lambda_1^2\lambda_2 + 3\lambda_1\lambda_2^2\mu_2) = 0 \\
\delta_1(\lambda_2^3\mu_2 + \lambda_1^3\mu_1 - \mu_2\nu_2 - 3\lambda_1^2\lambda_2\mu_2 + \mu_1\nu_1 - 3\lambda_1\lambda_2^2\mu_1) = 0 \\
\delta_2(\mu_3\nu_3 + 3\lambda_1\lambda_3^3\mu_1 + \mu_1\nu_1 + \lambda_3^3\mu_3 + \lambda_1^2\lambda_3\mu_3 + \lambda_1^3\mu_1) = 0 \\
\delta_2(\lambda_3^3\mu_1 + \mu_1\nu_3 + \mu_3\nu_1 + 3\lambda_1^2\lambda_3\mu_1 + \lambda_1^2\mu_3 + 3\lambda_1\lambda_3^2\mu_3) = 0 \\
-4\delta_1\delta_2\delta_3(\lambda_1\lambda_2\lambda_3^2\mu_1 + \lambda_1^2\lambda_2\lambda_3\mu_3 - \lambda_2^2\lambda_3^2\mu_2 + \lambda_2^3\lambda_3\mu_3 - \lambda_1^2\lambda_2^2\mu_2 + \lambda_1\lambda_2^3\mu_1 - 2\lambda_1^2\lambda_3^2\mu_2) = 0 \\
4\delta_1\delta_2\delta_3(\lambda_1\lambda_2^2\mu_3 + 2\lambda_1\lambda_2\lambda_3^2\mu_3 - \lambda_2\lambda_3\mu_1 + 2\lambda_1^2\lambda_2\lambda_3\mu_1 - \lambda_1^3\lambda_2\mu_3 - \lambda_1^3\lambda_3\mu_2 - 2\lambda_1\lambda_2^2\lambda_3\mu_2 - \lambda_1\lambda_3^3\mu_2 + \lambda_2^3\lambda_3\mu_1) = 0 \\
4\delta_1\delta_2\delta_3(2\lambda_2^2\lambda_3^2\mu_1 + \lambda_1^2\lambda_3^2\mu_1 - \lambda_1\lambda_2\lambda_3^2\mu_2 - \lambda_1\lambda_2^2\lambda_3\mu_3 - \lambda_1^3\lambda_3\mu_3 + \lambda_1^3\lambda_2\mu_2 - \lambda_1^2\lambda_2^2\mu_1) = 0 \\
4\delta_1\delta_2\delta_3(-\lambda_1\lambda_2^2\lambda_3\mu_1 + 2\lambda_1^2\lambda_2^2\mu_3 - \lambda_1\lambda_3^3\mu_1 + \lambda_1^2\lambda_3^2\mu_3 - \lambda_1^2\lambda_2\mu_2 - \lambda_2^2\lambda_3^2 + \lambda_2\lambda_3^3\mu_2) = 0 \\
4\delta_1^2\delta_3(-\lambda_1^2\lambda_2^2\mu_1 + \lambda_1\lambda_2^3\mu_2 + \lambda_1^3\lambda_2\mu_2 - \lambda_2^4\mu_1) + 4\delta_2^2\delta_3(-\lambda_1^2\lambda_3^2\mu_1 - \lambda_1\lambda_3^3\mu_3 + \lambda_1^3\lambda_3\mu_3 + \lambda_3^4\mu_1) = 0 \\
4\delta_1\delta_2^2(-\lambda_3^4\mu_2 - \lambda_2^2\lambda_3^2\mu_1 + \lambda_2^3\lambda_3\mu_3 + \lambda_2\lambda_3^3\mu_3) + 4\delta_1\delta_3(-\lambda_1^3\lambda_2\mu_1 + \lambda_1^2\lambda_2^2\mu_2 + \lambda_1^4\mu_2 - \lambda_1\lambda_2^3\mu_1) = 0 \\
4\delta_1^2\delta_2(\lambda_2^3\lambda_3\mu_2 - \lambda_2^4\mu_3 + \lambda_2\lambda_3^3\mu_2) + 4\delta_2\delta_3(\lambda_1^3\lambda_3\mu_1 - \lambda_1^4\mu_3 - \lambda_1\lambda_3^3\mu_1 + \lambda_1^2\lambda_3^2\mu_3) = 0 \\
4\delta_1\delta_2(2\lambda_1^2\lambda_3^2\mu_2 - \lambda_2^3\lambda_3\mu_3 - \lambda_1^2\lambda_2\lambda_3\mu_3 + \lambda_1^2\lambda_2^2\mu_1 - \lambda_1\lambda_3^3\mu_1 - \lambda_1\lambda_2\lambda_3^2\mu_1 + \lambda_2^2\lambda_3^2\mu_2) = 0 \\
4\delta_1\delta_2(\lambda_1^3\lambda_3\mu_2 + \lambda_2\lambda_3^3\mu_1 - 2\lambda_1\lambda_2\lambda_3^2\mu_3 + 2\lambda_1\lambda_2^2\lambda_3\mu_2 + \lambda_1\lambda_2 + 3^3\mu_2 + \lambda_1^3\lambda_2\mu_3 - 2\lambda_1^2\lambda_2\lambda_3\mu_1 - \lambda_1\lambda_2^3\mu_3 - \lambda_2^3\lambda_3\mu_1) = 0 \\
4\delta_1\delta_2(\lambda_1^2\lambda_2^2\mu_1 - \lambda_1^3\lambda_2\mu_2 - 2\lambda_2^2\lambda_3^2\mu_1 + \lambda_1^3\lambda_3\mu_3 + \lambda_1\lambda_2^2\lambda_3\mu_3 + \lambda_1\lambda_2\lambda_3^2\mu_2 - \lambda_1^2\lambda_3^2\mu_1) = 0 \\
4\delta_1\delta_2(\lambda_1\lambda_2^2\lambda_3\mu_1 + \lambda_2^2\lambda_3^2\mu_3 + \lambda_1^2\lambda_2\lambda_3\mu_2 - \lambda_2\lambda_3^3\mu_2 - \lambda_1^2\lambda_3^2\mu_3 + \lambda_1\lambda_3^3\mu_1 - 2\lambda_1^2\lambda_2^2\mu_3) = 0 \\
\delta_1^2(\lambda_1\lambda_2^3\mu_2 + \lambda_1^2\lambda_2^2\mu_1 - \lambda_1^3\lambda_2\mu_2 + \lambda_2^4\mu_1) + \delta_2^2(\lambda_1^2\lambda_3^2\mu_1 + \lambda_1\lambda_3^3\mu_3 - \lambda_1^3\lambda_3\mu_3 - \lambda_3^4\mu_1) = 0.
\end{cases} \tag{18}$$

Solving the system of Eqs. (18) with the aid of Maple, we get the following results:
Case(III):

$$\lambda_1 = \lambda_3, \quad \lambda_2 = i\lambda_3, \quad \mu_1 = 0, \quad \nu_1 = -4\lambda_3^3, \quad \nu_2 = -4i\lambda_2^3, \quad \nu_3 = -4\lambda_3^3, \tag{19}$$

where $\lambda_3, \mu_3, \delta_1, \delta_2, \delta_3$ are some free real constants, μ_2 is a free constants. Substituting Eq. (19) into Eq. (17) and taking $\mu_2 = iB_2, \delta_3 > 0$, we have

$$g_3 = 2\sqrt{\delta_3}\cosh(\xi + \frac{1}{2}\ln(\delta_3)) + \delta_1\cosh(\eta) + \delta_2\sinh(\gamma), \tag{20}$$

where B_2 is a real number, $\xi = \lambda_3 x - 4\lambda_3^3 t, \eta = \lambda_3 x + B_2 y - 4\lambda_3^3 t, \gamma = \lambda_3 x + \mu_3 y - 4\lambda_3^3 t$. Substituting Eq. (20) into Eq. (2) yields the kink three-solitary solutions of BS equation as follows:

$$\varphi_3 = \frac{2\lambda_3(2\sqrt{\delta_3}\sinh(\xi + \frac{1}{2}\ln(\delta_3)) + \delta_1\sinh(\eta) - \delta_2\cosh(\gamma))}{2\sqrt{\delta_3}\cosh(\xi + \frac{1}{2}\ln(\delta_3)) + \delta_1\cosh(\eta) + \delta_2\sinh(\gamma)}. \tag{21}$$

Case(IV):

$$\lambda_1 = i\lambda_2, \quad \lambda_3 = i\lambda_2, \quad \nu_1 = i4\lambda_2^3, \quad \nu_2 = -i4\lambda_2^3, \quad \nu_3 = i4\lambda_2^3, \tag{22}$$

where $\mu_1, \mu_3, \delta_1, \delta_2, \delta_3$ are some free real constants, λ_2, μ_2 are some free constants. Substituting Eq. (22) into Eq. (17) and taking $\lambda_2 = iA_2, \mu_2 = iB_2, \delta_3 > 0$, we have

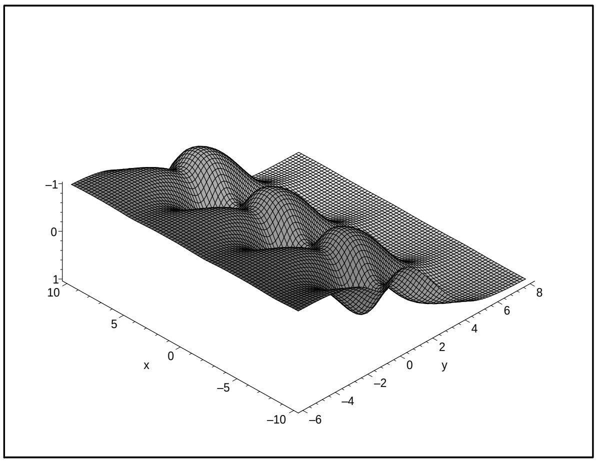
$$g_4 = 2\sqrt{\delta_3}\cosh(\xi + \frac{1}{2}\ln(\delta_3)) + \delta_1\cosh(\eta) + \delta_2\sinh(\gamma), \tag{23}$$

where A_2, B_2 are real number, $\xi = -A_2 x + \mu_1 y + 4A_2^3 t, \eta = A_2 x + B_2 y - 4A_2^3 t, \gamma = -A_2 x + \mu_3 y + 4A_2^3 t$. Substituting Eq. (23) into Eq. (2) yields the cross-kink three-solitary solutions of BS equation as follows:

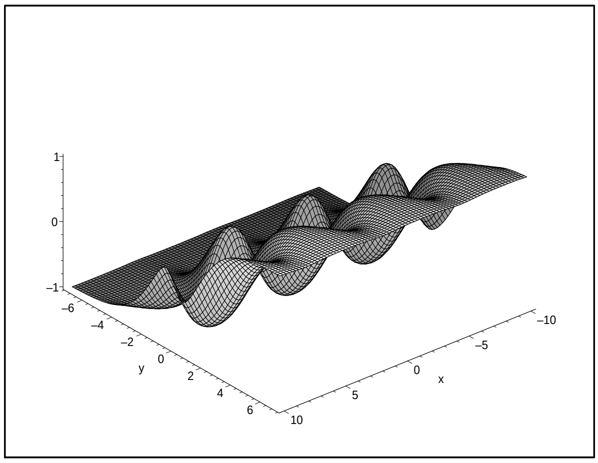
$$\varphi_4 = -\frac{2A_2(2\sqrt{\delta_3}\sinh(\xi + \frac{1}{2}\ln(\delta_3)) - \delta_1\sinh(\eta) + \delta_2\cosh(\gamma))}{2\sqrt{\delta_3}\cosh(\xi + \frac{1}{2}\ln(\delta_3)) + \delta_1\cosh(\eta) + \delta_2\sinh(\gamma)}. \tag{24}$$

4. Conclusion

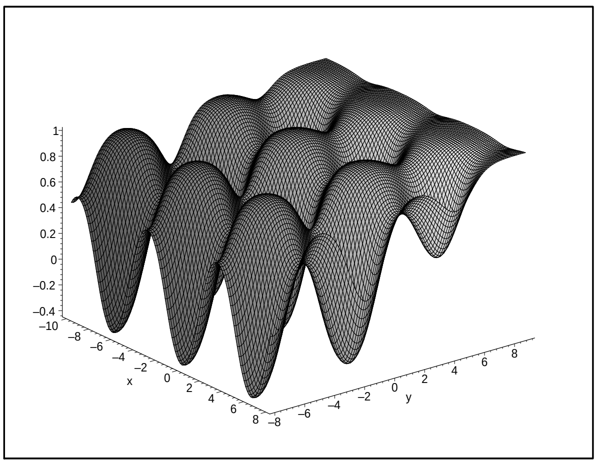
By means of modification of extended homoclinic test approach, we discuss the (2+1)-D Bogoyavlenskii-Schiff equation and find some new kink breather soliton and kink multi-solitons solutions. A modification of extended homoclinic test approach may provide us with a straightforward and effective mathematical tool for seeking soliton solutions of higher dimensional nonlinear evolution equations which cann't be converted to a complete form of Hirota's bilinear operator, so that it can be applied to other nonlinear partial differential equations.



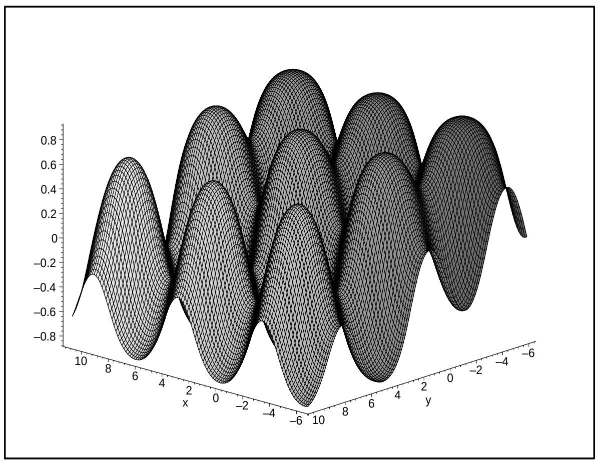
Fig(a). The figure of φ_1 as $\lambda_1 = \frac{1}{3}, \lambda_2 = 1, \mu_2 = 5, \delta_1 = 1, t = 1$.



Fig(b). The figure of φ_2 as $A_2 = \frac{1}{2}, \mu_1 = 2, B_2 = 2, \delta_1 = 1, \delta_2 = 3, t = 1$.



Fig(c).The figure of φ_3 as $\lambda_3 = \frac{1}{2}, B_2 = \frac{1}{2}, \mu_3 = \frac{1}{2}, \delta_1 = 1, \delta_2 = 1, \delta_3 = 3, t = 1$.



Fig(d).The figure of φ_4 as $A_2 = \frac{1}{2}, B_2 = \frac{1}{2}, \mu_1 = \frac{1}{2}, \mu_3 = 0.2, \delta_1 = 1, \delta_2 = 1, \delta_3 = 1, t = 1$.

References

Dai, Z. D., Liu, J., & Li, D. (2009). Applications of HTA and EHTA to YTSF equation extended three-soliton method. *Applied Mathematics and Computation, 207*, 360. http://dx.doi.org/10.1016/j.amc.2008.10.042

Dai, Z. D., Liu, J., & Liu, Z. J. (2010). Exact periodic kink-wave and degenerative soliton solutions for potential Kadomtsev-Petviashvili equation. *Commum. Nonlinear Sci. Numer. Simulat., 15*(9), 2331-2336. http://dx.doi.org/10.1016/j.cnsns.2009.09.037

Dai, Z. D., & Li, Z. T. (2011). Exact periodic cross-kink wave solutions and breather type of two-solitary wave solutions for the (3 + 1)-dimensional potential-YTSF equation. *Computers and Mathematics with Applications, 61*, 1939-1945.

Darvishi, M. T., & Najafi, M. (2011). A modification of extended homoclinic test approach to solve the (3+1)-dimensional potential-YTST equation. *Chin. Phys. Lett., 28*, 040202.

Schiff, J. (1992). Their Asymtopics and Physical Application (New York: Plenum). *Painlevé Transendents,* 393.

Song, J. Y., Kouichi, T., Narimasa, S., & Takeshi, F. (1998). N soliton to the Bogoyavlenskii-Schiff equation and a quest for the sliton solution in (3+1)dimensions. *J Phys A: Math. Gen., 31,* 3337-3347.

Wang, C. J., Dai, Z.D., & Liang, L. (2010). Exact three-wave solution for higher dimensional KdV-type equation. *Applied Mathematics and Computation, 216*, 501-505. http://dx.doi.org/10.1016/j.amc.2010.01.057

Xu, Z. H., & Xian, D. Q. (2010). New periodic solitary-wave solutions for the Benjiamin Ono equation. *Applied Mathematics and Computation, 215*, 4439-4442. http://dx.doi.org/10.1016/j.amc.2009.11.009

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