Iteration Distribution of Prime Numbers

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Abstract

The distribution of prime numbers is given iteration function, iterated function transformation, series of transformations, the general term of iteration, calculation examples and numerical control.

Keywords: Iterative, Transform, Series, Control

1. Introduction

Should the number theory is the mathematical crown, then the prime number theorem is the crown jewel. Mathematicians say, to find a prime number theorem is very difficult and requires long-term joint efforts of many mathematicians to get.

In 1737, the mathematician Leonhard Euler published a Euler product formula:

$$\sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_p \left(1 - \frac{1}{p^s}\right)^{-1}, \ s > 1. \tag{1}$$

French mathematician A.M.Legendre, by (1) study, the numerical statistics, in 1798, made a surprising asymptotic formula $[6]_7$:

$$\pi(x) \sim \frac{x}{\ln x - 1.08366}, \ (x \to \infty). \tag{2}$$

Where π (x) of that not more than x number of prime numbers.

Mathematicians, by (1) , logarithmic obtained and (2) a similar asymptotic formula:

$$\pi(x) \sim \frac{x}{\ln x}, \ (x \to \infty). \tag{3}$$

Here (3) known as: the prime number theorem $[6]_8$.

We present the distribution of prime numbers iterated function:

$$s_n(x) = \frac{1}{\ln x(1 - k_n)}, \ k_n = \frac{s_i(y)}{s_i(x)}. \tag{4}$$

By (4), the first transformation:

$$s_1(x) = \frac{x}{\ln x - 1}. \tag{5}$$

By (5), the second transformation:

$$s_2(x) = \frac{x(\ln x - 1)}{\ln x(\ln x - 2)}. \tag{6}$$

For (5), (6), where proof is not given, only the given function and transformation, calculation examples and data control.

2. The first iteration function transformation

Set Located not more than x, y number of prime numbers approximate $s_i(x), s_i(y)$, table are:

$$s_n(x) = \frac{1}{\ln x(1-k_n)},\ k_n = \frac{s_i(y)}{s_i(x)}. \tag{7}$$

Here (7) is called: iterated function distribution of prime numbers.$s_i(x), s_i(y)$ is an arbitrary prime number function.

If the prime function of the input approximate $s_i(x)$, The more accurate the output is the prime function of $s_n(x)$, As: Iterative transform the distribution of prime numbers.

For example:

Arbitrary function: $s_i(x) = \frac{x}{\ln x+8},\ s_i(y) = \frac{y}{\ln y+8}$,

Set $x = 100000000$£$y = x - 1$. Substituting, we get:

$$s_i(10^8) = \frac{10^8}{\ln(10^8)+8} \approx 3784913.83205142341291392397063 57.$$

$$s_i(10^8-1) = \frac{10^8-1}{\ln(10^8-1)+8} \approx 3784913.795634842357384533102814 7.$$

By (7), Calculation k_1 ≈0.99999999037849138145594114617225, get:

$$s_1(10^8) \approx 5642234.$$

Comparison $s_i(10^8)$ ≈3784913, $s_1(10^8)$ ≈5642234, $\pi(10^8)$ =5761455.

No matter how rough the input, will be the more correct calculation of the output form.

May wish to set:

$$s_i(x) = \frac{x}{\ln x},\ s_i(y) = \frac{y}{\ln y}. \tag{8}$$

By (7), (8) set:

$$s_1(x) = \frac{1}{\ln x(1-k_1)}.\quad k_1 = \frac{y\ln x}{x\ln y}. \tag{9}$$

Set $y = x - 1, x \to \infty$, by (9) get:

$$\begin{aligned} s_1(x) &= \frac{1}{\ln x}\frac{1}{1-\frac{y\ln x}{x\ln y}} \\ &= \frac{1}{\ln x}\frac{x\ln y}{x\ln y - y\ln x} \\ &= \frac{1}{\ln x}\frac{x\ln(x-1)}{x\ln(x-1)-(x-1)\ln x} \\ &= \frac{1}{\ln x}\frac{x\ln x - 1}{x\ln x - 1 - x\ln x + \ln x} \\ &= \frac{1}{\ln x}\frac{\ln x(x-1/\ln x)}{\ln x - 1} \\ &= \frac{x-1/\ln x}{\ln x - 1}. \end{aligned} \tag{10}$$

Ignore $1/\ln x$, by (10) get:

$$s_1(x) = \frac{x}{\ln x - 1}. \tag{11}$$

Here (11) is the first iteration function transformation.

Xiamen University, Professor Lu Yuanhong from (11) to prove:

$$\frac{x}{\ln x - 1} = \frac{x}{\ln x} + \frac{x}{\ln^2 x} + \cdots\cdots + \frac{x}{\ln^k x} \tag{12}$$

Here (12) is a series form.

3. The second transformation

By (11), get:

$$s_1(x) = \frac{x}{\ln x - 1},\ s_1(y) = \frac{y}{\ln y - 1}. \tag{13}$$

By (7), (13) get:

$$k_2 = \frac{s_1(y)}{s_1(x)} = \frac{y(\ln x - 1)}{x(\ln y - 1)}. \tag{14}$$

By (7), (14) get:

$$s_2(x) = \frac{1}{\ln x(1 - k_2)}. \quad k_2 = \frac{y(\ln x - 1)}{x(\ln y - 1)}. \tag{15}$$

Set $y = x - 1,\ x \to \infty$, by (15) get:

$$\begin{aligned} s_2(x) &= \frac{1}{\ln x}\frac{1}{1 - \frac{y(\ln x-1)}{x(\ln y-1)}} \\ &= \frac{1}{\ln x}\frac{x\ln y - x}{x\ln y - x - y\ln x + y} \\ &= \frac{1}{\ln x}\frac{x\ln(x-1) - x}{x\ln(x-1) - x - (x-1)\ln x + x - 1} \\ &= \frac{1}{\ln x}\frac{x\ln x - 1 - x}{x\ln x - 1 - x - x\ln x + \ln x + x - 1} \\ &= \frac{1}{\ln x}\frac{x\ln x - x - 1}{\ln x - 2} \\ &= \frac{x(\ln x - 1 - 1/x)}{\ln x(\ln x - 2)}. \end{aligned} \tag{16}$$

Ignore $1/x$, by (16) get:

$$s_2(x) = \frac{x(\ln x - 1)}{\ln x(\ln x - 2)}. \tag{17}$$

Here (17) is the second transformation. Can continue to transform. However, the more the number of transformations, but also more complex.

By (17), set $a = \frac{\ln x-2}{\ln x-1}$, get:

$$s_2(x) = \frac{x}{a\ln x},\ a = \frac{\ln x - 2}{\ln x - 1} \tag{18}$$

4. Series transformation

By (18) get:

$$s_2(x) = \frac{x}{a\ln x}$$

$$= \frac{x}{\ln x - \ln x + a\ln x}$$

$$= \frac{x}{\ln x - \ln x(1-a)}$$

$$= \frac{x}{\ln x - \ln x\left(1 - \frac{\ln x - 2}{\ln x - 1}\right)}$$

$$= \frac{x}{\ln x - \ln x\left(\frac{\ln x - 1 - \ln x + 2}{\ln x - 1}\right)}$$

$$= \frac{x}{\ln x - \frac{\ln x}{\ln x - 1}} \tag{19}$$

By (19), set $\lambda = \frac{\ln x}{\ln x - 1}$, get:

$$s_2(x) = \frac{x}{\ln x - \lambda}. \tag{20}$$

By (12), (20) get:

$$\frac{x}{\ln x - \lambda} = \frac{x}{\ln x} + \frac{\lambda x}{\ln^2 x} + \cdots\cdots + \frac{\lambda^{k-1} x}{\ln^k x}. \tag{21}$$

Here (21) is a series transformation.

5. Iterations of the general computing

By (20) set:

$$s_i(x) = \frac{x}{\ln x - \lambda_x}, \quad s_i(y) = \frac{y}{\ln y - \lambda_y}. \tag{22}$$

By (9), (22) get:

$$s_n(x) = \frac{1}{\ln x(1-k_n)}, \quad k_n = \frac{y(\ln x - \lambda_x)}{x(\ln y - \lambda_y)}. \tag{23}$$

Set $y = x - 1, \quad x \to \infty$, by (4,2) get:

$$s_n(x) = \frac{1}{\ln x}\frac{1}{1 - \frac{y(\ln x - \lambda_x)}{x(\ln y - \lambda_y)}}$$

$$= \frac{1}{\ln x}\frac{1}{\frac{x\ln y - \lambda_y x - y\ln x + \lambda_x y}{x(\ln y - \lambda_y)}}$$

$$= \frac{1}{\ln x}\frac{1}{\frac{x\ln(x-1) - \lambda_y x - (x-1)\ln x + \lambda_x(x-1)}{x\ln(x-1) - \lambda_y x)}}$$

$$= \frac{1}{\ln x} \frac{1}{1 - \frac{x \ln x - 1 - \lambda_y x - x \ln x + \ln x + \lambda_x x - \lambda_x}{x \ln x - 1 - \lambda_y x}}$$

$$= \frac{1}{\ln x} \frac{x \ln x - \lambda_y x - 1}{\ln x + \lambda_x x - \lambda_y x - \lambda_x - 1}$$

$$= \frac{x(\ln x - \lambda_y - 1/x)}{\ln x(\ln x + \lambda_x x - \lambda_y x - \lambda_x - 1)}. \tag{24}$$

Ignore $1/x$, by (24) get:

$$s_n(x) = \frac{x(\ln x - \lambda_y)}{\ln x(\ln x + \lambda_x x - \lambda_y x - \lambda_x - 1)}. \tag{25}$$

Here (25) is called: iterative calculation of the general.

$$n = 1, \quad \lambda_x = 0,\ \lambda_y = 0,$$

$$n = 2, \quad \lambda_x = 1,\ \lambda_y = 1,$$

$$n = 3, \quad \lambda_x = \frac{\ln x}{\ln x - 1},\ \lambda_y = \frac{\ln y}{\ln y - 1}.$$

$$n = 4, \quad \lambda_x = 1 + \frac{\ln x}{(\ln x - 1)(\ln x - 2)},\ \lambda_y = 1 + \frac{\ln y}{(\ln y - 1)(\ln y - 2)}.$$

Example calculation: set $x = 10^{16}$, by $n = 4$, get:

$$\lambda_x = 1.0295023122497644185998716004798.$$

$$\lambda_y = 1.0295023122497644186867819893334.$$

By (25), get: $s_4(10^{16})$ =279238 194508031, $\pi(10^{16})$ =279238 341033925.

By (25), set $a = \frac{\ln x + \lambda_x x - \lambda_y x - \lambda_x - 1}{\ln x - \lambda_y}$, get:

$$s_n(x) = \frac{x}{\ln x - \ln x(1 - a)}. \tag{26}$$

Set $\lambda = \ln x(1 - a)$, by (21), (26) get:

$$s_n(x) = \frac{x}{\ln x} + \frac{\lambda x}{\ln^2 x} + \cdots\cdots + \frac{\lambda^{k-1} x}{\ln^k x}. \tag{27}$$

Here (27) is the iteration function of the General series.

6. Calculation comparison

Asymptotic formula:

$$\pi(x) \sim \frac{x}{\ln x},\ (x \to \infty).$$

Referred to as: the prime number theorem $[6]_8$.

Prime number theorem and $s_1(x)$, $s_2(x)$ Comparison.

x,	$\pi(10^x)$,	$10^x / \ln 10^x$,	$s_1(10^x)$,	$s_2(10^x)$.
1,	4,	4,	8,	18,
2,	25,	22,	28,	30,
3,	168,	145,	169,	174,
4,	1229,	1086,	1218,	1236,
5,	9592,	8685,	9512,	9599,
6,	78498,	72382,	78031,	78509,
7,	664579,	620421,	661459,	664365,
8,	5761455,	5428681,	5740304,	5759281,
9,	50847534,	48254942,	50701543,	50832214,
10,	455052511,	434294482,	454011971,	454949745,
11,	4118054813,	3948131654,	4110416301,	4117372817,
12,	37607912018,	36191206825,	37550193649,	37603214823,
13,	346065536839,	334072678387,	345618860221,	346032203981,
14,	3204941750802,	3102103442166,	3201414635781,	3204699149779,
15,	29844570422669,	28952965460217,	29816233849001,	29842764299264,
16,	279238341033925,	271434051189532,	279007258230821,	279224620776882,

7. Summarizes

From (11) are:

$$\pi(x) \sim \frac{x}{\ln x - 1}, \; (x \to \infty). \tag{28}$$

by (17) get:

$$\pi(x) \sim \frac{x(\ln x - 1)}{\ln x(\ln x - 2)}, \; (x \to \infty). \tag{29}$$

Here (28), (29) is the distribution of prime number theorem, obtained by iterative transformation, but the distribution of prime numbers require strict proof of iterated function in order to become theorems.

To find a prime number theorem is very difficult. Usually put forward a conjecture, and then discussed by the rest of the mathematician

Riemann hypothesis, Jie Bofu conjecture, twin prime conjecture, Goldbach conjecture, the distribution of prime numbers are well-known conjecture.

This distribution of prime numbers of the iteration function has to "guess" the form, there is no proof, only the iteration function is given, iterative transformation, series transformation, iteration through items, calculation examples and data control. If necessary, continue the discussion later.

References

Dan Liu. (2010). *Approximate calculation of the distribution of prime numbers*. Beijing: modern technology.

Dan Liu. (2010). *Distribution of prime numbers and other times the function*. Sichuan: Sichuan College of Education.

Hua. (1979). *Number of guidance*. Beijing: Science Press.

Hua, Wang. (1963). *Numerical integration and its application*. Beijing: Science Press.

Illinois Gish (Germany). (2007). *Algebraic number theory*. Beijing: Science Press. 2007.

Manning (Russia). (2006). *Modern number theory guided*. Beijing: Science Press.

Pan Cheng-dong, Pan Chengbiao. (1988). *Elementary Proof of the prime number theorem*. Shanghai: Shanghai Science and Technology Press.

Si-Min He, Yan Shijian. (1958). *Elementary number theory.* Beijing: Science Press.