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New Approach of Generalized $\exp(-\phi(\xi))$ Expansion Method and Its Application to Som Nonlinear Partial Differential Equations

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Abstract

In this article, the new approach of generalized $\exp(-\phi(\xi))$expansion method has been successfully implemented to seek traveling wave solutions of the Korteweg-de vries equation and the modified Zakharov-Kuznetsov equation. The result reveals that the method together with the new ordinary differential equation is a very influential and effective tool for solving nonlinear partial differential equations in mathematical physics and engineering. The obtained solutions have been articulated by the hyperbolic functions, trigonometric functions and rational functions with arbitrary constants.

Keywords: generalized $\exp(-\phi(\xi))$ expansion method, exact solutions, Kdv equation, modified ZK equation

1. Introduction

In recent years, the nonlinear partial deferential equations (NPDEs) are widely used to describe many important phenomena and dynamic processes in physics, mechanics, chemistry and biology, etc. With the development of soliton theory, many powerful methods have been presented, such as the Jacobi elliptic function expansion method (M.Inc & M. Ergut, 2005), The $\left(\frac{\acute{G}}{G}\right)$ expansion method (M. Wang, X. Li & j. Zhang, 2008; M. Bashir & A. Moussa 2014; L.X.Li, E.q.Li & M. Wang 2010),the sine.cosine method (E. Yusufoglu & A. Bekir, 2006), the tanh-coth method (M. Bashir & A. Moussa 2014), the F-expansion method (M. Bashir & L. Alhakim 2013), the Exp-function method (He. J. H & Wu. x. H 2006), the $\exp(-\phi(\xi))$ expansion method and others (M. A. Akbar & N. H. Ali, 2014; K. Khan & M. A. Akbar, 2013; N. Rahman, M. N. Alam, H. Roshid, S. Akter & M. Ali Akbar, 2014; M.j. Ablowitz, G. Biondini & S. Lillo, 1997; M. Mirzazadeh, S. Khaleghizadeh, 2013; S.T. Mohyud- Din & M.A. Noor, 2008). the $\exp(-\phi(\xi))$ expansion method is powerful to solve nonlinear partial deferential equations (NPDEs) and can help to get many new exact solutions which we have never seen before. The present work is motivated by the desire to generate many new and more general exact traveling wave solutions. For this purpose, we propose new approach of $\exp(-\phi(\xi))$-expansion method for investigating NLEEs. To depict the novelty and advantages of the methods, we apply these to the KdV equation and modified ZK Equation.

2. Description of New Approach of Generalized $\exp(-\phi(\xi))$ Expansion Method

Suppose that we have a nonlinear PDE in the following form :

$$F(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, u_{xxt}, ...) = 0 \tag{2.1}$$

where $u = u(x, t)$ is an unknown function F is a polynomial in $u = u(x, t)$ and its partial derivatives, in which the highest order derivatives and nonlinear terms are involved.The main steps of this method are as follows

Step 1: Use the traveling wave transformation :

$$u(x, t) = u(\xi), \qquad \xi = k_1 x + k_2 t \tag{2.2}$$

where k_1, k_2 are a constants to be determined latter, permits us reducing (2.1) to an ODE for $u = u(\xi)$ in the form

$$P\left(u, k_1\acute{u}, k_2\acute{u}, k_1k_2u'', ...\right) = 0 \tag{2.3}$$

where P is a polynomial of $u = u(\xi)$ and its total derivatives.

Stepe 2: Balancing the highest derivative term with the nonlinear terms in (2.3),we find the value of the positive integer (m).

Step 3 : Suppose that the solution of (2.3) can be expressed as follows:

$$u(\xi) = \sum_{i=0}^{m} \alpha_i \left[\exp\left(-\frac{A_1\phi(\xi) + A_2}{A_3\phi(\xi) + A_4}\right)\right]^i \tag{2.4}$$

where, α_i $(i = 0, 1, ..m)$ are constants to be determined, such that $\alpha_i \neq 0$ and $\phi(\xi)$ satisfies the following differential equation:

$$\acute{\phi}(\xi) = \frac{(A_3\phi(\xi) + A_4)^2}{(A_1A_4 - A_2A_3)}\left(\exp\left(-\frac{A_1\phi(\xi) + A_2}{A_3\phi(\xi) + A_4}\right) + \mu\exp\left(\frac{A_1\phi(\xi) + A_2}{A_3\phi(\xi) + A_4}\right) + \lambda\right) \tag{2.5}$$

where $(A_1A_4 - A_2A_3) \neq 0$ Eq. (2.5) gives the following solutions:
Family 1:when

$$(A_1A_4 - A_2A_3) \neq 0, \mu \neq 0, \left(\lambda^2 - 4\mu\right) > 0, A_2 = 0$$

$$\phi_1(\xi) = \frac{A_4 \ln\left(\frac{-\sqrt{(\lambda^2-4\mu)}\tanh\left(\frac{\sqrt{(\lambda^2-4\mu)}}{2}(\xi+c)\right)-\lambda}{2\mu}\right)}{A_1 - A_3\ln\left(\frac{-\sqrt{(\lambda^2-4\mu)}\tanh\left(\frac{\sqrt{(\lambda^2-4\mu)}}{2}(\xi+c)\right)-\lambda}{2\mu}\right)} \tag{2.6}$$

Family 2 :when

$$(A_1A_4 - A_2A_3) \neq 0, \mu \neq 0, \left(\lambda^2 - 4\mu\right) < 0, A_2 = 0$$

$$\phi_2(\xi) = \frac{A_4 \ln\left(\frac{\sqrt{(4\mu-\lambda^2)}\tan\left(\frac{\sqrt{(4\mu-\lambda^2)}}{2}(\xi+c)\right)-\lambda}{2\mu}\right)}{A_1 - A_3\ln\left(\frac{\sqrt{(4\mu-\lambda^2)}\tan\left(\frac{\sqrt{(4\mu-\lambda^2)}}{2}(\xi+c)\right)-\lambda}{2\mu}\right)} \tag{2.7}$$

Family 3: when

$$(A_1A_4 - A_2A_3) \neq 0, \mu \neq 0, \lambda \neq 0, \left(\lambda^2 - 4\mu\right) = 0, A_2 = 0$$

$$\phi_3(\xi) = \frac{A_4 \ln\left(-\frac{2(\lambda(\xi+c)+2)}{\lambda^2(\xi+c)}\right)}{A_1 - A_3\ln\left(-\frac{2(\lambda(\xi+c)+2)}{\lambda^2(\xi+c)}\right)} \tag{2.8}$$

*Family 4:*when

$$(A_1A_4 - A_2A_3) \neq 0, \mu \neq 0, \lambda \neq 0, \left(\lambda^2 - 4\mu\right) > 0, A_3 = 0$$

$$\phi_4(\xi) = -2\left(\frac{A_2}{A_1}\right) + \left(\frac{A_4}{A_1}\right)\ln\left(\frac{-\tanh\left(\sqrt{(\lambda^2-4\mu)}\frac{(\xi+c)}{2}\right)\sqrt{e^{\left(\frac{2A_2}{A_4}\right)}(\lambda^2-4\mu)} - \lambda e^{\left(\frac{A_2}{A_4}\right)}}{2\mu}\right) \tag{2.9}$$

*Family5:*when

$$(A_1A_4 - A_2A_3) \neq 0, \mu \neq 0, \lambda \neq 0, \left(\lambda^2 - 4\mu\right) < 0, A_3 = 0$$

$$\phi_5(\xi) = -2\left(\frac{A_2}{A_1}\right) + \left(\frac{A_4}{A_1}\right)\ln\left(\frac{\tan\left(\sqrt{(4\mu-\lambda^2)}\frac{(\xi+c)}{2}\right)\sqrt{e^{\left(\frac{2A_2}{A_4}\right)}(4\mu-\lambda^2)} - \lambda e^{\left(\frac{A_2}{A_4}\right)}}{2\mu}\right) \tag{2.10}$$

Family 6: when

$$(A_1A_4 - A_2A_3) \neq 0, \mu \neq 0, \lambda \neq 0, \left(\lambda^2 - 4\mu\right) = 0, A_3 = 0$$

$$\phi_6(\xi) = -\left(\frac{A_2}{A_1}\right) + \left(\frac{A_4}{A_1}\right)\ln\left(-\frac{2(\lambda(\xi+c)+2)}{\lambda^2(\xi+c)}\right) \tag{2.11}$$

*Family 7:*when

$$(A_1A_4 - A_2A_3) \neq 0, \mu = 0, \lambda \neq 0, \left(\lambda^2 - 4\mu\right) > 0$$

$$\phi_7(\xi) = -\frac{A_2 + A_4\ln\left(\frac{\lambda}{\exp(\lambda(\xi+c))-1}\right)}{A_1 + A_3\ln\left(\frac{\lambda}{\exp(\lambda(\xi+c))-1}\right)} \tag{2.12}$$

Family 8: when

$$(A_1A_4 - A_2A_3) \neq 0, \mu = 0, \lambda = 0, \left(\lambda^2 - 4\mu\right) = 0$$

$$\phi_8(\xi) = -\frac{A_2 - A_4\ln(\xi+c)}{A_1 - A_3\ln(\xi+c)} \tag{2.13}$$

Family 9: when

$$(A_1A_4 - A_2A_3) \neq 0, \mu \neq 0, \lambda \neq 0, \left(\lambda^2 - 4\mu\right) = 0, A_i \neq 0\,(i = 1,2,3,4)$$

$$\phi_9(\xi) = -\frac{A_2 - A_4\ln\left(-\frac{2(\xi+c)}{\lambda(\xi+c)-2}\right)}{A_1 - A_3\ln\left(-\frac{2(\xi+c)}{\lambda(\xi+c)-2}\right)} \tag{2.14}$$

Family 10: when

$$(A_1A_4 - A_2A_3) \neq 0, \mu < 0, \lambda = 0$$

$$\phi_{10}(\xi) = -\frac{A_2 - A_4\ln\left(-\frac{\exp\left(-2\sqrt{-\mu}(\xi+c)\right)+1}{\sqrt{-\mu}\exp\left(-2\sqrt{-\mu}(\xi+c)\right)-1}\right)}{A_1 - A_3\ln\left(-\frac{\exp\left(-2\sqrt{-\mu}(\xi+c)\right)+1}{\sqrt{-\mu}\exp\left(-2\sqrt{-\mu}(\xi+c)\right)-1}\right)} \tag{2.15}$$

Stepe 4 : Substituting (2.4) into (2.3) and using (2.5), and then setting all the cofficients of $\left(\exp\left(-\frac{A_1\phi(\xi)+A_2}{A_3\phi(\xi)+A_4}\right)\right)^i$ of the resulting systems to zero, yields a system of algebraic equations for k_1, k_2, λ, μ and α_i $(i = 0, 1, 2, ...m)$.

Step 5 : Suppose that the value of the constants k_1, k_2, λ, μ and α_i $(i = 0, 1, 2, ...m)$ can be found by solving the algebraic equations which are obtained in step 4. Since the general solutions of (5) have been well known for us, substituting $k_1, k_2, \lambda, \mu, \alpha_i$ and the general solutions of (2.5) into (2.4), we have the exact solutions of the nonlinear PDEs (2.1).

3. Korteweg - De Vries Equation

In this section, we will apply the The Generalized of $\exp(-\phi(\xi))$ expansion method to find the exact solutions of the Kdv equation. Let us consider Kdv equation :

$$u_t + \delta u u_x + u_{xxx} = 0 \tag{3.1}$$

We may choose the following traveling wave transformation

$$u(x,t) = u(\xi); \quad \xi = k_1 x + k_2 t \tag{3.2}$$

where k_1, k_2 are costants to be determined later. Eq. (3.3) becomes

$$k_2 u_\xi + \delta k_1 u.u_\xi + k_1^2 u_{\xi\xi\xi} = 0 \tag{3.3}$$

By balancing the height order derivative term $\left(u_{\xi\xi\xi}\right)$ with the nonlinear term $\left(u.u_\xi\right)$ in (3.3), gives $(m = 2)$. Therefore, the generalized of $\exp(-\phi(\xi))$ expansion method allows us to use the solution in the following form:

$$u(\xi) = \alpha_0 + \alpha_1 \exp\left(-\frac{A_1\phi(\xi) + A_2}{A_3\phi(\xi) + A_4}\right) + \alpha_2 \exp\left(-\frac{A_1\phi(\xi) + A_2}{A_3\phi(\xi) + A_4}\right)^2 \tag{3.4}$$

Substituting (3.4)and(2.5) into(3.3), the left-hand side is converted into polynomials in $\left(\exp\left(-\frac{A_1\phi(\xi)+A_2}{A_3\phi(\xi)+A_4}\right)\right)^j$,$(j = 0, 1, 2, ...)$. We collect each coefficient of these resulted polynomials to zero, yields a set of simultaneous algebraic equations (for simplicity,which are not presented) for $\alpha_0, \alpha_1, , \alpha_2, k_1, k_2, \lambda, \mu$ and δ. Solving these algebraic equations with the help of algebraic software Maple, we obtain

$$\begin{aligned} \alpha_0 &= \alpha_0, \alpha_1 = -\frac{12\lambda k_1^2}{\delta}, \alpha_2 = -\frac{12k_1^2}{\delta}, \lambda = \lambda \\ k_1 &= k_1, k_2 = -k_1\left(k_1^2\lambda^2 + \alpha_0\delta + 8\mu k_1^2\right), \mu = \mu \end{aligned} \tag{3.5}$$

Substituting (3.5) into(3.4), we have :

$$u(\xi) = \alpha_0 - \frac{12k_1^2}{\delta}\left(\lambda \exp\left(-\frac{A_1\phi(\xi) + A_2}{A_3\phi(\xi) + A_4}\right) + \exp\left(-\frac{A_1\phi(\xi) + A_2}{A_3\phi(\xi) + A_4}\right)^2\right) \tag{3.6}$$

where

$$\xi = k_1 x - k_1\left(k_1^2\lambda^2 + \alpha_0\delta + 8\mu k_1^2\right)t$$

Consequently,the exact solution of the Kdv equation (3.1) with the help of Eq. (2.6) to Eq. (2.15) are obtained in

the following form:

Case (3-1): when

$$(A_1A_4 - A_2A_3) \neq 0, \mu \neq 0, \left(\lambda^2 - 4\mu\right) > 0, A_2 = 0$$

$$
\begin{aligned}
u_1(\xi) &= \alpha_0 - \frac{12k_1^2}{\delta}\left[\lambda \exp\left(-\frac{A_1\left(\dfrac{A_4 \ln\left(\dfrac{-\sqrt{(\lambda^2-4\mu)}\tanh\left(\frac{\sqrt{(\lambda^2-4\mu)}}{2}(\xi+c)\right)-\lambda}{2\mu}\right)}{A_1 - A_3 \ln\left(\dfrac{-\sqrt{(\lambda^2-4\mu)}\tanh\left(\frac{\sqrt{(\lambda^2-4\mu)}}{2}(\xi+c)\right)-\lambda}{2\mu}\right)}\right)}{A_3\left(\dfrac{A_4 \ln\left(\dfrac{-\sqrt{(\lambda^2-4\mu)}\tanh\left(\frac{\sqrt{(\lambda^2-4\mu)}}{2}(\xi+c)\right)-\lambda}{2\mu}\right)}{A_1 - A_3 \ln\left(\dfrac{-\sqrt{(\lambda^2-4\mu)}\tanh\left(\frac{\sqrt{(\lambda^2-4\mu)}}{2}(\xi+c)\right)-\lambda}{2\mu}\right)}\right)+A_4}\right) \right. \\
&\qquad \left. + \exp\left(-\frac{A_1\left(\dfrac{A_4 \ln\left(\dfrac{-\sqrt{(\lambda^2-4\mu)}\tanh\left(\frac{\sqrt{(\lambda^2-4\mu)}}{2}(\xi+c)\right)-\lambda}{2\mu}\right)}{A_1 - A_3 \ln\left(\dfrac{-\sqrt{(\lambda^2-4\mu)}\tanh\left(\frac{\sqrt{(\lambda^2-4\mu)}}{2}(\xi+c)\right)-\lambda}{2\mu}\right)}\right)}{A_3\left(\dfrac{A_4 \ln\left(\dfrac{-\sqrt{(\lambda^2-4\mu)}\tanh\left(\frac{\sqrt{(\lambda^2-4\mu)}}{2}(\xi+c)\right)-\lambda}{2\mu}\right)}{A_1 - A_3 \ln\left(\dfrac{-\sqrt{(\lambda^2-4\mu)}\tanh\left(\frac{\sqrt{(\lambda^2-4\mu)}}{2}(\xi+c)\right)-\lambda}{2\mu}\right)}\right)+A_4}\right)^2\right] \qquad (3.7) \\
\xi &= k_1 x - k_1\left(k_1^2\lambda^2 + \alpha_0\delta + 8\mu k_1^2\right)t
\end{aligned}
$$

Case (3-2) : when

$$(A_1A_4 - A_2A_3) \neq 0, \mu \neq 0, \left(\lambda^2 - 4\mu\right) < 0, A_2 = 0$$

$$
\begin{aligned}
u_2(\xi) &= \left[\alpha_0-\frac{12k_1^2}{\delta}\left(\lambda\exp\left(-\frac{A_1\left(\frac{A_4\ln\left(\frac{\sqrt{\left(4\mu-\lambda^2\right)}\tan\left(\frac{\sqrt{\left(4\mu-\lambda^2\right)}}{2}(\xi+c)\right)-\lambda}{2\mu}\right)}{A_1-A_3\ln\left(\frac{\sqrt{\left(4\mu-\lambda^2\right)}\tan\left(\frac{\sqrt{\left(4\mu-\lambda^2\right)}}{2}(\xi+c)\right)-\lambda}{2\mu}\right)}\right)}{A_3\left(\frac{A_4\ln\left(\frac{-\sqrt{\left(\lambda^2-4\mu\right)}\tan\left(\frac{\sqrt{\left(\lambda^2-4\mu\right)}}{2}(\xi+c)\right)-\lambda}{2\mu}\right)}{A_1-A_3\ln\left(\frac{\sqrt{\left(4\mu-\lambda^2\right)}\tan\left(\frac{\sqrt{\left(4\mu-\lambda^2\right)}}{2}(\xi+c)\right)-\lambda}{2\mu}\right)}\right)+A_4}\right)\right.\right.\\
&\qquad\left.\left.+\exp\left(-\frac{A_1\left(\frac{A_4\ln\left[\frac{\sqrt{\left(4\mu-\lambda^2\right)}\tan\left(\frac{\sqrt{\left(4\mu-\lambda^2\right)}}{2}(\xi+c)\right)-\lambda}{2\mu}\right]}{A_1-A_3\ln\left[\frac{-\sqrt{\left(4\mu-\lambda^2\right)}\tan\left(\frac{\sqrt{\left(4\mu-\lambda^2\right)}}{2}(\xi+c)\right)-\lambda}{2\mu}\right]}\right)}{A_3\left(\frac{A_4\ln\left[\frac{\sqrt{\left(4\mu-\lambda^2\right)}\tan\left(\frac{\sqrt{\left(4\mu-\lambda^2\right)}}{2}(\xi+c)\right)-\lambda}{2\mu}\right]}{A_1-A_3\ln\left[\frac{\sqrt{\left(4\mu-\lambda^2\right)}\tan\left(\frac{\sqrt{\left(4\mu-\lambda^2\right)}}{2}(\xi+c)\right)-\lambda}{2\mu}\right]}\right)+A_4}\right)\right)^2\right]\Bigg] \qquad (3.8)\\
\xi &= k_1x-k_1\left(k_1^2\lambda^2+\alpha_0\delta+8\mu k_1^2\right)t
\end{aligned}
$$

Case (3-3) : when

$$(A_1A_4-A_2A_3)\neq 0,\mu\neq 0,\lambda\neq 0,\left(\lambda^2-4\mu\right)=0,A_2=0$$

$$
\begin{aligned}
u_3(\xi) &= \left[\alpha_0-\frac{12k_1^2}{\delta}\left(\lambda\exp\left(-\frac{A_1\left(\frac{A_4\ln\left(-\frac{2(\lambda(\xi+c)+2)}{\lambda^2(\xi+c)}\right)}{A_1-A_3\ln\left(-\frac{2(\lambda(\xi+c)+2)}{\lambda^2(\xi+c)}\right)}\right)}{A_3\left(\frac{A_4\ln\left(-\frac{2(\lambda(\xi+c)+2)}{\lambda^2(\xi+c)}\right)}{A_1-A_3\ln\left(-\frac{2(\lambda(\xi+c)+2)}{\lambda^2(\xi+c)}\right)}\right)+A_4}\right)+\exp\left(-\frac{A_1\left(\frac{A_4\ln\left(-\frac{2(\lambda(\xi+c)+2)}{\lambda^2(\xi+c)}\right)}{A_1-A_3\ln\left(-\frac{2(\lambda(\xi+c)+2)}{\lambda^2(\xi+c)}\right)}\right)}{A_3\left(\frac{A_4\ln\left(-\frac{2(\lambda(\xi+c)+2)}{\lambda^2(\xi+c)}\right)}{A_1-A_3\ln\left(-\frac{2(\lambda(\xi+c)+2)}{\lambda^2(\xi+c)}\right)}\right)+A_4}\right)\right)^2\right]\Bigg] \qquad (3.9)\\
\xi &= k_1x-k_1\left(12\mu k_1^2+\alpha_0\delta\right)t
\end{aligned}
$$

Case (3-4): when

$$(A_1A_4-A_2A_3)\neq 0,\mu\neq 0,\lambda\neq 0,\left(\lambda^2-4\mu\right)>0,A_3=0$$

$$\begin{aligned} u_4(\xi) &= \left[\alpha_0 - \frac{12k_1^2}{\delta}\left(\begin{array}{c} \lambda \exp\left(\left(\frac{A_2}{A_4}\right) - \ln\left(\frac{-\tanh\left(\sqrt{(\lambda^2-4\mu)}\frac{(\xi+c)}{2}\right)\sqrt{e^{\left(\frac{2A_2}{A_4}\right)}(\lambda^2-4\mu)}-\lambda e^{\left(\frac{A_2}{A_4}\right)}}{2\mu}\right)\right) \\ + \exp\left(\left(\frac{A_2}{A_4}\right) - \ln\left(\frac{-\tanh\left(\sqrt{(\lambda^2-4\mu)}\frac{(\xi+c)}{2}\right)\sqrt{e^{\left(\frac{2A_2}{A_4}\right)}(\lambda^2-4\mu)}-\lambda e^{\left(\frac{A_2}{A_4}\right)}}{2\mu}\right)\right)^2 \end{array}\right)\right] \\ \xi &= k_1 x - k_1\left(k_1^2\lambda^2 + \alpha_0\delta + 8\mu k_1^2\right)t \end{aligned} \tag{3.10}$$

Case (3-5): when

$$(A_1A_4 - A_2A_3) \neq 0, \mu \neq 0, \lambda \neq 0, \left(\lambda^2 - 4\mu\right) < 0, A_3 = 0$$

$$\begin{aligned} u_5(\xi) &= \left[\alpha_0 - \frac{12k_1^2}{\delta}\left(\begin{array}{c} \lambda \exp\left(\left(\frac{A_2}{A_4}\right) - \ln\left(\frac{\tan\left(\sqrt{(4\mu-\lambda^2)}\frac{(\xi+c)}{2}\right)\sqrt{e^{\left(\frac{2A_2}{A_4}\right)}(4\mu-\lambda^2)}-\lambda e^{\left(\frac{A_2}{A_4}\right)}}{2\mu}\right)\right) \\ + \exp\left(\left(\frac{A_2}{A_4}\right) - \ln\left(\frac{\tan\left(\sqrt{(4\mu-\lambda^2)}\frac{(\xi+c)}{2}\right)\sqrt{e^{\left(\frac{2A_2}{A_4}\right)}(4\mu-\lambda^2)}-\lambda e^{\left(\frac{A_2}{A_4}\right)}}{2\mu}\right)\right)^2 \end{array}\right)\right] \\ \xi &= k_1 x - k_1\left(k_1^2\lambda^2 + \alpha_0\delta + 8\mu k_1^2\right)t \end{aligned} \tag{3.11}$$

Case (3-6): when

$$(A_1A_4 - A_2A_3) \neq 0, \mu \neq 0, \lambda \neq 0, \left(\lambda^2 - 4\mu\right) = 0, A_3 = 0$$

$$\begin{aligned} u_6(\xi) &= \alpha_0 - \frac{12k_1^2}{\delta}\left(\left(-\frac{\lambda^3(\xi+c)}{2(\lambda(\xi+c)+2)}\right) + \left(\frac{\lambda^2(\xi+c)}{2(\lambda(\xi+c)+2)}\right)^2\right) \\ \xi &= k_1 x - k_1\left(\alpha_0\delta + 12\mu k_1^2\right)t \end{aligned} \tag{3.12}$$

Case (3-7): when

$$(A_1A_4 - A_2A_3) \neq 0, \mu = 0, \lambda \neq 0, \left(\lambda^2 - 4\mu\right) > 0$$

$$\begin{aligned} u_7(\xi) &= \left[\alpha_0 - \frac{12k_1^2}{\delta}\left(\begin{array}{c} \lambda \exp\left(-\frac{A_1\left(-\frac{A_2+A_4\ln\left(\frac{\lambda}{\exp(\lambda(\xi+c))-1}\right)}{A_1+A_3\ln\left(\frac{\lambda}{\exp(\lambda(\xi+c))-1}\right)}\right)+A_2}{A_3\left(-\frac{A_2+A_4\ln\left(\frac{\lambda}{\exp(\lambda(\xi+c))-1}\right)}{A_1+A_3\ln\left(\frac{\lambda}{\exp(\lambda(\xi+c))-1}\right)}\right)+A_4}\right) \\ + \exp\left(-\frac{A_1\left(-\frac{A_2+A_4\ln\left(\frac{\lambda}{\exp(\lambda(\xi+c))-1}\right)}{A_1+A_3\ln\left(\frac{\lambda}{\exp(\lambda(\xi+c))-1}\right)}\right)+A_2}{A_3\left(-\frac{A_2+A_4\ln\left(\frac{\lambda}{\exp(\lambda(\xi+c))-1}\right)}{A_1+A_3\ln\left(\frac{\lambda}{\exp(\lambda(\xi+c))-1}\right)}\right)+A_4}\right)^2 \end{array}\right)\right] \\ \xi &= k_1 x - k_1\left(k_1^2\lambda^2 + \alpha_0\delta + 8\mu k_1^2\right)t \end{aligned} \tag{3.13}$$

Case (3-8): when

$$(A_1A_4 - A_2A_3) \neq 0, \mu = 0, \lambda = 0, \left(\lambda^2 - 4\mu\right) = 0$$

$$\begin{aligned} u_8(\xi) &= \alpha_0 - \frac{12k_1^2}{\delta}\exp\left(-\frac{A_1\left(-\frac{A_2-A_4\ln(\xi+c)}{A_1-A_3\ln(\xi+c)}\right)+A_2}{A_3\left(-\frac{A_2-A_4\ln(\xi+c)}{A_1-A_3\ln(\xi+c)}\right)+A_4}\right)^2 \\ \xi &= k_1 x - k_1\alpha_0\delta + 12\mu k_1^2 t \end{aligned} \tag{3.14}$$

Case (3-9): when

$$(A_1A_4 - A_2A_3) \neq 0, \mu \neq 0, \lambda \neq 0, \left(\lambda^2 - 4\mu\right) = 0, A_i \neq 0\,(i = 1, 2, 3, 4)$$

$$
\begin{aligned}
u_9(\xi) &= \left[\alpha_0 - \frac{12k_1^2}{\delta}\left(\begin{array}{l}
\lambda \exp\left(-\dfrac{A_1\left(-\dfrac{A_2-A_4\ln\left(-\frac{2(\xi+c)}{\lambda(\xi+c)-2}\right)}{A_1-A_3\ln\left(-\frac{2(\xi+c)}{\lambda(\xi+c)-2}\right)}\right)+A_2}{A_3\left(-\dfrac{A_2-A_4\ln\left(-\frac{2(\xi+c)}{\lambda(\xi+c)-2}\right)}{A_1-A_3\ln\left(-\frac{2(\xi+c)}{\lambda(\xi+c)-2}\right)}\right)+A_4}\right) \\
+\exp\left(-\dfrac{A_1\left(-\dfrac{A_2-A_4\ln\left(-\frac{2(\xi+c)}{\lambda(\xi+c)-2}\right)}{A_1-A_3\ln\left(-\frac{2(\xi+c)}{\lambda(\xi+c)-2}\right)}\right)+A_2}{A_3\left(-\dfrac{A_2-A_4\ln\left(-\frac{2(\xi+c)}{\lambda(\xi+c)-2}\right)}{A_1-A_3\ln\left(-\frac{2(\xi+c)}{\lambda(\xi+c)-2}\right)}\right)+A_4}\right)^2
\end{array}\right)\right] \qquad (3.15)\\
\xi &= k_1x - k_1\left(\alpha_0\delta + 12\mu k_1^2\right)t
\end{aligned}
$$

Case (3-10): when

$$(A_1A_4 - A_2A_3) \neq 0, \mu < 0, \lambda = 0$$

$$
\begin{aligned}
u_{10}(\xi) &= \left[\alpha_0 - \frac{12k_1^2}{\delta}\left(\begin{array}{l}
\lambda \exp\left(-\dfrac{A_1\left(-\dfrac{A_2-A_4\ln\left(-\frac{\exp(-2\sqrt{-\mu}(\xi+c))+1}{\sqrt{-\mu}\exp(-2\sqrt{-\mu}(\xi+c))-1}\right)}{A_1-A_3\ln\left(-\frac{\exp(-2\sqrt{-\mu}(\xi+c))+1}{\sqrt{-\mu}\exp(-2\sqrt{-\mu}(\xi+c))-1}\right)}\right)+A_2}{A_3\left(-\dfrac{A_2-A_4\ln\left(-\frac{\exp(-2\sqrt{-\mu}(\xi+c))+1}{\sqrt{-\mu}\exp(-2\sqrt{-\mu}(\xi+c))-1}\right)}{A_1-A_3\ln\left(-\frac{\exp(-2\sqrt{-\mu}(\xi+c))+1}{\sqrt{-\mu}\exp(-2\sqrt{-\mu}(\xi+c))-1}\right)}\right)+A_4}\right) \\
+\exp\left(-\dfrac{A_1\left(-\dfrac{A_2-A_4\ln\left(-\frac{\exp(-2\sqrt{-\mu}(\xi+c))+1}{\sqrt{-\mu}\exp(-2\sqrt{-\mu}(\xi+c))-1}\right)}{A_1-A_3\ln\left(-\frac{\exp(-2\sqrt{-\mu}(\xi+c))+1}{\sqrt{-\mu}\exp(-2\sqrt{-\mu}(\xi+c))-1}\right)}\right)+A_2}{A_3\left(-\dfrac{A_2-A_4\ln\left(-\frac{\exp(-2\sqrt{-\mu}(\xi+c))+1}{\sqrt{-\mu}\exp(-2\sqrt{-\mu}(\xi+c))-1}\right)}{A_1-A_3\ln\left(-\frac{\exp(-2\sqrt{-\mu}(\xi+c))+1}{\sqrt{-\mu}\exp(-2\sqrt{-\mu}(\xi+c))-1}\right)}\right)+A_4}\right)^2
\end{array}\right)\right] \qquad (3.16)\\
\xi &= k_1x - k_1\left(k_1^2\lambda^2 + \alpha_0\delta + 8\mu k_1^2\right)t
\end{aligned}
$$

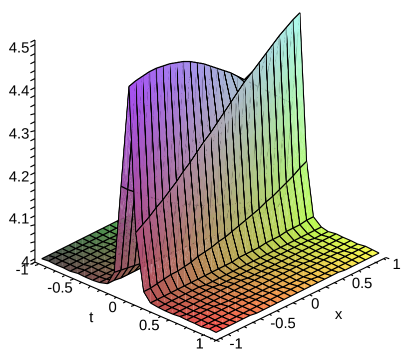

Figure (3.1)
3D plot of $u_1(\xi)$
$\xi = x - 25t$

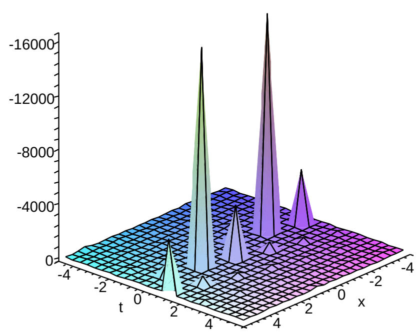

Figure (3.2)
3D plot of $u_2(\xi)$
$\xi = x - 11t$

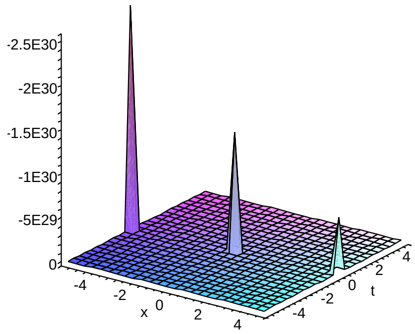

Figure (3.3)
3D plot of $u_3(\xi)$
$\xi = x - 12t$

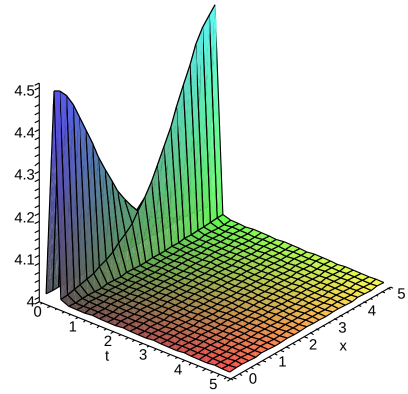

Figure (3.4)
3D plot of $u_4(\xi)$
$\xi = x - 25t$

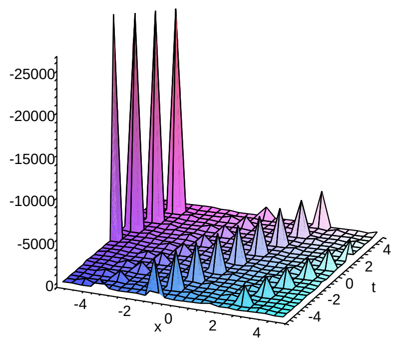

Figure (3.5)
3D plot of $u_5(\xi)$
$\xi = \sqrt{6}(x - 66t)$

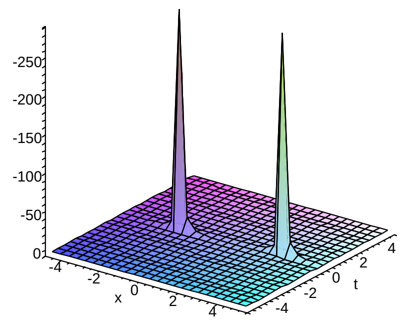

Figure (3.6)
3D plot of $u_6(\xi)$
$\xi = x - 12t$

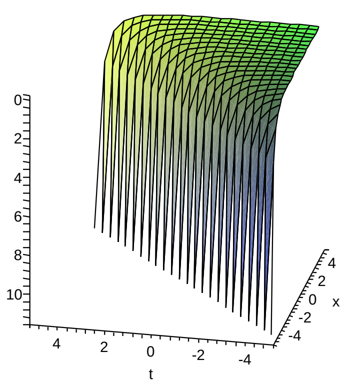

Figure (3.7)
3D plot of $u_7(\xi)$
$\xi = x - t$

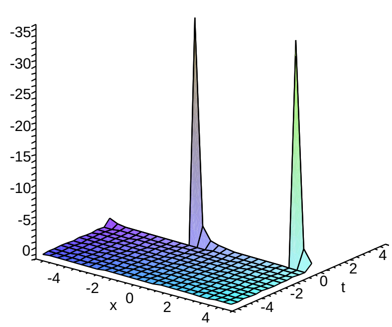

Figure (3.8)
3D plot of $u_8(\xi)$
$\xi = x - 12t$

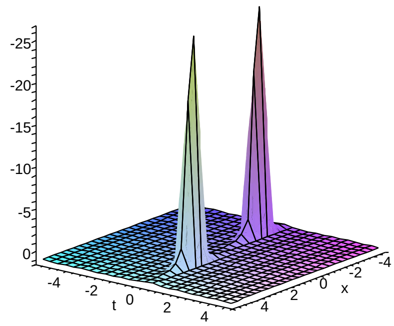

Figure (3.9)
3D plot of $u_9(\xi)$
$\xi = x - 12t$

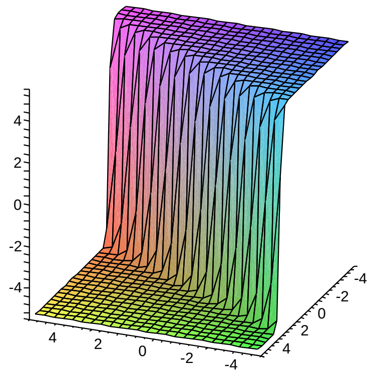

Figure (3.10)
3D plot of $u_{10}(\xi)$
$\xi = x + 8t$

4. Modified Zakharov-Kuznetsov Equation

In this section, we will apply the The Generalized of $\exp(-\phi(\xi))$ expansion method to find the exact solutions of the modified ZK equation, Let us consider modified ZK equation :

$$v_t + \delta v^2 v_x + v_{xxx} + v_{xyy} = 0 \tag{4.1}$$

We may choose the following traveling wave transformation

$$v(x,t) = v(\xi); \quad \xi = k_1 x + k_2 y + k_3 t \tag{4.2}$$

where k_1, k_2 and k_3 are costants to be determined later.

Eq. (4.1) becomes

$$k_3 v_\xi + k_1 \delta v^2 v_\xi + \left(k_1^3 + k_1 k_2^2\right) v_{\xi\xi\xi} = 0 \tag{4.3}$$

By balancing the height order derivative term $\left(v_{\xi\xi\xi}\right)$ with the nonlinear term $\left(v^2 v_\xi\right)$ in (4.3), gives $(m = 1)$. Therefore, the generalized of $\exp(-\phi(\xi))$ expansion method allows us to use the solution in the following form:

$$v(\xi) = \alpha_0 + \alpha_1 \exp\left(-\frac{A_1\phi(\xi) + A_2}{A_3\phi(\xi) + A_4}\right) \tag{4.4}$$

Substituting (4.4)and(2.5) into(4.3), the left-hand side is converted into polynomials in $\left(\exp\left(-\frac{A_1\phi(\xi)+A_2}{A_3\phi(\xi)+A_4}\right)\right)^j$,$(j = 0, 1, 2, ...)$. We collect each coefficient of these resulted polynomials to zero, yields a set of simultaneous algebraic equations (for simplicity,which are not presented) for $\alpha_0, \alpha_1, k_1, k_2, \lambda$and μ. Solving these algebraic equations with the help of algebraic software Maple, we obtain

$$\begin{aligned} \alpha_0 &= \frac{-3\lambda\left(k_1^2 + k_2^2\right)}{\sqrt{-6\delta\left(k_1^2 + k_2^2\right)}}, \alpha_1 = \frac{\sqrt{-6\delta\left(k_1^2 + k_2^2\right)}}{\delta}, \mu = \mu \\ k_1 &= k_1, k_2 = k_2, k_3 = \frac{1}{2}k_1\left(k_1^2 + k_2^2\right)\left(\lambda^2 - 4\mu\right), \lambda = \lambda \end{aligned} \tag{4.5}$$

Substituting (4.5) into(4.4), we have :

$$v(\xi) = -3\sqrt{\frac{\left(k_1^2+k_2^2\right)}{-6\delta}}\left(\lambda + 2\exp\left(-\frac{A_1\phi(\xi)+A_2}{A_3\phi(\xi)+A_4}\right)\right) \tag{4.6}$$

where

$$\xi = k_1x + k_2y + \frac{1}{2}k_1\left(k_1^2+k_2^2\right)\left(\lambda^2-4\mu\right)t \tag{4.7}$$

Consequently,the exact solution of the modified ZK equation (4.1) with the help of Eq. (2.6) to Eq. (2.15) are obtained in the followin form:

Case (4-1): when

$$(A_1A_4 - A_2A_3) \neq 0, \mu \neq 0, \left(\lambda^2-4\mu\right) > 0, A_2 = 0$$

$$\begin{aligned} v_1(\xi) &= -3\sqrt{\frac{\left(k_1^2+k_2^2\right)}{-6\delta}}\left(\lambda + 2\exp\left(-\frac{A_1\left(\frac{A_4\ln\left(\frac{-\sqrt{\left(\lambda^2-4\mu\right)}\tanh\left(\frac{\sqrt{\left(\lambda^2-4\mu\right)}}{2}(\xi+c)\right)-\lambda}{2\mu}\right)}{A_1-A_3\ln\left(\frac{-\sqrt{\left(\lambda^2-4\mu\right)}\tanh\left(\frac{\sqrt{\left(\lambda^2-4\mu\right)}}{2}(\xi+c)\right)-\lambda}{2\mu}\right)}\right)}{A_3\left(\frac{A_4\ln\left(\frac{-\sqrt{\left(\lambda^2-4\mu\right)}\tanh\left(\frac{\sqrt{\left(\lambda^2-4\mu\right)}}{2}(\xi+c)\right)-\lambda}{2\mu}\right)}{A_1-A_3\ln\left(\frac{-\sqrt{\left(\lambda^2-4\mu\right)}\tanh\left(\frac{\sqrt{\left(\lambda^2-4\mu\right)}}{2}(\xi+c)\right)-\lambda}{2\mu}\right)}\right)+A_4}\right)\right) \\ \xi &= k_1x + k_2y + \frac{1}{2}k_1\left(k_1^2+k_2^2\right)\left(\lambda^2-4\mu\right)t \end{aligned} \tag{4.8}$$

Case (4-2) : when

$$(A_1A_4 - A_2A_3) \neq 0, \mu \neq 0, \left(\lambda^2-4\mu\right) < 0, A_2 = 0$$

$$\begin{aligned} v_2(\xi) &= -3\sqrt{\frac{\left(k_1^2+k_2^2\right)}{-6\delta}}\left(\lambda + 2\exp\left(-\frac{A_1\left(\frac{A_4\ln\left(\frac{\sqrt{\left(4\mu-\lambda^2\right)}\tan\left(\frac{\sqrt{\left(4\mu-\lambda^2\right)}}{2}(\xi+c)\right)-\lambda}{2\mu}\right)}{A_1-A_3\ln\left(\frac{\sqrt{\left(4\mu-\lambda^2\right)}\tan\left(\frac{\sqrt{\left(4\mu-\lambda^2\right)}}{2}(\xi+c)\right)-\lambda}{2\mu}\right)}\right)}{A_3\left(\frac{A_4\ln\left(\frac{\sqrt{\left(4\mu-\lambda^2\right)}\tan\left(\frac{\sqrt{\left(4\mu-\lambda^2\right)}}{2}(\xi+c)\right)-\lambda}{2\mu}\right)}{A_1-A_3\ln\left(\frac{\sqrt{\left(4\mu-\lambda^2\right)}\tan\left(\frac{\sqrt{\left(4\mu-\lambda^2\right)}}{2}(\xi+c)\right)-\lambda}{2\mu}\right)}\right)+A_4}\right)\right) \\ \xi &= k_1x + k_2y + \frac{1}{2}k_1\left(k_1^2+k_2^2\right)\left(\lambda^2-4\mu\right)t \end{aligned} \tag{4.9}$$

Case (4-3) : when

$$(A_1A_4 - A_2A_3) \neq 0, \mu \neq 0, \lambda \neq 0, \left(\lambda^2 - 4\mu\right) = 0, A_2 = 0$$

$$v_3(\xi) = -3\sqrt{\frac{\left(k_1^2 + k_2^2\right)}{-6\delta}}\left(\lambda + 2\exp\left(-\frac{A_1\left(\frac{A_4 \ln\left(-\frac{2(\lambda(\xi+c)+2)}{\lambda^2(\xi+c)}\right)}{A_1 - A_3 \ln\left(-\frac{2(\lambda(\xi+c)+2)}{\lambda^2(\xi+c)}\right)}\right)}{A_3\left(\frac{A_4 \ln\left(-\frac{2(\lambda(\xi+c)+2)}{\lambda^2(\xi+c)}\right)}{A_1 - A_3 \ln\left(-\frac{2(\lambda(\xi+c)+2)}{\lambda^2(\xi+c)}\right)}\right) + A_4}\right)\right) \tag{4.10}$$

$$\xi = k_1x + k_2y$$

Case (4-4): when

$$(A_1A_4 - A_2A_3) \neq 0, \mu \neq 0, \lambda \neq 0, \left(\lambda^2 - 4\mu\right) > 0, A_3 = 0$$

$$v_4(\xi) = -3\sqrt{\frac{\left(k_1^2 + k_2^2\right)}{-6\delta}} \times$$

$$\left(\lambda + 2\exp\left(\left(\frac{A_2}{A_4}\right) - \ln\left(\frac{-\tanh\left(\sqrt{(\lambda^2 - 4\mu)}\frac{(\xi+c)}{2}\right)\sqrt{e^{\left(\frac{2A_2}{A_4}\right)}(\lambda^2 - 4\mu)} - \lambda e^{\left(\frac{A_2}{A_4}\right)}}{2\mu}\right)\right)\right)$$

$$\xi = k_1x + k_2y + \frac{1}{2}k_1\left(k_1^2 + k_2^2\right)\left(\lambda^2 - 4\mu\right)t \tag{4.11}$$

Case (4-5): when

$$(A_1A_4 - A_2A_3) \neq 0, \mu \neq 0, \lambda \neq 0, \left(\lambda^2 - 4\mu\right) < 0, A_3 = 0$$

$$v_5(\xi) = -3\sqrt{\frac{\left(k_1^2 + k_2^2\right)}{-6\delta}} \times$$

$$\left(\lambda + 2\exp\left(\left(\frac{A_2}{A_4}\right) - \ln\left(\frac{\tan\left(\sqrt{(4\mu - \lambda^2)}\frac{(\xi+c)}{2}\right)\sqrt{e^{\left(\frac{2A_2}{A_4}\right)}(4\mu - \lambda^2)} - \lambda e^{\left(\frac{A_2}{A_4}\right)}}{2\mu}\right)\right)\right)$$

$$\xi = k_1x + k_2y \tag{4.12}$$

Case (4-6): when

$$(A_1A_4 - A_2A_3) \neq 0, \mu \neq 0, \lambda \neq 0, \left(\lambda^2 - 4\mu\right) = 0, A_3 = 0$$

$$v_6(\xi) = -3\sqrt{\frac{\left(k_1^2 + k_2^2\right)}{-6\delta}}\left(\lambda + 2\exp\left(-\ln\left(-\frac{2\left(\lambda(\xi + c) + 2\right)}{\lambda^2(\xi + c)}\right)\right)\right) \tag{4.13}$$

$$\xi = k_1x + k_2y$$

Case (4-7): when

$$(A_1A_4 - A_2A_3) \neq 0, \mu = 0, \lambda \neq 0, \left(\lambda^2 - 4\mu\right) > 0$$

$$v_7(\xi) = -3\sqrt{\frac{\left(k_1^2 + k_2^2\right)}{-6\delta}}\left(\lambda + 2\exp\left(-\frac{A_1\left(-\frac{A_2 + A_4 \ln\left(\frac{\lambda}{\exp(\lambda(\xi+c)) - 1}\right)}{A_1 + A_3 \ln\left(\frac{\lambda}{\exp(\lambda(\xi+c)) - 1}\right)}\right) + A_2}{A_3\left(-\frac{A_2 + A_4 \ln\left(\frac{\lambda}{\exp(\lambda(\xi+c)) - 1}\right)}{A_1 + A_3 \ln\left(\frac{\lambda}{\exp(\lambda(\xi+c)) - 1}\right)}\right) + A_4}\right)\right) \tag{4.14}$$

$$\xi = k_1x + k_2y + \frac{1}{2}k_1\left(k_1^2 + k_2^2\right)\lambda^2 t$$

Case (4-8): when

$$(A_1A_4 - A_2A_3) \neq 0, \mu = 0, \lambda = 0, \left(\lambda^2 - 4\mu\right) = 0$$

$$\begin{aligned} v_8(\xi) &= -6\sqrt{\frac{\left(k_1^2 + k_2^2\right)}{-6\delta}} \exp\left(-\frac{A_1\left(-\frac{A_2 - A_4\ln(\xi+c)}{A_1 - A_3\ln(\xi+c)}\right) + A_2}{A_3\left(-\frac{A_2 - A_4\ln(\xi+c)}{A_1 - A_3\ln(\xi+c)}\right) + A_4}\right) \\ \xi &= k_1x + k_2y \end{aligned} \tag{4.15}$$

Case (4-9): when

$$(A_1A_4 - A_2A_3) \neq 0, \mu \neq 0, \lambda \neq 0, \left(\lambda^2 - 4\mu\right) = 0, A_i \neq 0\,(i = 1, 2, 3, 4)$$

$$\begin{aligned} v_9(\xi) &= -3\sqrt{\frac{\left(k_1^2 + k_2^2\right)}{-6\delta}} \left(\lambda + 2\exp\left(-\frac{A_1\left(-\frac{A_2 - A_4\ln\left(-\frac{2(\xi+c)}{\lambda(\xi+c)-2}\right)}{A_1 - A_3\ln\left(-\frac{2(\xi+c)}{\lambda(\xi+c)-2}\right)}\right) + A_2}{A_3\left(-\frac{A_2 - A_4\ln\left(-\frac{2(\xi+c)}{\lambda(\xi+c)-2}\right)}{A_1 - A_3\ln\left(-\frac{2(\xi+c)}{\lambda(\xi+c)-2}\right)}\right) + A_4}\right)\right) \\ \xi &= k_1x + k_2y \end{aligned} \tag{4.16}$$

Case (4-10): when

$$(A_1A_4 - A_2A_3) \neq 0, \mu < 0, \lambda = 0$$

$$\begin{aligned} v_{10}(\xi) &= \left[\begin{array}{c} \left(-3\sqrt{\frac{\left(k_1^2+k_2^2\right)}{-6\delta}}\right) \\ \times\left(\lambda + 2\exp\left(-\frac{A_1\left(-\frac{A_2 - A_4\ln\left(-\frac{\exp(-2\sqrt{-\mu}(\xi+c))+1}{\sqrt{-\mu}\exp(-2\sqrt{-\mu}(\xi+c))-1}\right)}{A_1 - A_3\ln\left(-\frac{\exp(-2\sqrt{-\mu}(\xi+c))+1}{\sqrt{-\mu}\exp(-2\sqrt{-\mu}(\xi+c))-1}\right)}\right)+A_2}{A_3\left(-\frac{A_2 - A_4\ln\left(-\frac{\exp(-2\sqrt{-\mu}(\xi+c))+1}{\sqrt{-\mu}\exp(-2\sqrt{-\mu}(\xi+c))-1}\right)}{A_1 - A_3\ln\left(-\frac{\exp(-2\sqrt{-\mu}(\xi+c))+1}{\sqrt{-\mu}\exp(-2\sqrt{-\mu}(\xi+c))-1}\right)}\right)+A_4}\right)\right) \end{array}\right] \\ \xi &= k_1x + k_2y + \frac{1}{2}k_1\left(k_1^2 + k_2^2\right)\left(\lambda^2 - 4\mu\right)t \end{aligned} \tag{4.17}$$

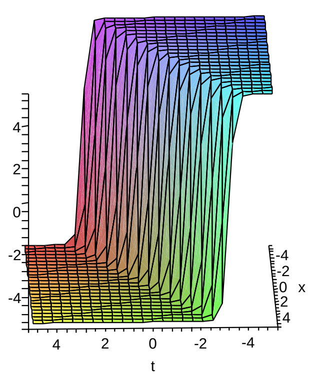

Figure (4.1)
3D plot of $v_1(\xi)$
$\xi = x + \sqrt{3}t; y = 0$

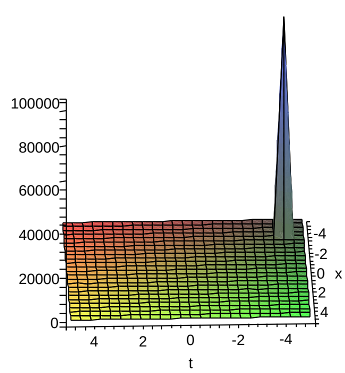

Figure (4.2)
3D plot of $v_2(\xi)$
$\xi = x + 56t; y = 0$

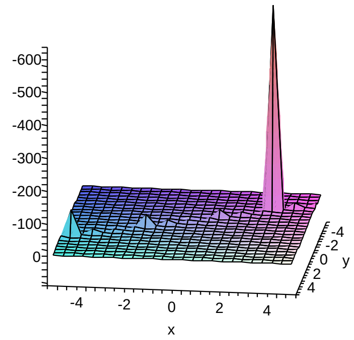

Figure (4.3)
3D plot of $v_3(\xi)$
$\xi = x + \sqrt{3}y$

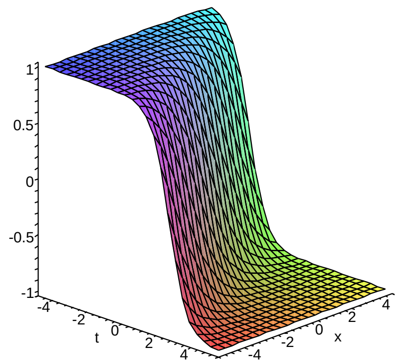

Figure (4.4)
3D plot of $v_4(\xi)$
$\xi = x + 2t; y = 0$

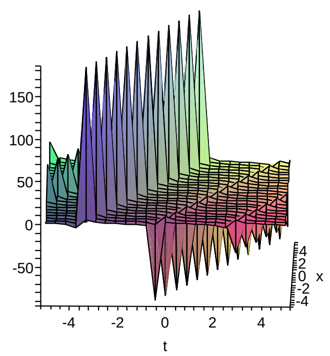

Figure (4.5)
3D plot of $v_5(\xi)$
$\xi = x - 2t$

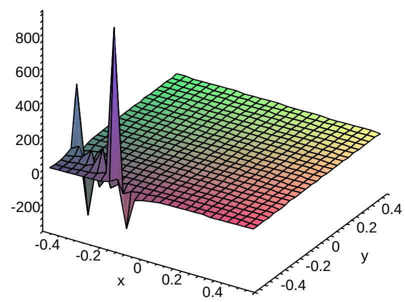

Figure (4.6)
3D plot of $v_6(\xi)$
$\xi = x + \sqrt{3}y$

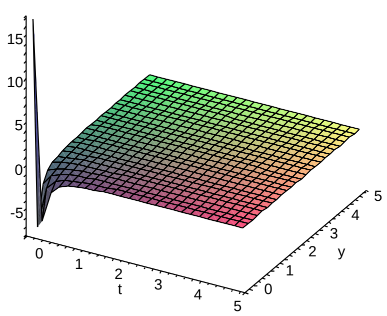

Figure (4.7)
3D plot of $v_7(\xi)$
$\xi = \sqrt{3}y + 2t; x = 0$

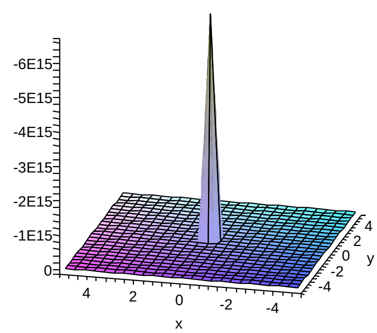

Figure (4.8)
3D plot of $v_8(\xi)$
$\xi = x + \sqrt{3}y$

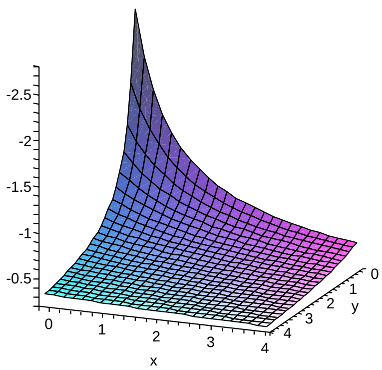

Figure (4.9)
3D plot of $v_9(\xi)$
$\xi = x + \sqrt{3}y$

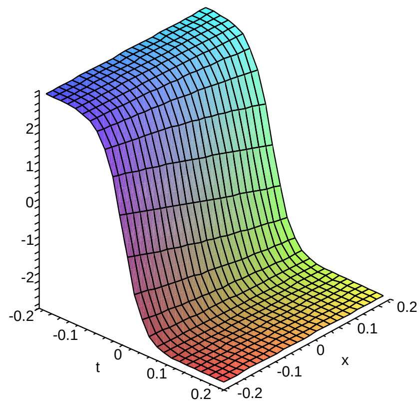

Figure (4.10)
3D plot of $v_{10}(\xi)$
$\xi = x + 16t; y = 0$

5. Conclusion

In this paper, we have proposed the new approach of $\exp(-\phi(\xi))$ expansion method to construct more general exact solutions of NLEEs. With the help of Maple, the method provides a straightforward mathematical tool for obtaining more general exact solutions of NLEEs in mathematical physics. Applying this method to the Kdv Equation and modified ZK Equation, we have successfully obtained many new and more general exact solutions, these solutions have rich local structures, It may be important to explain some physical phenomena . This work shows that, the new approach of $\exp(-\phi(\xi))$ expansion method is direct, effective and can be used for many other NLPDEs in mathematical physics.

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