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Physical Meaning and a Duality of Concepts of Wave Function, Action Functional, Entropy, the Pointing Vector, the Einstein Tensor

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Abstract

Physical meaning and a duality of concepts of wave function, action functional, entropy, the Pointing vector, the Einstein tensor and so on can be disclosed by investigating the state of material systems such as thermodynamic and gas dynamic systems, systems of charged particles, cosmologic systems and others. These concepts play a same role in mathematical physics. They are quantities that specify a state of material systems and also characteristics of physical fields. The duality of these concepts reveals in the fact that they can at once be both functionals and state functions or potentials. As functionals they are defined on nonintegrable manifold (for example, on tangent one), and as a state function they are defined on integrable manifold (for example, on cotangent one). The transition from functionals to state functions describes the mechanism of physical structure origination. The properties of these concepts can be studied by the example of entropy and action. The role of these concepts in mathematical physics and field theory will be demonstrated.

Such results have been obtained by using skew-symmetric forms. In addition to exterior forms, the skew-symmetric forms, which are obtained from differential equations and, in distinction to exterior forms, are evolutionary ones and are defined on nonintegrable manifolds, were used.

Keywords: Functionals, State functions, Skew-symmetric forms, Transition from nonintegrable manifold o integrable one, Degenerate transformations

1. Introduction

Such concepts as wave function, entropy, action, the Pointing vector, the Einstein tensor and others are those on which the fundamental branches of mathematical physics and field theories are based. In the present paper we try to show that these concepts have a same physical meaning.

The properties of conservation laws lie at the basis of the present investigation. Before beginning this investigation, it should be said a little about the concept of ‘conservation laws’.

The concept of ‘conservation laws’ assumes a different meaning in various branches of physics and mechanics.

In areas of physics related to the field theory and in theoretical mechanics ‘the conservation laws’ are those according to which there exist conservative physical quantities or objects. These are the conservation laws that above were named ‘exact’.

In mechanics and physics of continuous media the concept of ‘conservation laws’ relates to the conservation laws for energy, linear momentum, angular momentum, and mass that establish the balance between the change of physical quantities and the external action. These conservation laws can be named ‘the balance conservation laws’. They are described by differential (or integral) equations. It may be pointed out that all continuous media such as thermodynamic, gas dynamical, cosmic systems and others (which can be referred to as material systems), are subject to the balance conservation laws.

It turns out that the balance and exact conservation laws are mutually connected. All physical processes are controlled by interaction between the balance and exact conservation laws. The concepts pointed above are functionals that describe the controlling role of conservation laws in evolutionary processes and in processes of generating physical fields. It appears to be possible to disclose the physical meaning and duality of these concepts by studying

the relation that follows from the equations of the balance conservation laws and describes the mechanism of transition from balance conservation laws to exact ones.

The investigation has been carried out by using skew-symmetric differential forms whose properties reflect the properties of conservation laws. Exact conservation laws are described by closed exterior skew-symmetric forms (Cartan, E., 1945; Schutz, B. F., 1982). (It is known that the differential of closed exterior form equals zero, that is, the closed form is a conservative quantity). The Noether theorem, which is expressed as $d\omega = 0$, is an example. While studying the balance conservation laws, the skew-symmetric differential forms also arise. The relation that follows from the equations of balance conservation laws is expressed in terms of skew-symmetric forms. However, in contrast to exterior forms, these skew-symmetric forms are defined on nonintegrable manifolds.

The physical meaning and duality of these concepts can be demonstrated while analyzing the equations of balance conservation laws for energy and linear momentum.

2. Properties of the Balance Conservation Law Equations

In mechanics and physics of material systems the equations of balance conservation laws are used for description of physical quantities, which specify the behavior of material systems. However, the equations of balance conservation laws not only describe the variation of physical quantities. Their role in mathematical physics is much wider.

The required functions for the equations of balance conservation laws are usually functions which relate to such physical quantities as the particle velocity (of elements), temperature or energy, pressure and density (Tolman, R. C., 1969; Fock, V. A., 1955; Clark, J. F. & Machesney, M., 1964). Since these functions relate to one material system, it has to be a connection between them. The concepts analyzed (entropy, action and others) are quantities that describe this connection and characterize the system state. The system state appears to be an equilibrium one if these concepts are functions of state. The relation that follows from the balance conservation laws is just a relation for such function of state. However, as it will be shown later, this relation turns out to be nonidentical. And this means that the concepts described are functionals (this corresponds to nonequilibrium system state). As it will be shown, they become state functions only at realization of some conditions. And to this case the degenerate transformation and the transition from nonintegrable manifold to integrable one are assigned.

2.1 Evolutionary Nonidentical Relation

Let us analyze the equations that describe the balance conservation laws for energy and linear momentum.

We introduce two frames of reference: the first is an inertial one (this frame of reference is not connected with the material system), and the second is an accompanying one (this system is connected with the manifold made up by the trajectories of material system elements).

As one can see, for various material systems the energy equation in the inertial frame of reference can be reduced to the form:

$$\frac{D\psi}{Dt} = A \tag{1}$$

where D/Dt is the total derivative with respect to time, ψ is the function of state that specifies the material system, A is the quantity that depends on specific features of the system and external energy actions on the system.

It is well known that the equations for entropy and action have such a form.

So, the equation for energy presented in terms of the action S has a form:

$$\frac{DS}{Dt} = L \tag{2}$$

Here $\psi = S, A = L$ where L is the Lagrange function.

The equation for energy of an ideal gas can be presented in the form (Clark, J. F. & Machesney, M., 1964):

$$\frac{Ds}{Dt} = 0 \tag{3}$$

where s is the entropy. In this case $\psi = s, A = 0$.

It is worth noting that the examples presented show that the action functional and entropy play the common role.

At this point, the following should be noted. It is known that entropy can be a function of the state of thermodynamic system. In this case it depends on thermodynamic variables. Besides of this, entropy can be a function of the state of gas dynamic system. In this case it depends on the space-time coordinates. The entropy in equation (3) is a characteristics of gas dynamical system.

In the accompanying frame of reference (connected with the manifold formed by the trajectories of the system elements) the total derivative with respect to time is transformed into the derivative along the trajectory. Equation (1) is now written in the form

$$\frac{\partial \psi}{\partial \xi^1} = A_1 \tag{4}$$

here ξ^1 is the coordinate along the trajectory, A_1 is equal A in this case (see equation (1)). In the general case A_1 is a quantity that depends on the energetic action to the material system and on the system characteristics.

What is a peculiarity of the energy equation?

From this equation one can seemingly obtain a required state function.

However, it turns out that the required state function in addition has to satisfy the equation of the conservation law for linear momentum, which in the accompanying frame of reference is written as

$$\frac{\partial \psi}{\partial \xi^\nu} = A_\nu, \quad \nu = 2, ... \tag{5}$$

where ξ^ν are the coordinates in the direction normal to the trajectory, A_ν are the quantities that depend on the specific features of the system and external force actions.

The function that simultaneously obeys both equations can be obtained only in the case when these equations appear to be consistent. Since these equations are for derivatives of the same function, they can be consistent only if the derivatives made up a differential, i.e. the mixed derivatives appear to be commutative.

Equation (4) and (5) can be convoluted into the relation

$$d\psi = A_\mu \, d\xi^\mu, \quad (\mu = 1, \nu) \tag{6}$$

where $d\psi$ is the differential expression $d\psi = (\partial\psi/\partial\xi^\mu)d\xi^\mu$. Relation (6) can be written as

$$d\psi = \omega \tag{7}$$

here $\omega = A_\mu \, d\xi^\mu$ is the differential form of the first degree.

This relation is an evolutionary one since it has been obtained from the evolutionary equations.

The relation obtained possesses a peculiarity, namely, it turns out to be nonidentical. The skew-symmetric form in the right-hand side of this relation is not a closed form, i. e. a differential, like the left-hand side. The differential form $\omega = A_\mu d\xi^\mu$ appears to be unclosed since the commutator of the form ω, and hence a differential, are nonzero. The components of the commutator of such a form can be written as follows:

$$K_{\alpha\beta} = \left(\frac{\partial A_\beta}{\partial \xi^\alpha} - \frac{\partial A_\alpha}{\partial \xi^\beta} \right)$$

(here the term connected with the manifold metric form has not yet been taken into account).

The coefficients A_μ of the form ω have been obtained either from the equation of the balance conservation law for energy or from that for linear momentum. This means that in the first case the coefficients depend on the energetic action and in the second case they depend on the force action. In actual processes the energetic and force actions have different nature and appear to be inconsistent. The commutator of the form ω made up by the derivatives of such coefficients is nonzero. This means that the differential of the form ω is nonzero as well. Thus, the form ω proves to be unclosed and cannot be a differential like the left-hand side of relation (7). The evolutionary relation (7) appears to be nonidentical: the left-hand side of this relation is a differential, whereas the right-hand side is not a differential. [Nonidentity of such relation has been pointed out in the monograph (Synge, J. L., 1936). In that case a possibility of using a symbol of differential in the left-hand side of this relation is allowed.]

The nonidentity of the evolutionary relation points out to the fact that the state function differential does not exist. From this it follows that the function desired is not a state function but is a functional, which depends on the

nonzero commutator of unclosed form in the right-hand side of the evolutionary relation. The functional depends on the quantities that make a contribution to the commutator such as the energetic or force actions on the system. (If the commutator be zero, the form ω would be closed and be a differential. In this case one would be able to define the differential $d\psi$ and obtain the state function ψ.)

Thus we have that the quantity, which characterizes the material system state and for which entropy and action are the examples, turns out to be a functional. This points out to the fact that the material system state is a nonequilibrium one. (The nonidentity of the evolutionary relation obtained from the equations of balance conservation laws means that the equations of balance conservation laws are inconsistent. This points out to a noncommutativity of the conservation laws. It is a noncommutativity of the conservation laws that is just a cause of the nonequilibrium state of material systems.)

The evolutionary relation possesses one more peculiarity, namely, this relation is a selfvarying relation. (The evolutionary form entering into this relation is defined on the deforming manifold made up by trajectories of material system elements. This means that the evolutionary form basis varies. In turn, this leads to variation of evolutionary form, and the process of intervariation of the evolutionary form and the basis is repeated.) It should be emphasized that under selfvariation the evolutionary relation remains to be nonidentical since the evolutionary form in the right-hand side of the relation remains to be unclosed.

The selfvariation of evolutionary relation points out to the fact that the required function, which characterizes the material system state, changes but remains to be a functional. This means that the material system state changes but keeps to be nonequilibrium.

From the properties of nonidentical relation it follows that under degenerate transformation the identical relation can be obtained from that. The degrees of freedom of material systems (translational, rotating, oscillating and others) are the conditions of degenerate transformation. The conditions of degenerate transformation specify the pseudostructures (the integral surfaces): the characteristics (the determinant of coefficients at the normal derivatives vanishes), the singular points (Jacobian is equal to zero), the envelopes of characteristics of the Euler equations and so on. (Sections of the cotangent bundles, cohomologies by de Rham, singular cohomologies, potential surfaces, eikonals are examples of pseudostructures and relevant surfaces.)

The realization of the conditions of degenerate transformation leads to realization of some pseudostructure π (the closed dual form) and formatting the closed inexact form ω_π. On the pseudostructure π from evolutionary relation (7) it is obtained the relation

$$d\psi_\pi = \omega_\pi \tag{8}$$

which turns out to be identical since the form in the right-hand side is closed and hence is a differential. (It should be noted that such a differential is an interior one: it asserts only on pseudostructure, which is defined by the condition of degenerate transformation). Since the relation (8) is an identical one, from that one can obtain the differential $d\psi_\pi$, and this points to the availability of the state function desired.

Under realization additional conditions, which may be caused by the degrees of freedom of material system, the identical on pseudostructure relation is obtained from the nonidentical evolutionary relation. In this case the original relation remains to be nonidentical. At this point it should be noted that the original relation is defined on the manifold made up by the trajectories of material system elements, whereas the identical relation is defined on pseudostructure. The manifold made up by the trajectories of material system elements is a deforming nonintegrable manifold, and the pseudostructures form an integrable manifold. That is, the degenerate transformation is a transition from nonintegrable manifold to integrable one. On pseudostructure the function desired (entropy and action are examples), which characterizes the system state, appears to be a state function (or a potential). But in this case remains to be a functional on the original manifold.

The availability on pseudostructure of the state function points out to a locally-equilibrium state of the system, whereas the total state of material system remains to be nonequilibrium.

One can see that the transition from the functional to state function describes the process of realization of the locally-equilibrium state of the system.

Such a transition is accompanied by the origination in material system of any observable formations such as waves, vortices, turbulent pulsations and so on. The intensity of these formations is defined by the commutator of unclosed evolutionary form.

This follows from the analysis of the identical relation (8).

Identical relation (8) holds the duality. The left-hand side of this relation includes the differential, which specifies material system and whose availability points to the locally-equilibrium state of material system. And the right-hand side includes a closed inexact form, which is a characteristics of physical structures. Physical fields are formatted of such physical structures. (Closed inexact form and relevant closed dual form describe a quantity being conservative on pseudostructure. Physical structures that create physical fields are pseudostructures with a conservative physical quantity.)

Thus we have obtained that the transition from the nonequilibrium state to the locally equilibrium one is accompanied by emergence of physical structure, which reveals in material system as an emergence of certain observable formations (which develop spontaneously). (Massless particles, structures made up by eikonal surfaces and wave fronts, and so on are examples of physical structures.) [The observed formation and the physical structure are not identical objects. If the wave be such a formation, the element of wave front made up the physical structure at its motion. Structures of physical fields and the formations of material systems observed are a manifestation of the same phenomena. The light is an example of such a duality. The light manifests itself in the form of a massless particle (photon) and as a wave.]

Thus, one can see that the nonidentical evolutionary relation for functionals and the transition from functionals to state functions disclose the mechanism of emergence of physical structures, which made up physical fields, and the process of emergence of various observable formations in material media.

Relation (8) has been obtained from the equations of balance conservation law for energy and linear momentum. In this relation the form ω is that of the first degree. If the equations of the balance conservation law for angular momentum be added to the equations for energy and linear momentum, this form will be a form of the second degree. And, in combination with the equation of the balance conservation law for mass, this form will be a form of degree 3. In general case the evolutionary relation can be written as

$$d\psi = \omega^p \tag{9}$$

where the form degree p takes the values $p = 0, 1, 2, 3$. (The relation for $p = 0$ is an analog to that in the differential forms, and it has been obtained from the interaction of energy and time.)

The relation (9), as well as the relation (8), is nonidentical. This relation discloses the properties of such concepts as wave function ($p = 0$), the Pointing vector ($p = 2$), the Christoffel and Einstein tensors ($p = 3$).

Below we analyze some peculiarities of entropy and action.

3. Peculiarities of Entropy

In the next subsections the properties of entropy as a functional and a state function of thermodynamic system and gas dynamic system are described.

3.1 Entropy as a Functional and a State Function of Thermodynamic System

It is known that the first and second principles of thermodynamics, which have been introduced as postulates, form the basis of thermodynamics (Haywood, R. W., 1980).

The first principle of thermodynamics is an example of the evolutionary nonidentical relation. It follows from the balance conservation laws for energy and linear momentum and is valid in the case when the heat influx is the only external action. The second principle of thermodynamics with equality is an example of identical relation. This relation follows from the first principle of thermodynamics when the condition of integrability, i.e. the realization of the integrating multiplier, namely, temperature, is satisfied. The condition of integrability is a condition of degenerate transformation at which the entropy as state function is realized. (The second principle of thermodynamics with inequality takes into account the availability in real processes another actions on the system in addition to the heat influx and is an example of nonidentical relation. In this case entropy is a functional.)

Is is well known that the first principle of thermodynamics can be written in the form

$$dE + \delta w = \delta Q$$

where dE is the variation of the thermodynamic system energy, δw is the work made by the system (this means that δw is expressed in terms of the system parameters), δQ is the heat delivered to the system (i.e. the external action). Since the term δw must be expressed in terms the system parameters and characterizes a real (rather then

virtual) variation, it can be denoted as dw, the first principle of thermodynamics can be written as

$$dE + dw = \delta Q \tag{10}$$

What is a distinction of the first principle of thermodynamics from the conservation laws?

The balance conservation law for the energy of thermodynamic system can be written in the form

$$dE = \delta Q + \delta G \tag{11}$$

where by G we denote others (in addition to the heat influx) power actions. For thermodynamic system the balance conservation law for linear momentum (the variation of the system momentum as a function of force and mechanical actions on the system) can be written as

$$dw = \delta W \tag{12}$$

Here the force (mechanical) action on the system (external compression of the system, effect of boundaries and others for example) is denoted by δW.

If to sum relations (11) and (12), we obtain the relation

$$dE + dw = \delta Q + \delta G + \delta W \tag{13}$$

which is just the evolutionary relation for thermodynamic system.

By comparing the relation (13), which follows from the balance conservation laws for energy and linear momentum, with the relation (10), one can see that they are identic if the heat influx is the only external action on thermodynamic system ($\delta W = 0$ and $\delta G = 0$).

Thus, the first principle of thermodynamics follows from the balance conservation laws (and not only corresponds to the conservation law for energy). This is analogous to the evolutionary relation.

Since δQ is not a differential (closed form), the relation (10), which corresponds to the first principle of thermodynamics, as well as the evolutionary relation, appears to be a nonidentical nonintegrable relation. The form $dE + p\,dV$, even it is made up of differentials, in the general case without the integrating multiplier, is not a differential, since its terms depend on different variables, namely, the first term is defined by variables that specify the internal structure of the element, and the second term is defined by the variables, which characterize the interaction of elements, for example, such as the pressure.

In the this case under consideration entropy doesn't explicitly enter in relation (10). In this case the state of thermodynamic system is defined by the expression $dE + p\,dV$, which is not a differential, and this points out that there is no the state function.

As it follows from the analysis of evolutionary relation, the state function can be obtained only under degenerate transformation. To this it has to correspond the realization of the additional condition.

Consider the case when the work performed by system proceeds through a compression. Then $dw = p\,dV$ (here p and V are the pressure and the volume) and $dE + dw = dE + p\,dV$. As it is well known, the form $dE + p\,dV$ can become a differential (a closed form) only if there is an integrating multiplier $1/\theta = pV/R$, where T is a quantity that depends only on the system characteristics and is named the thermodynamic temperature (Haywood, R. W., 1980). In this case the form $(dE + p\,dV)/T$ proves to be a differential (interior) of a certain quantity. Such a quantity is the entropy S:

$$(dE + p\,dV)/T = dS \tag{14}$$

If the integrating multiplier $1/\theta = T$ is realized (this is just a condition of degenerate transformation), that is, the relation (14) is satisfied, from the relation (10), which corresponds to the first principle of thermodynamics, it follows that

$$dS = \delta Q/T \tag{15}$$

This is just the second principle of thermodynamics for reversible processes. This takes place only when the heat influx is the only action on the system.

In the case analyzed, the differential of entropy, rather then the entropy itself, becomes a closed form. The entropy reveals as the thermodynamic potential, i. e. the state function.

(For the entropy becomes a closed form, one more condition has to be realized. As such a condition it can be the realization of integrating direction, the example of which is the sound speed: $a^2 = \partial p/\partial\rho = \gamma\, p/\rho$. In this case it is fulfilled the equality $ds = d(p/\rho^\lambda) = 0$, from which it follows that the entropy $s = p/\rho^\lambda = const$ itself is a closed form (of zero degree). The transition from variables E and V to variables p and ρ at the realization of integrating direction is a degenerate transformation.

If, in addition to the heat influx, the system experiences any mechanical actions δW (for example, the influence of boundaries) or the additional power action δG, according to relation (13) from relation (14) it follows

$$dS = (dE + p\,dV)/T = (\delta Q + \delta W + \delta G)/T$$

from which one finds

$$dS > \delta Q/T \tag{16}$$

that corresponds to the second principle of thermodynamics for irreversible processes. In this case the entropy reveals as the functional.

3.2 Entropy as a Functional and a State Function of Gas Dynamic System

Above we analyzed the peculiarities of entropy as a functional and a state function of thermodynamic system. Such entropy depends of the thermodynamic variables. Entropy is also a functional and a state function of gas dynamic system. However, in this case entropy depends on the space-time coordinates.

Equations and the relation for entropy of gas dynamic system have been investigated and presented in the paper (Petrova, L. I., 2008). Below we will consider the simplest case, namely, a flow of ideal (inviscous, heat nonconductive) gas.

Assume that the gas is a thermodynamic system in the state of local equilibrium (whenever the gas dynamic system itself may be in nonequilibrium state), that is, the following relation is fulfilled (Haywood, R. W., 1980):

$$Tds = de + pdV \tag{17}$$

where T, p and V are the temperature, the pressure and the gas volume, s and e are entropy and internal energy per unit volume. The entropy s in relation (17) is a thermodynamic state function and depends on the thermodynamic variables. For the gas dynamical system the thermodynamic state function describes only the state of the gas dynamical element (a gas particle). As it has been noted before and will be shown below, for the gas dynamical system the entropy is also a state function. However, in this case the entropy is a function of space-time coordinates.

The equation of conservation law for energy of an ideal gas can be presented as the equation (3). In the accompanying frame of reference (that is connected with the manifold made up by the trajectories of the system elements) this equation take the form:

$$\frac{\partial s}{\partial \xi^1} = A_1 \tag{18}$$

where ξ^1 is the coordinate along the trajectory, $A_1 = 0$ (see the equation (3)).

In the accompanying frame of reference the equation of the conservation law for linear momentum can be presented as (Clark, J. F. & Machesney, M., 1964)

$$\frac{\partial s}{\partial \xi^\nu} = A_\nu \tag{19}$$

where ξ^ν is the coordinate in the direction normal to the trajectory, and in two-dimensional case A_ν has the form (Clark, J. F. & Machesney, M., 1964; Liepman, H. W. & Roshko, A., 1957):

$$A_\nu = \frac{\partial h_0}{\partial \xi^\nu} + ({u_1}^2 + {u_2}^2)^{1/2}\zeta - F_\nu + \frac{\partial U_\nu}{\partial t} \tag{20}$$

where $\zeta = \partial u_2/\partial x - \partial u_1/\partial y$.

Equations (18) and (19) can be convoluted into the relation

$$ds = \omega \tag{21}$$

where $\omega = A_\mu d\xi^\mu$ is the first degree skew-symmetric differential form and $\mu = 1,\ \nu$.

Relation (21) is an example of the nonidentical evolutionary relation (7). While describing actual processes relation (21) turns out to be not identical, since the evolutionary form ω is not closed and is not a differential, its commutator is nonzero. From the analysis of the expression A_ν and with taking into account that $A_1 = 0$ one can see that the terms related to the multiple connectedness of the flow domain (the second term of expression (20)), the nonpotentiality of the external forces (the third term in (20)) and the nonstationarity of the flow (the forth term in (20)) contribute to the commutator. All these factors lead to the emergence of internal forces, the nonequilibrium state and developing the instability.

The nonidentity of relation (21) points to the fact that the entropy is a functional since the differential of entropy does not exist.

From the properties of nonidentical relation it follows that under degenerate transformation the identical relation (from which the differential of entropy is defined) can be obtained from that. The degrees of freedom of gas dynamical systems (translational, rotating, oscillating and others) are the conditions of degenerate transformation. The conditions of degenerate transformation specify the integral surfaces (pseudostructures): the characteristics (the determinant of coefficients at the normal derivatives vanishes), the singular points (Jacobian is equal to zero), the envelopes of characteristics of the Euler equations and so on. As it was already noted, the realization of the conditions of degenerate transformation leads to realization of pseudostructure π (the closed dual form) and formatting the closed inexact form ω_π. On the pseudostructure π from evolutionary relation (21) it is obtained the identical relation

$$ds_\pi = \omega_\pi \tag{22}$$

from which the differential ds_π can be obtained. This means that we have the realization of entropy as a state function of gas dynamic system, whose availability points to the locally-equilibrium state of the gas dynamic system. On pseudostructure entropy appears to be a state function (or a potential). But in this case entropy remains to be a functional on the original manifold.

[In the text-books on gas dynamics it is assumed that from equation (18) one can obtain the entropy along the trajectory. However, it occurs that the entropy as a function of space-time coordinates (that is, as a function of the state) has in addition to satisfy equation (19)].

The realization of gas dynamic state function (entropy as a function of space-time coordinates) points out to the transition from the nonequilibrium state to the locally equilibrium one. This process is accompanied by emergence of the gas dynamic observable formations such as waves, vortices and so on. In this case the quantity, which is described by the commutator of unclosed form ω and acts as an internal force (producing the nonequilibrium system state), defines the intensity of these formations.

[Studying the instability on the basis of the analysis of entropy behavior was carried out in publications by Prigogine and co-authors (Prigogine, I., 1955; Glansdorff, P. & Prigogine, I., 1971). In their papers the entropy was considered as a thermodynamic function of state (though its behavior along the trajectory was analyzed). By means of such state function one can trace the development (in gas fluxes) of the thermodynamic instability only (Prigogine, I., 1955). To investigate the gas dynamic instability it is necessary to consider the entropy as a gas dynamic state function, i.e. as a function of the space-time coordinates.]

4. Peculiarities of 'Action'

As it has been already noted, equation (2) for the action S and equation (3) for entropy of gas dynamical system obtained from the balance conservation law for energy have the identic form.

The action, as well as the entropy, can be both a functional and a state function.

In classic mechanics the action may be written in two forms: in the Lagrangian form

$$S = \int L(q_j, \dot{q}_j)\, dt \tag{23}$$

and in the Hamiltonian one

$$S = \int H(t, q_j, p_j)\, dt \tag{24}$$

Here L is the Lagrangian function and H is the Hamiltonian function $H(t, q_j, p_j) = p_j \dot{q}_j - L$, $p_j = \partial L/\partial \dot{q}_j$.

It is assumed that both forms are equivalent. However, these forms are not equivalent. In expression (23) the action is defined on the Lagrangian manifold q_j, $\dot{q}_j$, which is a tangent nonintegrable manifold. The action defined on

such a manifold is a functional. In expression (24) the action is defined on the Hamiltonian manifold q_j, p_j, which is a cotangent integrable manifold. The action defined on such a manifold is a function of state.

Here the degenerate transformation (the transition from the functional to the state function) is a transition from the Lagrangian function to the Hamiltonian function. The transition from the Lagrangian function L to the Hamiltonian function H (the transition from variables q_j, $\dot{q}_j$ to variables q_j, $p_j = \partial L/\partial \dot{q}_j$) is a transition from the tangent manifold to the cotangent one. This is a Legendre transformation, which is a degenerate transformation.

In the equations of field theory

$$\frac{\partial s}{\partial t} + H\left(t,\, q_j,\, \frac{\partial s}{\partial q_j}\right) = 0, \quad \frac{\partial s}{\partial q_j} = p_j \tag{25}$$

the field function s is an action, which is a function of state and is obtained from the action functional $S = \int L\, dt$. To equation (25) it is assigned the differential (a closed form) $ds = -H\, dt + p_j\, dq_j$ (the Poincare invariant). This points out to that the action S, which obeys the equation (24), is a function of state.

In quantum mechanics (where to the coordinates q_j, p_j the operators are assigned) the Schrődinger equation for wave function serves as an analog to equation (25). Wave function in quantum mechanics plays the same role as the action in field theory.

The duality and physical meaning of concepts of entropy and action were described above. It has been shown that these concepts relate to the properties of the balance conservation laws for energy and momentum, which are described by the evolutionary nonidentical relation (7). As it was already noted, an account for another balance conservation laws (of angular momentum and mass) leads to relation (9), which is also evolutionary and non-identical. The relation (9) with skew-symmetric form ω^p of appropriate degree p discloses the properties of such concepts as wave function ($p = 0$), the Pointing vector ($p = 2$), Christoffel's and Einstein's tensors ($p = 3$). All these concepts possess a duality. Due to this duality all these concepts play a fundamental role in the description of evolutionary processes in material systems and the processes of generation of physical fields.

5. A Role of Functionals in Mathematical Physics and Field Theory

The functionals such as wave function, action functional, entropy, the Pointing vector and others are concepts that enable one to describe the regulating role of the conservation laws in the evolutionary processes in material media and the processes of generation of physical fields. The peculiarity of such functional consists in that they are used both in the theories that describe material systems (in mechanics and physics of continuous media) and in field theory. As it is seen from the analysis of the evolutionary nonidentical relation obtained from the equations of balance conservation laws for material systems, in the theories that describe material systems these functionals specify the state of material system. And in the field theory they describe physical fields. One can see that the equations of field theory are those in these functionals.

This duality of functionals just allows to disclose a connection between the equations for material systems and the field theory equations, which describe physical fields. And this connection is realized with the help of nonidentical evolutionary relations.

The peculiarity of the field theory equations consists in the fact that all these equations have the form of relations. They can be relations in differential forms or in the forms of their tensor or differential analogs (i. e. expressed in terms of derivatives).

The Einstein equation is a relation in differential forms. This equation relates the differential of the form of first degree (Einstein's tensor) and the differential form of second degree, namely, the energy-momentum tensor. (It should be noted that Einstein's equation is obtained from differential forms of third degree).

The Dirac equation relates Dirac's *bra-* and *cket-* vectors, which made up the differential forms of zero degree.

The Maxwell equations have the form of tensor relations.

Field equation and Schrődinger's equation have the form of relations expressed in terms of derivatives and their analogs.

All equations of existing field theories have the form of nonidentical relations for the above pointed functional. From these equations the identical relations follow, and from these relations the differentials of functionals and closed exterior forms assigned to the exact conservation laws for physical fields are obtained. The identical relations that include closed exterior forms or their tensor or differential analogs are the solutions to the field theory

equations.

As one can see, from the field theory equations it follows such identical relation as

1) The Dirac relations made up of Dirac's *bra*- and *cket*- vectors, which connect a closed exterior form of zero degree;

2) The Poincare invariant, which connects a closed exterior form of first degree;

3) The relations $d\theta^2 = 0, d^*\theta^2 = 0$ for closed exterior forms of second degree obtained from the Maxwell equations;

4) The Bianchi identical.

From the Einstein equation the identical relation is obtained in the case when the covariant derivative of the energy-momentum tensor vanishes.

It turns out that all equations of existing field theories are, in essence, relations that connect skew-symmetric forms or their analogs. In this case one can see that the equations of field theories have the form of relations for functionals such as wave function (the relation corresponding to differential form of zero degree), action functional (the relation corresponding to differential form of first degree), the Pointing vector (the relation corresponding to differential form of second degree). The tensor functionals that correspond to Einstein's equation are obtained from the relation connecting the differential forms of third degree.

The nonidentical evolutionary relation derived from the equations for material media unites the relations for all these functionals. That is, all equations of field theories are an analog of the nonidentical evolutionary relation obtained from the equations of balance conservation laws. From this it follows that the nonidentical evolutionary relation can play a role of the equation of general field theory that discloses common properties and peculiarities of existing equations of field theory.

The correspondence between the equations of existing field theories and the nonidentical evolutionary relations for the functionals under consideration has the mathematical and physical meaning. Firstly, this discloses the internal connection between all physical theories, and, secondly, this enables one to understand the basic principles of field theories, namely, their connections with the equations for material systems generating relevant physical fields.

6. Conclusion

The duality of such concepts as wave function, action functional, entropy, the Pointing vector and others makes itself evident in the fact that, firstly, they can at once be both functionals and state functions (or potentials) and, secondly, as functionals they are used both in the theories that describe material systems (in mechanics and physics of continuous medium) and in field theory.

Such a duality leads to that they serve not only for the description of material media and physical fields but they also disclose the mechanism of origination of various discrete structures and formations, which connect physical fields with material media. The process of transition from functionals to state functions (or potentials) describes the mechanism of origination of various structures.

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