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Perception of Polynomial for Weighted Directed Graph

Hend El- Morsy

Correspondence: Hend El-Morsy, College of the Preparatory Year, Department of Mathematics, Umm Al-Qura University, Mecca, Kingdom of Saudi Arabia. E-mail: heebrahim@uqu.edu.sa, hendelmorsy@yahoo.com

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Abstract

In this paper we will apply a polynomial for directed weighted graph. We will introduce notion of deletion and contraction in directed weighted graph. Some examples and propositions will be illustrated.

Keywords: directed graph, weighted graph, contraction, deletion, polynomials

1. Introduction

In mathematics, a polynomial is an expression consisting of variables (also called indeterminates) and coefficients, that involves only the operations of addition, subtraction, multiplication, and non-negative integer exponents of variables. An example of a polynomial of a single indeterminate, *x*, is $x^2 - 4x + 7$. An example in three variables is $x^3 + 2xyz^2 - yz + 1$.

Polynomials appear in many areas of mathematics and science. For example, they are used to form polynomial equations, which encode a wide range of problems.

Let $\vec{G}_w$(V, A) be a directed weighted graph, a directed weighted polynomial of $\vec{G}$ maping C from V to the set of X_n satisfying :

i. $\forall (x, y) \in A \ , \overrightarrow{xy} \neq \overrightarrow{yx}.$

ii. A weighted graph consists of finite graph $\vec{G}$ with vertex set { $v_1, v_2, \ldots\ldots v_n$ }, edge set E together with weight function W: $V \rightarrow Z^+$ then $W(V_i)$ the weight of V_i.

iii. If $U \subset V$ we define weight of U, W(U) to be $\sum_{v \in V} W(v)$

2. Polynomial for Weighted Directed Graph

We need to introduce notion of deletion and contraction in directed weighted graph $\vec{G}_w$ as follows:

* If e is edge of ($\vec{G}$, w), then let ($\vec{G}_e^{\backslash}$, w) denote the graph obtained from $\vec{G}$ by deleting e and leaving weight unchanged, see Fig.(1)

*If e is an edge of simple directed weighted graph ($\vec{G}$, w), then ($\vec{G}^c_e$, w) is graph formed from ($\vec{G}^{\backslash\backslash}_e$, w) by replacing every parallel class by single edge. Fig. (1)

*If e is not loop of (G, w), then let ($\vec{G}^{\backslash\backslash}_e$, w) be a graph obtained by contracting e that is deleting identifying its end points V, $V^{\backslash}$ into a single vertex $V^{\backslash\backslash}$ and setting $W(V^{\backslash\backslash}) = W(V) + W(V^{\backslash})$ if the edges in the same direction Fig.(2-a), and $W(V^{\backslash\backslash}) = W(V) - W(V^{\backslash})$ if the edges in opposite directions. Fig.(2,b).

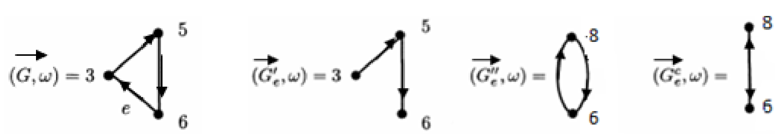


Figure 1.

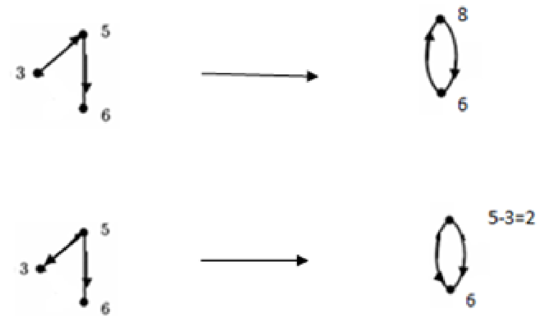


Figure 2-a.

We associate with any directed weighted graph $(\vec{G}, W)$, a multivariate polynomial $W_G(x, y)$which define as follows:

Let $y_1, x_1 x_2, \ldots\ldots x_n$ be commuting indeterminates.

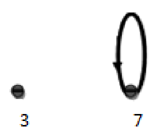
Now let $W_G(x,y)$ be defined recursively by the following rules:

i. If $\vec{G}_W$ consists of m isolated vertices with weights $w_1, w_2 \ldots\ldots w_m$ then $W_G(x, y) = X_{w1} \ldots\ldots X_{Wm}$.

ii. If $\vec{G}_w$ has loop, then $W_G(x, y) = y\, W_{G\backslash e}(x, y)$.

iii. The polynomial take the form: $X_n X_m + X_z + X_z\, y$, $(z=n+m)$.

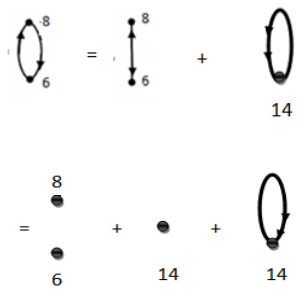
Example2.1:



If $(\vec{G}, W)$=

Then $W_G(x, y) = X_3\, X_7\, y$ ($\exists$ a loop)

b. If $(\vec{G}, W)$=



$= X_8\, X_6 + X_{14} + X_{14}Y$.

Theorem 2.2:

Let $\vec{G}$ (V, W) be weighted directed graph, and let $\vec{G}_1$, $\vec{G}_2$ be two non-empty subsets of G, such that $\vec{G} = \vec{G}_1 \cup \vec{G}_2$, and if

$X_{n1}, X_{n2, \ldots\ldots} X_{ni} \in \vec{G}_1$, $X_{m1}, X_{m2,} X_{mj} \in \vec{G}_2$ then:

$X_{wn} X_{wm} = \sum_{i=w}^{n} \sum_{j=w}^{m} x_i x_{j \epsilon G}$

Then we have :

$P(\vec{G}, W) = P(\vec{G}_{1,} W_1) \odot P(\vec{G}_2, W_2)$. (Where P is the related polynomial).

Proof:

Let $\vec{G}$ (V, A) be a weighted directed graph, $\vec{G}_1$, $\vec{G}_2$ are subsets of G such that $\vec{G} = \vec{G}_1 \cup \vec{G}_2$

If $\vec{G}_1 = X_{n1}\, X_{n2} \ldots . X_{ni} + X_{n1\backslash} X_{n2\backslash} + X_{n\backslash\backslash} + X_{n\backslash\backslash} y_i$

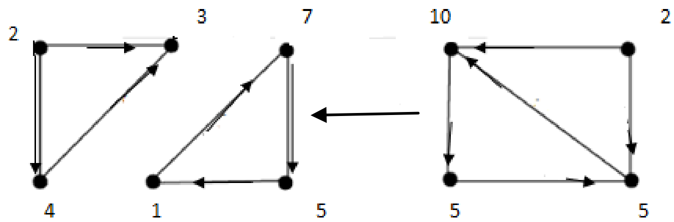
$\vec{G}_2 = X_{m1}\, X_{m2} \ldots . X_{mj} + X_{m1\backslash} X_{m2\backslash} + X_{m\backslash\backslash} + X_{m\backslash\backslash} y_j$

Then $\vec{G} = Xn_{1+}m_1Xn_{2+}m_2 \quad \ldots Xn_{i+}m_j + Xn_{1\backslash+}m_{1\backslash} \ Xn_{2\backslash+}m_{2\backslash} \ + Xn_{\backslash\backslash+}m_{\backslash\backslash} + Xn_{\backslash\backslash+}m_{\backslash\backslash}y_{i+j}$

It follows that this polynomial can be found in its factorial form by taken the factorial forms of X_n and X_m and adding there as if the factorials were weights.

This process that we denoted symbolically by $\vec{G}_1 \Theta \ \vec{G}_2$.

Example 2.3:

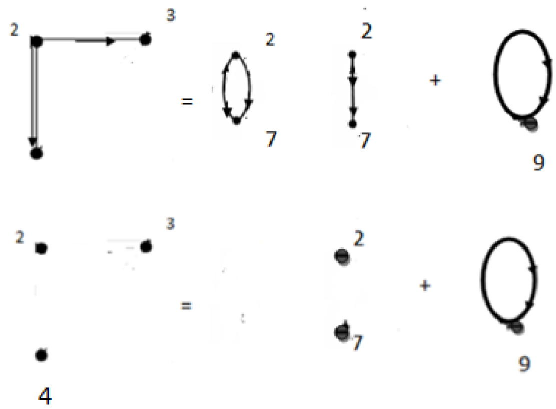


$\vec{G}$ $\vec{G}_2$ $\vec{G}_1$

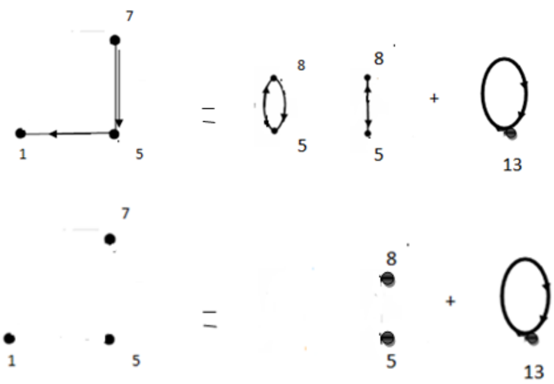
Weighted directed graph $\vec{G}$, $\vec{G}_1$, $\vec{G}_2$

$\ni \vec{G} = \vec{G}_1 \cup \vec{G}_2$

First we find polynomial of $\vec{G}_1$ and $\vec{G}_2$



P $(\vec{G}_1) = X_4X_2X_3 + X_2X_7 + X_9 + X_9Y$.

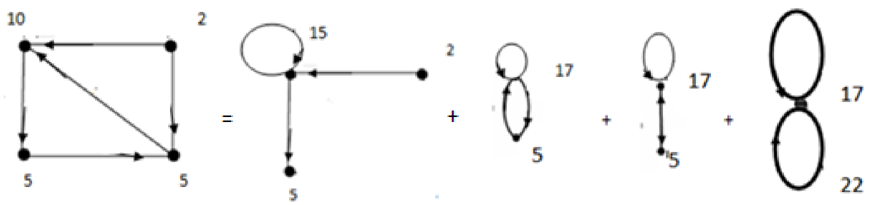


$P(\vec{G}_2) = X_7X_5X_1 + X_5X_8 + X_{13} + X_{13}Y$.

Then $P(\vec{G}) = P(\vec{G}_1) + P(\vec{G}_2)$

$P(\vec{G}) = X_4X_2X_3 + X_2X_7 + X_9 + X_9Y + X_7X_5X_1 + X_5X_8 + X_{13} + X_{13}Y$

$= X_5X_7X_{10} + X_7X_{15} + X_{22} + X_{22}Y_2$.



$P(\vec{G}) = X_2X_{15}X_5 + X_{17}X_5 + X_{22} + X_{22}Y_2$.

The weights of X in all terms are equal.

Proposition 2.4:

For any weighted directed graph $\vec{G}$ (V, W) with n vertices, we have:

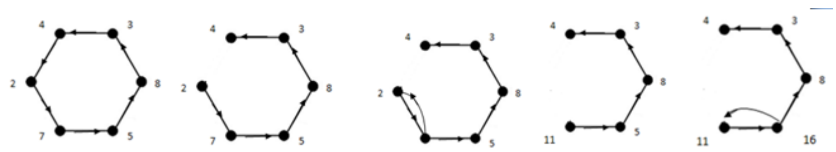
The coefficient of $X_nX_m.....+ X_nY_{mm}$ are 1.

ii. Polynomial P ($\vec{G}$, W) has no constant term.

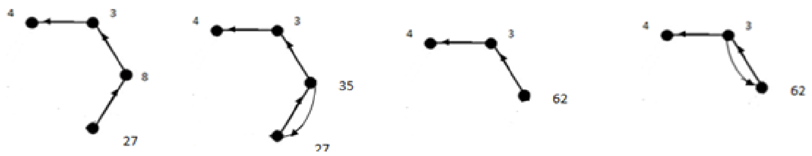
iii. Loop write only on the last term with the final X_n remaining.

Example 2.5:

For weighted directed cycle graph C_6 We can compute polynomial as follows:



1. $X_2X_9 + X_{11} + X_{11}Y$ 2. $X_{11}X_{16} + X_{27} + X_{27}Y$



3. $X_{27}X_{35} + X_{62} + X_{62}Y$ 4. $X_{62}X_3 + X_{65} + X_{65}Y$

5. $X_4X_{65} + X_{69} + X_{69}Y$

By adding the five equations we obtain the polynomial of C_6 as follows:

$X_{106}X_{128} + X_{234} + X_{234}Y$.

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