Journal of Mathematics Research; Vol. 11, No. 6; December 2019
ISSN 1916-9795 E-ISSN 1916-9809
Published by Canadian Center of Science and Education

Some Series and Mathematic Constants Arising in Radioactive Decay

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Received: October 2, 2019 Accepted: October 23, 2019 Online Published: October 28, 2019

doi:10.5539/jmr.v11n6p14 URL: https://doi.org/10.5539/jmr.v11n6p14

Abstract

In this paper we show the construction of 32 infinite series based on the law of decay of radioactive isotopes, which indicates that a radioactive parent isotope is reduced by 1/2 and 1/*e* of its initial value during each half-life and mean life, respectively. We found that the ratios among the values of the radioactive parent isotope and the radiogenic daughter isotope for each half-life's and mean life's decay can be used to construct 16 half-life related (or 2-related) and 16 mean life related (or *e*-related) infinite series. There are 8 divergent series, 4 previously known convergent series and 2 series converging to the Erdős-Borwein constant. The remaining 18 series are found to converge to 18 mathematical constants and the divergent and alternating mean life related series leads to another 2 mathematical constants. A few interesting mathematical relations exist among these convergent series and 5 sequences are also attained from the convergent half-life related series.

Keywords: infinite series, mathematical constant, radioactive decay, radioactive isotope, sequence

1. Introduction

Mathematical constants were found or arise in a variety of ways. Many mathematical constants are associated with number theory, analytic inequalities, enumerating discrete structures, functional iteration, complex analysis and geometry, and so on (Finch, 2003; Choi, 2012). $\sqrt{2}=1.414\cdots$; which is sometime called the Pythagoras' constant, is associated with geometry when the length of the diagonal of a unit square was determined (Finch, 2003). Quite a few mathematical constants are the sums of convergent infinite series or have series representations (Alzer, Karayannakis, & Srivastava, 2006). The Apéry's constant, for instance, is the sum of the convergent infinite series: $1+1/2^3+1/3^3+1/4^3+1/5^3+\cdots=1.202056\cdots$ (M. Chen, & S. Chen, 2016; R. Chen, 2011). A divergent infinite series may also lead to mathematical constants if its partial sum is bounded. The Marvin Ray Burns' (MRB) constant is the upper bounded value of the partial sum of the divergent and alternating infinite series: $-1^{1/1}+2^{1/2}-3^{1/3}+4^{1/4}-5^{1/5}+6^{1/6}-\cdots=0.187859\cdots$ (M. Chen, & S. Chen, 2016). Thus, construction of new infinite series has the possibility of leading to new mathematical constants.

2. Half-Life Related (or 2-Related) Infinite Series

Radioactive isotopes (e. g., ^{14}C) are used to determine the age of some geological materials, such as travertines or tufas depositing from hot springs or ordinary-temperature springs. Radioactive parent isotopes spontaneously radiate rays and decays continually, and radiogenic daughter isotopes grow simultaneously (Faure, 1979; Lutgens & Tarbuck, 2014; Turcotte & Schubert, 2002). During a single-component decay, the value of atom of the radioactive parent isotope decays in a negative exponential function, and the value of atom of the radiogenic daughter isotope grows, respectively, as follows (Faure, 1979; Turcotte & Schubert, 2002) :

$$P(t)=P(0)e^{-\lambda t}, \tag{1}$$

and

$$D(t)=P(0)(1-e^{-\lambda t}), \tag{2}$$

where $P(0)$ is the initial value of atom of the radioactive parent isotope, $P(t)$ and $D(t)$ are the values of atoms of the radioactive parent isotope and the radiogenic daughter isotope at time t, respectively, and λ is the decay constant of the radioactive parent isotope. The term, half-life ($T_{1/2}$), which is defined to be the time required for one-half of the atoms of the parent isotope present at the initial moment to decay, is used to describe the velocity of decay, and $T_{1/2}=\ln 2/\lambda$.

Therefore, Equations (1) and (2) can be rewritten using half-life as follows:

$$P(i) = P(0)e^{-\lambda T_{1/2}(i)}, \tag{3}$$

and

$$D(i) = P(0)(1 - e^{-\lambda T_{1/2}(i)}), \tag{4}$$

where i is the number of half-life of decay of the radioactive parent isotope, $P(i)$ and $D(i)$ are the values of atoms of the radioactive parent isotope and the radiogenic daughter isotope after ith half-life's decay, respectively. Both the decay of $P(i)$ and the increase of $D(i)$ with i when $P(0)$ is assumed to be 120 are shown in Figure 1. It is easy to see that $P(i)=D(i)=60$ when $i=1$.

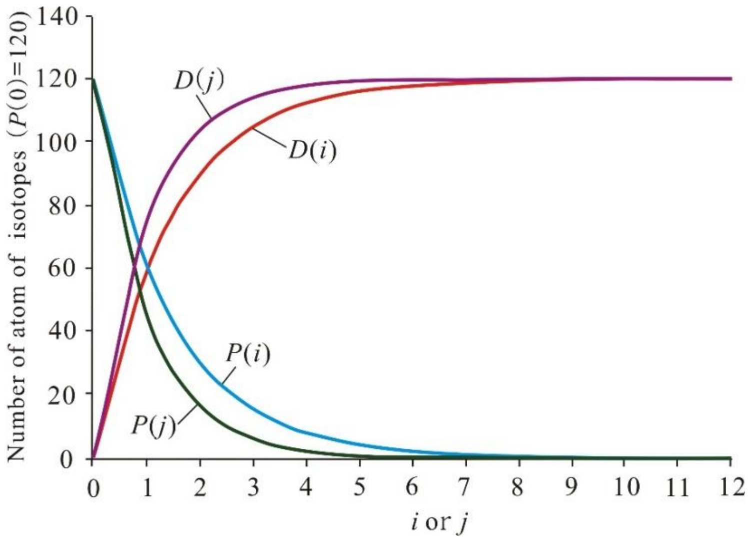

Figure 1. Changes of $P(i)$ and $D(i)$ with i and $P(j)$ and $D(j)$ with j, and curves of $P(i)$ and $D(i)$ are modified after Faure (1977)

Obviously, the radioactive parent isotope is reduced by a factor of 1/2 during each half-life (Faure, 1977). The remaining values of the radioactive parent isotope and the accumulating values of the radiogenic daughter isotope from 1 to 10 half-life's decay when $P(0)=120$ are listed in Table 1. Also listed in Table 1 are the ratios of $P(i)/P(0)$, $D(i)/P(0)$, $P(i)/D(i)$ and $(P(i)/P(0)) \cdot (P(i)/D(i))$ at ith half-life, $T_{1/2}(i)$, $i=1, 2, 3, \cdots; 10, \cdots$:

Table 1. Decrease in the radioactive parent isotope and increase in the radiogenic daughter isotope during each half-life in a single-component decay

C1	C2	C3	C4	C5	C6	C7	C8
i	$T_{1/2}(i)$	$P(i)$	$D(i)$	$\frac{P(i)}{P(0)}$	$\frac{D(i)}{P(0)}$	$\frac{P(i)}{D(i)}$	$\frac{P(i)}{P(0)} \cdot \frac{P(i)}{D(i)}$
1	ln2/λ	60	60	1/2	1/2	1	1/2
2	2ln2/λ	30	90	1/4	3/4	1/3	1/12
3	3ln2/λ	15	105	1/8	7/8	1/7	1/56
4	4ln2/λ	7.5	112.5	1/16	15/16	1/15	1/240
5	5ln2/λ	3.75	116.25	1/32	31/32	1/31	1/992
6	6ln2/λ	1.875	118.125	1/64	63/64	1/63	1/4032
7	7ln2/λ	0.9375	119.0625	1/128	127/128	1/127	1/16256
8	8ln2/λ	0.46875	119.53125	1/256	255/256	1/255	1/65280
9	9ln2/λ	0.234375	119.765625	1/512	511/512	1/511	1/261632
10	10ln2/λ	0.1171875	119.8828125	1/1024	1023/1024	1/1023	1/1047552
…	…	…	…	…	…	…	…

Note. $P(0)=120$.

When 1 is added into column C5 in Table 1, from the fractions we can construct the previously known infinite geometric series, including the following positive term series XA, alternating series XB, odd term series XC and even term series XD, and give their sums:

$$\text{XA}=1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\frac{1}{64}+\frac{1}{128}+\frac{1}{256}+\frac{1}{512}+\frac{1}{1024}+\cdots=\sum_{i=0}^{\infty}\frac{1}{2^i}=2\,;$$
$$\text{XB}=1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\frac{1}{16}-\frac{1}{32}+\frac{1}{64}-\frac{1}{128}+\frac{1}{256}-\frac{1}{512}+\frac{1}{1024}-\cdots=\sum_{i=0}^{\infty}(-1)^i\frac{1}{2^i}=\frac{2}{3}\,;$$
$$\text{XC}=1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\frac{1}{256}+\frac{1}{1024}+\cdots=\sum_{i=0}^{\infty}\frac{1}{2^{2i}}=\frac{4}{3}\,;$$
$$\text{XD}=\frac{1}{2}+\frac{1}{8}+\frac{1}{32}+\frac{1}{128}+\frac{1}{512}+\cdots=\sum_{i=1}^{\infty}\frac{1}{2^{2i-1}}=\frac{1}{2}\sum_{i=0}^{\infty}\frac{1}{2^{2i}}=\frac{2}{3}\,.$$

It is easy to find that XB=XA/3, XC=2XA/3, XC=2XB, XC=XA-XB, XC=(XA+XB)/2, XC=XA·XB, XD=XA/3, XD=XB, XD=XC/2, XD=XA-2XB, XD=XA-XC and XD=XC-XB.

When 1 is added into column C6 in Table 1, the following positive term series XE, alternating series XF, odd term series XG and even term series XH can be constructed:

$$\text{XE}=1+\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+\frac{31}{32}+\frac{63}{64}+\frac{127}{128}+\frac{255}{256}+\frac{511}{512}+\frac{1023}{1024}+\cdots=1+\sum_{i=1}^{\infty}\frac{2^i-1}{2^i}=\infty\,;$$
$$\text{XF}=1-\frac{1}{2}+\frac{3}{4}-\frac{7}{8}+\frac{15}{16}-\frac{31}{32}+\frac{63}{64}-\frac{127}{128}+\frac{255}{256}-\frac{511}{512}+\frac{1023}{1024}-\cdots=1+\sum_{i=1}^{\infty}(-1)^i\frac{2^i-1}{2^i}=\begin{cases}\frac{4}{3}\\ \frac{1}{3}\end{cases};$$
$$\text{XG}=1+\frac{3}{4}+\frac{15}{16}+\frac{63}{64}+\frac{255}{256}+\frac{1023}{1024}+\cdots=1+\sum_{i=1}^{\infty}\frac{2^{2i}-1}{2^{2i}}=\infty\,;$$
$$\text{XH}=\frac{1}{2}+\frac{7}{8}+\frac{31}{32}+\frac{127}{128}+\frac{511}{512}+\cdots=\sum_{i=1}^{\infty}\frac{2^{2i-1}-1}{2^{2i-1}}=\infty\,.$$

Series XE, XG and XH are obviously divergent. Series XF also diverges since when $i\to\infty$, its ith term is not equal to 0 and the partial sum approaches alternatively to the upper bounded value XFU=4/3 and the lower bounded value XFL=1/3. It is easy to obtain: XFU-XFL=1.

From column C7 in Table 1, i.e., the ratios of the remaining numbers of the radioactive parent isotope to the accumulating numbers of the radiogenic daughter isotope for each half-life (Lutgens & Tarbuck, 2014), we can write the following positive term series XI, alternating series XJ, odd term series XK and even term series XL:

$$\text{XI}=1+\frac{1}{3}+\frac{1}{7}+\frac{1}{15}+\frac{1}{31}+\frac{1}{63}+\frac{1}{127}+\frac{1}{255}+\frac{1}{511}+\frac{1}{1023}+\cdots=\sum_{i=1}^{\infty}\frac{1}{2^i-1}=1.606695\cdots;$$
$$\text{XJ}=1-\frac{1}{3}+\frac{1}{7}-\frac{1}{15}+\frac{1}{31}-\frac{1}{63}+\frac{1}{127}-\frac{1}{255}+\frac{1}{511}-\frac{1}{1023}+\cdots=\sum_{i=1}^{\infty}(-1)^{i+1}\frac{1}{2^i-1}=0.764499\cdots;$$
$$\text{XK}=1+\frac{1}{7}+\frac{1}{31}+\frac{1}{127}+\frac{1}{511}+\frac{1}{2047}+\cdots=\sum_{i=1}^{\infty}\frac{1}{2^{2i-1}-1}=1.185597\cdots;$$
$$\text{XL}=\frac{1}{3}+\frac{1}{15}+\frac{1}{63}+\frac{1}{255}+\frac{1}{1023}+\cdots=\sum_{i=1}^{\infty}\frac{1}{2^{2i}-1}=0.421097\cdots.$$

The sum of series XI is exactly the sum of the reciprocals of the Mersenne's numbers. XI=1.606695 ⋯is known as the Erdős-Borwein constant, whose irrationality was proved by Erdős (1948) and Borwein (1992). XI is also known as one of digital search tree constants (Finch, 2003). It is easy to get: XI=XK+XL and XJ=XK-XL.

When 1 is added into column C8 in Table 1, the following positive term series XM, alternating series XN, odd term series XO and even term series XP can be constructed:

$$\text{XM}=1+\frac{1}{2}+\frac{1}{12}+\frac{1}{56}+\frac{1}{240}+\frac{1}{992}+\frac{1}{4032}+\frac{1}{16256}+\cdots=1+\sum_{i=1}^{\infty}\frac{1}{2^i(2^i-1)}=1.606695\cdots;$$
$$\text{XN}=1-\frac{1}{2}+\frac{1}{12}-\frac{1}{56}+\frac{1}{240}-\frac{1}{992}+\frac{1}{4032}-\frac{1}{16256}+\cdots=1+\sum_{i=1}^{\infty}(-1)^i\frac{1}{2^i(2^i-1)}=0.568834\cdots;$$
$$\text{XO}=1+\frac{1}{12}+\frac{1}{240}+\frac{1}{4032}+\frac{1}{65280}+\frac{1}{1047552}+\cdots=1+\sum_{i=1}^{\infty}\frac{1}{2^{2i}(2^{2i}-1)}=1.087764\cdots;$$

$$XP=\frac{1}{2}+\frac{1}{56}+\frac{1}{992}+\frac{1}{16256}+\frac{1}{261632}+\cdots=\sum_{i=1}^{\infty}\frac{1}{2^{2i-1}(2^{2i-1}-1)}=0.518930\cdots.$$

The numerical value of the sum of series XM is equal to that of the sum of series XI, that is, the sum of the series XM also converges to the same value as the Erdös-Borwein constant (M. Chen, & S. Chen, 2016). The Erdös-Borwein constant appears surprisingly twice in the half-life related series. Although the ith term of series XM decreases more rapidly when $i\to\infty$ than the ith term of series XI does, XM=XI still holds since the second term of series XM is larger than the second term of series XI. It is easy to attain: XM=XO+XP and XN=XO-XP.

It is also found that one-half of the numerical value of the sum of series XN plus the sum of series XJ is equal to 2/3, i.e., the sum of series XB or XD, leading to the relations: (XJ+XN)/2=2/3=XB=XD, XJ+XN=4/3=XC, and 3(XJ+XN)/2=XB+XC=XA. So, we further obtain the following interesting relations among the above series (except the divergent series XE to XH): (XK-XL+XO-XP)/2=2/3=XB=XD, XK-XL+XO-XP=4/3=XC, 3(XK-XL+XO-XP)/2=XB+XC=XA, (XJ+XO-XP)/2=XB=XD, XJ+XO-XP=XC, 3(XJ+XO-XP)/2=XA, (XK-XL+XN)/2=XB=XD, XK-XL+XN=XC, and 3(XK-XL+XN)/2=XA, and so on.

From the denominators of series XK, XL, XM, XO and XP, we can construct the following 5 increasing sequences:

1, 7, 31, 127, 511, 2047, 8191, $\cdots$; $2^{2i-1}-1$, $\cdots$;

(1), 3, 15, 63, 255, 1023, 4095, $\cdots$; $2^{2i}-1$, $\cdots$;

(1), 2, 12, 56, 240, 992, 4032, 16256, $\cdots$; $2^{i}(2^{i}-1)$, $\cdots$;

(1), 12, 240, 4032, 65280, 1047552, 16773120, $\cdots$; $2^{2i}(2^{2i}-1)$, $\cdots$;

(1), 2, 56, 992, 16256, 261632, 4192256, $\cdots$; $2^{2i-1}(2^{2i-1}-1)$, $\cdots$.

The above sequences are neither arithmetic sequences nor geometric sequences. When i increases, they grow rapidly, especially the latter two sequences. The limits of (i+1) term/ ith term when $i\to\infty$ for these sequences are 4, 4, 4, 16, and 16, respectively.

3. Mean Life Related (or *e*-Related) Infinite Series

Another parameter that is sometimes used to describe the decay of a radioactive nuclide is the mean life, τ, defined as the average life expectancy of a radioactive atom (Faure, 1977):

$$\tau=\frac{1}{P(0)}\int_0^\infty \lambda P(t)tdt=\lambda\int_0^\infty te^{-\lambda t}dt=\frac{1}{\lambda}. \tag{5}$$

We also obtain $\tau=T_{1/2}/\ln 2$. Therefore Equations (1) and (2) can also be rewritten using mean life as follows:

$$P(j)=P(0)e^{-\lambda\tau(j)}, \tag{6}$$

and

$$D(j)=P(0)(1-e^{-\lambda\tau(j)}), \tag{7}$$

where j is the number of mean life of decay of the radioactive parent isotope, $P(j)$ and $D(j)$ are the values of atoms of the radioactive parent isotope and the radiogenic daughter isotope after jth mean life's decay, respectively. Both the decay of $P(j)$ and the increase of $D(j)$ with j are also shown in Figure 1 with the same scale on the abscissa when $P(0)$=120. It is also easy to see that $P(j)=D(j)=60$ when $j=\ln 2$.

Obviously a radioactive nuclide is reduced by a factor equal to 1/e of its initial value during each mean life (Faure, 1977). Similarly, if the initial value of the radioactive parent isotope is $P(0)$, we can give the information on decrease in the radioactive parent isotope and increase in the radiogenic daughter isotope in each mean life's decay during a single-component decay, as listed in Table 2. Also listed in Table 2 are the similar ratios as those in Table 1.

Table 2. Decrease in the radioactive parent isotope and increase in the radiogenic daughter isotope during each mean life in a single-component decay.

D1	D2	D3	D4	D5	D6	D7	D8
j	$\tau(j)$	$P(j)$	$D(j)$	$\frac{P(j)}{P(0)}$	$\frac{D(j)}{P(0)}$	$\frac{P(j)}{D(j)}$	$\frac{P(j)}{P(0)}\cdot\frac{P(j)}{D(j)}$
1	$1/\lambda$	$P(0)/e$	$P(0)[(e\text{-}1)/e]$	$1/e$	$(e\text{-}1)/e$	$1/(e\text{-}1)$	$1/[e(e\text{-}1)]$
2	$2/\lambda$	$P(0)/e^2$	$P(0)[(e^2\text{-}1)/e^2]$	$1/e^2$	$(e^2\text{-}1)/e^2$	$1/(e^2\text{-}1)$	$1/[e^2(e^2\text{-}1)]$
3	$3/\lambda$	$P(0)/e^3$	$P(0)[(e^3\text{-}1)/e^3]$	$1/e^3$	$(e^3\text{-}1)/e^3$	$1/(e^3\text{-}1)$	$1/[e^3(e^3\text{-}1)]$
4	$4/\lambda$	$P(0)/e^4$	$P(0)[(e^4\text{-}1)/e^4]$	$1/e^4$	$(e^4\text{-}1)/e^4$	$1/(e^4\text{-}1)$	$1/[e^4(e^4\text{-}1)]$
5	$5/\lambda$	$P(0)/e^5$	$P(0)[(e^5\text{-}1)/e^5]$	$1/e^5$	$(e^5\text{-}1)/e^5$	$1/(e^5\text{-}1)$	$1/[e^5(e^5\text{-}1)]$
6	$6/\lambda$	$P(0)/e^6$	$P(0)[(e^6\text{-}1)/e^6]$	$1/e^6$	$(e^6\text{-}1)/e^6$	$1/(e^6\text{-}1)$	$1/[e^6(e^6\text{-}1)]$
7	$7/\lambda$	$P(0)/e^7$	$P(0)[(e^7\text{-}1)/e^7]$	$1/e^7$	$(e^7\text{-}1)/e^7$	$1/(e^7\text{-}1)$	$1/[e^7(e^7\text{-}1)]$
8	$8/\lambda$	$P(0)/e^8$	$P(0)[(e^8\text{-}1)/e^8]$	$1/e^8$	$(e^8\text{-}1)/e^8$	$1/(e^8\text{-}1)$	$1/[e^8(e^8\text{-}1)]$
9	$9/\lambda$	$P(0)/e^9$	$P(0)[(e^9\text{-}1)/e^9]$	$1/e^9$	$(e^9\text{-}1)/e^9$	$1/(e^9\text{-}1)$	$1/[e^9(e^9\text{-}1)]$
10	$10/\lambda$	$P(0)/e^{10}$	$P(0)[(e^{10}\text{-}1)/e^{10}]$	$1/e^{10}$	$(e^{10}\text{-}1)/e^{10}$	$1/(e^{10}\text{-}1)$	$1/[e^{10}(e^{10}\text{-}1)]$
…	…	…	…	…	…	…	…

When 1 is added into column D5 in Table 2, from the fractions we can construct the infinite geometric series, including the following positive term series ZA, alternating series ZB, odd term series ZC and even term series ZD, and obtain their sums:

$$\text{ZA}=1+\frac{1}{e}+\frac{1}{e^2}+\frac{1}{e^3}+\frac{1}{e^4}+\frac{1}{e^5}+\frac{1}{e^6}+\frac{1}{e^7}+\frac{1}{e^8}+\cdots=\sum_{j=0}^{\infty}\frac{1}{e^j}=\frac{e}{e-1}=1.581976\cdots;$$

$$\text{ZB}=1-\frac{1}{e}+\frac{1}{e^2}-\frac{1}{e^3}+\frac{1}{e^4}-\frac{1}{e^5}+\frac{1}{e^6}-\frac{1}{e^7}+\frac{1}{e^8}-\cdots=\sum_{j=0}^{\infty}(-1)^j\frac{1}{e^j}=\frac{e}{e+1}=0.731058\cdots;$$

$$\text{ZC}=1+\frac{1}{e^2}+\frac{1}{e^4}+\frac{1}{e^6}+\frac{1}{e^8}+\frac{1}{e^{10}}+\cdots=\sum_{j=0}^{\infty}\frac{1}{e^{2j}}=\frac{e^2}{e^2-1}=1.156517\cdots;$$

$$\text{ZD}=\frac{1}{e}+\frac{1}{e^3}+\frac{1}{e^5}+\frac{1}{e^7}+\frac{1}{e^9}+\cdots=\sum_{j=1}^{\infty}\frac{1}{e^{2j-1}}=\frac{1}{e}\sum_{j=0}^{\infty}\frac{1}{e^{2j}}=\frac{e}{e^2-1}=0.425459\cdots.$$

It is found that ZA=ZC+ZD, ZB=ZC-ZD, ZC=ZA·ZB, ZC=(ZA+ZB)/2, ZD=ZC/e, ZD=(ZA·ZB)/e and ZD=(ZA-ZB)/2.

When 1 is added into column D6 in Table 2, the following positive term series ZE, alternating series ZF, odd term series ZG and even term series ZH can be constructed:

$$\text{ZE}=1+\frac{e-1}{e}+\frac{e^2-1}{e^2}+\frac{e^3-1}{e^3}+\frac{e^4-1}{e^4}+\frac{e^5-1}{e^5}+\frac{e^6-1}{e^6}+\cdots=1+\sum_{j=1}^{\infty}\frac{e^j-1}{e^j}=\infty;$$

$$\text{ZF}=1-\frac{e-1}{e}+\frac{e^2-1}{e^2}-\frac{e^3-1}{e^3}+\frac{e^4-1}{e^4}-\frac{e^5-1}{e^5}+\cdots=1+\sum_{j=1}^{\infty}(-1)^j\frac{e^j-1}{e^j}=\begin{cases}1.268941\cdots\\0.268941\cdots\end{cases};$$

$$\text{ZG}=1+\frac{e^2-1}{e^2}+\frac{e^4-1}{e^4}+\frac{e^6-1}{e^6}+\frac{e^8-1}{e^8}+\frac{e^{10}-1}{e^{10}}+\cdots=1+\sum_{j=1}^{\infty}\frac{e^{2j}-1}{e^{2j}}=\infty;$$

$$\text{ZH}=\frac{e-1}{e}+\frac{e^3-1}{e^3}+\frac{e^5-1}{e^5}+\frac{e^7-1}{e^7}+\frac{e^9-1}{e^9}+\cdots=\sum_{j=1}^{\infty}\frac{e^{2j-1}-1}{e^{2j-1}}=\infty.$$

Series ZE, ZG and ZH are obviously divergent. Series ZF also diverges since when $j\to\infty$, its jth term is not equal to 0 and the partial sum approaches alternatively to the upper bounded value ZFU=1.268941 ⋯ and the lower bounded value ZFL=0.268941 ⋯(Figure 2). It is easy to obtain: ZFU-ZFL=1.

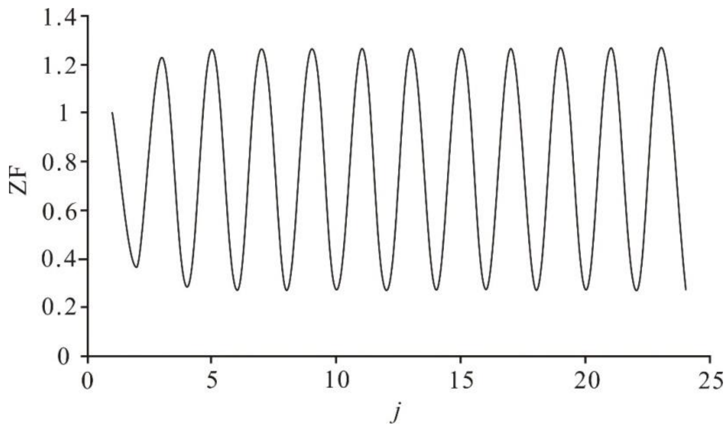

Figure 2. Change in the partial sum of series ZF with *j* ($1 \le j \le 24$)

From column D7 in Table 2, i.e., the ratios of the remaining numbers of the radioactive parent isotope to the accumulating numbers of the radiogenic daughter isotope for each mean life's decay, we can write the following positive term series ZI, alternating series ZJ, odd term series ZK and even term series ZL:

$$\mathrm{ZI}=\frac{1}{e-1}+\frac{1}{e^2-1}+\frac{1}{e^3-1}+\frac{1}{e^4-1}+\frac{1}{e^5-1}+\frac{1}{e^6-1}+\frac{1}{e^7-1}+\cdots=\sum_{j=1}^{\infty}\frac{1}{e^j-1}=0.820259\cdots;$$

$$\mathrm{ZJ}=\frac{1}{e-1}-\frac{1}{e^2-1}+\frac{1}{e^3-1}-\frac{1}{e^4-1}+\frac{1}{e^5-1}-\frac{1}{e^6-1}+\frac{1}{e^7-1}-\cdots=\sum_{j=1}^{\infty}(-1)^{j+1}\frac{1}{e^j-1}=0.464163\cdots;$$

$$\mathrm{ZK}=\frac{1}{e-1}+\frac{1}{e^3-1}+\frac{1}{e^5-1}+\frac{1}{e^7-1}+\frac{1}{e^9-1}+\cdots=\sum_{j=1}^{\infty}\frac{1}{e^{2j-1}-1}=0.642211\cdots;$$

$$\mathrm{ZL}=\frac{1}{e^2-1}+\frac{1}{e^4-1}+\frac{1}{e^6-1}+\frac{1}{e^8-1}+\frac{1}{e^{10}-1}+\cdots=\sum_{j=1}^{\infty}\frac{1}{e^{2j}-1}=0.178047\cdots.$$

It is found that ZI=ZK+ZL and ZJ=ZK-ZL.

From column D8 in Table 2, the following positive term series ZM, alternating series ZN, odd term series ZO and even term series ZP can be constructed:

$$\mathrm{ZM}=\frac{1}{e(e-1)}+\frac{1}{e^2(e^2-1)}+\frac{1}{e^3(e^3-1)}+\frac{1}{e^4(e^4-1)}+\frac{1}{e^5(e^5-1)}+\cdots=\sum_{j=1}^{\infty}\frac{1}{e^j(e^j-1)}=0.238282\cdots;$$

$$\mathrm{ZN}=\frac{1}{e(e-1)}-\frac{1}{e^2(e^2-1)}+\frac{1}{e^3(e^3-1)}-\frac{1}{e^4(e^4-1)}+\cdots=\sum_{j=1}^{\infty}(-1)^{j+1}\frac{1}{e^j(e^j-1)}=0.195222\cdots;$$

$$\mathrm{ZO}=\frac{1}{e(e-1)}+\frac{1}{e^3(e^3-1)}+\frac{1}{e^5(e^5-1)}+\frac{1}{e^7(e^7-1)}+\cdots=\sum_{j=1}^{\infty}\frac{1}{e^{2j-1}(e^{2j-1}-1)}=0.216752\cdots;$$

$$\mathrm{ZP}=\frac{1}{e^2(e^2-1)}+\frac{1}{e^4(e^4-1)}+\frac{1}{e^6(e^6-1)}+\frac{1}{e^8(e^8-1)}+\cdots=\sum_{j=1}^{\infty}\frac{1}{e^{2j}(e^{2j}-1)}=0.021530\cdots.$$

It is also found that ZM=ZO+ZP and ZN=ZO-ZP.

4. Summary

Sixteen half-life related infinite series and 16 mean life related infinite series can be constructed by examining the changes in atoms of the radioactive parent isotope and the radiogenic daughter isotope for each half-life's and mean life's decay, respectively. Four half-life related series (XE, XF, XG and XH) and 4 mean life related series (ZE, ZF, ZG and ZH) are divergent. Four convergent half-life related series (XA, XB, XC and XD) are previously known, and 2 half-life related series (XI and XM) converge to the Erdős-Borwein constant. From the remaining convergent series, 18 mathematical constants (XJ, XK, XL, XN, XO, XP, ZA, ZB, ZC, ZD, ZI, ZJ, ZK, ZL, ZM, ZN, ZO and ZP) are acquired. From the upper and lower bounded values of the divergent and alternating mean life related series ZF, 2 more mathematical constants (ZFU and ZFL) are obtained. Five sequences are also constructed from the convergent half-life related series (XK, XL, XM, XO and XP). It is also found that there is a close link and exist a few interesting mathematical relations among the sums of the convergent series. The half-life related and mean life related series are 2-related and *e*-related, respectively.

Acknowledgements

This work was supported by the National Natural Science Foundation of China (41772261).

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