

# $\alpha$ -level Fuzzy Soft Sets

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## Abstract

In this paper, based on soft lattices, with the help of fuzzy level (cut) set,  $\alpha$ -level fuzzy soft sets and  $\alpha$ -level fuzzy soft lattices are defined, and the structure and characteristics of our definitions are explained with examples, at the same time, their differences and relations are compared with classic soft set.

**Keywords:** fuzzy soft sets (lattices), lattice structure, fuzzy level sets,  $\alpha$ -level fuzzy soft set (lattices)

Molodtsov (Molodtsov, 1999; Maji, Biswas & Roy, 2003) initiated the theory of soft set which is widely used in the uncertainty theory and very popular in computer sciences, economics, social sciences and more and more researchers study the related field from the different aspects. For example, authors studied the soft set relations and functions in Babitha and Sunil (2010), discussed the soft equality in Qin and Hong (2010), and so on. More information can be seen in Babitha and Sunil (2011), Muhamad (2011), Zhang and Wang (2014), Roy and Maji (2007), Qin, Yang and Liu (2017).

Many researchers studied soft sets combined with fuzzy sets (Zadeh, 1965), such as, the authors introduced the notion of fuzzy soft set based on Maji, Biswas and Roy (2001), and presented some more properties of fuzzy soft union and fuzzy soft intersection in Ahmad and Athar Kharal (2009), and in Majumdar and KSamanta (2010), authors defined generalized fuzzy soft sets and studied some of their properties and application. In Fu (2010), the author gave the concept for soft lattices, and in Shao and Qin (2012), the notion of fuzzy soft lattice was defined and some related properties were derived, which extended the notion of a fuzzy lattice to include the algebraic structures of soft sets.

In this article, we mainly give the definition of the  $\alpha$ -level fuzzy soft sets and  $\alpha$ -level fuzzy soft lattices. The rest of this article is listed as followings. In section 1, some basic notions about fuzzy sets, soft sets, fuzzy soft sets, and soft lattices are reviewed concisely. In section 2, we define the  $\alpha$ -level fuzzy soft sets. In section 3, the  $\alpha$ -level fuzzy soft lattices soft basis is defined, and give some examples to understand the concept of  $\alpha$ -level fuzzy soft sets and lattices, respectively. Meanwhile, in section 4 we point their differences and relations with classic soft set. The conclusion is in section 5.

## 1. Related Basic Theoretical Knowledge

Assume the reader is familiar with the knowledge of fuzzy sets, soft sets, orders and lattices. And we only review the most basic contents about them.

**Definition 1.1.** (molodtsov, 1999) Let  $U$  be an initial universe set and  $E$  be a parameters set. Let  $\mathcal{P}(U)$  be the power set of  $U$ ,  $A \subset E$ . Then a pair  $(F, A)$  is called a **soft set** over  $U$ , where  $F : A \rightarrow \mathcal{P}(U)$  is a mapping.

That is, the soft set is a parameterized family of subsets of the set  $U$ . Every set  $F(e), \forall e \in E$ , from this family may be considered as the set of  **$e$ -elements** of the soft set  $(F, E)$ , or considered as the set of  **$e$ -approximate** elements of the soft set. According to this manner, we can view a soft set  $(F, E)$  as consisting of collection of approximations:  $(F, E) = \{F(e) \mid e \in E\} = \{(F(e), e) \mid e \in E\}$ .

**Definition 1.2.** (Zadeh, 1965) Let  $U$  be a nonempty set which is universe.

(i) A **fuzzy set** is a class of objects  $U$  with a continuum of grades of membership. Such a set is characterized by a membership (characteristic) function which assigns to each object a grade of membership ranging between zero and one.

(ii) A **fuzzy subset**  $\mu$  of  $U$  is defined by the **membership function**  $\mu : U \rightarrow [0, 1]$ . For  $x \in U$ , the **membership value**  $\mu(x)$  essentially specifies the degree to which  $x$  belongs to the fuzzy subset  $\mu$ .

**Definition 1.3.** (Davey & Priestley, 1999) Let  $P$  be a set. An **order (partial order)** on  $P$  is a binary relation  $\leq$  such that, for all  $x, y, z \in P$ , (i)  $x \leq x$ ; (ii)  $x \leq y, y \leq x \Rightarrow x = y$ ; (iii)  $x \leq y, y \leq z \Rightarrow x \leq z$ .

And  $(P, \leq)$  is a **partial order set**.

**Definition 1.4.** (Davey & Priestley, 1999) Let  $P$  be a none-empty ordered set.

- (i) If  $x \vee y$  and  $x \wedge y$  exist for all  $x, y \in P$ , then  $P$  is called a **lattice**;
- (ii) If  $\vee S$  and  $\wedge S$  exist for all  $S \subseteq P$ , then  $P$  is called a **complete lattice**.

**Denotation** (i)  $x \leq y$  means  $x$  is less than  $y$ , then there is a line between  $x$  and  $y$ , and  $x$  is below of  $y$  in their diagram.

(ii)  $x \parallel y$  mean  $x$  and  $y$  is **non-comparability** and no line between them.

(iii) In the following, we will use the operators  $\vee$  and  $\wedge$ , for fuzzy set,  $x \vee y = \max\{x, y\}$  and  $x \wedge y = \min\{x, y\}$ ; for lattice,  $x \vee y = \sup\{x, y\}$ , and  $x \wedge y = \inf\{x, y\}$ .

**Definition 1.5.** (Maji, Biswas, & Roy, 2001) Let  $U$  be an initial universe set and  $E$  be a parameters set. Let  $\mathcal{F}(U)$  be the power set of all fuzzy subsets of  $U$ ,  $A \subset E$ . Then a pair  $(F, A)$  is called a **fuzzy soft set** over  $U$ , where  $F : A \rightarrow \mathcal{F}(U)$  is a mapping.

**Example 1.1.** Suppose a fuzzy soft set  $(F, E)$  describes attractiveness of the shirts with respect to the given parameters, which the authors are going to wear.

$U = \{x_1, x_2, x_3, x_4, x_5\}$  which is the set of all shirts under consideration.  $\mathcal{F}(U)$  is the collection of all fuzzy subsets of  $U$ ,  $E = \{e_1 = 'colorful', e_2 = 'bright', e_3 = 'cheap', e_4 = 'warm'\}$ . Let

$F(e_1) = \{\frac{x_1}{0.5}, \frac{x_2}{0.9}, \frac{x_3}{0}, \frac{x_4}{0}, \frac{x_5}{0}\} = \frac{x_1}{0.5} + \frac{x_2}{0.9} + \frac{x_3}{0} + \frac{x_4}{0} + \frac{x_5}{0}$  which means the membership  $x_1$  belongs to the colorful shirt is 0.5 (simply denoted as  $m(F(e_1))(x_1) = 0.5$ ), the membership  $x_2$  belongs to the colorful shirt is 0.9 (i.e.  $m(F(e_1))(x_2) = 0.9$ ), and so on. Similarly, for fuzzy soft set  $(F, B)$ , we use the symbol  $m(F, B)(x) = \{m(F(e))(x) | x \in U, e \in E\}$  represents the membership of object  $x$  belongs to  $(F, B)$ .

$$F(e_2) = \{\frac{x_1}{1.0}, \frac{x_2}{0.8}, \frac{x_3}{0.7}, \frac{x_4}{0}, \frac{x_5}{0}\} = \frac{x_1}{1.0} + \frac{x_2}{0.8} + \frac{x_3}{0.7} + \frac{x_4}{0} + \frac{x_5}{0},$$

$$F(e_3) = \{\frac{x_1}{0}, \frac{x_2}{0}, \frac{x_3}{0}, \frac{x_4}{0.6}, \frac{x_5}{0}\} = \frac{x_1}{0} + \frac{x_2}{0} + \frac{x_3}{0} + \frac{x_4}{0.6} + \frac{x_5}{0},$$

$$F(e_4) = \{\frac{x_1}{0}, \frac{x_2}{1.0}, \frac{x_3}{0}, \frac{x_4}{0}, \frac{x_5}{0.3}\} = \frac{x_1}{0} + \frac{x_2}{1.0} + \frac{x_3}{0} + \frac{x_4}{0} + \frac{x_5}{0.3}.$$

Then the fuzzy soft set  $(F, E) = \{F(e_i) | e_i \in E, i = 1, 2, 3, 4\}$ .

We can also represent the fuzzy soft set  $(F, E)$  using the following Table 1.

Table 1. The fuzzy soft set  $(F, E)$  for Example 1.1

	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$
$e_1$	0.5	0.9	0	0	0
$e_2$	1.0	0.8	0.7	0	0
$e_3$	0	0	0	0.6	0
$e_4$	0	1.0	0	0	0.3

**Definition 1.6.** (Maji, Biswas, & Roy, 2001) Let  $U$  be an initial universe set and  $E$  be a parameters set. Let  $\mathcal{F}(U)$  be the power set of all fuzzy subsets of  $U$ ,  $A, B \subset E$ ,  $(F, A)$  and  $(G, B)$  be two fuzzy soft sets over  $U$ .

(i) If  $A \subseteq B$ , and  $\forall e \in A$ , having  $F(e) \subseteq G(e)$ , then  $(F, A)$  is a **fuzzy soft subset** of  $(G, B)$ , denoted as  $(F, A) \widetilde{\subset} (G, B)$ .

(ii)  $(F, A)$  and  $(G, B)$  are said **fuzzy soft equal**, if  $(F, A) \widetilde{\subset} (G, B)$ , and  $(G, B) \widetilde{\subset} (F, A)$ . We simply denote by  $(F, A) = (G, B)$ .

(iii) The **complement** of  $(F, A)$  is denoted by  $(F, A)^c$  and is defined by  $(F, A)^c = (F^c, \neg A)$ , where  $F^c : \neg A \rightarrow \mathcal{F}(U)$ , and  $F^c(e) = F(\neg e), \forall e \in \neg A$ .

**In here, we define**  $F^c(e) = F(\neg e) = 1 - F(e), \forall e \in A$ .

For example, in the above example,

$$\begin{aligned}
 F(e_1) &= \{\frac{x_1}{0.5}, \frac{x_2}{0.9}, \frac{x_3}{0}, \frac{x_4}{0}, \frac{x_5}{0}\} \\
 &= \frac{x_1}{0.5} + \frac{x_2}{0.9} + \frac{x_3}{0} + \frac{x_4}{0} + \frac{x_5}{0}, e_1 \in E, \\
 F^c(e_1) &= F(\neg e_1) = \{\frac{x_1}{0.5}, \frac{x_2}{0.1}, \frac{x_3}{1}, \frac{x_4}{1}, \frac{x_5}{1}\} \\
 &= \frac{x_1}{0.5} + \frac{x_2}{0.1} + \frac{x_3}{1} + \frac{x_4}{1} + \frac{x_5}{1}, e_1 \in \neg E (e_1 \notin E).
 \end{aligned}$$

(iv) The **fuzzy soft union** of  $(F, A)$  and  $(G, B)$  is the fuzzy soft set  $(H, C)$ , where  $C = A \cup B$ , and  $\forall e \in C$ , denoted as  $(F, A) \cup (G, B) = (H, C) = (H, A \cup B)$ , where

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B \\ G(e), & \text{if } e \in B - A \\ F(e) \cup G(e), & \text{if } e \in A \cap B \end{cases}$$

(v) The **fuzzy soft intersection** of  $(F, A)$  and  $(G, B)$  is the fuzzy soft set  $(H, C)$  is denoted as  $(F, A) \cap (G, B)$  and is defined as  $(F, A) \cap (G, B) = (H, C)$ , where  $C = A \cap B$ , and  $\forall e \in C, H(e) = F(e) \cap G(e)$ .

**Definition 1.7.** (Fu, 2010) Let triplet  $M = (F, E, L)$ , where  $L$  is a complete lattice,  $E$  is an attributes set, and  $F : E \rightarrow L$  is a mapping, that is,  $\forall e \in E, F(e) \subseteq L$  and  $F(e)$  is a sublattice of  $L$ , then the pair  $(e, F(e))$  is a **soft lattice** over  $L$ , simply speaking, the triple  $M$  is called **the soft lattice**.

**Remark 1.1.** (i) The operations between soft lattices are similar to those between soft sets and fuzzy soft sets, and they are not repeated. We will not describe them here.

(ii) In the above definition, the condition that  $L$  is a complete lattice is too strong. In literature Shao, & Qin, (2012), as long as the soft lattice defined by the author is a lattice, but if there is no completeness, there is no guarantee that the soft operation between the soft lattices can meet the closeness. That is to say, the completeness of  $L$  is the condition to ensure the closeness of soft lattice operation.

**Remark 1.2.** The soft union of two soft lattices does not have to be the soft lattice. For example:

Let  $L = \{0, a, b, c, d, e, f, 1\}$  be a lattice (its figure as Figure 1),  $E = \{e_1, e_2, e_3, e_4\}$  be attributes set.  $B_1 = \{e_1, e_2, e_3\}, B_2 = \{e_1, e_2, e_4\} \subseteq E$ , suppose that  $(F_1, B_1), (F_2, B_2)$  are soft lattices over  $L$ , in which

$$(F_1, B_1) = \{(\{0, a, d\}, e_1), (\{0, c, e\}, e_2), (\{0, a, e, 1\}, e_3)\},$$

$$(F_2, B_2) = \{(\{0, c, f\}, e_1), (\{0, c, f\}, e_2), (\{0, b, e\}, e_4)\},$$

$$\text{then } (F_1, B_1) \cup (F_2, B_2) = (H, C) = \{(\{0, a, c, d, f\}, e_1), (\{0, c, e, f\}, e_2), (\{0, a, e, 1\}, e_3), (\{0, b, e\}, e_4)\},$$

And  $H(e_1) = \{0, a, c, d, f\} \subseteq L, H(e_2) = \{0, c, e, f\} \subseteq L$ , they are not lattices because  $d \wedge f = b \notin \{0, a, c, d, f\}, e \vee f = 1 \notin \{0, c, e, f\}$ .

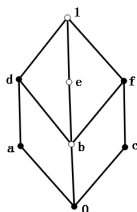


Figure 1

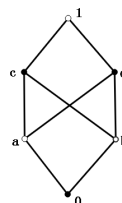


Figure 2

**Remark 1.3.** The soft intersection of two soft lattices does not have to be the soft lattice. For example:

Let  $L = \{0, a, b, c, d, 1\}$  be a lattice (its figure as Figure 2),  $E = \{e_1, e_2, e_3\}$  be an attributes set.  $B_1 = \{e_1, e_2\}, B_2 = \{e_1, e_3\} \subseteq E$ . Suppose that  $(F_1, B_1), (F_2, B_2)$  are soft lattices over  $L$ , in which  $(F_1, B_1) = \{(\{0, a, c, d\}, e_1), (\{0, a, d\}, e_2)\}, (F_2, B_2) = \{(\{0, b, c, d\}, e_1), (\{0, a, b\}, e_3)\}$ , then

$$(F_1, B_1) \cap (F_2, B_2) = (H, C) = \{(\{0, c, d\}, e_1)\}, H(e_1) = \{0, c, d\} \text{ is subset of } L \text{ but not lattice because } c \wedge d = \{0, a, d\} \notin \{0, c, d\}.$$

**Proposition 1.1.** Let triplet  $M = (F, E, L)$  be a soft lattice, where  $L$  is a lattice,  $E$  is an attributes set, and  $B_1, B_2 \subseteq E$ .  $(F, B_1)$  and  $(F, B_2)$  are two soft lattices over  $L$ .  $(F, B_1) \cap (F, B_2) = (H, C)$  is a soft lattice **if and only if**  $\forall e \in B_1 \cap B_2$ , having  $F_1(e) = F_2(e)$ .

*Proof.*

$$\left. \begin{aligned} H(e) = \begin{cases} F_1(e), & \text{if } e \in B_1 - B_2 \\ F_2(e), & \text{if } e \in B_2 - B_1 \\ F_1(e) = F_2(e), & \text{if } e \in B_1 \cap B_2 \end{cases} \\ (H, C) \text{ is a soft lattice} \end{aligned} \right\} \iff (F_1, B_1) \text{ and } (F_2, B_2) \text{ are two lattices.}$$

□

### 2. $\alpha$ -level Fuzzy Soft Sets

**Definition 2.1.** (Maji, Biswas, & Roy, 2001; Ahmad & Kharal, 2009) Let  $U$  be an initial universal set and  $E$  be a set of parameters.  $\mathcal{F}(U)$  be the class of all fuzzy subsets of  $U$ . Let  $A \subseteq E$ , the pair  $(F, A)$  is called a **fuzzy soft set** over  $U$ , where  $F : A \rightarrow \mathcal{F}(U)$ . That is,  $\forall e \in A, F(e)$  is a fuzzy subset of  $U$ .

**Definition 2.2.** (Shao & Qin, 2012) Let  $(F, A)$  be a fuzzy soft set over  $L$ ,  $(F, A)$  is called a **fuzzy soft lattice** if  $F(e)$  is a fuzzy sublattice of  $L$  for each  $e \in A$ .

**Remark 2.1.** In reference Shao & Qin (2012), the authors defined the fuzzy soft lattice as the above definition and discussed their properties. The fuzzy soft lattice given in this definition was a very abstract definition. The author did not give an example to describe their conclusion. In this paper, we will define fuzzy soft lattice from another point of view.

**Denotation:** In the following,

(i)  $(G, M, F)$  is a soft formal context, in which  $G$  is an objects set,  $M$  is an attributes set,  $B \subseteq M, F : B \rightarrow L$ , the pair  $(F, B)$  is a soft set over  $(G, M, L)$ , simply speaking,  $(F, B)$  is a soft set.

(ii)  $(L, M, F)$  is a fuzzy soft formal context, in which  $L$  is universe,  $\mathcal{F}(L)$  is the class of all fuzzy subsets of  $L$ ,  $M$  is an attributes set,  $B \subseteq M, F : B \rightarrow L$ , the pair  $(F, B)$  is a fuzzy soft set over  $(L, M, F)$ , simply speaking,  $(F, B)$  is a fuzzy soft set.

**Definition 2.3.** Let  $(L, M, F)$  be a fuzzy soft formal context in which  $L$  is the universe,  $\mathcal{F}(L)$  be the class of all fuzzy subsets of  $L$ , and  $(F, B)$  be a fuzzy soft set over  $L$ , that is,  $\forall e \in B, F(e) \in \mathcal{F}(L)$ . For all  $\alpha \in [0, 1]$ , define

$$(F, B)_\alpha = \{((F(e))_\alpha, e) | e \in B\},$$

where  $(F(e))_\alpha = \{x \in L | m(F(e))(x) \geq \alpha\}$  is  $\alpha$ -level set, in other words,  $(F(e))_\alpha$  is some objects set whose membership is bigger than a given  $\alpha \in [0, 1]$ . Then  $(F, B)_\alpha$  is called as the  **$\alpha$ -level fuzzy soft set**, also called the  **$\alpha$ -cut fuzzy soft set**.

**Example 2.1.** Consider Example 1.1, for different  $\alpha$ ,  $(F, E)_\alpha$  as Table 2.

Table 2.  $(F, E)_\alpha$  for different  $\alpha$  of Example 1.1

$\alpha$	$(F, E)_\alpha$
0.9	$\{(\{x_2\}, e_1), (\{x_1\}, e_2), (\emptyset, e_3), (\{x_2\}, e_4)\}$
0.8	$\{(\{x_2\}, e_1), (\{x_1, x_2\}, e_2), (\emptyset, e_3), (\{x_2\}, e_4)\}$
0.7	$\{(\{x_2\}, e_1), (\{x_1, x_2, x_3\}, e_2), (\emptyset, e_3), (\{x_2\}, e_4)\}$
0.6	$\{(\{x_2\}, e_1), (\{x_1, x_2, x_3\}, e_2), (\{x_4\}, e_3), (\{x_2\}, e_4)\}$
0.5	$\{(\{x_1, x_2\}, e_1), (\{x_1, x_2, x_3\}, e_2), (\{x_4\}, e_3), (\{x_2\}, e_4)\}$
0.3	$\{(\{x_1, x_2\}, e_1), (\{x_1, x_2, x_3\}, e_2), (\{x_4\}, e_3), (\{x_2, x_5\}, e_4)\}$

### 3. $\alpha$ -level Fuzzy Soft Lattices

**Definition 3.1.** Let  $(L, M, F)$  be a fuzzy soft formal context in which the universe  $L$  is a lattice.  $(F, B)$  is a fuzzy soft set over  $L$ , define the preference order  $\leq$  on  $L$  as:  $\forall h, k \in L, h \leq k$  **if and only if**  $m(F(e))_h \leq m(F(e))_k, \forall e \in B$ , clearly,  $\leq$  is a partial order, and stipulate: if  $h \leq k$ , then  $h$  is in the below of  $k$  in the lattice structure.

(i) If  $\forall e \in B, \exists \alpha \in [0, 1]$ , such that  $F(e)$  is a fuzzy subset of  $L$ ,  $(F(e))_\alpha$  is a sublattice of  $L$ , then  $(F, B)$  is a  **$\alpha$ -level fuzzy soft lattice** over  $L$ .

(ii) If  $\forall e \in B, \forall \alpha \in [0, 1]$ , such that  $F(e)$  is a fuzzy subset of  $L$ ,  $(F(e))_\alpha$  is a sublattice of  $L$ , then  $(F, B)$  is simply called a **fuzzy soft lattice** over  $L$ .

**Example 3.1.** Let  $L = \{h, i, j, k\}$  be the clothes set, defined a preference relation (order) between these clothes which forms the lattice (its figure as Figure 3).  $E = \{e_1, e_2, e_3, e_4, e_5\}$  which is the attributes set, and  $e_1$ : 'very costly',  $e_2$ : 'costly',  $e_3$ : 'corolful',  $e_4$ : 'cheap',  $e_5$ : 'fashion'.

Suppose that  $B_1 = \{e_1, e_2, e_4\} \subseteq E$  which means the cost of the given clothes,  $B_2 = \{e_3, e_4, e_5\} \subseteq E$  which represents the attractiveness of the given clothes,  $(F_1, B_1), (F_2, B_2)$  are fuzzy soft sets, in which,  $F_1(e_1) = \{\frac{h}{0.3}, \frac{i}{0.4}, \frac{j}{0.3}, \frac{k}{0.7}\} = \frac{h}{0.3} + \frac{i}{0.4} + \frac{j}{0.3} + \frac{k}{0.7}, \dots$ , we simply represent them using the Table 3.

Let  $\alpha$  take the different values, the  $\alpha$ -level sets of fuzzy soft sets  $(F_1, B_1)$  and  $(F_2, B_2)$  are represented in the Table 4.

Obviously, for all  $\alpha$ ,  $(F_1(e_i))_\alpha$  and  $(F_2(e_i))_\alpha (i = 1, 2, 3, 4, 5)$  are sublattices of  $L$ , then  $(F_1, B_1)$  and  $(F_2, B_2)$  are fuzzy soft lattices over  $L$  (their lattice structure as Figure 3).

Table 3. The fuzzy soft sets  $(F_1, B_1), (F_2, B_2)$  for Example 3.1

	$B_1$			$B_2$		
	$e_1$	$e_2$	$e_4$	$e_3$	$e_4$	$e_5$
$h$	0.3	0.5	0.3	0.3	0.3	0.4
$i$	0.4	0.5	0.7	0.5	0.3	0.4
$j$	0.3	0.7	0.3	0.3	0.4	0.6
$k$	0.7	0.9	0.9	0.7	0.6	0.9

Table 4. The fuzzy soft sets  $(F_1, B_1), (F_2, B_2)$  for different  $\alpha$  of Example 3.1

	$(F_1, B_1)_\alpha$			$(F_2, B_2)_\alpha$		
	$(F_1(e_1))_\alpha$	$(F_1(e_2))_\alpha$	$(F_1(e_4))_\alpha$	$(F_2(e_3))_\alpha$	$(F_2(e_4))_\alpha$	$(F_2(e_5))_\alpha$
$\alpha = 0.9$	$\emptyset$	$\{k\}$	$\{k\}$	$\emptyset$	$\emptyset$	$\{k\}$
$\alpha = 0.7$	$\{k\}$	$\{j, k\}$	$\{i, k\}$	$\{k\}$	$\emptyset$	$\{k\}$
$\alpha = 0.6$	$\{k\}$	$\{j, k\}$	$\{i, k\}$	$\{k\}$	$\{k\}$	$\{j, k\}$
$\alpha = 0.5$	$\{k\}$	$L$	$\{i, k\}$	$\{i, k\}$	$\{k\}$	$\{j, k\}$
$\alpha = 0.4$	$\{i, k\}$	$L$	$\{i, k\}$	$\{i, k\}$	$\{j, k\}$	$L$
$\alpha = 0.3$	$L$	$L$	$L$	$L$	$L$	$L$

**Example 3.2.** Let  $L = \{h_1, h_2, h_3, h_4, h_5, h_6\}$  be an objects set which includes all the considerate houses, and there is a partial order between them (its figure as Figure 4).

$E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$  be the set of parameters for houses, in which  $e_1$ : 'expensive',  $e_2$ : 'beautiful',  $e_3$ : 'wooden',  $e_4$ : 'in the green surroundings',  $e_5$ : 'modern',  $e_6$ : 'in good repair'.

Suppose that  $B_1 = \{e_1, e_2, e_3, e_4\} \subseteq E, B_2 = \{e_2, e_4, e_5, e_6\} \subseteq E, (F_1, B_1), (F_2, B_2)$  are fuzzy soft sets as the Table 5.

Table 5.  $(F_1, B_1), (F_2, B_2)$  for Example 3.2

	$B_1$				$B_2$			
	$e_1$	$e_2$	$e_3$	$e_4$	$e_2$	$e_4$	$e_5$	$e_6$
$h_1$	0.3	0.4	0.3	0.5	0.3	0.3	0.4	0.5
$h_2$	0.3	0.5	0.4	0.5	0.4	0.4	0.4	0.7
$h_3$	0.4	0.6	0.5	0.6	0.5	0.5	0.5	0.7
$h_4$	0.5	0.6	0.6	0.7	0.5	0.6	0.6	0.7
$h_5$	0.6	0.7	0.7	0.8	0.6	0.6	0.7	0.8
$h_6$	0.8	0.8	0.8	0.9	0.7	0.7	0.8	0.9

Assume  $\alpha$  takes the different values, the  $\alpha$ -level sets of fuzzy soft sets  $(F_1, B_1)$  and  $(F_2, B_2)$  are represented in the Table 6 and Table 7, respectively.

Table 6.  $(F_1, B_1)_\alpha$  for different  $\alpha$

$\alpha$	$(F_1, B_1)_\alpha$			
	$(F_1(e_1))_\alpha$	$(F_1(e_2))_\alpha$	$(F_1(e_3))_\alpha$	$(F_1(e_4))_\alpha$
0.9	$\emptyset$	$\emptyset$	$\emptyset$	$\{h_6\}$
0.8	$\{h_6\}$	$\{h_6\}$	$\{h_6\}$	$\{h_5, h_6\}$
0.7	$\{h_6\}$	$\{h_5, h_6\}$	$\{h_5, h_6\}$	$\{h_4, h_5, h_6\}$
0.6	$\{h_5, h_6\}$	$\{h_3, h_4, h_5, h_6\}$	$\{h_4, h_5, h_6\}$	$\{h_3, h_4, h_5, h_6\}$
0.5	$\{h_4, h_5, h_6\}$	$\{h_2, h_3, h_4, h_5, h_6\}$	$\{h_3, h_4, h_5, h_6\}$	$L$
0.4	$\{h_3, h_4, h_5, h_6\}$	$L$	$\{h_2, h_3, h_4, h_5, h_6\}$	$L$
0.3	$L$	$L$	$L$	$L$

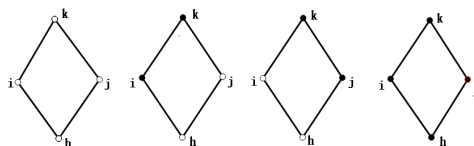


Figure 3

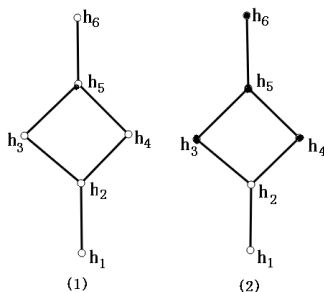


Figure 4

Table 7.  $(F_2, B_2)_\alpha$  for different  $\alpha$  of Example 3.2

$\alpha$	$(F_2, B_2)_\alpha$			
	$(F_2(e_2))_\alpha$	$(F_2(e_4))_\alpha$	$(F_2(e_5))_\alpha$	$(F_2(e_6))_\alpha$
0.9	$\emptyset$	$\emptyset$	$\emptyset$	$\{h_6\}$
0.8	$\emptyset$	$\emptyset$	$\{h_6\}$	$\{h_5, h_6\}$
0.7	$\{h_6\}$	$\{h_6\}$	$\{h_5, h_6\}$	$\{h_2, h_3, h_4, h_5, h_6\}$
0.6	$\{h_5, h_6\}$	$\{h_4, h_5, h_6\}$	$\{h_4, h_5, h_6\}$	$\{h_2, h_3, h_4, h_5, h_6\}$
0.5	$\{h_3, h_4, h_5, h_6\}$	$\{h_3, h_4, h_5, h_6\}$	$\{h_3, h_4, h_5, h_6\}$	$L$
0.4	$\{h_2, h_3, h_4, h_5, h_6\}$	$\{h_2, h_3, h_4, h_5, h_6\}$	$L$	$L$
0.3	$L$	$L$	$L$	$L$

From the above table, we can find that when  $\alpha = 0.9, 0.8, 0.7, 0.3$ ,  $(F_1(e_i))_\alpha$  and  $(F_2(e_i))_\alpha (i = 1, 2, 3, 4, 5, 6)$  are sublattice of  $L$  (their lattice structure as Figure 4(1)). When  $\alpha = 0.4, 0.5, 0.6$ ,  $(F_1(e_i))_\alpha$  and  $(F_2(e_i))_\alpha (i = 1, 2, 3, 4, 5, 6)$  are not sublattices of  $L$  because  $h_3 \wedge h_4 = h_2 \notin \{h_3, h_4, h_5, h_6\} \subseteq L$  (see Figure 4(2)).

#### 4. The Relationship Between the $\alpha$ -level Fuzzy Soft Set and Classical Soft Set

**Result 4.1.** the  $\alpha$ -level fuzzy soft set is the classical soft set, and the membership of  $x$  belongs to  $\alpha$ -level fuzzy soft set  $(F, B)$  is as the following:

$$m(F, B)(x) = \begin{cases} 1, & \text{if } F(e)(x) \geq \alpha \\ 0, & \text{if } F(e)(x) < \alpha \end{cases}$$

**Example 4.1.** Consider the Example 2.1, from the Table 2, we can find that for every  $\alpha$ ,  $(F, E)_\alpha$  is the classical soft set. e.g.  $\alpha = 0.7$ ,  $(F, E)_{0.7} = \{(\{x_2\}, e_1), (\{x_1, x_2, x_3\}, e_2), (\emptyset, e_3), (\{x_2\}, e_4)\}$  is the classical soft set.

**Result 4.2.**  $\alpha$ -level fuzzy soft lattices are the special case of  $\alpha$ -level fuzzy soft sets which embody two aspects.

Firstly, the universe  $L$  includes the partial (preference) order and forms the lattice structure.

Secondly,  $(F, B)_\alpha = \{((F(e))_\alpha, e) | e \in B\}$  in which  $\forall e \in B, (F(e))_\alpha$  is sublattice of  $L$ .

The Example 3.1 and 3.2 can help us to understand this result.

**Result 4.3.** Although the universe  $L$  is a lattice,  $\alpha$ -level fuzzy soft set over  $L$  does not have to be the  $\alpha$ -level fuzzy soft lattice. See the Example 3.2, when  $\alpha = 0.4, 0.5, 0.6$ ,  $(F_1(e_i))_\alpha$  and  $(F_2(e_i))_\alpha (i = 1, 2, 3, 4, 5, 6)$  are not sublattices of  $L$  because  $h_3 \wedge h_4 = \{h_1, h_2\} \notin \{h_3, h_4, h_5, h_6\} \subseteq L$  (see Figure 4(2) and the red parts in the Table 7).

## 5. Conclusion

From the above discussion, this paper mainly define the  $\alpha$ -level fuzzy soft sets and  $\alpha$ -level fuzzy soft lattices, and at the same time, it gives some examples to understand the concept of  $\alpha$ -level fuzzy soft sets (lattices). We also point the relationship between the  $\alpha$ -level fuzzy soft sets (lattices) and the classical soft set (lattice). And we will further study the properties of  $\alpha$ -level fuzzy soft sets (lattices) in the other paper.

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