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On a New Optimization Method With Constraints

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Abstract

We consider the constrained optimization problem defined by:

$$f(x^*) = \min_{x \in X} f(x) \tag{1}$$

where the function $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ is convex on a closed convex set X.

In this work, we will give a new method to solve problem (1) without bringing it back to an unconstrained problem. We study the convergence of this new method and give numerical examples.

Keywords: nonlinear optimization

1. Introduction

In many fields of science we need to solve some non-linear optimization problems. Several authors have studied the resolution of non-linear optimization problems, for example (Dennis & Schnabel, 1983; Ortega & Rheinboldt, 1970; Gilbert, 2009) and (Rhanizar, 2002). Our goal is to propose a new constrained optimization method without turning in to an unconstrained optimization problem. Among the methods used to solve the problem (1) by transforming it to an unconstrained problem, we can cite the projection methods defined by:

$$x_{k+1} = P_X(x_k - \alpha_k \nabla f(x_k))$$

with

$$||x - P_X(x)|| = \min_{y \in X} ||x - y||$$

Calculating the projection on X can sometimes be more difficult than the initial problem. In this paper, we propose a new method to solve the problem (1) based on defining a descent direction and a step while staying in the convex domain X.

In order to present conveniently our results, let us introduce the setting used throughout this paper. We use the following notations:

$||x|| = (\sum_{i=1}^{n} x_i^2)^{\frac{1}{2}}$, the Euclidean norm.

$(\, , \,)$ the corresponding inner product.

$\nabla f(x) = (\frac{\partial f}{\partial x_i}(x))_{1 \le i \le n}$: the gradient of the function f.

$\nabla^2 f(x) = (\frac{\partial^2 f}{\partial^2 x_i x_j}(x))_{\substack{1 \le i \le n \\ 1 \le j \le n}}$: the Hessian matrix.

This work is organized as follows:

The introduction is in section 1. In section 2, we give how to choose the direction of descent and a new algorithm that solves problem (1). Section 3 is devoted to results of convergence of the new method. In section 4, we study the speed of convergence. In section 5, some numerical examples are elaborated. Finally, the conclusions form section 6.

2. Searching for a Direction of Descent

At step k, we will define a direction d_k which checks $(\nabla f(x_k) \, , \, d_k) \le 0$.

We look for x_{k+1} of the form $x_{k+1} = x_k + \alpha_k d_k$ such that:

1). $x_{k+1} \in X$, this condition for not using the projection on X

2). $\alpha_k > 0$

3). $f(x_{k+1}) - f(x_k) \leq 0$

On the other hand, we have $f(x_{k+1}) - f(x_k) \approx \nabla f(x_k)(x_{k+1} - x_k) = \nabla f(x_k)\alpha_k d_k$

To get 1), just take $0 < \alpha_k < 1$ and $d_k = y_k - x_k$ with $y_k \in X$

For 3) to be verified, it is enough $\nabla f(x_k)(y_k - x_k) \leq 0$ which gives $\nabla f(x_k)y_k \leq \nabla f(x_k)x_k$

It is thus finally enough to choose y_k such as:

$$(\nabla f(x_k), y_k) = \min_{y\in X}(\nabla f(x_k), y) \tag{2}$$

Condition 3) is not sufficient for convergence (Rondepierre, A. (2017)). This is why we are going to impose that:

$$f(x_k + \alpha_k d_k) - f(x_k) \leq \frac{1}{2}\alpha_k(\nabla f(x_k)\ ,\ d_k)$$

For this direction $d_k = y_k - x_k$, two cases arise:

First case: If $d_k = 0$, we have $y_k = x_k$ so:

$$(\nabla f(x_k)\ ,\ x_k) \leq (\nabla f(x_k)\ ,\ y)\ \forall y \in X$$

and thereafter:

$$(\nabla f(x_k)\ ,\ x_k - y) \leq\ 0\ \forall y \in X$$

therefore x_k is the solution to problem (1) according to the optimization conditions.

Second case: If $d_k \neq 0$

we have $(\nabla f(x_k)\ ,\ y_k) \leq (\nabla f(x_k)\ ,\ x_k)$ therefore $(\nabla f(x_k)\ ,\ d_k) \leq 0$

which gives d_k is a direction of descent.

So we have the following algorithm:

Algorithm

1. Choose $x_0 \in X$, $\epsilon \in]0,1[$, $\alpha_0 \in]0,1[$ and $k := 0$

2. At step k

    2.1. Compute $\nabla f(x_k)$

    2.2. Compute $y_k \in X$ by: (2)

    2.3. Set $d_k = y_k - x_k$

3. If $|(\nabla f(x_k)\ ,\ d_k)| \leq \epsilon$, then:

    3.1. Set $x^* = x_k$

    3.2. Stop

end If

4. Compute $x_k + \alpha_k d_k$

5. If $f(x_k + \alpha_k d_k) - f(x_k) \leq \frac{1}{2}\alpha_k(\nabla f(x_k)\ ,\ d_k)$

    5.1. $x_{k+1} = x_k + \alpha_k d_k$

    5.2. Set $k := k + 1$ and go to 2.

end If

6. $\alpha_k = \frac{1}{2}\alpha_k$ and go to 4.

3. Convergence Study

The possible choice of α_k and the convergence of the sequence $(x_k)_k$ to the solution x^* are given by the following theorem:

Theorem 1 *Let f be of class C^2 on X convex bounded, and suppose that there exists $m > 0$ and $M > 0$, such that:*

$$m\|y\|^2 \leq (\nabla^2 f(x)y, y) \leq M\|y\|^2 \quad \forall x,\ y \in X \tag{3}$$

Then:

1. $x_k \in X \quad \forall k \geq 0$
2. *Condition* $f(x_k + \alpha_k d_k) - f(x_k) \leq \frac{1}{2}\alpha_k(\nabla f(x_k)\ ,\ d_k)$ *is checked for* $0 < \alpha_k \leq -\frac{1}{2M\|d_k)\|^2}(\nabla f(x_k)\ ,\ d_k)$
3. $f(x_k)$ *is a convergent sequence*
4. $(\nabla f(x_k)\ ,\ d_k) \longrightarrow 0$ *when* $k \longrightarrow +\infty$
5. *The sequence* $(x_k)_k$ *converges to the* x^* *solution of problem* (1).

Proof.

1) $x_0 \in X$, we suppose $x_k \in X$, we have $y_k \in X$ and $\alpha_k = 2^{-i}\alpha_0 \in]0.1[$, then $x_{k+1} = x_k + \alpha_k(y_k - x_k) \in X$

2) By applying Taylor formula to f , we have:

$$f(x_k + \alpha_k d_k) - f(x_k) = \alpha_k(\nabla f(x_k)\ , d_k) + \frac{1}{2}\alpha_k^2(\nabla^2 f(t_k)d_k, d_k)$$

with $t_k = x_k + s\alpha_k d_k$ and $0 < s < 1$

Then, by using (3) we get :

$$f(x_k + \alpha_k d_k) - f(x_k) \leq \alpha_k(\nabla f(x_k)\ ,\ d_k) + \frac{1}{2}\alpha_k^2 M\|d_k)\|^2$$

So the condition:

$$f(x_{k+1}) - f(x_k) \leq \frac{1}{2}\alpha_k(\nabla f(x_k)\ ,\ d_k)$$

is checked for

$$\frac{1}{2}\alpha_k^2 M\|d_k)\|^2 + \alpha_k(\nabla f(x_k)\ ,\ d_k) \leq \frac{1}{2}\alpha_k(\nabla f(x_k)\ ,\ d_k)$$

which gives that the condition is checked for

$$\frac{1}{2}\alpha_k^2 M\|d_k)\|^2 \leq -\frac{1}{2}\alpha_k(\nabla f(x_k)\ ,\ d_k)$$

so α_k verifies

$$\alpha_k \leq -\frac{1}{M\|d_k)\|^2}(\nabla f(x_k)\ ,\ d_k)$$

We can choose $\alpha_k = \alpha_0 2^{-i}$ with i as the first clue that verifies:

$$2^{-i}\alpha_0 \leq -\frac{1}{M\|\|d_k)\|^2}(\nabla f(x_k)\ ,\ d_k)$$

It's always possible indeed

$$-\frac{1}{M|d_k)\|^2}(\nabla f(x_k)\ ,\ d_k) > 0$$

and $2^{-n}\alpha_0 \longrightarrow 0$ when $n \longrightarrow +\infty$.

3) We have:

$$f(x_{k+1}) - f(x_k) \leq \frac{1}{2}\alpha_k(\nabla f(x_k)\ ,\ d_k) \leq 0$$

then $(f(x_k))$ is a declining sequence, so it converges.

4) The condition on α_k, gives:

$$2^{-i+1}\alpha_0 > -\frac{1}{M||||d_k)||^2}(\nabla f(x_k)\ ,\ d_k)$$

then

$$2\alpha_k > -\frac{1}{M||d_k||^2}(\nabla f(x_k)\ ,\ d_k) \tag{4}$$

So

$$\frac{1}{2}\alpha_k(\nabla f(x_k)\ ,\ d_k) < \frac{-1}{4M||d_k||^2}(\nabla f(x_k)\ ,\ d_k)^2$$

but

$$f(x_{k+1}) - f(x_k) \leq \frac{1}{2}\alpha_k(\nabla f(x_k)\ ,\ d_k),$$

then

$$f(x_{k+1}) - f(x_k) \leq -\frac{1}{4M||d_k||^2}(\nabla f(x_k)\ ,\ d_k)^2$$

As a result:

$$0 \leq (\nabla f(x_k)\ ,\ d_k)^2 \leq 4M||d_k||^2(f(x_k) - f(x_{k+1})) \leq 4MD^2(f(x_k) - f(x_{k+1}))$$

with $D = \max_{(x,y)\in X^2} ||x - y||$, and by 3) we have:

$$(\nabla f(x_k)\ ,\ d_k) \xrightarrow[k\to+\infty]{} 0.$$

5) By Taylor formula we have

$$f(x^*) - f(x_k) = (\nabla f(x_k), x^* - x_k) + \frac{1}{2}(\nabla^2 f(x_k + t_k(x^* - x_k))(x^* - x_k), x^* - x_k)$$

with $t_k \in]0, 1[$. The relation (3) implies

$$\frac{m}{2}||x^* - x_k||^2 + (\nabla f(x_k), x^* - x_k) \leq f(x^*) - f(x_k) \leq 0$$

then

$$\frac{m}{2}||x^* - x_k||^2 \leq (\nabla f(x_k), x_k - x^*)$$

but

$$(\nabla f(x_k), x_k - x^*) = (\nabla f(x_k), x_k - y_k) + (\nabla f(x_k), y_k - x^*)$$

we also have by (2)

$$(\nabla f(x_k), y_k - x^*) \leq 0$$

then

$$\frac{m}{2}||x^* - x_k||^2 \leq (\nabla f(x_k), x_k - y_k)$$

and

$$||x^* - x_k||^2 \leq -\frac{2}{m}(\nabla f(x_k), y_k - x_k) = -\frac{2}{m}(\nabla f(x_k), d_k).$$

Using 4), we obtain

$$||x^* - x_k||^2 \longrightarrow 0 \text{ when } k \longrightarrow +\infty$$

.

4. Convergence Speed Assesment

The demonstration of the theorem that gives the speed of convergence of the sequence x requires the following lemmas:

Lemma 1

We consider $x_0 \in X$ and $E = \{x \in X$ such that $f(x) \leq f(x_0)\}$.

We suppose that there exists $\lambda > 0$ and $R > 0$ such that:

1. $f(\frac{x+y}{2}) \leq \frac{1}{2}(f(x) + f(y)) - \lambda \|x - y\|^2 \;\; \forall x, \; \forall y \in X$
2. $\|\nabla f(x)\| \leq R \;\; \forall x \in X$

Then:

$$\frac{x_k+y_k}{2} + B(0, \frac{\lambda}{R}\|y_k - x_k\|^2) \subset E.$$

Proof.

For $t \in B(0, \frac{\lambda}{R}\|y_k - x_k\|^2)$, applying the mean value theorem to f, we have:

$$f(\frac{x_k + y_k}{2} + t) = f(\frac{x_k + y_k}{2}) + (\nabla f(t_k), t)$$

with $t_k = \frac{x_k+y_k}{2} + st$ and $0 < s < 1$,

then
$$\begin{aligned} f(\frac{x_k+y_k}{2} + t) &\leq \frac{1}{2}(f(x_k) + f(y_k)) - \lambda\|x_k - y_k\|^2 + (\nabla f(t_k), t) \\ &\leq f(x_0) - \lambda\|x_k - y_k\|^2 + \|\nabla f(t_k)\|\|t\| \\ &\leq f(x_0) - \lambda\|x_k - y_k\|^2 + R\|t\| \end{aligned}$$

Using $t \in B(0, \frac{\lambda}{R}\|y_k - x_k\|^2)$ we obtain:

$$f(\frac{x_k + y_k}{2} + t) \leq f(x_0) - \lambda\|x_k - y_k\|^2 + R\frac{\lambda}{R}\|y_k - x_k\|^2 = f(x_0)$$

when therefore $\frac{x_k+y_k}{2} + t \in E$.

Lemma 2

Under the same assumptions as in Lemma 1, we have:

$$(\nabla f(x_k), y_k - x_k) \leq -\frac{2\lambda}{R}\|y_k - x_k\|^2\|\nabla f(x_k)\| \tag{5}$$

Proof.

By (2), we have

$$(\nabla f(x_k), y_k - x_k) \leq (\nabla f(x_k), y - x_k) \; \forall y \in E$$

using Lemma 1

$$\forall t \in B(0, \frac{\lambda}{R}\|y_k - x_k\|^2)$$

we have:

$$(\nabla f(x_k), y_k - x_k) \leq (\nabla f(x_k), \frac{x_k + y_k}{2} + t - x_k) = (\nabla f(x_k), \frac{y_k - x_k}{2}) + (\nabla f(x_k), t)$$

So, for $t = -\frac{\lambda}{R\|\nabla f(x_k)\|}\|y_k - x_k\|^2 \nabla f(x_k) \in B(0, \frac{\lambda}{R}\|y_k - x_k\|^2)$, we get :

$$(\nabla f(x_k), y_k - x_k) \leq \frac{1}{2}(\nabla f(x_k), y_k - x_k) - \frac{\lambda}{R}\|y_k - x_k\|^2\|\nabla f(x_k)\|$$

and then we have:

$$(\nabla f(x_k), y_k - x_k) \leq -\frac{2\lambda}{R}\|y_k - x_k\|^2\|\nabla f(x_k)\|.$$

Theorem 2 *Let f be of class C^2 on X convex bounded and suppose that there exists $m > 0$, $M > 0$, $\lambda > 0$, $R > 0$, and $K > 0$ such that:*

1. $m\|y\|^2 \leq (\nabla^2 f(x)y, y) \leq M\|y\|^2 \;\; \forall x,\; y \in X$
2. $f(\frac{x+y}{2}) \leq \frac{1}{2}(f(x) + f(y)) - \lambda\|x - y\|^2 \;\; \forall x,\; \forall y \in X$
3. $K \leq \|\nabla f(x)\| \leq R \;\; \forall x \in X$

Then :
1. $(f(x_k) - f(x^*)) \leq q^k(f(x_0) - f(x^*))$
2.$\|x_k - x^*\|^2 \leq Lq^k$
with $q = \left(1 - \frac{K}{2MR}\right)$ *and* $L = 2\frac{f(x_0)-f(x^*)}{m}$.

Proof.

By (4), we have:

$$2\alpha_k > -\frac{1}{M\|d_k\|^2}(\nabla f(x_k)\;,\; d_k)$$

then

$$\alpha_k > -\frac{1}{2M\|d_k\|^2}(\nabla f(x_k)\;,\; d_k)$$

and

$$\frac{1}{2}\alpha_k(\nabla f(x_k)\;,\; d_k) < \frac{-1}{4M\|d_k\|^2}(\nabla f(x_k)\;,\; d_k)^2$$

By the choice of α_k, we have :

$$f(x_{k+1}) - f(x_k) \leq \frac{1}{2}\alpha_k(\nabla f(x_k)\;,\; d_k)$$

then

$$f(x_{k+1}) - f(x_k) \leq \frac{-1}{4M\|d_k\|^2}(\nabla f(x_k)\;,\; d_k)^2$$

So

$$(f(x_{k+1}) - f(x^*)) - (f(x_k) - f(x^*)) \leq \frac{-1}{4M\|d_k\|^2}(\nabla f(x_k)\;,\; d_k)^2$$

$$\leq -(\nabla f(x_k)\;,\; d_k)(\frac{1}{4M\|d_k\|^2}(\nabla f(x_k)\;,\; d_k))$$

By the lemma 2, we have

$$(\nabla f(x_k), y_k - x_k) \leq -\frac{2\lambda}{R}\|y_k - x_k\|^2\|\nabla f(x_k)\|$$

then

$$(\nabla f(x_k), y_k - x_k)\frac{1}{4M\|d_k\|^2} \leq -\frac{\lambda}{2MR}\|\nabla f(x_k)\|$$

and

$$-(\nabla f(x_k)\;,\; d_k)(\frac{1}{4M\|d_k\|^2}(\nabla f(x_k)\;,\; d_k) \leq (\nabla f(x_k)\;,\; d_k)\frac{\lambda}{2MR}\|\nabla f(x_k)\|$$

from where

$$(f(x_{k+1}) - f(x^*)) - (f(x_k) - f(x^*)) \leq \frac{\lambda}{2RM}\|\nabla f(x_k)\|(\nabla f(x_k), y_k - x_k)$$

By Taylor formula and (3) we have:

$$\frac{m}{2}\|x_k - x^*\|^2 + (\nabla f(x_k), x^* - x_k) \leq f(x^*) - f(x_k)$$

then

$$(\nabla f(x_k), x^* - x_k) \leq f(x^*) - f(x_k)$$

By (2), we also have

$$(\nabla f(x_k)\;,\; y_k - x_k) \leq (\nabla f(x_k)\;,\; x^* - x_k)$$

therefore

$$(f(x_{k+1}) - f(x^*)) - (f(x_k) - f(x^*)) \leq \frac{\lambda}{2MR} \|\nabla f(x_k)\|(f(x^*) - f(x_k))$$

We deduce thus:

$$(f(x_{k+1}) - f(x^*)) - (f(x_k) - f(x^*)) \leq \frac{\lambda}{2RM} K(f(x^*) - f(x_k))$$

from where

$$(f(x_{k+1}) - f(x^*)) \leq (1 - \frac{\lambda K}{2MR})(f(x_k) - f(x^*))$$

Thus, we deduce

$$(f(x_k) - f(x^*)) \leq (1 - \frac{\lambda K}{2MR})^k (f(x_0) - f(x^*))$$

2) By applying Taylor's theorem and (3), we have:

$$\frac{m}{2}\|x_k - x^*\|^2 + (\nabla f(x^*),\ x_k - x^*) \leq f(x_k) - f(x^*)$$

Using (1), we have:

$$(\nabla f(x^*),\ x_k - x^*) \geq 0$$

then

$$\frac{m}{2}|\|x_k - x^*\|^2 \leq f(x_k) - f(x^*)$$

we deduce

$$\|x_k - x^*\|^2 \leq \frac{2}{m}(1 - \frac{\lambda K}{2MR} K)^k (f(x_0) - f(x^*))$$

we finally obtain : $\|x_k - x^*\|^2 \leq Lq^k$

with $q = \left(1 - \frac{\lambda K}{2MR}\right)$ **and** $L = 2\frac{f(x_0) - f(x^*)}{m}$**.**

5. Numerical Experiments

In this section, we present some numerical experiments. We compare the new method **N.M.** with a quadratic programming method **Q.M.**. This comparison is summarized in the tables which give the number of iterations, the associated residual norms for each method and the convergence time.

Example 1:

We consider the following problem: $$\begin{cases} \text{Minimize } 0.5x_1^2 + 0.5x_2^2 - 2x_1x_2 - x_1 - 2x_2 \\ \text{subject to : } \begin{cases} x_1 + x_2 \leq 1 \\ x_1 \geq 0, \text{ and } x_2 \geq 0. \end{cases} \end{cases}$$

where $x_0 = (0.2, 0.8)$

Table 1. Numerical result of example 1

Iterations	N.M.	Q.M.
1	0.035555555555556	0.035555
2	0.008888888878264	3.000000e+00
3	0.002222221994011	1.500000e-03
4	5.555551107565639e-04	5.379119e-06
5	1.388888222550025e-04	1.159073e-12
6	3.472221628893632e-05	
7	8.680564444023085e-06	
8	2.170143031633367e-06	
9	1.356348304996638e-07	
10	3.390935719608362e-08	
11	8.477501597801585e-09	
12	2.119537653982382e-09	
13	5.299249731595960e-10	
14	1.325218040089173e-10	
15	3.314058366479270e-11	
16	8.295162550844132e-12	
CPU	0.141647s	1.584391

$$x^* = (0.333333333463281, 0.666666666355120)$$

Example 2:

We consider the following problem:

$$\begin{cases} \text{Minimize } x_1^2 + x_2^2 + x_3^2 + x_4^2 - 2x_1 - x_2 - 3x_4 \\ \text{subject to : } \begin{cases} 2x_1 + x_2 + x_3 + 4x_4 \leq 7 \\ x_1 + x_2 + 2x_3 + x_4 \leq 6 \\ x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0 \text{ and } \ x_4 \geq 0. \end{cases} \end{cases}$$

where $x_0 = (1; 1; 0; 1)$.

Table 2. Numerical result of example 2

Iterations	N.M.	Q.M.
1	0.392827780807241	0.392827
2	0.140918977309031	3.370370e+00
3	0.023263204565265	3.809960e-01
4	0.005240614092247	1.904980e-04
5	0.002297400115120	6.923784e-07
6	5.220697158989774e-04	3.461880e-10
7	8.492267636464691e-05	1.740830e-13
8	2.220525070921954e-05	
84 9	9.298572946438025e-06	
10	1.703274996615137e-06	
11	3.159987804839340e-07	
12	2.305206106734471e-07	
13	4.409278531393869e-08	
14	1.538823542774516e-08	
15	5.064006381347326e-09	
16	1.336291322194917e-09	
17	6.632606629537780e-10	
CPU	0.268475 s	0.689212s

$$x^* = (0.857142846607700, 0.428571423312525, 0.000000110524998, 1.214285693208593)$$

Example 3:

We consider the following problem:

$$\begin{cases} \text{Minimize } 2x_1^2 + 2x_2^2 + x_1x_2 - 6x_1 - 6x_2 + 15 \\ \text{subject to : } \begin{cases} x_1 + 2x_2 \le 5 \\ 4x_1 \le 7 \\ x_2 \le 2 \\ -2x_1 + 2x_2 = -1 \\ x_1 \ge 0, \ x_2 \ge 0. \end{cases} \end{cases}$$

where $x_0 = (0.5, 0)$.

Table 3. Numerical result of example 3

Iterations	N.M.	Q.M.
1	1.804999935863038	1.804
2	0.488071966640245	2.351e+00
3	0.131974651411347	3.267e-02
4	0.035685941396897	9.112e-04
5	0.009649476122220	5.051e-07
6	0.002609217148293	2.526e-10
7	7.055317223436338e-04	
8	1.907754546998011e-04	
9	5.158548814694946e-05	
10	1.394863229703878e-05	
11	3.771666855020771e-06	
12	1.019836139137253e-06	
13	2.757520022330317e-07	
14	7.455724534172043e-08	
15	2.015711684713288e-08	
16	5.448966664930066e-09	
17	6.632606629537780e-10	
18	1.472544491803056e-09	
19	3.977165539178516e-10	
20	1.072955802546778e-10	
21	2.889284321440711e-11	
22	7.759896562204401e-12	
CPU	0.607344 s	0.689212s

$$x^* = (1.449999999545066, 0.949999999545066)$$

6. Conclusions

1. The method is simple to describe.

2. The method allows to solve a large class of optimization problems with constraints:

$$\begin{cases} Min f(x) \\ Ax \le b \\ x \ge 0 \end{cases}$$

where A the matrix of constraint coefficients, x a variable vector, b the vector of variables and 0 a vector made up only of zeros.

At each iteration x_k the direction $d_k = y_k - x_k$ is determined by solving the following problem:

$$\begin{cases} Min(c_k, y) = (c_k, y_k) \\ Ay \le b \\ y \ge 0 \end{cases}$$

where $c_k = \nabla f(x_k)$ will be the vector of coefficients of the objective function.

3. At each step, only the objective function is changed. The constraints remain the same.

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