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On Periodic Maintenance of a Coherent System

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Abstract

In this paper, we study a coherent system that has periodic maintenance performed at regular intervals. The exact analytical expressions are obtained for some important maintenance performance measures such as mean time between failures, average availability and mean fractional dead time. The CHA algorithm is used to do the relevant calculations. Some s-coherent structures viz., series, parallel, 2-out-of-3:G and a fire-detector system are considered to illustrate the method.

Keywords: mean time between failures, average availability, the CHA algorithm, periodic maintenance, system reliability.

1. Introduction

Consider a system under a periodic maintenance performed at regular intervals. This type of maintenance policy calls for repairing/replacing a failed component/sub-system prior to a failure of the system. The system is supposed to be highly reliable and mission-oriented so that a testing is performed once the mission is completed.

Specifically, suppose that the system starts operating at time t = 0. At regular intervals of length η, the system is subjected to a testing and after testing, the system is brought back to a level which is "as good as new". During testing, if necessary, a component is repaired or replaced. We strictly follow the treatment given in Hoyland & Rausand 1994.

Since after each testing, a component that undergoes maintenance is as good as new, all the test intervals can be assumed to be "equal" from a stochastic point of view. Therefore, without loss of generality, we consider only the test interval $(0, \eta]$. Then, the following important maintenance performance measures are easy to derive (see Hoyland & Rausand 1994):

$$A_{av} = \frac{1}{\eta}\int_0^{\eta} R(t)dt \tag{1.1}$$

$$\text{MFDT} = 1 - \frac{1}{\eta}\int_0^{\eta} R(t)dt \tag{1.2}$$

$$\text{MTBF} = \frac{\int_0^{\eta} R(t)dt}{1 - R(\eta)} \tag{1.3}$$

$$E(D_1 / X(\eta) = 0) = \frac{1}{1 - R(\eta)}\left(\eta - \int_0^{\eta} R(t)dt\right) \tag{1.4}$$

$$E(U) = \frac{1 - \beta\eta MFDT}{\beta\eta MFDT} \tag{1.5}$$

$$P(0,\eta) = \beta\eta MFDT \tag{1.6}$$

For notational details see the Table 1.1.

We notice that each expression above involves $R(t)$ and/or $\int_0^\eta R(t)dt$. Calculations of these quantities for a large complex coherent system are prohibitive; hence in the literature, bounds and approximations are suggested to get simplified expressions (see Hoyland & Rausand 1994 and Mondro 2002). Though these approximate methods work quite well for a small system, no exact method is available to study a large complex system.

Table 1.1. Notations used in the paper

t	system operational time
η	length of periodic maintenance interval
R(t)	reliability function of the system
MTBF	mean time between failures
A_{av}	average availability
MFDT	mean fractional dead time
$E(D_1)$	expected length of time in which the system is in a failed state
E(U)	expected number of test intervals until a critical situation occurs
$P(0,\eta)$	probability of at least one critical event in $(0,\eta]$. A critical event occurs if a fire occurs when the fire detector is in the failed state.
β	mean number of fires per unit time
$X(t)$	state of the system at time t
p_i	reliability of component i
λ_i	failure rate of a component with exponential life
M	number of minimal path sets of the system
N	number of components of the system

Moreover, the associated drawbacks with the these existing methods are: (1) data at the system level is not available for a highly reliable systems, (2) no tractable expression exits for R(t) so that one can easily evaluate the integral $\int_0^\eta R(t)dt$.

In this paper, we have tried to overcome these drawbacks with the help of the CHA algorithm to obtain R(t) which is described below; the algorithm developed by us (Chaudhuri et al (2001) is referred to as the CHA algorithm in the literature.

With *n* being the number of components of the system under consideration, let x =($x_1,\ldots, x_n$) denote the states of the components such that x_i, state of component *i*, = 1, if component *i* is working, = 0, otherwise. We assume that the state of the system, denoted by the structure function $\Phi(x)$, is completely determined by the states of its components so that $\Phi(x)$=1, if system is working, and, = 0, otherwise. We also assume that the system is coherent. The m will denote number of minimal path sets for the system and the p_i, i=1,…,n , will denote the component reliabilities.

Result (Chaudhuri et al. 2001): For a coherent structure with m minimal path sets and Design matrix D =((D(i,j))), the structure function of the system is given by:

$$\Phi(x) = \sum_{j=1}^{2^m-1} 1(j).\prod_{i=1}^{n} x_i^{D(i,j)} \quad (1.7)$$

where D(i,j) = element (i,j) of D and 1(j) = element j of 1, a vector of ones.

Some notational details such as the Design matrix D and **1,** the vector of 1s, will be explained through some examples of present context in Section 3 below.

Hence, for a coherent structure with m minimal path sets and Design matrix D =((D(i,j))), the system reliability, under the assumption of independent components, is given by (see Chaudhuri et al 2001):

$$\text{R(t)} = \sum_{j=1}^{2^m-1} 1(j).\prod_{i=1}^{n} p_i^{D(i,j)}. \quad (1.8)$$

The inputs to the CHA algorithm are the component reliabilities p_i, i=1,…,n and the minimal path sets of the system under consideration. The basic features of the algorithm will be highlighted for some simple systems in Section 3. For further details, see Chaudhuri et al 2001. The CHA algorithm is successfully applied to the study of many important reliability situations (see Chaudhuri 2004, Chaudhuri 2004, Chaudhuri 2009, Chaudhuri 2011).

The paper is organized as follows. In Section 2, we obtain exact expressions for the performance measures. Section 3 contains some examples to illustrate the method and we conclude in Section 4.

2. Method

Consider a system composed of independent exponential components with

$$p_i = e^{-\lambda_i t}, i = 1,...,n \quad (2.1)$$

Then (1.8) reduces to

$$R(t) = \sum_{j=1}^{2^m-1} 1(j) e^{-\sum_{i=1}^{n} \lambda_i D(i,j) t} \quad (2.2)$$

Thus, we have,

$$\int_0^\eta R(t)dt = \sum_{j=1}^{2^m-1} 1(j) \frac{1-e^{-\sum_{i=1}^{n} \lambda_i D(i,j)\eta}}{\sum_{i=1}^{n} \lambda_i D(i,j)} \quad (2.3)$$

Given the CHA algorithm, therefore, the exact computations of the expressions listed in (1.1) through (1.6) become simple. The CHA algorithm is simple and easy to use.

3. Illustrative Examples

3.1 Series System

For a series system with two s-independent exponential components, we have

$D = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $1 = (1)$, and m = 1, n = 2, so that

$$R(t) = \sum_{j=1}^{2^m-1} 1(j) e^{-\sum_{i=1}^{n} \lambda_i D(i,j) t} = 1(1) e^{-\sum_{i=1}^{n} \lambda_i D(i,1) t} = e^{-(\lambda_1 D(1,1) + \lambda_2 D(2,1))t} = e^{-(\lambda_1+\lambda_2)t}.$$

Hence (2.3) simplifies to

$$\int_0^\eta R(t)dt = \frac{1-e^{-(\lambda_1+\lambda_2)\eta}}{\lambda_1+\lambda_2} \quad (3.1)$$

3.2 Parallel System

Consider a parallel system with two s-independent exponential components. Then,

$$D=\begin{pmatrix}1 & 0 & 1\\ 0 & 1 & 1\end{pmatrix}, \quad 1=\begin{pmatrix}1\\ 1\\ -1\end{pmatrix}, \text{ and m=2, n=2,}$$

so that (2.3) reduces to

$$\int_0^{\eta} R(t)dt=\frac{1-e^{-\lambda_1\eta}}{\lambda_1}+\frac{1-e^{-\lambda_2\eta}}{\lambda_2}-\frac{1-e^{-(\lambda_1+\lambda_2)\eta}}{\lambda_1+\lambda_2} \tag{3.2}$$

3.3 2-out-of-3: G system

For this system,

$$D=\begin{pmatrix}1 & 1 & 0 & 1 & 1 & 1 & 1\\ 1 & 0 & 1 & 1 & 1 & 1 & 1\\ 0 & 0 & 1 & 1 & 1 & 1 & 1\end{pmatrix}, \quad 1=\begin{pmatrix}1\\ 1\\ 1\\ -1\\ -1\\ -1\\ 1\end{pmatrix}, \quad \text{m = 3, n = 3.}$$

Thus, we have,

$$\int_0^{\eta} R(t)dt=\frac{1-e^{-(\lambda_1+\lambda_2)\eta}}{\lambda_1+\lambda_2}+\frac{1-e^{-\lambda_1\eta}}{\lambda_1}+\frac{1-e^{-(\lambda_2+\lambda_3)\eta}}{\lambda_2+\lambda_3}-2\frac{1-e^{-(\sum_{i=1}^{3}\lambda_i)\eta}}{\sum_{i=1}^{3}\lambda_i} \tag{3.3}$$

3.4 Fire Detector System

We have given a description of this system in Chaudhuri et al 2001. The system has 13 components and 8 minimal path sets. The Design matrix D matrix for this system has 255 columns. Let the components have exponential distributions with the following failure rates:

Table 3.1. Components and their failure rates for Fire Detector System

Component	Failure Rate (failures/month)
1	3.0 x e^{-5}
2	0.1 x e^{-7}
3, 4	5.0 x e^{-6}
5	5.0 x e^{-5}
6, 7, 8, 9	2.0 x e^{-4}
10, 11, 12, 13	4.0 x e^{-6}

Then, the CHA algorithm yields the following values for maintenance performance parameters (the value of η is taken as 1 month):

Table 3.2. Maintenance performance parameters for the Fire Detector system

Parameters	Value
MTBF	18.3867
A_{av}	0.9734
MFDT	0.0266
$E(D_1)$	0.5030
$E(U)$	3.7554 x e^{4}
$P(0,\eta)$	2.6628 x e^{-5}

4. Conclusion

Periodic maintenance is a very common type of maintenance policy conveniently performed on highly reliable, mission-oriented systems after completion of the mission. The exact computations of the performance measures such as the MTBF (as noted by Mondro (2002)) of a complex system with many components is extremely difficult and sometimes impossible. That is why, several bounds/approximations are proposed in the literature to obtain estimates of various performance measures of a system having a periodic maintenance. In this paper, we have overcome this problem with the help of the CHA algorithm and provided an analytical expression suitable for computer implementation

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References

Chaudhuri, G. (2004). A lower confidence limit for reliability of a coherent system with independent exponential components. *Calcutta Statistical Association Bulletin, 55*, 119–127.

Chaudhuri, G. (2004). Some important reliability results via the CHA Representation /algorithm, Indian Association for Productivity. *Quality and Reliability Transactions, 29*, 75-79.

Chaudhuri, G. (2009). On the reliability function of a coherent structure of components sharing a common environment. *Journal of Probability and Statistical Science, 7*, 19-27.

Chaudhuri, G. (2011). A Lower Confidence Limit for Reliability of a Coherent system with Independent Components via the CHA Algorithm. *Journal of Statistical Research, 4,* 71–76.

Chaudhuri, G., Hu, K., & Afshar, N. (2001). A new approach to system reliability. *IEEE Transactions on Reliability, 50*, 75-84. http://dx.doi.org/10.1109/24.935020

Hoyland, A., & Rausand, M. (1994). System Reliability-- Theory, Models, and Statistical Methods, Wiley.

Mondro, M. (2002). Approximation of mean time between failure when a system has periodic maintenance. *IEEE Transaction on Reliability, 51*, 166-167. http://dx.doi.org/10.1109/TR.2002.1011521