Weak Convergence of Ishikawa Iteration with Error for Pseudo Contractive Mappings in Hilbert Spaces

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Abstract

Our purpose in this paper is to consider a sequence $\{x_n\}$ defined as (2) and a asymptotically k_n- strict pseudocontractive mapping in the intermediate sense Ishikawa iterative process with errors. The results presented in this paper mainly improved and extend the corresponding results announced in [D.R.Sahu, 2009; T.H.Kim, 2008; K.Nammanee, 2000; T.H.Kim, 2008].

Keywords: Demiclosedness principle, Asymptotically strict pseudocontrative mapping, Fixed point, Ishikawa iterative process

1. Introduction

The class of asymptotically nonexpansive mappings was introduced as an important generalization of the class of nonexpansive mappings£and the existence of fixed points of asymptotically nonexpansive mappings was proved by Goebel and Kirk [1972] as below:

Theorem 1. *If C is a nonempty closed convex bounded subset of a uniformly convex Banach Space, then every asymptotically nonexpansive mapping $T : C \longrightarrow C$ has a fixed point in C.*

Mann iteration process:

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n T^n x_n, \quad n \in \mathbb{N}., \tag{1}$$

for (1) the approximation of fixed points of asymptotically nonexpansive mappings was developed and it's weak convergence was obtained by Schu [1991].

Iterative methods for approximation of fixed points of asymptotically nonexpansive mappings have been widely studied by authors [C.E.Chidume, 2003; S.S.Chang, 2001; S.H.Khan, 2005; Z.Q.Liu, 2004; K.Nammanee, 2000; B.E.Rhoades, 1994; M.O.Osilike, 2000; J.Schu, 1991; L.Wang, 2006].

The class of asymptotically nonexpansive mappings in the intermediate sense was introduced by Bruck, Kuczumow and Reich [1993] .

In 2008, Kim and Xu [2008] introduced the concept of asymptotically k-strict pseudocontractive mappings in Hilbert space.

In 2009, Sahu and Xu [2009] study some properties and convergence of (1) for the class of asymptotically k-strict pseudocontractive mappings in the intermedidate sense which are not necessarily Lipschitzian.

Our purpose in this paper is to consider a sequence $\{x_n\}$ defined as (2) and a asymptotically k_n- strict pseudocontractive mapping in the intermediate sense Ishikawa iterative process with errors. The results presented in this paper mainly improved and extend the corresponding results announced in [D.R.Sahu, 2009; T.H.Kim, 2008; K.Nammanee, 2000; T.H.Kim, 2008].

$$\begin{cases} x_{n+1} = (1-\alpha_n)x_n + \alpha_n T^n y_n + \sigma_n \\ y_n = (1-\beta_n)x_n + \beta_n T^n x_n + \tau_n \end{cases} \tag{2}$$

2. Preliminaries

Let C be a nonempty subset of a normed space X and $T : C \to C$ a mapping. We need the following concepts,

(i) T is nonexpansive if,

$$\|Tx - Ty\| \le \|x - y\|$$

for all $x, y \in C$.

(ii) T is asymptotically nonexpansive(cf.[K.Goebel, 1972])if there exists a sequence k_n of positive numbers satisfying the property

$$lim_{n\to\infty} k_n = 1 \textit{ and } \|T^n x - T^n y\| \le k_n \|x - y\|$$

for all integers $n \ge 1$ and $x, y \in C$.

(iii) T is uniformly Lipschitzian if there exists a constant $L > 0$ such that,

$$\|T^n x - T^n y\| \le L\|x - y\|$$

for all integers $n \ge 1$ and $x, y \in C$.

(iv) T is asymptotically nonexpansive in the intermediate sense [R.E.Bruck, 1993] provided T is uniformly continuous and,

$$limsup_{n\to\infty} sup_{x,y\in C}(\|T^n x - T^n y\| - \|x - y\|) \le 0.$$

It is clear that every nonexpansive mapping is asymptotically nonexpansive and every asymptotically nonexpansive mapping is uniformly Lipschitzian.

Let H be a real Hilbert space with inner product $\langle \cdot, \cdot \rangle$ and norm $\| \cdot \|$, respectively and let C be a closed convex subset of H, $\omega_\omega(\{x_n\})$ denotes the weak $\omega - limit$ set of $\{x_n\}$.

Definition 1 Let C be a nonempty subset of a Hilbert space H. A mapping $T : C \longrightarrow C$ is said to be an asymptotically k_n-strict pseudocontractive mapping in the intermediate sense with sequence $\{\gamma_n\}$ if there exist sequences $\{k_n\}$ in $[0, 1)$ and $\{\gamma_n\}$, $\{c_n\}$ in $[0, \infty)$ with $limsup_{n\to\infty} k_n = k < 1$, $lim_{n\to\infty}\gamma_n = lim_{n\to\infty} c_n = 0$ such that

$$\|T^n x - T^n y\|^2 \le (1 + \gamma_n)\|x - y\|^2 + k_n\|x - T^n x - (y - T^n y)\|^2 + c_n, \tag{3}$$

for all $x, y \in C$ and $n \in \mathbb{N}$.

we need some facts and tools which are listed as follows,

Lemma 1. *([M.O.Osilike, 2000; K.K.Tan, 1992]) Let $\{\gamma_n\}, \{\beta_n\}, \{\delta_n\}$ be three sequences of nonnegative numbers satisfying the recursive inequality,*

$$\delta_{n+1} \le \delta_n \beta_n + \gamma_n, \forall n \in \mathbb{N}$$

If $\beta_n \ge 1, \Sigma_{n=1}^{\infty}(\beta_n - 1) < \infty$ and $\Sigma_{n=1}^{\infty}\gamma_n < \infty$ then $\lim_{n\to\infty} \delta_n$ exists.

Lemma 2. *(Agarwal, O'Regon. [2007]) Let $\{x_n\}$ be a bounded sequence in a reflexive Banach space X. if $\omega_\omega(\{x_n\}) = \{x\}$, then x_n converges weakly to x.*

Lemma 3. *Let H be a real Hilbert space. Then the following hold:*

(a) $\|x - y\|^2 = \|x\|^2 - \|y\|^2 - 2\langle x - y, y\rangle, \forall x, y \in H;$

(b) $\|(1 - t)x + ty\|^2 = (1 - t)\|x\|^2 + t\|y\|^2 - t(1 - t)\|x - y\|^2, \forall x, y \in H;$

(c) let $\{x_n\} \in H$ such that x_n weak converges to x, then $\limsup_{n\to\infty} \|x_n - y\|^2 = \limsup_{n\to\infty} \|x_n - x\|^2 + \|x - y\|, \forall y \in H.$

Lemma 4. *([D.R.Sahu, 2009]) Let C be a nonempty subset of a Hilbert space H ,$T : C \to C$ an asymptotically k− strict pseudocontractive mapping in the intermediate sense with sequence $\{\gamma_n\}$. Then*

$$\|T^n x - T^n y\| \le \frac{1}{1-k}(k\|x - y\| + \sqrt{(1 + (1 - k)\gamma_n)\|x - y\|^2 + (1 - k)c_n})$$

for all $x, y \in C$ and $n \in \mathbb{N}$.

Lemma 5. *([D.R.Sahu, 2009]) Let C be a nonempty subset of a Hilbert space H, $T : C \to C$ a uniformly continuous asymptotically k− strict pseudocontractive mapping in the intermediate sense with sequence$\{\gamma_n\}$,let $\{x_n\}$ be a bounded sequence in C such that $\|x_{n+1} - x_n\| \to 0$ and $\|x_n - T^n x_{n+1}\| \to 0$, $(n \to \infty)$ Then$\|x_n - Tx_n\| \to 0$,as$(n \to \infty)$.*

3. Main results

Theorem 2. *Let C be be a nonempty closed convex subset of a Hilbert space H, $T : C \to C$ a continuous asymptotically k_n− strict pseudo -contractive mapping in the intermediate sense with sequence $\{\gamma_n\}$ .Then $F(T)$ is closed and convex and $I - T$ is demiclosed at zero.*

Proof. It is easy to see that $F(T)$ is closed and convex.

We prove that $I - T$ is demiclosed at zero. Assume that $\{x_n\}$ be a sequence in C. By Lemma 4. we obtain

$$\|T^m x_n - T^m x\| < M$$

for all $n, m \in N$ and some constant $M > 0$.
Let $f(x) = \limsup_{n\to\infty} \|x_n - x\|^2$, for all $x \in H$. Since $\{x_n\}$ weakly converges to x,

$$f(y) = f(x) + \|x - y\|^2 \quad for\ all\ y \in H.$$

Hence we have

$$\begin{aligned}
f(T^m x) &= \limsup_{n\to\infty} \|x_n - T^m x\|^2 \\
&\le \limsup_{n\to\infty}((1 + \gamma_m)\|x_n - x\|^2 + k_m\|x_n - T^m x_n - (x - T^m x)\|^2 + c_m) + \\
&\quad \limsup_{n\to\infty}(\|x_n - T^m x_n\|^2 + 2\|x_n - T^m x_n\|M) \\
&\le f(x) + k \limsup_{n\to\infty} \|x_n - T^m x_n - (x - T^m x)\|^2 + \limsup_{n\to\infty}(\|x_n - T^m x_n\|^2 \\
&\quad +2\|x_n - T^m x_n\|M) + f(x)\gamma_m + c_m
\end{aligned}$$

for each $m \in N$. Since $f(x) + \|x - T^m x\|^2 = f(T^m x)$,

$$\|x - T^m x\|^2 \le f(x)\gamma_m + c_m + k \limsup_{n\to\infty} \|x_n - T^m x_n - (x - T^m x)\|^2 + \limsup_{n\to\infty}(\|x_n - T^m x_n\|^2 + 2\|x_n - T^m x_n\|M).$$

By $\limsup_{n\to\infty} \|x_n - T^m x_n\| = 0$, we have

$$\limsup_{m\to\infty} \|x - T^m x\|^2 \le k \limsup_{m\to\infty} \|x - T^m x\|^2.$$

It follows that $T^m x \to x(m \to \infty)$ and $(I - T)x = 0$. □

Theorem 3. *Let C be a nonempty subset of a Hilbert space H and $T : C \to C$ be uniformly continuous asymptotically k_n− pseudocontracitive in the intermediate sense with sequence γ_n such that $F(T) \neq \phi$. Assume $\{\sigma_n\}, \{\tau_n\} \subset C$, $\{\alpha_n\}, \{\beta_n\} \subset (0, 1)$ such that $0 < \delta \le \alpha_n, \beta_n \le 1 - k_n - \delta < 1$ and $\{x_n\}$ defined as (2) satisfy the following conditions:*

(i) $\Sigma_{n=1}^{\infty}\gamma_n < \infty$, $\Sigma_{n=1}^{\infty}c_n < \infty$, $\Sigma_{n=1}^{\infty}k_n < \infty$, $\Sigma_{n=1}^{\infty}\|\sigma_n\|^2 < \infty$ and $\Sigma_{n=1}^{\infty}\|\tau_n\|^2 < \infty$;

(ii) $\{x_n\}$ be bounded.

Then $\{x_n\}$ converges weakly to an element of $F(T)$.

Proof. First we show that $\{\|y_n - T^n y_n\|\}$ be bounded.

$$\begin{aligned}
\|y_n - p\|^2 &= \|(1 - \beta_n)(x_n + \tau_n - p) + \beta_n(T^n x_n + \tau_n - p)\|^2 \\
&= (1 - \beta_n)\|x_n + \tau_n - p\|^2 + \beta_n\|T^n x_n + \tau_n - p\|^2 - \beta_n(1 - \beta_n)\|T^n x_n - x_n\|^2 \\
&\le (1 - \beta_n)[\|x_n - p\|^2 + 2\langle\tau_n, x_n - p\rangle + \|\tau_n\|^2] + \beta_n[\|T^n x_n - p\|^2 \\
&\quad +2\langle\tau_n, T^n x_n - p\rangle + \|\tau_n\|^2] - \beta_n(1 - \beta_n)\|T^n x_n - x_n\|^2 \\
&= (1 - \beta_n)\|x_n - p\|^2 + \beta_n\|T^n x_n - p\|^2 + \|\tau_n\|^2 - \beta_n(1 - \beta_n)\|T^n x_n - x_n\|^2 + \sigma. \qquad (4)
\end{aligned}$$

where $\sigma := 2\langle(1-\beta_n)(x_n-p)+\beta_n(T^nx_n-p),\tau_n\rangle$, we simply σ as follows.

$$\begin{aligned}\sigma &= 2\langle(1-\beta_n)(x_n-p)+\beta_n(T^nx_n-p),\tau_n\rangle\\ &\le \|(1-\beta_n)(x_n-p)+\beta_n(T^nx_n-p)\|^2+\|\tau_n\|^2\\ &= (1-\beta_n)\|x_n-p\|^2+\beta_n\|T^nx_n-p\|^2-\beta_n(1-\beta_n)\|T^nx_n-x_n\|^2+\|\tau_n\|^2. \end{aligned}\tag{5}$$

hence

$$\begin{aligned}\|y_n-p\|^2 &\le 2(1-\beta_n)\|x_n-p\|^2+2\beta_n\|T^nx_n-p\|^2+2\|\tau_n\|^2-2\beta_n(1-\beta_n)\|T^nx_n-x_n\|^2\\ &\le 2(1-\beta_n)\|x_n-p\|^2+2\beta_n[(1+\gamma_n)\|x_n-p\|^2+k_n\|T^nx_n-x_n\|^2+c_n]\\ &\quad +2\|\tau_n\|^2-2\beta_n(1-\beta_n)\|T^nx_n-x_n\|^2\\ &= 2(1+\beta_n\gamma_n)\|x_n-p\|^2+2\beta_nc_n+2\|\tau_n\|^2-2\beta_n(1-\beta_n-k_n)\|T^nx_n-x_n\|^2\\ &\le 2(1+\beta_n\gamma_n)\|x_n-p\|^2+2\beta_nc_n+2\|\tau_n\|^2-2\delta^2\|T^nx_n-x_n\|^2. \end{aligned}\tag{6}$$

Which implies that

$$\|y_n-p\|^2 \le 2(1+\beta_n\gamma_n)\|x_n-p\|^2+2\beta_nc_n+2\|\tau_n\|^2. \tag{7}$$

By condition (i), (ii) and (7), we have $\{\|y_n-p\|\}$ be bounded, and so $\{y_n\}$.
From lemma 4, we obtained

$$\|T^ny_n-p\|^2 \le \frac{1}{1-k_n}(k_n\|y_n-p\|+\sqrt{1+(1-k_n)\gamma_n\|y_n-p\|^2+(1-y_n)c_n}), \tag{8}$$

hence $\{\|T^ny_n-p\|\}$ be bounded. By $\|y_n-T^ny_n\|\le\|T^ny_n-p\|+\|y_n-p\|$, we have $\{\|y_n-T^ny_n\|\}$ be bounded.

Next we prove that $\lim_{n\to\infty}\|x_n-p\|$, for each $p\in F(T)$ exists. Indeed,

$$\begin{aligned}\|x_{n+1}-p\|^2 &= \|(1-\alpha_n)(x_n+\sigma_n-p)+\alpha_n(T^ny_n+\sigma_n-p)\|^2\\ &\le (1-\alpha_n)\|x_n+\sigma_n-p\|^2+\alpha_n\|T^ny_n+\sigma_n-p\|^2-\alpha_n(1-\alpha_n)\|T^ny_n-x_n\|^2\\ &\le (1-\alpha_n)\|x_n-p\|^2+2(1-\alpha_n)\langle\sigma_n,x_n+\sigma_n-p\rangle+\alpha_n\|T^ny_n-p\|^2\\ &\quad +2\alpha_n\langle\sigma_n,T^ny_n+\sigma_n-p\rangle-\alpha_n(1-\alpha_n)\|T^ny_n-x_n\|^2\\ &\le (1-\alpha_n)\|x_n-p\|^2+2(1-\alpha_n)\langle x_n-p,\sigma_n\rangle+\alpha_n\|T^ny_n-p\|^2\\ &\quad +2\alpha_n\langle T^ny_n-p,\sigma_n\rangle+2\langle\sigma_n,\sigma_n\rangle-\alpha_n(1-\alpha_n)\|T^ny_n-x_n\|^2\\ &= (1-\alpha_n)\|x_n-p\|^2+\alpha_n\|T^ny_n-p\|^2+2\langle(1-\alpha_n)(x_n-p)+\alpha_n(T^ny_n-p),\sigma_n\rangle\\ &\quad +2\|\sigma_n\|^2-\alpha_n(1-\alpha_n)\|T^ny_n-x_n\|^2. \end{aligned}\tag{9}$$

In the other hand,

$$\begin{aligned}2\langle(1-\alpha_n)(x_n-p)+\alpha_n(T^ny_n-p),\sigma_n\rangle &\le \|(1-\alpha_n)(x_n-p)+\alpha_n(T^ny_n-p)\|^2+\|\sigma\|^2\\ &\le (1-\alpha_n)\|x_n-p\|^2+\alpha_n\|T^ny_n-x_n\|^2\\ &\quad -\alpha_n(1-\alpha_n)\|T^ny_n-x_n\|^2+\|\sigma_n\|^2 \end{aligned}\tag{10}$$

By substituting (10) into (9), we have

$$\begin{aligned}\|x_{n+1}-p\|^2 &\le 2(1-\alpha_n)\|x_n-p\|^2+2\alpha_n\|T^ny_n-p\|^2-2\alpha_n(1-\alpha_n)\|T^ny_n-x_n\|^2+3\|\sigma_n\|^2\\ &\le 2(1-\alpha_n)\|x_n-p\|^2+2\alpha_n[(1+\gamma_n)\|y_n-p\|^2+k_n\|y_n-T^ny_n\|^2+c_n]\\ &\quad -2\alpha_n(1-\alpha_n)\|T^ny_n-x_n\|^2+3\|\sigma_n\|^2. \end{aligned}\tag{11}$$

Substituting (6) into (11), we have,

$$\begin{aligned}\|x_{n+1}-p\|^2 &\le 2[(1-\alpha_n)+\alpha_n(1+\gamma_n)(1+\beta_n\gamma_n)]\|x_n-p\|^2+2\alpha_n(1+\gamma_n)(\beta_nc_n+2\|\tau_n\|^2)\\ &\quad +2\alpha_nc_n+3\|\sigma_n\|^2+2\alpha_nk_n\|y_n-T^ny_n\|-2\delta^2\|T^ny_n-x_n\|^2-2\delta^2\|x_n-T^nx_n\|^2\\ &\le 2\varepsilon_n\|x_n-p\|^2+\theta_n-2\delta^2\|T^ny_n-x_n\|^2-2\delta^2\|x_n-T^nx_n\|^2. \end{aligned}\tag{12}$$

where $\varepsilon_n=[(1-\alpha_n)+\alpha_n(1+\gamma_n)(1+\beta_n\gamma_n)]$, $\theta_n=2\alpha_n(1+\gamma_n)(2\beta_nc_n+3\|\tau_n\|^2)+2\alpha_nc_n+3\|\sigma_n\|^2+2\alpha_nk_n\|y_n-T^ny_n\|$.
Hence,

$$\|x_{n+1}-p\|^2 \le 2\varepsilon_n\|x_n-p\|^2+\theta_n, \tag{13}$$

By the conditions (i), $\Sigma_{n=1}^{\infty}(\varepsilon_n - 1) < \infty$, $\Sigma_{n=1}^{\infty}\theta_n < \infty$, and Lemma 1, we have

$$\lim_{n\to\infty} \|x_n - p\| \ \ exists. \tag{14}$$

On the other hand, from (12) we have,

$$\delta^2\|T^n y_n - x_n\|^2 \le \varepsilon_n\|x_n - p\|^2 - \|x_{n+1} - p\|^2 + \theta_n.$$

By the conditions (i) and the boundeness of $\{x_n\}$, we have

$$\lim_{n\to\infty} \|T^n y_n - x_n\| = 0. \tag{15}$$

Again by (12),

$$\delta^2\|T^n x_n - x_n\|^2 \le \varepsilon_n\|x_n - p\|^2 - \|x_{n+1} - p\|^2 + \theta_n.$$

Hence,

$$\lim_{n\to\infty} \|T^n x_n - x_n\| = 0 \tag{16}$$

and

$$\|x_{n+1} - x_n\| = \|\alpha_n(T^n y_n - x_n) + \sigma_n\| \le \alpha_n\|T^n y_n - x_n\| + \|\sigma_n\|.$$

From (15) and condition (i),

$$\lim_{n\to\infty} \|x_{n+1} - x_n\| = 0 \tag{17}$$

By (16), (17) and uniformly continous of T^n , by Theorem 2 we obtain that

$$\lim_{n\to\infty} \|x_n - Tx_n\| = 0. \tag{18}$$

By the boundedness of $\{x_n\}$, there exists a subsequence $\{x_{n_j}\} \subseteq \{x_n\}$ such that $\{x_{n_j}\}$ converges weakly to x.
From (16), uniform continuits of T and theorem 2, we see that

$$\lim_{n\to\infty} \|x_n - T^m x_n\| = 0$$

for all $m \in \mathbb{N}$. We obtain $x \in F(T)$.

Next we prove the $\omega_\omega(\{x_n\}) = \{x\}$. Suppose there exists another subsequence $\{x_{n_k}\} \subseteq \{x_n\}$ which converges to $z \ne x$. As in the case of x, we must have $z \in F(T)$ It follows from (11) that $\lim_{n\to\infty}\|x_n - x\|$ and $\lim_{n\to\infty}\|x_n - z\|$ exist. Since $\{x_n\}$ satisfies the Opial condition, we have

$$\begin{aligned}\lim_{n\to\infty} \|x_n - x\| &= \lim_{k\to\infty} \|x_{n_k} - x\| < \lim_{k\to\infty} \|x_{n_k} - z\| = \lim_{n\to\infty} \|x_n - z\| \\ &= \lim_{j\to\infty} \|x_{n_j} - z\| < \lim_{j\to\infty} \|x_{n_j} - x\| = \lim_{n\to\infty} \|x_n - x\|,\end{aligned}$$

a contradiction. So that $z = x$ and $\omega_\omega(\{x_n\}) = \{x\}$. Thus, $\{x_n\}$ converges weakly to x . □

Remark 1. *Theorem 2 and Theorem 3 are more general than the results studied in D.R.Sahu [2009] and Schu [1991].*

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