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Optimized Dickey-Fuller Test Refines Sign and Boundary Problems Compare to Traditional Dickey-Fuller Test

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Abstract

Impede nonstationarity is vigorous to study performance of time series data and removes long-term components to expose any regular short-term regularity. So, we find miscellaneous unit root tests for instance Dickey-Fuller test, Augmented Dickey-Fuller plus DF-GLS Tests and identify that almost all unit root tests with the estimated model suffer from sign and boundary problems of the parameters to smooth the progress of the non-stationarity problem. In this paper, we usage Dickey-Fuller test and impose some limits on the parameter. Our proposed optimized DF test based on error sum of square (ESS). Monto Carlo simulation method is used to generate simulated critical values for different sample size. Our proposed optimized DF test gives better result than the ordinary DF test with effectiveness, uniformity and power properties. Also, optimized DF improves the sign and boundary problems through imposing some limit on error sum of squares and capture more nonstationarity of time related data.

Keywords: Optimized Dickey-Fuller test, Non-stationarity, ESS, Sign and boundary problems

1. Introduction

Socio-economic, statistical, time series, econometrics or econometric era are pedestal on a few exact postulations. Infringement of theories greatly influences the guess of the parameter over and above test of hypothesis (Akter, 2014). Nonstationary test is necessary for analyzing the activities of advance time series research. Usually non-stationarity can be tested by different unit root tests for example Dickey-Fuller (DF) test, Augmented Dickey-Fuller (ADF) test, DF-GLS Tests, Phillips-Perron (PP) test and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test discussed by Dickey and Fuller,1979; Kwiatkowski et al.,1992; Kodde and Palm, 1986). But all the unit root tests as well as the estimated model suffer from sign and boundary tribulations of the parameters (Akter and Majumder, 2013). So, appropriate testing procedure plays key role at the preliminary arena of any inquiries. In keeping with the assumption of the Dickey-Fuller test, $|\rho| < 1$ or $-2 < \delta < 0$ of the time series models, such as $w_i = \rho w_{i-1} + u_i$. Any estimated value of $\delta < -2$ or $\delta > 0$ may fallout in invalid model for making decision regarding nonstationarity (Naznin et al., 2014). To triumph over this condition, it is required to impose appropriate limits on the parameters, which is larger than zero. Very diminutive quantity of literatures is on this concern such as Majumder and King (1999) proposed one sided tests. Basak et al. (2005) and Rois et al. (2008) worked on distance based approach. Aktar and Majumder (2013) developed one sided DF testing procedure. Naznin et al (2014) showed the sign and boundary problems and solution by Augmented Dickey-Fuller (ADF) test. Hence the usual DF test for testing unit root is not always fit and we need to enlarge constrained parameter estimation on restricted DF test. So, we are provoked to expand a suitable testing technique. The principle of this paper is, firstly test the stationarity of some observed time series. Secondly, propose the testing approach due to arising some unit root problems. Finally, compare the proposed test with the usual test.

2. Method

2.1 Unit Root Tests

2.1.1 Dickey Fuller Test

Dickey and Fuller suggested that under the null hypothesis the estimated coefficient of w_{t-1} in the model (1) follows the τ statistic is known as Dickey-Fuller test. Here, errors are serially uncorrelated. In principle, three specifications can be tried, depending on whether the series show a trend or not. Allowing the various possibilities, DF test is estimated in three different forms under different null hypothesis for the following models,

$$\Delta w_t = \delta w_{t-1} + \varepsilon_t \,, \Delta w_t = \beta_1 + \delta w_{t-1} + \varepsilon_t \,, \Delta w_t = \beta_1 + \beta_2 t + \delta w_{t-1} + \varepsilon_t \tag{1}$$

where t is the time or trend variable. In each case, $H_0: \delta = 0$ means the time series is non-stationary and $H_1: \delta < 0$ means the time series is stationary. If H_0 is rejected, is w_t stationary with nonzero mean $\left[\frac{\beta_1}{(1-\rho)}\right]$ for the second model and w_t is stationary around a deterministic trend for third model. If the critical absolute value of the τ statistic exceeds the DF or Mackinnon DF absolute critical τ values, then we don't reject the hypothesis that the given time series is stationary. If it is less than the critical value, the time series is non-stationary (Gujarati, 2003).

2.1.2 Augmented Dickey-Fuller Test

Said and Dickey (1984) advised the autoregressive unit root test to accommodate general ARMA(p, q) models with unknown orders and their test is referred to as the Augmented Dickey-Fuller (ADF) test (Said and Dickey, 1984). For ADF, tests the null hypothesis, a time series Δw_t is $I(1)$ against the alternative is $I(1)$, assume the dynamics in the data have an ARMA structure. The ADF is based on estimating the test regression

$$\Delta w_t = \alpha_0 + \alpha_1 + \delta w_{t-1} + \sum_{i=1}^{m} \beta_i \Delta w_{t-i} + \varepsilon_t$$

where, t is the time or trend and $\rho - 1 = \delta$, ε_t is a white noise, α_0 is an intercept and $\alpha_1, \delta, \beta_i$ are coefficients(Gujarati, 2003).

2.1.3 DF-GLS Tests

DF-GLS tests perform the customized Dickey–Fuller t test proposed by Elliott et al. (1996). Basically, the assessment is similar to augmented Dickey–Fuller test and performed by Stata's dfuller command, except that the time series is transformed via GLS regression before carrying out the analysis. Elliott et al. later studies have revealed the unknown parameters β of the trend function are efficiently estimated under the alternative model with $\bar{\phi} = 1 + \bar{c}/T$ i.e., $\hat{\beta}_{\bar{\phi}} = (D'_{\phi} D_{\bar{\phi}})^{-1} D'_{\phi} y_{\bar{\phi}}$ (Dickey and Fuller, 1979). ERS use this insight to derive ADF t-statistic, which they call the DF-GLS test. They construct this t-statistic as follows. Firstly, using the trend parameters $\hat{\beta}_{\bar{\phi}}$ estimated under the alternative, define the detruded data $w_t^d = w_t - \hat{\beta}'_{\bar{\phi}} D_t$. Next, using GLS, estimate ADF test regression which omits the deterministic terms $\Delta w_t^d = \pi w_{t-1}^d + \sum_{j=1}^{p} \psi_j \Delta w_{t-j}^d + \varepsilon_t$ and compute the t-statistic for testing $\pi = 0$ (Chatfield, 2003).

2.2 Modified Dickey-Fuller Test

Dickey-Fuller test is the most commonly used unit root test for testing stationarity. The Dickey-Fuller test can be related to the three models as follows:

$$w_t = \rho w_{t-1} + \varepsilon_t \text{ or } \Delta w_t = \delta w_{t-1} + \varepsilon_t \tag{2}$$

$$w_t = \alpha + \rho w_{t-1} + \varepsilon_t \text{ or } \Delta w_t = \alpha + \delta w_{t-1} + \varepsilon_t \tag{3}$$

$$w_t = \alpha + \beta t + \rho w_{t-1} + \varepsilon_t \text{ or } \Delta w_t = \alpha + \beta t + \delta w_{t-1} + \varepsilon_t \tag{4}$$

where, t is the time or trend variable. For (4), when a time series is trend stationary, then the coefficient of time β may be either positive ($\beta > 0$ for upward trend) or negative ($\beta < 0$ for downward trend). Hence, the parameter β is also restricted. We observed that the ignoring of the two restrictions may result in three different problems, (i) estimated parameters values will be overestimated, (ii) test statistic based on this estimate may produce loss in power, (iii) estimated models may be invalid. So, if β is strictly affirmative or pessimistic the subsequent model will be more appropriate. In matrix notation (4) may written as,

$$W = X\beta + \varepsilon,$$

$$\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_T \end{bmatrix} = \begin{bmatrix} 1 & 1 & w_0 \\ 1 & 2 & w_1 \\ \vdots & \vdots & \vdots \\ 1 & T & w_{T-1} \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \rho \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_T \end{bmatrix}, \text{where } W = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_T \end{bmatrix}, \; X = \begin{bmatrix} 1 & 1 & w_0 \\ 1 & 2 & w_1 \\ \vdots & \vdots & \vdots \\ 1 & T & w_{T-1} \end{bmatrix}, \; \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_T \end{bmatrix}, \; \beta = \begin{bmatrix} \alpha \\ \beta \\ \rho \end{bmatrix}'$$

Or,

$$\Delta W = X\Gamma + \varepsilon$$

$$\begin{bmatrix} \Delta w_1 \\ \Delta w_2 \\ \vdots \\ \Delta w_T \end{bmatrix} = \begin{bmatrix} 1 & 1 & w_0 \\ 1 & 2 & w_1 \\ \vdots & \vdots & \vdots \\ 1 & T & w_{T-1} \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \rho \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_T \end{bmatrix}, \text{where } \Delta W = \begin{bmatrix} \Delta w_1 \\ \Delta w_2 \\ \vdots \\ \Delta w_T \end{bmatrix}, X = \begin{bmatrix} 1 & 1 & w_0 \\ 1 & 2 & w_1 \\ \vdots & \vdots & \vdots \\ 1 & T & w_{T-1} \end{bmatrix}, \; \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_T \end{bmatrix}, \Gamma = \begin{bmatrix} \alpha \\ \beta \\ \rho \end{bmatrix}$$

where, we impose some limits on parameters and guess the model using constraint optimization subroutine and appropriate modification. Thus, we minimize the ESS by subsequent approach to estimate the model appropriately. Minimizing, $ESS = (W - X\beta)'\Sigma^{-1}(W - X\beta), |\rho| < 1, \beta > 0 \; or \; \beta < 0,$ or

Minimizing, $ESS = (W - X\Gamma)'\Sigma^{-1}(W - X\Gamma), \; -2 < \tilde{\delta} < 0, \beta > 0 \; or \; \beta < 0$

where, $\beta > 0$ for upward trend or $\beta < 0$ for downward trend. If we can provide the Hessian or Jacobian, the optimization process will be quicker. The Hessian matrix can be gained by differencing ESS successively two times relating to the parameters,

$$\theta = (\alpha \;\; \beta \;\; \rho), \; H(\theta) = \frac{\partial^2 ESS}{\partial \theta^2} = \begin{bmatrix} \frac{\partial^2 ESS}{\partial \alpha^2} & \frac{\partial^2 ESS}{\partial \alpha \partial \beta} & \frac{\partial^2 ESS}{\partial \alpha \partial \rho} \\ \frac{\partial^2 ESS}{\partial \alpha \partial \beta} & \frac{\partial^2 ESS}{\partial \beta^2} & \frac{\partial^2 ESS}{\partial \beta \partial \rho} \\ \frac{\partial^2 ESS}{\partial \alpha \partial \rho} & \frac{\partial^2 ESS}{\partial \beta \partial \rho} & \frac{\partial^2 ESS}{\partial \rho^2} \end{bmatrix}, \qquad J(\theta) = \frac{\partial ESS}{\partial \theta} = \begin{bmatrix} \frac{\partial ESS}{\partial \alpha} \\ \frac{\partial ESS}{\partial \beta} \\ \frac{\partial ESS}{\partial \rho} \end{bmatrix}$$

and the information matrix,

$$I(\theta) = -E[H(\theta)] = -E \begin{bmatrix} \frac{\partial^2 ESS}{\partial \alpha^2} & \frac{\partial^2 ESS}{\partial \alpha \partial \beta} & \frac{\partial^2 ESS}{\partial \alpha \partial \rho} \\ \frac{\partial^2 ESS}{\partial \alpha \partial \beta} & \frac{\partial^2 ESS}{\partial \beta^2} & \frac{\partial^2 ESS}{\partial \beta \partial \rho} \\ \frac{\partial^2 ESS}{\partial \alpha \partial \rho} & \frac{\partial^2 ESS}{\partial \beta \partial \rho} & \frac{\partial^2 ESS}{\partial \rho^2} \end{bmatrix} \text{where, where, } \; \theta = (\alpha \;\; \beta \;\; \rho),$$

Here, we relate the constraint distance-based approach using information matrix. Shiparo (1988), Kodde and Palm's (1986), Majumder and King (1999) suggests that we should determine the closest point in the maintained hypothesis from the unconstrained point. The closest point is the solution of the following distance or optimal function of the parameter vector $\hat{\theta}$. $\left\|\tilde{\theta} - \hat{\theta}\right\|_{I} = \left(\tilde{\theta} - \hat{\theta}\right)' I(\theta) \left(\tilde{\theta} - \hat{\theta}\right)$, $\tilde{\theta} \varepsilon \beta$. Our proposed τ- statistic is based on the optimized estimates as follows, $\tilde{\tau} = \frac{\tilde{\delta}}{SE(\tilde{\delta})}$ where, $\tilde{\tau}$ is the optimized τ statistic and $\tilde{\delta}$, $\tilde{\beta}$ are the optimized estimate of the parameters. The τ- statistic follow the weighted mixture τ- distribution (Majumder, and King, 1999).

3. Results

To assess non-stationary, we used (1) diverse unit root tests, generating artificial data by Monto-Carlo simulation for early investigation, (Gujarati, 2003)

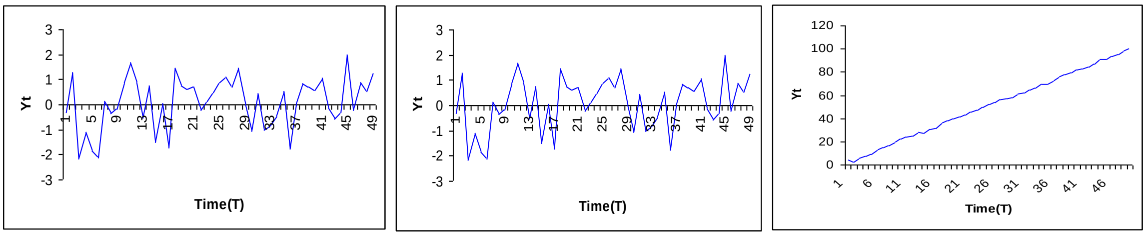

Fig 1.Time series plot for random walk model without drift, with drift and with drift and trend (Gujarati, 2003)

The above two figures facilitate there is no trend and more random movement in series w_t. Also, the series depends on time, so w_t is non-stationary. The last one shows a well-built upward trend component which is important for initial analysis. While the series is non-stationary because it enlarges with time. Now we consider unit root test of pretend series w_t and found that virtual random walk model with drift, without drift and with drift and trend shows nonstationary (Table-1). Since the complete assessment of the calculated τ is smaller than absolute DF critical values ($\alpha = 5\%$). Once more, we take some time series of sell overseas of Bangladesh such as pelt, clothing, Jute etc. and graphically observe the time series data and unit root test.

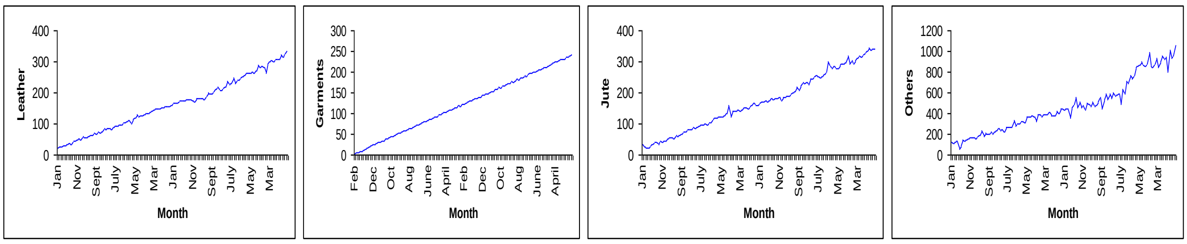

Fig 2. Plot for pelt, clothing, jute and additional rudiments from 2002 to 2015

Table 1. Unit root test of pelt, clothing, jute and additional rudiments

	Model	τ values	(5%) DF critical values	Decision
Pelt	$\Delta pl_i = \delta pl_{i-1} + u_i$	11.67*	-1.59	Stationary*
	$\Delta pl_i = \beta_1 + \delta pl_{i-1} + u_i$	-7.03*	-2.90	Stationary*
	$\Delta pl_i = \beta_1 + \beta_2 t + \delta pl_{i-1} + u_i$	-4.23*	-3.55	Stationary*
Clothing	$\Delta cl_i = \delta cl_{i-1} + u_i$	-8.65*	-1.97	Stationary*
	$\Delta cl_i = \beta_1 + \delta cl_{i-1} + u_i$	11.68*	-2.84	Stationary*
	$\Delta cl_t = \beta_1 + \beta_2 t + \delta cl_{t-1} + u_t$	-21.67*	-3.55	Stationary*
Jute	$\Delta Jl_i = \delta Jl_{i-1} + u_i$	-1.77*	-1.95	Nonstationary*
	$\Delta Jl_i = \beta_1 + \delta Jl_{i-1} + u_i$	-10.66*	-2.90	Stationary*
	$\Delta Jl_i = \beta_1 + \beta_2 t + \delta Jl_{i-1} + u_i$	-8.45*	-3.44	Stationary*
Additional rudiments	$\Delta ar_i = \delta ar_{i-1} + u_i$	19.32*	-1.95	Stationary*
	$\Delta ar_i = \beta_1 + \delta ar_{i-1} + u_i$	-1.26*	-2.90	Nonstationary*
	$\Delta ar_i = \beta_1 + \beta_2 t + \delta ar_{i-1} + u_i$	-9.20*	-3.23	Stationary*

Analyzing the above graphs and tables we perceive that standard testing approach is not suitable for all cases. In several cases, calculated τ is positive which describes the invalidity of the respective model. We also consider co-integration test between pelt and clothing as well as jute and additional rudiments. Surprising found that pelt and clothing are not co-integrated whereas jute and additional rudiments are co-integrated but some cases the τ statistic confirm positive value that mislead the model (Table-2). In these models, we then test the hypothesis $H_0: \beta = 0$, against $H_a: \beta > 0, \beta < 0$, and set $\alpha = 0.02$ and $\beta = 0.01$. We find that, as sample sizes increases, the generated critical values for any sample size, which are approximately same as the critical values in the Dickey-Fuller table (Table-3). Thus, standard unit root test shows some of the series stationary and cannot capture the non-stationary problem in the approved manner. The incorrect identification may occur due to ignoring the restrictions on the parameters of models. So, estimation procedure need restricted estimation taking error term of two non-stationary series based on minimizing the ESS. For restricted parameter under alternative, the constraint optimization requires sophisticated optimization subroutine. Hence, we introduce Modified Dickey-Fuller Test and estimate the parameters by using newly proposed restricted test based on ESS and compares power of the existing Dickey-Fuller tests and the newly proposed restricted test.

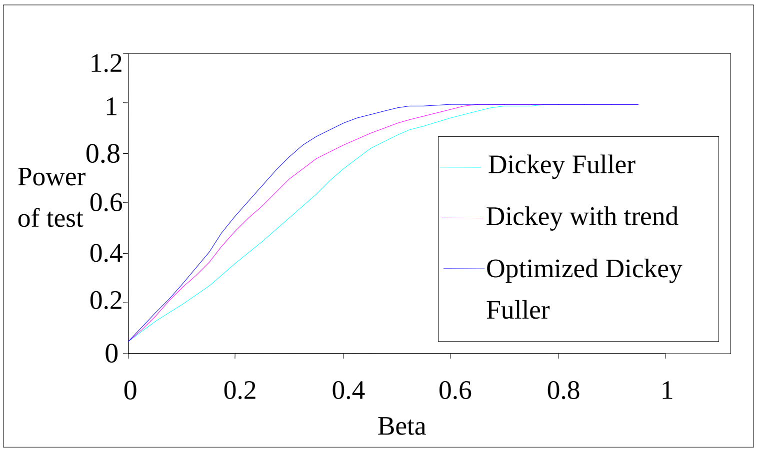

Fig 3. Power curve of standard Dickey-Fuller test and Optimized test

We observed from table-5 and figure-3 that the simulated power of the newly proposed test is always higher than the usual Dickey-Fuller test and the power of proposed optimized test converges to 1 more rapidly in compare with the usual test.

4. Conclusion

Usual Dickey-Fuller test suffer from sign and boundary problems of the parameters. This paper deals with optimized Dickey-Fuller test. So, we are wrapping up optimized method based on $\tilde{\tau} = (\tilde{\delta}/SE(\tilde{\delta}))$ restricted ESS of Dickey-Fuller test and observe that optimized Dickey-Fuller test gives better result than traditional Dickey-Fuller test with effectiveness, uniformity and power properties.

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Appendix

Table 2. Co-integration test of pelt, clothing, jute and additional rudiments

Co-integration	Model	τ values	DFcritical values (5%)	Decision
Pelt and Clothing	$\Delta u_t = u_{t-1} + \varepsilon_t$	40.5482	-1.948	Not cointegrated
	$\Delta u_t = \alpha + u_{t-1} + \varepsilon_t$	1.578	-2.891	Not cointegrated
	$\Delta u_t = \alpha + \beta t + u_{t-1} + \varepsilon_t$	15.236	-3.455	Not cointegrated
Jute and Others	$\Delta u_t = u_{t-1} + \varepsilon_t$	-1.284	-1.948	Cointegrated
	$\Delta u_t = \alpha + u_{t-1} + \varepsilon_t$	1.578	-2.891	Cointegrated
	$\Delta u_t = \alpha + \beta t + u_{t-1} + \varepsilon_t$	-2.623	-3.455	Cointegrated

Table 3. Simulated 1% and 5% critical Dickey-Fuller τ values for unit root tests

Sample Size	t_{nc}		t_c		t_{ct}	
	1%	5 %	1%	5 %	1%	5 %
25	-2.63	-1.95	-3.86	-3.09	-4.55	-3.79
50	-2.62	-1.97	-3.63	-2.97	-4.14	-3.54
100	-2.60	-1.97	-3.55	-2.92	-3.59	-2.91
250	-2.60	-1.96	-3.47	-2.88	-2.69	-2.00
500	-2.57	-1.94	-3.44	-2.86	-2.48	-1.79
∞	-2.55	-1.94	-3.45	-2.52	-2.48	-1.77

Table 4. Generated critical value of Dickey-Fuller table for Sample size $n = 20,21, \ldots, 1000$

Sample size	t_{nc}		t_c		t_{ct}	
	1 %	5 %	1 %	5 %	1 %	5 %
20	-2.6955	-1.9591	-3.8084	-3.0207	-4.4991	-3.6584
21	-2.6796	-1.9581	-3.7879	-3.0123	-4.4685	-3.6449
22	-2.6742	-1.9572	-3.7695	-3.0048	-4.4407	-3.6328
23	-2.6693	-1.9564	-3.7528	-2.9981	-4.4162	-3.6219
24	-2.6648	-1.9557	-3.7381	-2.9918	-4.3942	-3.6121
25	-2.6607	-1.9550	-3.7242	-2.9862	-4.3741	-3.6032
26	-2.6569	-1.9544	-3.7116	-2.9810	-4.3559	-3.5950
27	-2.6534	-1.9538	-3.7000	-2.9762	-4.3392	-3.5875
28	-2.6501	-1.9533	-3.6893	-2.9718	-4.3238	-3.5806
29	-2.6471	-1.9528	-3.6794	-2.9677	-4.3097	-3.5742
30	-2.6443	-1.9524	-3.6702	-2.9639	-4.2966	-3.5683
31	-2.6417	-1.9520	-3.6617	-2.9604	-4.2844	-3.5628
32	-2.6392	-1.9516	-3.6537	-2.9571	-4.2731	-3.5577

33	-2.6369	-1.9512	-3.6463	-2.9540	-4.2626	-3.5529
34	-2.6348	-1.9509	-3.6393	-2.9511	-4.2528	-3.5484
35	-2.6327	-1.9506	-3.6328	-2.9484	-4.2435	-3.5442
36	-2.6308	-1.9503	-3.6267	-2.9458	-4.2349	-3.5402
37	-2.6292	-1.9501	-3.6209	-2.943	-4.2267	-3.5365
38	-2.6275	-1.9497	-3.6155	-2.9411	-4.2190	-3.5330
39	-2.6258	-1.9495	-3.6103	-2.9389	-4.2118	-3.5297
40	-2.6243	-1.9493	-3.6055	-2.9369	-4.2049	-3.5265
41	-2.6228	-1.9490	-3.6009	-2.9350	-4.1984	-3.5236
42	-2.6214	-1.9488	-3.5965	-2.9331	-4.1923	-3.5207
43	-2.6201	-1.9486	-3.5923	-2.9314	-4.1864	-3.5180
44	-2.6188	-1.9484	-3.5884	-2.9297	-4.1808	-3.5155
45	-2.6176	-1.9483	-3.5846	-2.9281	-4.1758	-3.5130
46	-2.6164	-1.9481	-3.5810	-2.9266	-4.1707	-3.5107
47	-2.6153	-1.9479	-3.5776	-2.9251	-4.1659	-3.5084
48	-2.6142	-1.9478	-3.5743	-2.9237	-4.1613	-3.5063
49	-2.6132	-1.9476	-3.5712	-2.9224	-4.1569	-3.5043
50	-2.6122	-1.9475	-3.5682	-2.9211	-4.1526	-3.5023
55	-2.6079	-1.9468	-3.5549	-2.9155	-4.1339	-3.4936
60	-2.6043	-1.9463	-3.5439	-2.9108	-4.1185	-3.4865
65	-2.6012	-1.9458	-3.5347	-2.9069	-4.1056	-3.4804
70	-2.5986	-1.9455	-3.5269	-2.9035	-4.0946	-3.4753
75	-2.5963	-1.9451	-3.5202	-2.9006	-4.0851	-3.4708
80	-2.5943	-1.9449	-3.5143	-2.8981	-4.0769	-3.4669
85	-2.5926	-1.9446	-3.5092	-2.8959	-4.0697	-3.4635
90	-2.5911	-1.9444	-3.5046	-2.8939	-4.0633	-3.4605
95	-2.5897	-1.9442	-3.5006	-2.8921	-4.0575	-3.4578
100	-2.5884	-1.9440	-3.4970	-2.8906	-4.0524	-3.4554
150	-2.5806	-1.9429	-3.4742	-2.8807	-4.0203	-3.4400
200	-2.5767	-1.9424	-3.4631	-2.8758	-4.0045	-3.4324
250	-2.5743	-1.9421	-3.4564	-2.8729	-3.9951	-3.4279
300	-2.5728	-1.9418	-3.4520	-2.8709	-3.9888	-3.4249
400	-2.5708	-1.9416	-3.4465	-2.8685	-3.9810	-3.4211
500	-2.5696	-1.9414	-3.4433	-2.8671	-3.9764	-3.4188
1000	-2.5673	-1.9411	-3.4368	-2.8642	-3.9671	-3.4143

Table 5. Calculated power of usual DF test with trend and without trend and proposed optimized test.

δ	β	Usual DF	DF with trend	Optimized DF
0.0	0	0.05	0.05	0.05
	0.1	0.13	0.15	0.16
	0.5	0.20	0.26	0.28
	1.0	0.27	0.37	0.41
	5.0	0.36	0.49	0.55
	0	0.45	0.59	0.67
-0.5	0.1	0.54	0.70	0.79
	0.5	0.64	0.78	0.87
	1.0	0.79	0.84	0.92
	5.0	0.82	0.88	0.96
-1.0	0	0.88	0.92	0.98
	0.1	0.90	0.95	0.99
	0.5	0.95	0.97	1
	1.0	0.97	0.99	1
	5.0	0.99	1	1
-1.5	0	0.99	1	1
	0.1	1	1	1
	0.5	1	1	1
	1.0	1	1	1
	5.0	1	1	1