

## Modeling the Growth of Forage Legumes

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### Abstract

Modeling the growth curve of agricultural crops is of paramount importance so that management tasks such as fertilization and irrigation can be carried out at the appropriate time, increasing the vegetal yield. With this purpose, nonlinear models are commonly employed. The objective of this work was to fit some of the main nonlinear models that best describe the growth curve of some of the main species of forage legumes, namely: *Crotalaria juncea*, *Canavalia ensiformis*, *Cajanus cajan* and *Dolichos lablab* L. A randomized block experiment was conducted in field conditions between November 2015 and February 2016 in southeastern Goiás, Brazil. The variables plant height, stem diameter, fresh and dry mass were measured after 15, 30, 45, 60 and 90 days from sowing. The following models were fitted: Gompertz, Logistic, Brody and von Bertalanffy. The following goodness-of-fit criteria were calculated:  $R^2$  (normal and adjusted), AIC (Akaike Information Criterion) and absolute mean error. The growth curves of morphological variables are easier to model than the biomass curves. The von Bertalanffy and Gompertz models presented in general the best fit. The species *C. juncea* has an expressive biomass accumulation rate.

**Keywords:** fabaceae, Gompertz equation, growth curve, von Bertalanffy equation

### 1. Introduction

Forage legumes species have several uses, as human and animal food source and source of fertility for enrichment of soil. For the latter, they serve as nitrogen fixers contributing to soil fertility. They are relevant to pasture yield, making atmospheric N available to the soil-plant system, reducing expenses with nitrogen fertilization and improving the herd's food. Legume species produce large amounts of dry matter and have a high concentration of nutrients in the aerial part, have a deep and branched root system and are easily decomposed (Giacomini et al., 2003; Erasmo et al., 2004; Perin et al., 2007). The use of legumes is also a recommended practice for recovering degraded areas, as they protect the soil from erosion. Herbaceous and tree legumes may also be used in cropping systems to reduce soil erosion while also providing other products and services. They have the potential to contribute to soil erosion control because they have fast growth rates, high biomass production and some of them are drought-tolerant (Kaspar, Singer, Hatfield, & Sauer, 2011). Including sole crops of herbaceous species such as *Mucuna*, or tree hedgerows with mixtures of maize and grain legumes has the potential to reduce runoff and soil erosion in smallholder farming (Muoni et al., 2019).

To carry out managements such as fertilization and irrigation in the appropriate timing and to identify possible problems in the development of crops, a specific study of the growth phase is recommended. Plant growth dynamics can be described by means of mathematical formulas, making it possible to evaluate some parts of the plant in the final growth, taking the advantage of obtaining information at regular intervals without the need for sophisticated equipment, once the information for these analyzes is, in general, the mass of the dry matter and the size of the photosynthesizing apparatus. Nonlinear regression models are usually indicated. The nonlinear regression analysis has economic advantages in agriculture because it allows knowing the speed at which production increment occurs. Therefore, the most relevant models for measuring biological growth have been the function Brody, Von Bertalanffy, Gompertz, logistical, Weibull, as cited by Koya and Goshu (2013). However, Parks (1982) shows that simulation studies indicate that growth functions may adjust incorrectly because they are flexible, thus recommending care when selecting models.

In relation to linear models, the nonlinear models have some advantages such as the origin of the theory and biological, physical and chemical principles, being generally more parsimonious, that is, requiring a smaller number of parameters to give satisfactory modeling (Miguez, Archontoulis, & Dokoohaki, 2018). However, there are disadvantages: the parameter estimation process requires iterative searching algorithms (e.g., Gauss-Newton, Golub-Pereyra) and the use of initial estimates for the parameters; they allow only approximations rather than exact inferences; they require a solid understanding of the phenomenon under study (Schabenberger & Pierce, 2002).

Nonlinear models have been used in studies whose objective is to describe the growth curve and the accumulation of biomass of crops such as garlic (Reis et al., 2014), coffee (Fernandes, Pereira, Muniz, & Savian, 2014), banana (Maia, Siqueira, Silva, Peternelli, & Salomão, 2009) and onion (Pôrto, Cecílio-Filho, May, & Barbosa, 2006). However, none involving the legumes under study. In this sense, the objective of this work was to fit and to identify, among some of the main nonlinear models, those that best describe the growth of some of the main species of forage legumes.

## 2. Method

### 2.1 Field Experiment

The study was conducted from November 2015 to February 2016, in an experimental area at southeastern, Goiás, Brazil, at the geographical coordinates: 17°29'23" South and 48°13'02" West, with average altitude of 807 m.

A randomized complete block experiment with four replications was installed. Four species of forage legumes were sown in lines spaced 0.5 m, constituting plots of dimension 3 × 5 m. Species: *Crotalaria juncea* cv. IAC-KR1, *Canavalia ensiformis*, *Cajanus cajan* cv. IAPAR 43—Aratã and *Dolichos lablab* cv. Rongai. From selecting three plants in each plot, the following variables were measured: stem diameter or primary branch (cm), plant height (cm), dry mass (g) and fresh mass (g), in six periods: 15, 30, 45, 60, 75 and 90 days after sowing. The dry mass was obtained after drying the biomass in an oven at 110 °C for 24 hours.

### 2.2 Data Analysis and Modeling

Data were submitted to repeated measures analysis of variance. The least squares means were then estimated for each period. Afterwards, the growth models in Table 1 were fitted.

Table 1. Nonlinear regression models for growth curves

Model	Function	Eq.
Gompertz	$y_i = \beta_1 e^{-\beta_2 \exp(-\beta_3 x_i)} + \varepsilon_i$	(1)
Logistic	$y_i = \beta_1 (1 + \beta_2 e^{-\beta_3 x_i})^{-1} + \varepsilon_i$	(2)
Brody	$y_i = \beta_1 (1 - \beta_2 e^{-\beta_3 x_i}) + \varepsilon_i$	(3)
von Bertalanffy	$y_i = \beta_1 (1 - \beta_2 e^{-\beta_3 x_i})^3 + \varepsilon_i$	(4)

*Note.*  $\beta_1$  represents the maximum expected for the response, that is, the asymptote;  $\beta_2$  does not present practical interpretation, being just a parameter of fitting;  $\beta_3$  is related to the speed of growth;  $x_i$  represents the  $i$ -th time;  $y_i$  is the value of the response variable (least squares mean) in time  $x_i$ , and  $\varepsilon$  represents the random error.

In order to select the best model, the following goodness-of-fit criteria (Table 2) were used: multiple coefficient of determination ( $R^2$ ), adjusted multiple coefficient of determination ( $R^2_{aj}$ ), Akaike's Information Criterion (AIC) and the Absolute Mean Error (AME).

Table 2. Goodness-of-fit criteria for nonlinear regression models

Criterion	Function	Eq.
Coefficient of determination	$R^2 = 1 - \frac{SQR}{SQT}$	(1)
Adjusted coefficient of determination	$R^2_{aj} = \frac{R^2(n-1)-p}{n-p-1}$	(2)
Akaike Information Criterion	$AIC = 2[p - \log L(\hat{\theta})]$	(3)
Absolute Mean Error	$AME = \frac{\sum_{i=1}^n  y_i - \hat{y}_i }{n}$	(4)

Note.  $SQR$  = the residual sum of the squares;  $SQT$  = the total sum of squares. The higher the coefficient of determination, the better the fit of the model;  $p$  is the number of model parameters and  $L(\hat{\theta})$  is the maximum of the likelihood function; The lower the  $AIC$ , the better the adjustment.

The coefficient of determination ( $R^2$ ) represents the proportion of the variation of the response variable that is explained by the variation of the predictor variable (the time in this case). The AIC (Akaike, 1974) is a value for comparing the goodness-of-fit of models based on the maximum of the likelihood function, which is dependent on the number of observations ( $n$ ) and parameters ( $p$ ) of the model, as well as the adjusted coefficient of determination.

All analyzes were performed with software R version 3.2.4 (www.R-project.org). For fitting models, the package 'easynls' was used. The iterative algorithm used for obtaining least squares estimates was Gauss-Newton, limited to 6.000 iterations.

### 3. Results and Discussion

According to Bassanezi (2002), the mathematical model is the interpretation of something real, used to understand natural phenomena and the way in which changes are made to it. The model chosen is based on the nature of the phenomenon, so it is necessary to study the model that best suits each species under study and the parameters observed.

Goodness-of-fit indicators of the models for each variable in each species are presented in Table 3, highlighting in bold those that indicate the highest degree of adjustment. In general, for morphological variables (height and diameter of the primary branch), there was more convergence, that is, those variables allowed easier fitting, excepting for the Brody model, which converged only for *C. juncea* and *D. lablab* height. Similar results were found by Reis et al. (2014), identifying that the Gompertz and Logistic models converged for all morphological and production variables of garlic. The Brody model only converged for one variable, dry mass.

The adjusted coefficient of determination ( $R^2_{aj}$ ) for aerial fresh and dry matter weight were above 0.93 for all four species. Batista et al. (2013) achieved in their work a good performance in the adjustment of the Gompertz and Logistic models for *Saccharum officinarum*. The values of  $R^2_{aj}$  were higher than 0.92. They argue that these values signal that the models have been able to account for most of the fresh mass accumulation.

The values of coefficient of determination were above 0.94 in all combinations of variables and species. Reis et al. (2014) and Puiatti et al. (2013) observed that this is a common feature in nonlinear models, as they are built specifically to describe the biological phenomena. Thus, Oliveira et al. (2000) highlight the importance of using other adjustment criterion.

It was observed that the four criteria are concordant with each other, except for little variations for dry matter and stem diameter of *C. ensiformis* and *D. lablab*, respectively. In summary, the choice of the model was based on the largest number of indicators in agreement with each other.

Table 3. Goodness-of-fit of nonlinear models for morpho-agronomic variables of *C. juncea* (CJ), *C. ensiformis* (CE), *C. cajan* (CC) and *D. lablab* (DL), according to the following criteria: AIC (Akaike’s information criterion), AME (mean absolute error), R<sup>2</sup> (coefficient of determination) and R<sup>2</sup><sub>aj</sub> (adjusted coefficient of determination)

Crop	Model	Height				Stem diameter				Fresh matter				Dry matter			
		AIC	AME	R <sup>2</sup>	R <sup>2</sup> <sub>aj</sub>	AIC	AME	R <sup>2</sup>	R <sup>2</sup> <sub>aj</sub>	AIC	AME	R <sup>2</sup>	R <sup>2</sup> <sub>aj</sub>	AIC	AME	R <sup>2</sup>	R <sup>2</sup> <sub>aj</sub>
CJ	Gompertz	<b>42.75</b>	<b>3.67</b>	<b>0.9973</b>	<b>0.9955</b>	<b>13.51</b>	<b>0.36</b>	<b>0.9958</b>	<b>0.9930</b>	51.96	8.49	0.9981	0.9968	54.11	8.60	0.9770	0.9616
	Logístico	43.48	4.03	0.9969	0.9949	15.94	0.39	0.9937	0.9895	<b>33.79</b>	<b>1.69</b>	<b>0.9999</b>	<b>0.9998</b>	55.71	10.58	0.9699	0.9498
	Brody	55.84	11.49	0.9761	0.9601	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
	von Bertalanffy	46.10	4.59	0.9953	0.9921	15.00	0.39	0.9946	0.9911	64.85	22.95	0.9834	0.9723	<b>53.50</b>	<b>8.53</b>	<b>0.9792</b>	<b>0.9653</b>
CE	Gompertz	46.61	5.70	0.9836	0.9727	17.81	0.4298	0.9711	0.9518	<b>57.89</b>	<b>14.12</b>	<b>0.9936</b>	<b>0.9894</b>	52.69	<b>7.55</b>	0.9621	0.9369
	Logístico	48.88	6.35	0.9761	0.9601	18.50	0.46	0.9676	0.9459	60.37	16.63	0.9904	0.9840	53.56	8.67	0.9562	0.9270
	Brody	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
	von Bertalanffy	<b>45.95</b>	<b>5.41</b>	<b>0.9853</b>	<b>0.9755</b>	<b>17.63</b>	<b>0.4205</b>	<b>0.9719</b>	<b>0.9532</b>	64.13	20.71	0.9820	0.9701	<b>52.38</b>	7.68	<b>0.9640</b>	<b>0.9400</b>
CC	Gompertz	53.13	9.21	0.9719	0.9532	17.41	0.48	0.9840	0.9734	42.61	3.36	0.9978	0.9963	41.55	2.98	0.9829	0.9714
	Logístico	54.94	11.37	0.9620	0.9367	19.06	0.5333	0.9790	0.9650	47.24	5.16	0.9952	0.9920	42.20	3.23	0.9809	0.9682
	Brody	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
	von Bertalanffy	<b>52.49</b>	<b>8.46</b>	<b>0.9748</b>	<b>0.9579</b>	<b>16.57</b>	<b>0.4541</b>	<b>0.9861</b>	<b>0.9769</b>	<b>41.82</b>	<b>3.26</b>	<b>0.9981</b>	<b>0.9968</b>	<b>41.18</b>	<b>2.80</b>	<b>0.9839</b>	<b>0.9731</b>
DL	Gompertz	49.72	7.46	0.9845	0.9741	12.79	<b>0.27</b>	0.9837	0.9728	55.77	11.63	0.9950	0.9916	<b>36.90</b>	<b>2.42</b>	<b>0.9942</b>	<b>0.9903</b>
	Logístico	53.51	9.44	0.9708	0.9514	<b>11.63</b>	0.28	<b>0.9866</b>	<b>0.9776</b>	<b>43.04</b>	<b>4.13</b>	<b>0.9994</b>	<b>0.9990</b>	41.50	3.07	0.9874	0.9791
	Brody	57.65	12.37	0.9419	0.9031	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
	von Bertalanffy	<b>49.30</b>	<b>6.71</b>	<b>0.9855</b>	<b>0.9759</b>	13.43	0.29	0.9818	0.9697	64.81	23.31	0.9773	0.9622	42.17	3.31	0.9860	0.9766

The models that most stood out regarding the degree of adjustment were: von Bertalanffy and Gompertz, with the best adjustment in about 56% (9) and 25% (4) of the scenarios (species × variables), respectively. In the works of Reis et al. (2014), Puiatti et al. (2013), and Maia, Siqueira, Silva, Peternelli, and Salomão (2009), the logistic model presented the best adjustment for growth curves.

Figure 1 shows the fitted models that best describe the growth curves of each species for each variable. It is observed that the *C. juncea* is the crop that, although with medium height (Figure 1A), presents the greatest accumulation and the highest rate of accumulation of mass (Figures 1C and 1D), followed by *C. ensiformis*. This is explained by the significant increase in the diameter of the primary branch between 40 and 70 days after sowing (Figure 1B).

For plant height, the von Bertalanffy model was adjusted for all crops, although the Gompertz model presented a slightly better adjustment for *C. juncea* (Figure 1A). Considering the diameter of the primary branch, the Gompertz model presented the best overall adjustment (Figure 1B).

There was greater variation regarding the model for fresh matter, namely logistic for *C. juncea* and *D. lablab*, Gompertz for *C. ensiformis* and von Bertalanffy for *C. cajan*. For dry matter, the von Bertalanffy model was adjusted for all crops, except for the *D. lablab*.

Fernandes, Pereira, Muniz, and Savian (2014) obtained a good fit of Gompertz and logistic models for describing increments in fresh mass of coffee. Confalone et al. (2010) used the expolinear model to evaluate soybean growth due to different levels of water deficit. The model adjusted satisfactorily for soybean growth variation.

The models were able to describe the accumulation of fresh mass of the legumes under study, it is possible to verify that the maximum accumulation was reached at 85 days, being acquired as time increases (Figure 1C), however, there was greater variation regarding the models, being logistical for *C. juncea* and *D. lablab*, Gompertz for *C. ensiformis* and von Bertalanffy for *C. cajan*. Fernandes et al. (2014) obtained for coffee fresh matter growth curve a good fit with the Gompertz and Logistics models. For dry matter, the von Bertalanffy model was adjusted for all species, except for the *D. lablab*.

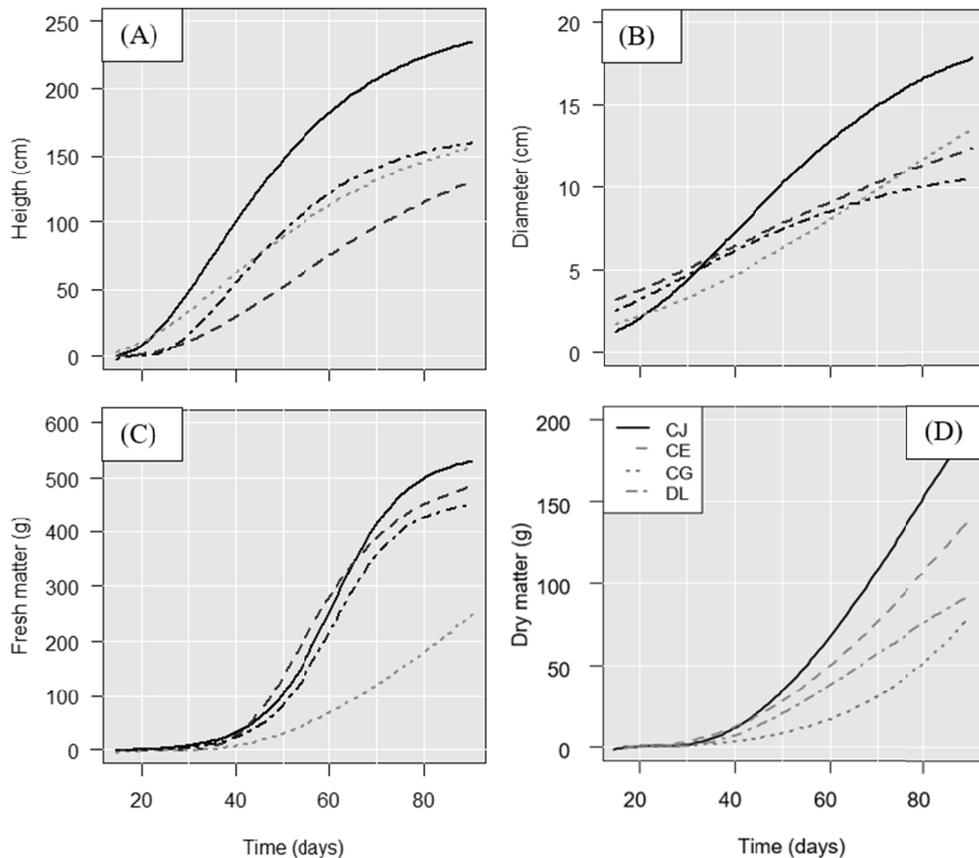


Figure 1. Fitted models for growth of *C. juncea* (CJ), *C. ensiformis* (CE), *C. cajan* (CC), *D. lablab* (DL), for: (A) plant height, (B) diameter of the primary branch, (C) aerial fresh matter weight and (D) aerial dry matter weight

#### 4. Conclusions

The models von Bertalanffy and Gompertz presented, in general, the highest goodness-of-fit for morphological and biomass growth of forage legumes.

The species *C. juncea* presented the highest biomass accumulation rate.

The adjustment of models in study proved to be promising to analyze the growth curve of morphological variables. However, it is understood that further studies are necessary, considering that results with the proposed objective are scarce in the literature.

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