Ranking Decision Making Unit with Stochastic Data Using Jam Model

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Abstract

Data envelopment analysis is a nonparametric technique checking efficiency of DMUs using math programming. Current models are just working with deterministic data. The point is that how would be ranking and checking efficiency of units if data were stochastic. In this article we will represent a method to rank decision making units with stochastic data using JAM Model of "Jahanshahloo, Mehrabian, Alirezaie" Finally a numerical example will applied to check the performance of purposed method.

Keywords: DEA, Ranking, Stochastic Programming

1. Introduction

DEA is a powerful tool in estimating efficiency of decision making units with multiple inputs and outputs. Charnes, Cooper, Rhodes (1978) were the pioneers of the field that introduced their first model named "CCR" in 1978. In 1997 Jahanshahloo and his coworkers proposed a technique, JAM to confront the AP techniques problem; these problems include infeasibility of covering form in case of data with special structure and instability that causes sudden mutation on performance of some under survey DMUs when removing some of DMUs."these problems include infeasibility when the data's have special structure and nonstability in case the elimination of some DMUs causes sudden mutation on efficiency of an evaluated DMU."

In recent years stochastic formulation of main models having nondeterministic inputs and outputs are under scope of researchers. Cooper and Huang (2002), Asgharian and Khodabakhshi (2010) are working in this issue. There are several method for ranking efficient units with stochastic data, including Hosseinzadeh Lotfi and Nematollahi ranking using coefficient variation(2010), also ranking using AP technique of Razavyan, and Tohidi (2008).

2. Stochastic Model

Suppose $\tilde{x}_j = (\tilde{x}_{1j}, \tilde{x}_{2j}, \cdots, \tilde{x}_{mj})$ is stochastic inputs vector like DMU_j, $\tilde{y}_j = (\tilde{y}_{1j}, \tilde{y}_{2j}, \cdots, \tilde{y}_{sj})$ is stochastic output vectors like DMU_i, suppose $\bar{x}_j = (\bar{x}_{1j}, \bar{x}_{2j}, \cdots, \bar{x}_{mj})$ and $\bar{y}_j = (\bar{y}_{1j}, \bar{y}_{2j}, \cdots, \bar{y}_{sj})$ respectively are deterministic inputs and outputs vectors. Assume that all inputs and outputs in DEA stochastic model independently use normal distribution. Following formulation represents JAM model with stochastic input and Output

$$\begin{aligned}
&Min\ \ 1 + w_p \\
&s.t\ \ P(\sum_{\substack{j=1\\j\neq p}}^{n} \lambda_j \tilde{x}_{ij} \leq \tilde{x}_{ip} + w_p) \geq 1-\alpha \quad i = 1,2,\cdots,m \\
&\qquad P(\sum_{\substack{j=1\\j\neq p}}^{n} \lambda_j \tilde{y}_{rj} \geq \tilde{y}_{rp}) \geq 1-\alpha \quad r = 1,2,\cdots,s \\
&\qquad \sum_{j=1}^{n} \lambda_j = 1 \quad \lambda_j \geq 0\ ,\ \ j = 1,2,...,n
\end{aligned}$$

Which P shows probability, α is a predefined number $0 < \alpha < 1$, $1-\alpha$ represents acceptance of constraints. We are explaining the steps of transforming to deterministic model now

$$P(\sum_{\substack{j=1\\j\neq p}}^{n} \lambda_j \tilde{x}_{ij} \leq \tilde{x}_{ip} + w_p) \geq 1-\alpha \Longrightarrow P(\sum_{\substack{j=1\\j\neq p}}^{n} \lambda_j \tilde{y}_{rj} =\geq \tilde{y}_{rp}) \geq 1-\alpha. \tag{1}$$

Assuming

$$h_i = \sum_{\substack{j=1\\j\neq p}}^{n} \lambda_j \tilde{x}_{ij} - \tilde{x}_{ip}\ ,\ \ i = 1,2,...,m \tag{2}$$

we can obtain variance and average of stochastic variable h_i through

$$E(h_i) = \sum_{\substack{j=1\\j\neq p}}^{n} \lambda_j \tilde{x}_{ij} - \tilde{x}_{ip}\ ,\ \ i = 1,2,...,m \tag{3}$$

$$\begin{aligned}
var(h_i) &= var(\lambda_1 \tilde{x}_{i1} + \lambda_2 \tilde{x}_{i2} + ... + \lambda_{p-1} \tilde{x}_{i(p-1)} - \tilde{x}_{ip} + \lambda_{p+1} \tilde{x}_{i(p+1)} + ... + \lambda_n \tilde{x}_{in}) \\
&= \sum_{\substack{j=1\\j\neq p}}^{n} \sum_{\substack{k=1\\k\neq p}}^{n} \lambda_j \lambda_k cov(\tilde{x}_{ij}, \tilde{x}_{ik}) + var(\tilde{x}_{ip}) - 2\sum_{\substack{j=1\\j\neq p}}^{n} \lambda_j cov(\tilde{x}_{ij}, \tilde{x}_{ip})\ ,\ \ i = 1,2,...,m
\end{aligned} \tag{4}$$

$$\sqrt{var(h_i)} = \sigma_i^1\ ,\ \ i = 1,2,...,m \tag{5}$$

Through equations (1) to (5), we have

$$P(h_i \leq w_p) \geq 1-\alpha \Longrightarrow P\Big(\frac{h_i - E(h_i)}{\sigma_i^1} \leq \frac{w_p - E(h_i)}{\sigma_i^1}\Big) \geq 1-\alpha. \tag{6}$$

Stochastic variable h_i will have standard normal distribution like this

$$\frac{h_i - E(h_i)}{\sigma_i^1} \sim N(0,1)\ ,\ \ i = 1,2,\cdots,m.$$

Through equation (6): $\Phi(\frac{w_p - E(h_i)}{\sigma_i^1}) \geq 1-\alpha$ where Φ has standard normal distribution. Assuming $\Phi(-K_\alpha) = 1-\alpha$, we will have $\Phi(\frac{w_p - E(h_i)}{\sigma_i^1}) \geq \Phi(-K_\alpha)$ considering Φ is an reversible we have

$$\begin{aligned}
&\frac{w_p - E(h_i)}{\sigma_i^1} \geq -K_\alpha \\
&\Longrightarrow w_p - E(h_i) \geq -K_\alpha \sigma_i^1 \\
&\Longrightarrow E(h_i) - K_\alpha \sigma_i^1 \leq w_p.
\end{aligned} \tag{7}$$

Considering $\Phi(-K_\alpha) + \Phi(K_\alpha) = 1$, we have

$$1 - \alpha = \Phi(-K_\alpha) = 1 - \Phi(k_\alpha) \Longrightarrow K_\alpha = \Phi^{-1}(\alpha). \tag{8}$$

So through equations (7), (8), we will have

$$\begin{aligned} & E(h_i) - \Phi^{-1}(\alpha)\sigma_i^1 \le w_p \\ \Rightarrow & E(h_i) \le w_p + \Phi^{-1}(\alpha)\sigma_i^1 \end{aligned} \tag{9}$$

From (3), (9)

$$\begin{aligned} & \sum_{\substack{j=1 \\ j\neq p}}^{n} \lambda_j \bar{x}_{ij} - \bar{x}_{ip} \le w_p + \Phi^{-1}(\alpha)\sigma_i^1 \\ \Longrightarrow & \sum_{\substack{j=1 \\ j\neq p}}^{n} \lambda_j \bar{x}_{ij} \le \bar{x}_{ip} + w_p + \Phi^{-1}(\alpha)\sigma_i^1. \end{aligned} \tag{10}$$

If the same explained procedure applied for outputs too, deterministic JAM model will be like this

$$Min \;\; 1 + w_p \tag{11}$$

$$s.t \;\; \sum_{\substack{j=1 \\ j\neq p}}^{n} \lambda_j \bar{x}_{ij} \le \bar{x}_{ip} + w_p + \Phi^{-1}(\alpha)\sigma_i^1 \quad i = 1, 2, \cdots, m$$

$$\sum_{\substack{j=1 \\ j\neq p}}^{n} \lambda_j \bar{y}_{rj} \ge \bar{y}_{rp} - \Phi^{-1}(\alpha)\sigma_i^0 \qquad r = 1, 2, \cdots, s \tag{12}$$

$$\sum_{j=1}^{n} \lambda_j = 1 \qquad \lambda_j \ge 0 \;\; , \;\; j = 1, 2, ..., n \tag{13}$$

Therefore in model (11), we have

$$(\sigma_i^1)^2 = \sum_{\substack{j=1 \\ j\neq p}}^{n} \sum_{\substack{k=1 \\ k\neq p}}^{n} \lambda_j \lambda_k cov(\tilde{x}_{ij}, \tilde{x}_{ik}) + var(\tilde{x}_{ip}) - 2\sum_{\substack{j=1 \\ j\neq p}}^{n} \lambda_j cov(\tilde{x}_{ij}, \tilde{x}_{ip}) \;\; , \;\; i = 1, 2, \cdots, m$$

$$(\sigma_r^0)^2 = \sum_{\substack{l=1 \\ l\neq p}}^{n} \sum_{\substack{m=1 \\ m\neq p}}^{n} \lambda_l \lambda_m cov(\tilde{y}_{rl}, \tilde{y}_{rm}) + var(\tilde{y}_{rp}) - 2\sum_{\substack{j=1 \\ j\neq p}}^{n} \lambda_j cov(\tilde{y}_{rj}, \tilde{y}_{rp}) \;\; , \;\; r = 1, 2, \cdots, s$$

Model (11) is a nonlinear programming; optimal value of above model provides ranking of DMUs with stochastic data. If $\Phi^{-1}(\alpha) = 0$ then model (11) will transform to JAM model with deterministic data.

3. Applied Example

An automobile manufacturing company decides to invest on a special model of its products in all over the world companies and tries to rank its efficient units. In order to do that 5 different units with parameters such as production cost, service cost, as inputs and acceleration of 0-100, maximum horse power per minute as outputs have been studied in 10 different working time. Assume that all data have normal distribution. Table 1 to 4 shows Average and variance of inputs and outputs.

[Table 1-4]

There is also covariance of inputs and outputs are noted in tables 5-8.

[Table 5-8]

Assuming $\alpha = 0.117$, B, C, D units will be stochastically efficient. Using (11) model and Lingo, we have

$$\theta_D^* > \theta_C^* > \theta_B^*.$$

4. Conclusion

To rank a unit stochastically efficient, first we need to remove it from possible production set and calculate efficiency of it regarding new possible production set using nonlinear deterministic model (11).

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Table 1. Average of production and service cost in 10 times

$E(\tilde{x}_{ij}) = \bar{x}_{ij}$	A	B	C	D	E
$E(\tilde{x}_{1j}) = \bar{x}_{1j}$	10011	9975.08	9981.57	10017.25	10014.39
$E(\tilde{x}_{2j}) = \bar{x}_{2j}$	9	9.41	8.51	9.06	10.7

Table 2. Average of acceleration of 0-100 and maximum horse power per minute in 10 times

$E(\tilde{y}_{ij}) = \bar{y}_{ij}$	A	B	C	D	E
$E(\tilde{y}_{1j}) = \bar{y}_{1j}$	11.94	11.18	11.94	11.42	10.05
$E(\tilde{y}_{2j}) = \bar{y}_{2j}$	96.86	103.7	105.02	105.62	98.15

Table 3. Variance of production and service cost in 10 times

$var(\tilde{x}_{ij})$	A	B	C	D	E
$var(\tilde{x}_{1j})$	653.91	1404.31	979.27	2997.27	1355.04
$var(\tilde{x}_{2j})$	0.26	0.28	0.6	0.64	0.01

Table 4. Variance of acceleration of 0-100 and maximum horse power per minute in 10 times

$var(\tilde{y}_{ij})$	A	B	C	D	E
$var(\tilde{y}_{1j})$	0.07	0.34	0.16	0.24	0.06
$var(\tilde{y}_{2j})$	106.7	117.35	39.16	20.35	15.86

Table 5. $cov(x_1, x_1)$

$cov(\tilde{x}_{1j}, \tilde{x}_{1j})$	$\tilde{x}_{11}$	$\tilde{x}_{12}$	$\tilde{x}_{13}$	$\tilde{x}_{14}$	$\tilde{x}_{15}$
$\tilde{x}_{11}$	588.52	424.69	54.62	-393.13	-291.78
$\tilde{x}_{12}$	424.69	1263.88	-80.83	301.92	-84.26
$\tilde{x}_{13}$	54.62	-8083	881.34	-982.21	-142.55
$\tilde{x}_{14}$	-393.13	301.92	-982.21	2697.54	226.01
$\tilde{x}_{15}$	-291.78	-84.26	-142.55	226.01	319.53

Table 6. $cov(x_2, x_2)$

$cov(\tilde{x}_{2j}, \tilde{x}_{2j})$	$\tilde{x}_{21}$	$\tilde{x}_{22}$	$\tilde{x}_{23}$	$\tilde{x}_{24}$	$\tilde{x}_{25}$
$\tilde{x}_{21}$	0.23	-0.13	-0.09	-0.21	-0.01
$\tilde{x}_{22}$	-0.13	0.25	-0.09	0.12	-0.003
$\tilde{x}_{23}$	-0.09	-0.09	0.54	0.26	-0.002
$\tilde{x}_{24}$	-0.21	0.12	0.26	0.58	0.003
$\tilde{x}_{25}$	-0.01	-0.003	-0.002	0.003	0.01

Table 7. $cov(y_1, y_1)$

$cov(\tilde{y}_{1j}, \tilde{y}_{1j})$	$\tilde{y}_{11}$	$\tilde{y}_{12}$	$\tilde{y}_{13}$	$\tilde{y}_{14}$	$\tilde{y}_{15}$
$\tilde{y}_{11}$	0.06	-0.004	0.03	-0.09	-0.01
$\tilde{y}_{12}$	-0.004	0.3	-0.02	0.06	-0.02
$\tilde{y}_{13}$	0.03	-0.02	0.15	-0.08	0.02
$\tilde{y}_{14}$	-0.09	0.06	-0.08	0.21	0.05
$\tilde{y}_{15}$	-0.01	-0.02	0.02	0.05	0.05

Table 8. $cov(y_2, y_2)$

$cov(\tilde{y}_{2j}, \tilde{y}_{2j})$	$\tilde{y}_{21}$	$\tilde{y}_{22}$	$\tilde{y}_{23}$	$\tilde{y}_{24}$	$\tilde{y}_{25}$
$\tilde{y}_{21}$	96.03	-58.82	2.63	0.52	19.95
$\tilde{y}_{22}$	-58.82	105.62	9.9	-13.82	-1.64
$\tilde{y}_{23}$	2.63	9.9	35.24	-4.16	-2.42
$\tilde{y}_{24}$	0.52	-13.82	-4.16	18.31	0.003
$\tilde{y}_{25}$	19.95	-1.64	-2.42	0.003	14.27