

# Criteria for H-Matrices Based on $\gamma$ -Diagonally Dominant Matrix

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## Abstract

In this paper, we present a new practical criteria for H-matrix based on  $\gamma$ -diagonally dominant matrix. In order to make the judgment conditions convenient and effective, we give two new definitions, one is called strong and weak diagonally dominant degree, the other is called the sum of non-principal diagonal element for the matrix. Further, we obtain a new practical method for the determination of the H-matrix by combining the properties of  $\gamma$ -diagonally dominant matrix, constructing positive diagonal matrix, and adding the appropriate parameters. Finally, we offer numerical examples to verify the validity of the judgment conditions, corresponding numerical examples compared the new criteria and the existing results are presented to verify the advantages of the new determination method.

**Keywords:** H-matrix,  $\gamma$ -diagonally dominant matrix, irreducible matrix, strong and weak diagonally dominant degree

## 1. Introduction

H-matrix is an important concept in computational mathematics. The definition of H-matrix was first given by A.M. Ostrowski in 1937 (Ostrowski, 1937). As a typical special matrix, H-matrix not only has important theoretical research value, but also has wide application in many fields such as numerical algebra, statistics and economic mathematics, etc (Gan, & Huang, 2004). For a long time, researchers' enthusiasm for the study of the properties of H-matrix, especially the criterion of H-matrix, has not diminished. For instance, Varga R.S. (Varga, 1976), Gao Y.M. (Gao, & Wang, 1996), Cvetkovic L. (Cvetkovic, & Kostic, 2005) gave some direct judgement methods of H-matrix by dividing matrix index sets and scaling inequalities. Huang T. Z. (Huang, Leng, Wachspress, & Tang, 2004), Liu J.Z. (Liu, & Huang, 2004) and other scholars have presented the relationship between strictly diagonally dominant matrices and H-matrices. In recent years, scholars such as Xie Q.M., He A.Q. (Xie, He, & Liu, 2007), Gao H.S. (Gao, & Han, 2017), Jeribi A. (Jeribi, Krichen, & Zitouni, 2019) have given some iterative discriminant algorithms for H-matrices on the basis of the existing theory. These algorithms speed up the determination and expand the scope of discrimination. However, until now, it has been still not convenient to check if a given matrix is an H-matrix.

Consider the structure of stationary linear large-scale systems

$$\frac{dX}{dt} = AX,$$

where  $A = (a_{ij}) \in \mathbb{C}^{n \times n}$ ,  $X = (x_1, x_2, \dots, x_n)^T \in \mathbb{C}^n$ .

For one thing, it is well known that H-matrices play an important role in matrix theory, numerical analysis, economic mathematics, etc (Li, & Sun, 2009). For example, solving the linear equation  $Ax = b$ , when the coefficient matrix  $A$  is H-matrix, then many classical iterative methods such as Jacobi, Gauss-Seidel, SOR and their improved algorithms are convergent. Hence, how to judge H-matrices is important and necessary.

For another, in the 1960s, people defined a kind of matrix class - generalized strictly diagonally dominant matrix class (Zhu, 1995), which is completely equivalent to H-matrix class in essence mathematics from different angles and different problem backgrounds. With the later  $\gamma$ -diagonally dominant matrix,  $\gamma$ -chain diagonally dominant matrix and other concepts were proposed (Liu, & Huang, 2010), we find that the properties of  $\gamma$ -diagonally dominant matrix is very ingenious for judging H-matrix conditions. Hence, how to determine H-matrix by using the properties of  $\gamma$ -diagonally dominant matrix is very useful.

Based on the above, studying the properties of  $\gamma$ -diagonally dominant matrix and seeking for the practical criterion of H-matrix may have important theoretical and practical significance. In this paper, we first present two new definitions, one is called strong and weak diagonally dominant degree, the other is called the sum of non-principal diagonal element

for the matrix. Compared with the pervious definitions such as diagonally dominant degree, they are not only more helpful, but also more general. Then, using two new definitions, combining the properties of  $\gamma$ -diagonally dominant matrix, constructing positive diagonal matrices and adding appropriate parameters, we obtain the criterion of H-matrix. It may be used to judge H-matrix more faster than several known methods in pervious article. Finally, we give numerical examples to verify the validity and illustrate the advantages of the criteria of nonsingular H-matrices.

In order to state the main results, we need the following symbols. Let  $\mathbb{C}^{n \times n}(\mathbb{R}^{n \times n})$  denote the set of all  $n \times n$  complex(real) matrices,  $N = \{1, 2, \dots, n\}$  and  $A = (a_{ij}) \in \mathbb{C}^{n \times n}$ . Denote  $|A| = (|a_{ij}|)$  and  $\forall i, j \in N$ ,

$$R_i = \sum_{j \in N, j \neq i} |a_{ij}|, Q_i = \sum_{j \in N, j \neq i} |a_{ji}|.$$

Now, we introduce some notations

$$N_1 = \{i | 0 < |a_{ii}| = \gamma R_i + (1 - \gamma) Q_i\},$$

$$N_2 = \{i | 0 < |a_{ii}| < \gamma R_i + (1 - \gamma) Q_i\},$$

$$N_3 = \{i | |a_{ii}| > \gamma R_i + (1 - \gamma) Q_i > 0\}.$$

Obviously,  $N = N_1 \cup N_2 \cup N_3$ .

## 2. Definitions and Lemmas

**Definition 2.1** (Liu, Zhang, Zhou, & Tu, 2018) Let  $A = (a_{ij}) \in \mathbb{R}^{n \times n}$ , if  $a_{ij} \leq 0$  for all  $i \neq j$ , then A is said to be a Z-matrix. If A can be expressed in the form

$$A = sI_n - B, s \geq 0, B \geq 0,$$

where  $B \geq 0$ , and  $s \geq \rho(B)$ ,  $I_n$  denotes a unit matrix of order n,  $\rho(B)$  stands for the spectral radius of B, then A is said to be an M-matrix.

**Definition 2.2** (Li., Liu, & Zhang, 2017) Let  $A = (a_{ij}) \in \mathbb{C}^{n \times n}$ , denoted by  $\mu(A) = (t_{ij}) \in \mathbb{R}^{n \times n}$ , is defined by

$$t_{ij} = \begin{cases} |a_{ij}|, & i = j, \\ -|a_{ij}|, & i \neq j, \end{cases}$$

then  $\mu(A)$  is said to be a comparison matrix of matrix A. If  $\mu(A) \in M_n$ , then A is called a general H-matrix.

**Definition 2.3** Let  $A = (a_{ij}) \in \mathbb{C}^{n \times n}$ . If for  $i \in N$ , such that

$$|a_{ii}| \geq R_i (i \in N),$$

then A is said to be a diagonally dominant matrix and is denoted by  $A \in D_0$ .

**Definition 2.4** Let  $A = (a_{ij}) \in \mathbb{C}^{n \times n}$ . If  $\exists \gamma \in [0, 1]$ , for  $\forall i \in N$ , such that

$$|a_{ii}| \geq \gamma R_i + (1 - \gamma) Q_i,$$

then A is said to be a  $\gamma$ -diagonally dominant matrix and is denoted by  $A \in D_{\gamma_0}$ . If for  $i \in N$ , such that

$$|a_{ii}| > \gamma R_i + (1 - \gamma) Q_i,$$

then A is said to be a strictly  $\gamma$ -diagonally dominant matrix and is denoted by  $A \in D_\gamma$ .

**Lemma 2.1** (Gao, & Wang, 1996) Let  $A = (a_{ij}) \in \mathbb{C}^{n \times n}$ . If there exists positive diagonally matrix  $X = \text{diag}(d_1, d_2, \dots, d_n)$ , where  $d_i > 0, i \in N$ , such that

$$AX \in SD,$$

then A is said to be a generalized strictly diagonally dominant matrix and is denoted by  $A \in GSD$ .

**Lemma 2.2** (Liu, & He, 2007) A is an H-matrix if and only if A is a generalized strictly diagonally dominant matrix.

**Lemma 2.3** (Gan, & Huang, 2004) Let  $A = (a_{ij}) \in \mathbb{C}^{n \times n}$  be  $\gamma$ -diagonally dominant matrix. If A satisfies either

- (i) A is an irreducible matrix,  $N_\gamma(A) = \{i \in N | |a_{ii}| > \gamma R_i + (1 - \gamma) Q_i\} \neq \emptyset$ ;
- (ii)  $N_\gamma(A) = \{i \in N | |a_{ii}| > \gamma R_i + (1 - \gamma) Q_i\} \neq \emptyset$  and  $\forall i \in N - N_\gamma(A)$ ;

there must exists a nonzero elements chain  $a_{i_1 i_1}, a_{i_1 i_2}, \dots, a_{i_s i_m}$ , and  $i \neq i_1, i_1 \neq i_2, \dots, i_s \neq i_m, i_m \in N_\gamma(A)$ , then A is a generalized strictly  $\gamma$ -diagonally dominant matrix.

If  $N_2 = \emptyset$ , then A is a strictly  $\gamma$ -diagonally dominant matrix; If  $N_3 = \emptyset$ , then A is not a strictly  $\gamma$ -diagonally dominant matrix.

So, we always assume that  $N_2, N_3 \neq \emptyset$  and all diagonal entries of A are nonzero.

### 3. Main Results

In this section, we propose two new definitions and present new judgment conditions for H-matrices. In order to verify the feasibility of judging conditions, we give a numerical example.

**Definition 3.1** Let  $A = (a_{ij}) \in \mathbb{C}^{n \times n}$ . Then A is a  $\gamma$ -diagonally dominant matrix, if there exists  $\gamma \in [0, 1]$ , such that

$$|a_{ii}| \geq \gamma R_i + (1 - \gamma) Q_i, \forall i \in N,$$

we denote  $S_i = \gamma R_i + (1 - \gamma) Q_i$  as the sum of non-principal diagonal element for the matrix A. If  $\gamma = 0$ , such that  $S_i = Q_i$ ,

$$|a_{ii}| > Q_i,$$

then  $S_i$  is said to be column sum of matrix A.

If  $\gamma = 1$ , such that  $S_i = R_i$ ,

$$|a_{ii}| > R_i,$$

then  $S_i$  is said to be row sum of matrix A.

**Definition 3.2** Let  $A = (a_{ij}) \in \mathbb{C}^{n \times n}$ . If exists  $\omega_i$ , for  $\forall i \in N_2$ , such that

$$\omega_i = \frac{S_i - |a_{ii}|}{S_i},$$

we called  $\omega_i$  the weak diagonally dominant degree of matrix A.

If exists  $\mu_j$ , for  $\forall j \in N_3$ , such that

$$\mu_j = \frac{S_j}{|a_{jj}|},$$

we called  $\mu_j$  the strong diagonally dominant degree of matrix A.

**Theorem 3.1** Let  $A = (a_{ij}) \in \mathbb{C}^{n \times n}$ , if  $N_1, N_2, N_3$  satisfy  $N = N_1 \cup N_2 \cup N_3$ , then for  $i \in N_2, j \in N_3$ ,

$$\begin{aligned} & (\omega_i |a_{ii}| - [\gamma \sum_{t \in N_1, t \neq i} |a_{it}| + \gamma \omega_t \sum_{t \in N_2, t \neq i} |a_{it}| + \omega_i (1 - \gamma) Q_i]) (S_j - [\mu_t \gamma \sum_{t \in N_3, t \neq j} |a_{jt}| + \mu_j (1 - \gamma) Q_j]) \\ & > (\mu_t \gamma \sum_{t \in N_3, t \neq i} |a_{it}|) (\gamma \sum_{t \in N_1, t \neq j} |a_{jt}| + \gamma \omega_t \sum_{t \in N_2, t \neq j} |a_{jt}|), \end{aligned}$$

then A is called an H-matrix.

*Proof.* Note that

$$\begin{aligned} d^{(1)} &= \min \frac{\omega_i |a_{ii}| - \gamma \sum_{t \in N_1, t \neq i} |a_{it}| - \gamma \omega_t \sum_{t \in N_2, t \neq i} |a_{it}| - \omega_i (1 - \gamma) Q_i}{\mu_t \gamma \sum_{t \in N_3, t \neq i} |a_{it}|}, \\ d^{(2)} &= \max \frac{\gamma (\sum_{t \in N_1, t \neq j} |a_{jt}| + \omega_t \sum_{t \in N_2, t \neq j} |a_{jt}|)}{S_j - [\mu_t \gamma \sum_{t \in N_3, t \neq j} |a_{jt}| + \mu_j (1 - \gamma) Q_j]}. \end{aligned}$$

We can get  $d^{(1)} > d^{(2)}$ , then we take any k and assume that  $\min d^{(1)} \geq k \geq \max d^{(2)}$ , we suppose a positive diagonally matrix

$$X = \text{diag}\{d_1, d_2, \dots, d_n\},$$

where

$$d_i = \begin{cases} 1, & i \in N_1 \\ \omega_i, & i \in N_2 \\ k\mu_i, & i \in N_3 \end{cases}$$

Let  $B = (b_{ij}) = AX$ , then we prove that B is a strictly  $\gamma$ -diagonally dominant matrix.

(i) For  $\forall i \in N_1$ ,

it is clear that  $|a_{ii}| = |b_{ii}|$ , we infer from the irreducibility of matrix A that

$$\sum_{t \in N_2, t \neq i} |a_{it}| \neq 0$$

or

$$\sum_{t \in N_3, t \neq i} |a_{it}| \neq 0,$$

thus, we know that

$$\gamma R_i(B) + (1 - \gamma) Q_i(B) = \gamma(1 \cdot \sum_{t \in N_1, t \neq i} |a_{it}| + \omega_t \cdot \sum_{t \in N_2, t \neq i} |a_{it}| + k\mu_t \cdot \sum_{t \in N_3, t \neq i} |a_{it}|) + (1 - \gamma) Q_i, \tag{1}$$

then, we have

$$|a_{ii}| = \gamma(\sum_{t \in N_1, t \neq i} |a_{it}| + \sum_{t \in N_2, t \neq i} |a_{it}| + \sum_{t \in N_3, t \neq i} |a_{it}|) + (1 - \gamma) Q_i, \tag{2}$$

thus, we arrive that

$$|b_{ii}| = |a_{ii}| \geq \gamma R_i(B) + (1 - \gamma) Q_i(B). \tag{3}$$

(ii) For  $\forall i \in N_2, k < d^{(1)}$ ,

we have

$$\gamma k\mu_t \sum_{t \in N_3, t \neq i} |a_{it}| \leq \omega_i |a_{ii}| - \gamma \sum_{t \in N_1, t \neq i} |a_{it}| - \gamma \omega_t \sum_{t \in N_2, t \neq i} |a_{it}| - \omega_i(1 - \gamma) Q_i, \tag{4}$$

then, we know that

$$\gamma R_i(B) + (1 - \gamma) Q_i(B) = \gamma(1 \cdot \sum_{t \in N_1, t \neq i} |a_{it}| + \omega_t \cdot \sum_{t \in N_2, t \neq i} |a_{it}| + k\mu_t \cdot \sum_{t \in N_3, t \neq i} |a_{it}|) + \omega_i(1 - \gamma) Q_i. \tag{5}$$

Thus, we have

$$\begin{aligned} |b_{ii}| = \omega_i |a_{ii}| &= \gamma \sum_{t \in N_1, t \neq i} |a_{it}| + \gamma \omega_t \sum_{t \in N_2, t \neq i} |a_{it}| + \omega_i(1 - \gamma) Q_i \\ &+ \omega_i |a_{ii}| - [\gamma(\sum_{t \in N_1, t \neq i} |a_{it}| + \omega_t \sum_{t \in N_2, t \neq i} |a_{it}|) + \omega_i(1 - \gamma) Q_i], \end{aligned} \tag{6}$$

we infer from (3.4) that

$$|b_{ii}| = \omega_i |a_{ii}| \geq \gamma R_i(B) + (1 - \gamma) Q_i(B). \tag{7}$$

(iii) For  $\forall i \in N_3$ , since  $k > \max d^{(2)}$ ,

we get

$$k(S_i - [\mu_t \gamma \sum_{t \in N_3, t \neq i} |a_{it}| + \mu_j(1 - \gamma) Q_i]) \geq \gamma(\sum_{t \in N_1, t \neq i} |a_{it}| + \omega_t \sum_{t \in N_2, t \neq i} |a_{it}|), \tag{8}$$

then

$$k(S_i - \mu_t \gamma \sum_{t \in N_3, t \neq i} |a_{it}|) - k\mu_j(1 - \gamma) Q_i \geq \gamma(\sum_{t \in N_1, t \neq i} |a_{it}| + \omega_t \sum_{t \in N_2, t \neq i} |a_{it}|), \tag{9}$$

it is clear that

$$k(S_i - \mu_t \gamma \sum_{t \in N_3, t \neq i} |a_{it}|) - \gamma(\sum_{t \in N_1, t \neq i} |a_{it}| + \omega_t \sum_{t \in N_2, t \neq i} |a_{it}|) \geq k\mu_j(1 - \gamma) Q_i, \tag{10}$$

we know that

$$\gamma R_i(B) + (1 - \gamma)Q_i(B) = \gamma(1 \cdot \sum_{t \in N_1, t \neq i} |a_{it}| + \omega_t \cdot \sum_{t \in N_2, t \neq i} |a_{it}| + k\mu_t \cdot \sum_{t \in N_3, t \neq i} |a_{it}|) + \mu_j k(1 - \gamma)Q_i, \tag{11}$$

we infer from (3.10) that

$$\begin{aligned} & \gamma(\sum_{t \in N_1, t \neq i} |a_{it}| + \omega_t \sum_{t \in N_2, t \neq i} |a_{it}| + k\mu_t \sum_{t \in N_3, t \neq i} |a_{it}|) + k(S_j - \mu_j \gamma \sum_{t \in N_3, t \neq i} |a_{jt}|) - \gamma(\sum_{t \in N_1, t \neq i} |a_{it}| + \omega_t \sum_{t \in N_2, t \neq i} |a_{it}|) \\ &= \gamma(\sum_{t \in N_1, t \neq i} |a_{it}| + \omega_t \sum_{t \in N_2, t \neq i} |a_{it}|) + \gamma k\mu_t \sum_{t \in N_3, t \neq i} |a_{it}| + kS_j - k\mu_j \gamma \sum_{t \in N_3, t \neq i} |a_{jt}| - \gamma(\sum_{t \in N_1, t \neq i} |a_{it}| + \omega_t \sum_{t \in N_2, t \neq i} |a_{it}|) \\ &= kS_j \geq \gamma R_i(B) + (1 - \gamma)Q_i(B). \end{aligned}$$

Thus, we arrive that,

$$|b_{ii}| = k\mu_i |a_{ii}| = k \frac{S_i}{|a_{ii}|} |a_{ii}| = kS_i \geq \gamma R_i(B) + (1 - \gamma)Q_i(B). \tag{12}$$

Combined (i) – (iii) show that B is a strictly  $\gamma$ -diagonally dominant matrix, then B is a generalized strictly diagonally dominant matrix. By Lemma2.2, A is a generalized strictly diagonally dominant matrix, thus A is an H-matrix. This completes the proof.

**Theorem 3.2** Let  $A = (a_{ij}) \in \mathbb{C}^{n \times n}$ , if  $N_1, N_2, N_3$  satisfy  $N = N_1 \cup N_2 \cup N_3$ , then for  $i \in N_2, j \in N_3$ , there exists

$$\begin{aligned} & (\omega_i |a_{ii}| - [\gamma(\sum_{t \in N_1, t \neq i} |a_{it}| + r_t \sum_{t \in N_2, t \neq i} |a_{it}|) + \omega_i(1 - \gamma)Q_i])(S_j - [\mu_t \gamma \sum_{t \in N_3, t \neq j} |a_{jt}| + \mu_j(1 - \gamma)Q_j]) \\ & > (\mu_t \gamma \sum_{t \in N_3, t \neq i} |a_{it}|)(\gamma(\sum_{t \in N_1, t \neq j} |a_{jt}| + \omega_t \sum_{t \in N_2, t \neq j} |a_{jt}|) \end{aligned}$$

and for  $\forall i \in N(A)$ , there exists a nonzero elements chain  $a_{i i_1}, a_{i_1 i_2}, \dots, a_{i_s i_m}$ , and  $i \neq i_1, i_1 \neq i_2, \dots, i_s \neq i_m, i_m \in N(A) \neq \emptyset$ , Thus  $A \in GSD$ , then A is a nonsingular H-matrix.

*Proof.* By the Theroem3.1, we also note that

$$\begin{aligned} d^{(1)} &= \min \frac{\omega_i |a_{ii}| - \gamma \sum_{t \in N_1, t \neq i} |a_{it}| - \omega_t \gamma \sum_{t \in N_2, t \neq i} |a_{it}| - \omega_i(1 - \gamma)Q_i}{\mu_t \gamma \sum_{t \in N_3, t \neq i} |a_{it}|}, \\ d^{(2)} &= \max \frac{\gamma(\sum_{t \in N_1, t \neq j} |a_{jt}| + \omega_t \sum_{t \in N_2, t \neq j} |a_{jt}|)}{S_j - [\mu_t \gamma \sum_{t \in N_3, t \neq j} |a_{jt}| + \mu_j(1 - \gamma)Q_j]}, \end{aligned}$$

we can get  $d^{(1)} > d^{(2)}$ , then we take any k and assume that  $\min d^{(1)} \geq k \geq \max d^{(2)}$ , we suppose

$$D = \text{diag}\{d_1, d_2, \dots, d_n\},$$

$$d_i = \begin{cases} 1, & i \in N_1 \\ \omega_i, & i \in N_2 \\ k\mu_i, & i \in N_3 \end{cases}$$

We suppose

$$B = AD = (b_{ij}),$$

In a similar way to the proof of Theorem3.1, we know that matrix B is a  $\gamma$ -diagonally dominant matrix.

For D is a positive diagonally matrix, when A satisfies the condition of lemma2.3, further  $\forall i \in N(A)$ , there exists a nonzero elements chain  $b_{i i_1}, b_{i_1 i_2}, \dots, b_{i_s i_m}$ , and  $i \neq i_1, i_1 \neq i_2, \dots, i_s \neq i_m, i_m \in N \setminus N(A) \neq \emptyset$ .

By the lemma2.2, thus B is a generalized diagonally dominant matrix, then A is a nonsingular H-matrix. The proof is completed.

**Example 1** Let

$$A = \begin{bmatrix} 10 & 4 & 2 & 1 \\ 3 & 10 & 1 & 2 \\ 4 & 5 & 9 & 4 \\ 8 & 4 & 5 & 10.6 \end{bmatrix}, \gamma = \frac{1}{2}.$$

Take

$$N_1 = \emptyset, N_2 = \{1, 3, 4\}, N_3 = \{2\},$$

Then

$$S_1 = 11, S_2 = 9.5, S_3 = 10.5, S_4 = 12.$$

According to Theorem3.1, we obtain

$$\begin{aligned} &(\omega_1|a_{11}| - [\frac{1}{2}(\omega_3|a_{13}| + \omega_4|a_{14}|) + \frac{1}{2}\omega_1Q_1])(S_2 - \frac{1}{2}\mu_2Q_2) = 1.2202 \\ &> (\frac{1}{2}\mu_2|a_{12}|)(\frac{1}{2}(\omega_1|a_{21}| + \omega_3|a_{23}| + \omega_4|a_{24}|)) = 0.6165, \end{aligned} \tag{13}$$

$$\begin{aligned} &(\omega_3|a_{33}| - [\frac{1}{2}(\omega_1|a_{31}| + \omega_4|a_{34}|) + \frac{1}{2}\omega_3Q_3])(S_2 - \frac{1}{2}\mu_2Q_2) = 1.945 \\ &> (\frac{1}{2}\mu_2|a_{32}|)(\frac{1}{2}(\omega_1|a_{21}| + \omega_3|a_{23}| + \omega_4|a_{24}|)) = 0.7706, \end{aligned} \tag{14}$$

$$\begin{aligned} &(\omega_4|a_{44}| - [\frac{1}{2}(\omega_1|a_{41}| + \omega_3|a_{43}|) + \frac{1}{2}\omega_4Q_4])(S_2 - \frac{1}{2}\mu_2Q_2) = 1.0365 \\ &> (\frac{1}{2}\mu_2|a_{42}|)(\frac{1}{2}(\omega_1|a_{21}| + \omega_3|a_{23}| + \omega_4|a_{24}|)) = 0.6165. \end{aligned} \tag{15}$$

Thus matrix A satisfies the conditions of the Theorem3.1, then By Theorem3.1, matrix A is an H-matrix. According to the method of comparing matrix element in reference[2], we have

$$\begin{aligned} &|a_{33}| = 9 \\ &< \frac{\Lambda_3(A)}{\Lambda_3(A) - |a_{33}|} [\frac{\Lambda_1(A) - |a_{11}|}{\Lambda_1(A)}|a_{31}| + \frac{\Lambda_4(A) - |a_{44}|}{\Lambda_4(A)}|a_{34}| + \frac{\Lambda_2(A) - |a_{22}|}{\Lambda_2(A)}|a_{32}|] \\ &= 9.0727. \end{aligned} \tag{16}$$

In reference[2], we can not obtain the conclusion that matrix A is an H-matrix. This shows that the method given in this paper is more practical in judging H-matrix.

**4. Conclusion**

In this paper, two new helpful definitions of strong and weak diagonal dominance degree and the sum of non-principal diagonal element for the matrix are given. Further, we present a new method of judging H-matrix. For making up for the deficiency of the existing literatures such as reference[2], as a contrast, our method can be used to judge and widen the range of conditions for judging H-matrix. Moreover, numerical example shows that the criterion is practical and feasible.

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