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Random Attractor Family for a Class of Stochastic Higher-Order Kirchhoff Equations

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Abstract

A class of stochastic dynamical systems with strong damped stochastic higher order Kirchhoff equation solutions with white noise is studied. Firstly, the equation is transformed into a stochastic equation with random variables as parameters and without noise by using Ornstein-Uhlenbeck process. Secondly, the bounded stochastic absorption set is obtained by estimating the solution of the equation. Finally, the stochastic dynamical system is obtained by using the isomorphic mapping method and the compact embedding theorem. It is progressively compact, thus proving the existence of random attractors.

Keywords: stochastic Kirchhoff equation, random attractor family, Wiener process, Ornstein-Uhlenbeck process

2010 MSC: 35K10, 35K25, 35K35

1. Introduction

In this paper, the stochastic higher-order Kirchhoff equation with strong damping and additive noise is studied.

$$u_{tt} + M(\|D^m u\|^2)(-\Delta)^m u + \beta(-\Delta)^m u_t + \Delta g(u) = q(x)\dot{W}, \tag{1.1}$$

$$u(x,t) = 0, \quad \frac{\partial^i u}{\partial v^i} = 0, \quad i = 1,2,\cdots,m-1, \quad x \in D, \quad t \in [0,+\infty), \tag{1.2}$$

$$u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x). \tag{1.3}$$

Where $m > 1$, $\Delta g(u)$ is a second-order non-linear source term, M is a real-valued function, $\beta > 0$. $u = u(x,t)$ is a real-valued function on $D \times [0,+\infty)$, D is a bounded open set with smooth boundary on $R^n (n \in N)$. qdW describes an additive white noise. $W(t)$ is a one-dimensional bilateral Wiener process on probability space (Ω, F, P), $\Omega = \{\omega \in C(R,R) : \omega(0) = 0\}$, F is a Borel algebra generated by compact expansion on Ω and P is a probability measure.

Chuangliang Qin, Jinji Du (2016) have studied the stochastic low-order Kirchhoff equation with strong damping and additive noise.

$$du_t + (-\alpha\Delta u_t + \beta u_t - (1 + (\int_\Omega |Du|^2 dx)^p)\Delta u + g(u))dt = f(x)dt + q(x)dW(t), \; x \in D, t \in [0,+\infty) \tag{1.4}$$

$$u(x,0)\big|_{x\in\partial D} = 0, u(x,0) = u_0(x), u_t(x,0) = u_1(x). \tag{1.5}$$

By using Ornstein-Uhlenbeck process and isomorphic mapping method, the existence and uniqueness of solutions and the existence of random attractors for stochastic Kirchhoff equation with strong damping are obtained.

Guigui Xu, Libo Wang and Guoguang Lin (2017) discussed the nonautonomous stochastic wave equation with dispersion term and dissipation term.

$$u_{tt} - \Delta u - \alpha\Delta u_t - \beta u_{tt} + h(u)u_t + \lambda u + f(x,u) = g(x,t)u + \varepsilon u \cdot \frac{dW}{dt} \tag{1.6}$$

$$u(x,\tau) = u_0(x), \; u_t(x,\tau) = u_1(x). \tag{1.7}$$

The existence of random attractors for Nonautonomous stochastic wave equations with product white noise is obtained by using the uniform estimation of solutions and the technique of decomposing solutions in a region.

Meng Wang (2017)analyzed autonomous and non-autonomous stochastic wave equation with strong damping

$$u_{tt}-\alpha\Delta u_t+u_t+f(u)-\Delta u=g(x)\frac{dW}{dt},t\in[0,+\infty), \tag{1.8}$$

$$u(x,t)\big|_{x\in\partial U}=0,t\geq 0,\quad u(x,0)=u_0(x),u_t(x,0)=u_1(x),x\in U. \tag{1.9}$$

and

$$u_{tt}-\alpha\Delta u_t+u_t+f(u)-\Delta u=g(x)+cu\cdot\frac{dW}{dt},t\in[0,+\infty), \tag{1.10}$$

$$u(x,t)\big|_{x\in\partial U}=0,t\geq 0,\quad u(x,0)=u_0(x),u_t(x,\tau)=u_1(x),x\in U. \tag{1.11}$$

The existence of random attractors and the upper bound of fractal dimension are discussed. First, they deal with random terms by using Ornstein-Uhlenbeck process and weak solutions of the established equation. Then, the solutions of the equation are estimated and bounded random absorption sets are obtained. Finally, the system is asymptotically compact by using the tight embedding theorem, which proves the existence of random attractors.

Donghong Cai and Xiaoming Fan (2014) considered the dissipative KdV equation with multiplicative noise

$$du=(au_{xxx}+u_{xx}+\beta uu_{xx}+ru)dt=f(x)dt+budW(t),x\in D,t>0, \tag{1.12}$$

$$u(x,0)=u_{t_0}x,x\in D, \tag{1.13}$$

$$u(0,t)=u(L,t)=0. \tag{1.14}$$

The equation is transformed into a stochastic KdV-type equation without white noise by appropriate transformation. In the new equation, the sample can be regarded as a common parameter. By using the methods and techniques of determining the KdV-type equation and the slowly increasing property of Wiener process, the absorptivity and asymptotic compactness of the dynamic system determined by the new equation are obtained, thus proving the existence of the stochastic attractor of the new equation.The stochastic attractor of the dynamic system determined by the dissipative KdV equation of multiplicative white noise also exists.

As a deterministic system, stochastic dynamical systems have been extensively studied. Specific cases can be referred to in references(Qin, C. L., Du, J. J., &Lin, G. G.2017; Lin, G. G., Chen, L., &Wang, W.2017; Guo, B. L.2000; Guo, B. L., & Pu, X. K.2009; Lin, G. G.2011; Massatt, P.1988; Guo, B. L., &Wang, B. X.1995; Zhao, C. D., &Zhou, S. F.2009;Yin, J. Y., Li, Y. R., &Zhao, H. J.2013; Wang, R., &Li, Y. R.2012; Hao, H. J., &Zhou, S. F.2010; Cheng, Y. Y., &Li, Y. R.2012).

On the basis of the correlation of random attractors for some low-order Kirchhoff equation with additive noise stochastic studied by previous scholars, the existence and uniqueness of solutions for high-order Kirchhoff equation with strong damping stochastic with additive noise are discussed, and the existence of attractors for stochastic Kirchhoff equation with strong damping is proved. This paper is organized as follows. Section 2, introduces the assumptionsbasic and basic knowledge needed in this paper. Section 3, discuss the existence and uniqueness of solutions of stochastic high-order Kirchhoff equation with strong damping and additive noise, and prove the existence of random attractors.

2. Preliminaries

In this section, we introduce some basic assumptions and knowledge of stochastic dynamical systems required in this paper.

For the sake of narrative convenience, we introduce the following symbols:

$\nabla=D$, $H=L^2(\Omega)$, $H_0^m(\Omega)=H^m(\Omega)\cap H_0^1(\Omega)$, $H_0^{m+k}(\Omega)=H^{m+k}(\Omega)\cap H_0^1(\Omega)$ $E_k=H_0^{m+k}(\Omega)\times H_0^k(\Omega),(k=0,1,2,\cdots m)$.

The following norms and inner products are defined:

$$(u,v)=\int_\Omega uvdx,\qquad \|u\|=(u,u)^{\frac{1}{2}},\quad \forall u,v\in H_0^k(\Omega). \tag{2.1}$$

$$(y_1,y_2)_{E_k}=(D^{m+k}u_1,D^{m+k}u_2)+(D^kv_1,D^kv_2).\quad \forall y_i=(u_i,v_i)\in E_k\quad i=1,2.\ (k=1,2,\cdots m). \tag{2.2}$$

(H1) Assume that the Kirchhoff type stress term satisfies:

$$\varepsilon+1<\mu_0\le M(s)\le\mu_1.\quad 0<\varepsilon<\min\{\frac{\sqrt{4+2\beta\lambda_1^m}-2}{2},\frac{\mu_0+\sqrt{\mu_0^2-\frac{2C_0^2}{\lambda_1^{2m-2}}}}{\beta},\frac{1}{\beta-1}\}. \tag{2.3}$$

Where μ_0,μ_1 are constants, λ_1 is the first eigenvalue of $-\Delta$ with homogeneous Dirichlet boundary conditions on Ω.

(H2) Assume that the non-linear source term satisfies:

$$g(s)\in C^3(R)\text{and}\|g'(s)\|_\infty=C_0\le\frac{\sqrt{2}}{2}\lambda_1^{m-1}.\ (\lambda_1>0). \tag{2.4}$$

Let (Ω,F,P) be a probabilistic space and define a family of measures-preserving and ergodic transformations $\{\theta_t,t\in R\}$:

$$\theta_t w(\cdot)=w(\cdot+t)-w(t), \tag{2.5}$$

$(\Omega,F,P,(\theta_t)_{t\in R})$ is an ergodic metric dynamical system.

Let $(X,\|\cdot\|_X)$ be a complete separable metric space and $B(X)$ be a *Borel* σ-algebra over X.

Definition 1[Chuangliang Qin, Jinji Du, &Guoguang Lin(2017)]Let $(\Omega,F,P,(\theta_t)_{t\in R})$ be a metric dynamical system, if $(B(R^+)\times F\times B(X),B(X))$ measurable mapping

$$S:R^+\times\Omega\times X\to X,\quad (t,w,x)\mapsto S(t,w,x). \tag{2.6}$$

satisfaction

(1) for all $s,t\ge 0$ and $w\in\Omega$,mapping $S(t,w):=S(t,w,\cdot)$ satisfies

$$S(0,w)=id,\quad S(t+s,w)=S(t,\theta_s w)\circ S(s,w), \tag{2.7}$$

(2) for each $w\in\Omega$,mapping $(t,w)\mapsto S(t,w,x)$ is continuous.

S is a continuous stochastic dynamical system on $(\Omega,F,P,(\theta_t)_{t\in R})$.

Definition 2[Chuangliang Qin. et al(2017)] It is said that random set $B(w)\subset X$ is tempered.If for $w\in\Omega$, $\beta\ge 0$, there is

$$\lim_{|s|\to\infty}\inf e^{-\beta s}d(B(\theta_{-s}w))=0. \tag{2.8}$$

Where $d(B)=\sup_{x\in B}\|x\|_X$,for $\forall x\in X$.

Definition 3[Chuangliang Qin. et al(2017)]Record $D(w)$ as the set of all random sets on X, a random set $B(w)$ is called an absorption set on $D(w)$.If for any $B(w)\in D(w)$ and $P_{a.e.w}\in\Omega$, there exists $T_{B(w)}>0$ such that

$$S(t,\theta_{-t}w)(B(\theta_{-t}w))\subset B_0(w). \tag{2.9}$$

Definition 4[Chuangliang Qin. et al(2017)]A random set $A(w)$ is called a random attractor on X for continuous stochastic dynamical system $S(t)$.If random set $A(w)$ satisfies

(1) $A(w)$ is a random compact set;

(2) $A(w)$ is invariant set $D(w)$,that is, to arbitrary $t>0,S(t,w)A(w)=A(\theta_t w)$;

(3) $A(w)$ attracts every sets in $D(w)$,that is, for any $B(w)\in D(w)$ and $P_{a.e.w}\in\Omega$, we have the limit formula

$$\lim_{t\to\infty}d(S(t,\theta_{-t}w)B(\theta_{-t}w),A(w))=0. \tag{2.10}$$

Where $d(A,B)=\sup_{x\in A}\inf_{y\in B}\|x-y\|_H$ is the Hausdorff semi-distance.(Here $A,B\subseteq H$).

Theorem 1[Chuangliang Qin. et al(2017)]Let random set $B(w)\in D(w)$ be the random absorption set of stochastic dynamical system $(S(t,w))_{t\ge0}$,and random set $B(w)$ satisfies

(1) random set $B(w)$ is a closed set on Hilbert space;

(2) for $P_{a.e.w}\in\Omega$, random set $B(w)$ satisfies the following asymptotic compactness conditions

for arbitrary sequence $x_n \in S(t_n, \theta_{-t_n} w) B_0(\theta_{-t_n} w)$, $t_n \to +\infty$,there is a convergent subsequence in space X , Then the stochastic dynamical system $(S(t,w))_{t\geq 0}$ has a unique global attractor.

$$A(w) = \bigcap_{\tau \geq t(w)} \overline{\bigcup_{t\geq\tau} S(t, \theta_{-t} w) B_0(\theta_{-t} w)}. \tag{2.11}$$

Ornstein-Uhlenbeck process [Chuangliang Qin. et al(2017)]

In this section, space is introduced. Ornstein-Uhlenbeck process on $H_0^{m+k}(\Omega)$,where O-U process is given by Wiener process on measurement system $(\Omega, F, P, (\theta_t)_{t\in R})$.

Set $z(\theta_t w) = -\alpha \int_{-\infty}^{0} e^{\alpha\tau} \theta_t w(\tau) d\tau,$ where $t \in R.$ It can be seen that for arbitrary $t \geq 0,$ the stochastic process $z(\theta_t w)$ satisfies the Itô equation.

$$dz + \alpha z dt = dW(t). \tag{2.12}$$

According to the properties of O-U process, there exists a probability measure P , $\theta_t -$invariant set $\Omega_0 \subset \Omega;$ and the above stochastic process $z(\theta_t w) = -\alpha \int_{-\infty}^{0} e^{\alpha\tau} \theta_t w(\tau) d\tau,$ satisfies the following properties

(1) for any given $w \subset \Omega_0,$ the mapping $s \to z(\theta_s w)$ is a continuous mapping;

(2) random variable $\|z(w)\|$ is slowly increasing;

(3) there is a slowly increasing $r(w) > 0,$ so that $\|z(\theta_t w)\| + \|z(\theta_t w)\|^2 \leq r(\theta_t w) \leq r(w) e^{\frac{\alpha}{2}|t|};$

(4) $\lim\limits_{t\to\infty} \frac{1}{t} \int_0^t |z(\theta_t w)|^2 d\tau = \frac{1}{2\alpha};$

(5) $\lim\limits_{t\to\infty} \frac{1}{t} \int_0^t |z(\theta_t w)| d\tau = \frac{1}{\sqrt{\pi\alpha}}.$

3. Existence of Random Attractor Family

In this section, we discuss the existence and uniqueness of solutions of stochastic high-order Kirchhoff equation with strong damping and additive noise, and prove the existence of random attractors.

For convenience, equation (1.1)-(1.3) can be transformed into

$$\begin{cases} du = u_t dt \\ du_t + [M(\|A^{\frac{m}{2}} u\|^2) A^m u + \beta A^m u_t + \Delta g(u)] dt = q(x) dW(t), \; t \in [0, +\infty) \\ u(x,0) = u_0(x), u_t(x,0) = u_1(x), x \in \Omega. \end{cases} \tag{3.1}$$

where $A = -\Delta$.

Let $\varphi = (u, v)^T$, $v = u_t + \varepsilon u$, then the problem (3.1) can be simplified to

$$\begin{cases} d\varphi + L\varphi dt = F(\theta_t \omega, \varphi), \\ \varphi_0(\omega) = (u_0, u_1 + \varepsilon u_0)^T. \end{cases} \tag{3.2}$$

where $\varphi = \begin{pmatrix} u \\ v \end{pmatrix}$, $L = \begin{pmatrix} \varepsilon I & -I \\ ((M(\|A^{\frac{m}{2}} u\|^2) - \beta\varepsilon) A^m + \varepsilon^2) I & (\beta A^m - \varepsilon) I \end{pmatrix}$, $F(\theta_t \omega, \varphi) = \begin{pmatrix} 0 \\ -\Delta g(u) + q(x) dW(t) \end{pmatrix}.$

If $z = v - q(x)\delta(\theta_t \omega)$, then equestion (3.1) can be written as

$$\begin{cases} \psi_t + L\psi = \overline{F}(\theta_t \omega, \psi), \\ \psi_0(\omega) = (u_0, u_1 + \varepsilon u_0 - q(x)\delta(\theta_t \omega))^T. \end{cases} \tag{3.3}$$

Where $\psi = \begin{pmatrix} u \\ z \end{pmatrix}$, $L = \begin{pmatrix} \varepsilon I & -I \\ ((M(\|A^{\frac{m}{2}}u\|^2) - \beta\varepsilon)A^m + \varepsilon^2)I & (\beta A^m - \varepsilon)I \end{pmatrix}$, $\overline{F}(\theta_t\omega, \psi) = \begin{pmatrix} q(x)\delta(\theta_t\omega) \\ -\Delta g(u) + (\varepsilon + 1 - \beta A^m)q(x)\delta(\theta_t\omega) \end{pmatrix}$.

Lemma 1 Assuming that the Kirchhoff stress term and the non-linear term satisfy the conditions (H1), (H2), respectively. $f \in H$, $(u_0, v_0) \in E_0$, Then the initial boundary value problem (1.1) - (1.3) has smooth solutions $(u, v) \in E_0$ and satisfies the following inequalities

$$\|(u,v)\|_{E_0}^2 = \|D^m u\|^2 + \|v\|^2 \le (\|v_0\|^2 + \mu\|D^m u_0\|^2 + \varepsilon^2\|u_0\|^2)e^{-\alpha_1 t} + \frac{C_1}{\alpha_1}(1 - e^{-\alpha_1 t}). \tag{3.4}$$

Where $v = u_t + \varepsilon u$, $\alpha_1 = \min\left\{a_1, \frac{a_2}{\mu}, 2\varepsilon\right\}$, there exiet $C(R_1)$ and $t = t_1(\Omega) > 0$, such that

$$\|(u,v)\|_{E_0}^2 = \|D^m u\|^2 + \|v\|^2 \le C(R_1). \qquad (t > t_1). \tag{3.5}$$

Proof Let $v = u_t + \varepsilon u$, v and the two sides of equation (1.1) are used as inner products, we can get

$$(u_{tt} + M(\|D^m u\|^2)(-\Delta)^m u + \beta(-\Delta)^m u_t + \Delta g(u), v) = (q(x)\dot{W}, v). \tag{3.6}$$

From Holder's inequality, Young's inequality and Poincare's inequality, we can get

$$(u_{tt}, v) = \frac{1}{2}\frac{d}{dt}\|v\|^2 - \varepsilon\|v\|^2 + \frac{\varepsilon^2}{2}\frac{d}{dt}\|u\|^2 + \varepsilon^3\|u\|^2. \tag{3.7}$$

From hypothesis (H1), we can get

$$(M(\|D^m u\|^2)(-\Delta)^m u, v) = (M(\|D^m u\|^2)\frac{1}{2}\frac{d}{dt}\|D^m u\|^2 + \varepsilon M(\|D^m u\|^2)\|D^m u\|^2. \tag{3.8}$$

The following two aspects are discussed

①When $\frac{1}{2}\frac{d}{dt}\|D^m u\|^2 \ge 0$, it can be obtained by hypothesis (H1).

$$(M(\|D^m u\|^2)\frac{1}{2}\frac{d}{dt}\|D^m u\|^2 + \varepsilon M(\|D^m u\|^2)\|D^m u\|^2 \ge \frac{\mu_0}{2}\frac{d}{dt}\|D^m u\|^2 + \varepsilon\mu_0\|D^m u\|^2. \tag{3.9}$$

Let $\mu = \mu_0$, there are

$$(M(\|D^m u\|^2)(-\Delta)^m u, v) \ge \frac{\mu}{2}\frac{d}{dt}\|D^m u\|^2 + \varepsilon\mu_0\|D^m u\|^2. \tag{3.10}$$

②When $\frac{1}{2}\frac{d}{dt}\|D^m u\|^2 < 0$, it can be obtained by hypothesis (H1).

$$(M(\|D^m u\|^2)\frac{1}{2}\frac{d}{dt}\|D^m u\|^2 + \varepsilon M(\|D^m u\|^2)\|D^m u\|^2 \ge \frac{\mu_1}{2}\frac{d}{dt}\|D^m u\|^2 + \varepsilon\mu_0\|D^m u\|^2. \tag{3.11}$$

Let $\mu = \mu_1$, there are

$$(M(\|D^m u\|^2)(-\Delta)^m u, v) \ge \frac{\mu}{2}\frac{d}{dt}\|D^m u\|^2 + \varepsilon\mu_0\|D^m u\|^2. \tag{3.12}$$

Comprehensive (3.9) - (3.12) , we have

$$(M(\|D^m u\|^2)(-\Delta)^m u, v) \ge \frac{\mu}{2}\frac{d}{dt}\|D^m u\|^2 + \varepsilon\mu_0\|D^m u\|^2. \tag{3.13}$$

$$(\beta(-\Delta)^m u_t, v) \ge \frac{\beta\lambda_1^{m-1}}{4}\|Dv\|^2 + \frac{\beta\lambda_1^m}{4}\|v\|^2 - \frac{\beta\varepsilon^2}{2}\|D^m u\|^2. \tag{3.14}$$

From hypothesis (H2), we can get

$$\left|(\Delta g(u),v)\right| \ge -\left\|g'(u)\right\|_\infty \cdot \|Du\| \cdot \|Dv\| \ge -\frac{\beta\lambda_1^{m-1}}{4}\|Dv\|^2 - \frac{C_0^2}{\beta\lambda_1^{2(m-1)}}\|D^m u\|^2. \tag{3.15}$$

$$(q(x)\dot{W},v) \le \left\|q(x)\dot{W}\right\| \cdot \|v\| \le \frac{1}{2\varepsilon^2}\left\|q(x)\dot{W}\right\|^2 + \frac{\varepsilon^2}{2}\|v\|^2. \tag{3.16}$$

From the above, we have

$$\frac{d}{dt}\left[\|v\|^2 + \mu\|D^m u\|^2 + \varepsilon^2\|u\|^2\right] + \left(\frac{\beta\lambda_1^m}{2} - 2\varepsilon - \varepsilon^2\right)\|v\|^2 + \left(2\varepsilon\mu_0 - \beta\varepsilon^2 - \frac{2C_0^2}{\beta\lambda_1^{2(m-1)}}\right)\|D^m u\|^2 + 2\varepsilon^3\|u\|^2 \le \frac{\left\|q(x)\dot{W}\right\|^2}{\varepsilon^2} = C_1. \tag{3.17}$$

From hypothesis (H1), (H2), we can get

$$a_1 = \frac{\beta\lambda_1^m}{2} - 2\varepsilon - \varepsilon^2 \ge 0, \qquad a_2 = 2\varepsilon\mu_0 - \beta\varepsilon^2 - \frac{2C_0^2}{\beta\lambda_1^{2(m-1)}} \ge 0. \tag{3.18}$$

Then take $\alpha_1 = \min\left\{a_1, \frac{a_2}{\mu}, 2\varepsilon\right\}$, then (3.17) can be transformed into

$$\frac{d}{dt}(\|v\|^2 + \mu\|D^m u\|^2 + \varepsilon^2\|u\|^2) + \alpha_1(\|v\|^2 + \mu\|D^m u\|^2 + \varepsilon^2\|u\|^2) \le C_1. \tag{3.19}$$

From Gronwall's inequality

$$(\|v\|^2 + \mu(\|D^m u\|^2) + \varepsilon^2\|u\|^2) \le (\|v_0\|^2 + \mu(\|D^m u_0\|^2) + \varepsilon^2\|u_0\|^2)e^{-\alpha_1 t} + \frac{C_1}{\alpha_1}(1 - e^{-\alpha_1 t}). \tag{3.20}$$

By hypothesis (H1), there are

$$\|(u,v)\|_{E_0}^2 = \|D^m u\|^2 + \|v\|^2 \le (\|v_0\|^2 + \mu\|D^m u_0\|^2 + \varepsilon^2\|u_0\|^2)e^{-\alpha_1 t} + \frac{C_1}{\alpha_1}\left(1 - e^{-\alpha_1 t}\right). \tag{3.21}$$

There exiet $C(R_1)$ and $t = t_1(\Omega) > 0$, such that

$$\|(u,v)\|_{E_0}^2 = \|D^m u\|^2 + \|v\|^2 \le C(R_1). \qquad (t > t_1). \tag{3.22}$$

Therefore, Lemma 1 is proved.

Lemma 2 Let $E_k = H_0^{m+k}(\Omega) \times H_0^k(\Omega), (k = 1,2,\cdots m)$, for $\forall y = (y_1, y_2)^T \in E_k$, we have

$$(Ly, y)_{E_k} \ge k_1\|y\|_{E_k}^2 + k_2\|D^{m+k} y_2\|^2. \tag{3.23}$$

Where $k_1 = \min\left\{\frac{\beta\varepsilon + \varepsilon - \varepsilon^2\lambda_1^{-m}}{2\beta}, \frac{\beta\lambda_1^m - \beta\varepsilon^2 - 2\varepsilon}{2}\right\}$, $k_2 = \frac{\beta(1 - \beta\varepsilon + \varepsilon)}{2}$.

Proof $$L = \begin{pmatrix} \varepsilon I & -I \\ ((M(\|A^{\frac{m}{2}}u\|^2) - \beta\varepsilon)A^m + \varepsilon^2)I & (\beta A^m - \varepsilon)I \end{pmatrix}, \quad \forall y = (y_1, y_2)^T \in E_k, \tag{3.24}$$

From hypothesis (H1), Holder inequality, Young inequality and Poincare inequality

$$(Ly, y)_{E_k} = (D^{m+k}(\varepsilon y_1 - y_2), D^{m+k} y_1) + (D^k(((M(\|A^{\frac{m}{2}}u\|^2) - \beta\varepsilon)A^m + \varepsilon^2)y_1 + (\beta A^m - \varepsilon)y_2), D^k y_2)$$

$$= \varepsilon\|D^{m+k} y_1\| - (D^{m+k} y_2, D^{m+k} y_1) + (M(\|A^{\frac{m}{2}}u\|^2)D^{m+k} y_1, D^{m+k} y_2) - \beta\varepsilon(D^{m+k} y_1, D^{m+k} y_2)$$
$$+ \varepsilon^2(D^k y_1, D^k y_2) + \beta\|D^{m+k} y_2\| - \varepsilon\|D^k y_2\|^2$$

$$= \varepsilon\|D^{m+k}y_1\| - (D^{m+k}y_2, D^{m+k}y_1) + (M(\|A^{\frac{m}{2}}u\|^2) - \varepsilon)(D^{m+k}y_1, D^{m+k}y_2) + (\varepsilon - \beta\varepsilon)(D^{m+k}y_1, D^{m+k}y_2)$$
$$+ \varepsilon^2(D^k y_1, D^k y_2) + \beta\|D^{m+k}y_2\| - \varepsilon\|D^k y_2\|^2$$
$$\geq \varepsilon\|D^{m+k}y_1\| - (D^{m+k}y_2, D^{m+k}y_1) + (D^{m+k}y_1, D^{m+k}y_2) - (\beta\varepsilon - \varepsilon)(D^{m+k}y_1, D^{m+k}y_2) + \varepsilon^2(D^k y_1, D^k y_2)$$
$$+ \beta\|D^{m+k}y_2\| - \varepsilon\|D^k y_2\|^2$$
$$\geq \varepsilon\|D^{m+k}y_1\| - \frac{(\beta-1)\varepsilon}{2\beta}\|D^{m+k}y_1\|^2 - \frac{\beta(\beta-1)\varepsilon}{2}\|D^{m+k}y_2\|^2 - \frac{\varepsilon^2}{2\beta}\|D^k y_1\|^2 - \frac{\beta\varepsilon^2}{2}\|D^k y_2\|^2 + \beta\|D^{m+k}y_2\| - \varepsilon\|D^k y_2\|^2$$
$$\geq \frac{\beta\varepsilon+\varepsilon}{2\beta}\|D^{m+k}y_1\|^2 + \frac{\beta(1-\beta\varepsilon+\varepsilon)}{2}\|D^{m+k}y_2\|^2 - \frac{\varepsilon^2\lambda_1^{-m}}{2\beta}\|D^{m+k}y_1\|^2 + \frac{\beta\lambda_1^m - \beta\varepsilon^2 - 2\varepsilon}{2}\|D^k y_2\|^2$$
$$= \frac{\beta\varepsilon+\varepsilon-\varepsilon^2\lambda_1^{-m}}{2\beta}\|D^{m+k}y_1\|^2 + \frac{\beta(1-\beta\varepsilon+\varepsilon)}{2}\|D^{m+k}y_2\|^2 + \frac{\beta\lambda_1^m - \beta\varepsilon^2 - 2\varepsilon}{2}\|D^k y_2\|^2$$

$$\geq k_1(\|D^{m+k}y_1\|^2 + \|D^k y_2\|^2) + k_2\|D^{m+k}y_2\|^2$$

$$= k_1\|y\|_{E_k}^2 + k_2\|D^{m+k}y_2\|^2. \tag{3.25}$$

Therefore, Lemma 2 is proved.

Lemma 3 Let φ be a solution of the problem (3.2), then there exists a bounded random compact set $\tilde{B}_{0k}(w) \in D(E_k)$, so that for any temperedly random set $B_k(w) \in D(E_k)$, there exists a random variable $T_{B_k(w)} > 0$, such that

$$\varphi(t, \theta_t w) B_k(\theta_{-t} w) \subset \tilde{B}_{0k}(w), \quad \forall t \geq T_{B_k(w)}, w \in \Omega. \tag{3.26}$$

Proof Let ψ be a solution of the problem (3.3). By taking the inner product of $\psi = (u,z)^T \in E_k$ and equation (3.3) on E_k, we can get.

$$\frac{1}{2}\frac{d}{dt}\|\psi\|_{E_k}^2 + (L\psi, \psi) = (\overline{F}(\theta_t w, \psi), \psi). \tag{3.27}$$

From Lemma 2

$$(L\psi, \psi)_{E_k} \geq k_1\|\psi\|_{E_k}^2 + k_2\|D^{m+k}z\|^2, \tag{3.28}$$

$$(\overline{F}(\theta_t w, \psi), \psi) = (D^{m+k}q(x)\delta(\theta_t w), D^{m+k}u) + (D^k(-\Delta g(u) + (\varepsilon+1-\beta A^m)q(x)\delta(\theta_t w)), D^k z). \tag{3.29}$$

$$(D^{m+k}q(x)\delta(\theta_t w), D^{m+k}u) \leq \frac{\varepsilon}{2}\|D^{m+k}u\|^2 + \frac{\lambda_1^{-m}}{2\varepsilon}\|A^{m+\frac{k}{2}}q(x)\|^2|\delta(\theta_t w)|^2, \tag{3.30}$$

$$(D^k \varepsilon q(x)\delta(\theta_t w)), D^k z) \leq \frac{\varepsilon\lambda_1^{-m}}{2}\|D^{m+k}z\|^2 + \frac{\varepsilon}{2}\|D^k q(x)\|^2|\delta(\theta_t w)|^2, \tag{3.31}$$

$$(D^k(1-\beta A^m)q(x)\delta(\theta_t w), D^k z) \leq \frac{\varepsilon\lambda_1^{-m}}{2}\|D^{m+k}z\|^2 + \frac{1}{2\varepsilon}(\|D^k q(x)\|^2 + \beta^2\|A^{m+\frac{k}{2}}q(x)\|^2)|\delta(\theta_t w)|^2. \tag{3.32}$$

According to hypothesis (H2), lemma 1 can be obtained

$$(D^k(-g\Delta(u)), D^k z) \leq \frac{1}{2}\|g''(u)\|_\infty^2\|Du\|_4^4 + \frac{1}{2}\|g'(u)\|_\infty^2\|\Delta u\|^2 + \frac{1}{2}\|D^{2k}z\|^2. \tag{3.33}$$

By interpolating inequalities, there are

$$\|Du\|_{L^4} \leq C_2\|D^m u\|_{L^2}^{\frac{n+4}{4m}}\|u\|_{L^2}^{\frac{4m-n-4}{4m}}. \qquad \begin{cases} 1 \leq r \leq \dfrac{n}{n-2m} \\ 1 \leq r \leq \infty \end{cases} \tag{3.34}$$

$$\left(D^k(-\Delta g(u)), D^k z\right) \leq C_3\left(\|g''(u)\|_\infty, \|g'(u)\|_\infty, \|D^m u\|\right) + \frac{1}{2}\|D^k z\|^2 \leq C_3 + \frac{\lambda_1^{-(m-k)}}{2}\|D^{m+k}z\|^2. \tag{3.35}$$

From the above, we have

$$\frac{d}{dt}\|\psi\|_{E_k}^2+2k_1\|\psi\|_{E_k}^2+(2k_2-2\varepsilon\lambda_1^{-m}-\lambda_1^{-(m-k)})\|D^{m+k}z\|^2\le\varepsilon\|D^{m+k}u\|^2+2C_3+\frac{\beta^2+\lambda_1^{-m}}{\varepsilon}\left\|A^{m+\frac{k}{2}}q(x)\right\|^2|\delta(\theta_t w)|^2$$

$$+(\varepsilon+\frac{1}{\varepsilon})\|D^k q(x)\|^2|\delta(\theta_t w)|^2. \tag{3.36}$$

where $\eta=2k_1$, $M=\frac{\beta^2+\lambda_1^{-m}}{\varepsilon}\left\|A^{m+\frac{k}{2}}q(x)\right\|^2+(\varepsilon+\frac{1}{\varepsilon})\|D^k q(x)\|^2$,we have

$$\frac{d}{dt}\|\psi\|_{E_k}^2+\eta\|\psi\|_{E_k}^2\le C_4+M|\delta(\theta_t w)|^2. \tag{3.37}$$

According to Gronwall inequation, $P_{a.e.w}\in\Omega$, we can get

$$\|\psi(t,w)\|_{E_k}^2\le e^{-\eta t}\|\psi_0(w)\|_{E_k}^2+\int_0^t e^{-\eta(t-r)}(C_4+M|\delta(\theta_r w)|^2)dr\ . \tag{3.38}$$

Because $\delta(\theta_t w)$ is tempered, and $\delta(\theta_t w)$ is continuous with respect to t, according to literature Massatt, P.(1988), a temper random variable $r_1:\Omega\to R^+$ can be obtained, so that for $\forall t\in R, w\in\Omega$, there is

$$|\delta(\theta_t w)|^2\le r_1(\theta_t w)\le e^{\frac{\eta}{2}|t|}r_1(w)\,. \tag{3.39}$$

Substituting w by $\theta_{-t}w$ in formula (3.38),we know

$$\|\psi(t,\theta_{-t}w)\|_{E_k}^2\le e^{-\eta t}\|\psi_0(\theta_{-t}w)\|_{E_k}^2+\int_0^t e^{-\eta(t-r)}(C_4+M|\delta(\theta_{r-t}w)|^2)dr. \tag{3.40}$$

Let $r-t=\tau$

$$\int_0^t e^{-\eta(t-r)}(C_4+M|\delta(\theta_{r-t}w)|^2)dr=\int_{-t}^0 e^{\eta\tau}(C_4+M|\delta(\theta_\tau w)|^2)d\tau\le\frac{C_4}{\eta}+\frac{2}{\eta}M\,r_1(w). \tag{3.41}$$

Because $\varphi_0(\theta_{-t}w)\in B_k(\theta_{-t}w)$ is tempered, and $|\delta(\theta_{-t}w)|$ is also tempered, hence we let

$$R_0^2(w)=\frac{C_4}{\eta}+\frac{2}{\eta}M\,r_1(w). \tag{3.42}$$

Then $R_0^2(w)$ is also tempered, $\hat{B}_{0k}=\{\psi\in E_k\big|\|\psi\|_{E_k}\le R_0(w)\}$ is called a random absorption set, because

$$\tilde{S}(t,\theta_{-t}w)\psi_0(\theta_{-t}w)=\varphi(t,\theta_{-t}w)(\psi_0(\theta_{-t}w)+(0,q(x)\delta(\theta_{-t}w))^T)-(0,q(x)\delta(\theta_{-t}w))^T. \tag{3.43}$$

so let

$$\tilde{B}_{0k}(w)=\{\varphi\in E_k\big|\|\varphi\|_{E_k}\le R_0(w)+\|D^k q(x)\delta(w)\|=\bar{R}_0(w)\}. \tag{3.44}$$

Then $\tilde{B}_{0k}(w)$ is the random absorption set of $\varphi(t,w)$, and $\tilde{B}_{0k}(w)\in D(E_k)$ .The proof is complete.

Lemma 4 When $k=m$, for $\forall B_m(w)\in D(E_m)$, let $\varphi(t)$ be the solution of equation (3.2) in initial value $\varphi_0=(u_0,u_1+\varepsilon u_0)^T\in B_m$. It can be decomposed into $\varphi=\varphi_1+\varphi_2$, where φ_1,φ_2 satisfy respectively

$$\begin{cases} d\varphi_1+L\varphi_1 dt=0,\\ \varphi_{01}(w)=(u_0,u_1+\varepsilon u_0)^T. \end{cases} \tag{3.45}$$

$$\begin{cases} d\varphi_2+L\varphi_2 dt=F(w,\varphi),\\ \varphi_{02}(w)=0. \end{cases} \tag{3.46}$$

Then

$$\|\varphi_1(t,\theta_{-t}w)\|_{E_m}^2 \to 0,(t\to\infty), \quad \forall\varphi_0(\theta_{-t}w)\in B_m(\theta_{-t}w), \tag{3.47}$$

and there exists a temper random radius $R_1(w)$, which satisfies for $w\in\Omega$.

$$\|\varphi_2(t,\theta_{-t}w)\|_{E_m} \le R_1(w). \tag{3.48}$$

Proof Let $\psi=\psi_1+\psi_2=(u_1,u_{1t}+\varepsilon u_1)^T+(u_2,u_{2t}+\varepsilon u_2-q(x)\delta(\theta_t w))^T$ be the solution of equation (3.3), then according to equation (3.45) and (3.46), we know that ψ_1,ψ_2 satisfy respectively

$$\begin{cases} \psi_{1t}+L\psi_1=0, \\ \psi_{01}=\psi_0=(u_0,u_1+\varepsilon u_0-q(x)\delta(\theta_t w))^T. \end{cases} \tag{3.49}$$

$$\begin{cases} \psi_{2t}+L\psi_2=\overline{F}(\psi_2,\theta_t w), \\ \psi_{02}=0. \end{cases} \tag{3.50}$$

By taking the inner product of equation $\psi_1=(u_1,u_{1t}+\varepsilon u_1)^T$ and equation (3.49) on E_m we can get

$$\frac{1}{2}\frac{d}{dt}\|\psi_1\|_{E_m}^2+(L\psi_1,\psi_1)=0. \tag{3.51}$$

According to lemma 2 and Gronwall inequality, we have

$$\|\psi_1(t,w)\|_{E_m}^2 \le e^{-2k_1t}\|\psi_{01}(w)\|_{E_m}^2. \tag{3.52}$$

Substituting w by $\theta_{-t}w$ in formula (3.52) , and because $\delta(\theta_{-t}w)\in B_m$ is tempered, then

$$\|\psi_1(t,\theta_{-t}w)\|_{E_m}^2 \le e^{-2k_1t}\|\psi_0(\theta_{-t}w)\|_{E_m}^2 \to 0,(t\to\infty), \quad \forall\psi_0(\theta_{-t}w)\in B_m. \tag{3.53}$$

Thus (3.47) is hold. The inner product of $\psi_2=(u_2,u_{2t}+\varepsilon u_2-q(x)\delta(\theta_t w))^T$ and equation (3.50) on E_m is obtained according to Lemma 1, Lemma 2 and Lemma 3

$$\frac{d}{dt}\|\psi_2\|_{E_m}^2+\eta\|\psi_2\|_{E_m}^2 \le C_4+M_1|\delta(\theta_t w)|^2. \tag{3.54}$$

Where $\eta=2k_1$, $M_1=\dfrac{\beta^2+\lambda_1^{-m}}{\varepsilon}\left\|A^{\frac{3m}{2}}q(x)\right\|^2+(\varepsilon+\dfrac{1}{\varepsilon})\|D^m q(x)\|^2$,

Substituting $\theta_{-t}w$ for w in Formula (3.54) is obtained by Gronwall inequality, we have

$$\|\psi_2(t,\theta_{-t}w)\|_{E_m}^2 \le e^{-\eta t}\|\psi_{02}(\theta_{-t}w)\|_{E_m}^2+\int_0^t e^{-\eta(t-r)}(C_4+M_1|\delta(\theta_{r-t}w)|^2)dr \le \frac{C_4}{\eta}+\frac{2}{\eta}M_1 r_1(w). \tag{3.55}$$

So there is a tempered random radius

$$R_1^2(w)=\frac{C_4}{\eta}+\frac{2}{\eta}M_1 r_1(w), \tag{3.56}$$

So that for $\forall w\in\Omega$, there is

$$\|\varphi_2(t,\theta_{-t}w)\|_{E_m} \le R_1(w). \tag{3.57}$$

Therefore, Lemma 4 is proved.

Lemma 5 The identified stochastic dynamic system $\{S(t,w),t\ge 0\}$ determined by equation (3.2) has a compact absorption set $K(w)\subset E_k$ C in $t=0, P_{a.e.w}\in\Omega$.

Proof Let $K(w)$ be a closed sphere with radius $R_1(w)$ in space E_k. According to embedding relation $E_k\subset E_0$, $K(w)$ is a compact set in E_k. For arbitrary temper random set $B_k(w)$ in E_k, for $\forall\varphi(t,\theta_{-t}w)\in B_k$, according to lemma 4,

$\varphi_2 = \varphi - \varphi_1 \in K(w)$, so for $\forall t \geq T_{B_k(w)} > 0$, we have

$$d_{E_k}(S(t,\theta_{-t} w) B_k(\theta_{-t} w), K(w)) = \inf_{\vartheta(t)\in K(w)} \left\|\varphi(t,\theta_{-t} w) - \vartheta(t)\right\|_{E_k}^2$$

$$\leq \left\|\varphi_1(t,\theta_{-t} w)\right\|_{E_k}^2$$

$$\leq e^{-2\eta t} \left\|\varphi_{01}(\theta_{-t} w)\right\|_{E_k}^2 \to 0, (t \to \infty). \quad (3.58)$$

Therefore, Lemma 5 is proved.

According to Lemma 1-Lemma 5, there are the following theorems.

Theorem 1 The random dynamical system $\{S(t,w), t \geq 0\}$ has a family of random attractors $A_k(w) \subset K(w) \subset E_k$, $w \in \Omega$ and there exists a tempered random set $K(w)$, so that $P_{a.e.w} \in \Omega$

$$A_k(w) = \bigcap_{t\geq 0, \tau \geq t} \overline{\bigcup S(t, \theta_{-\tau} w, K(\theta_{-\tau} w))}. \quad (3.59)$$

And $S(t,w) A_k(w) = A_k(\theta_t w)$. The proof is complete.

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